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A look inside Feynman route to gravitation

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14-18/09/2020 SIF 2020 Congresso Nazionale

Outline

- **Introduction: a timeline**
- **Gravity as Quantum Field Theory**
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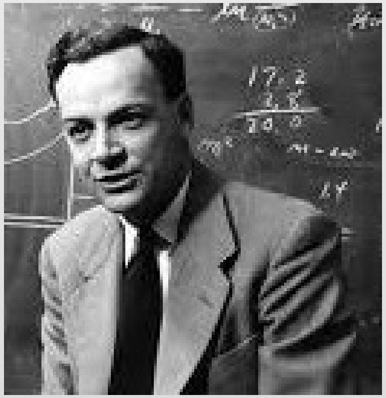
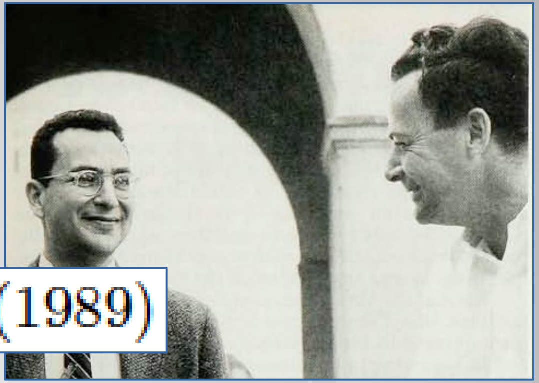


Introduction: a timeline

- *Mid '50s: Feynman starts thinking deeply about gravitation.*

Dick played a major part in working out the rules of quantum gravity in that approximation. It so happened that I was peripherally involved in the story of that research. We first discussed it when I visited Caltech during the Christmas vacation of 1954–55 and he was my host.

M. Gell-Mann, Phys. Today 42, 2, 50 (1989)



Letter from Bryce DeWitt to Agnew Bahnson, November 15, 1955

- *1957: Conference on The Role of Gravitation in Physics, Chapel Hill, North Carolina, January 18-23.*

C. DeWitt-Morette and D. Rickles, "The Rôle of Gravitation in Physics", Report from the 1957 Chapel Hill Conference, Edition Open Access (2011)

As you know, I am studying the problem of quantization of Einstein's General Relativity. I am still working out the details of handling divergent integrals which arise in problems in which some virtual momentum must be integrated over. But for cases of radiation, without radiation corrections, there is no difficulty (i.e., it is about where electrodynamics was in 1946).

- *1961: Letter to Wiki Weisskopf, January-February.*

- *1961: International Conference on the Theory of Weak and Strong Interactions, La Jolla, June 14-16.*

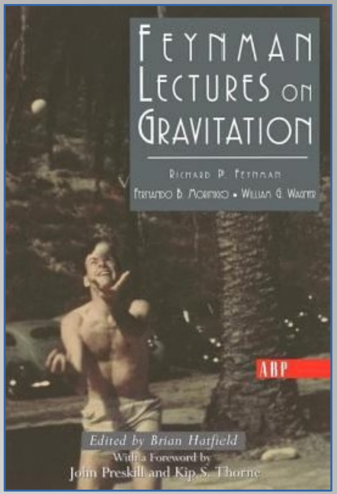
W. R. Frazer, Physics Today **14**(12) (1961) 80.

- *1962: Conference on Gravitation, Warsaw, July 25-31.*

R. P. Feynman, Acta Phys. Polon. **24** (1963) 697.

My subject is the quantum theory of gravitation. My interest in it is primarily in the relation of one part of nature to another. There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it;

- *1962-63: Caltech Lectures on Gravitation.*



- *1966-67: Hughes Lectures on "Astronomy, Astrophysics and Cosmology", delivered at Hughes Aircraft Company.*

- *1972: Feynman wrote a detailed account on his results in two articles for a volume in honor of John Wheeler's 60th birthday.*

R. P. Feynman, "Closed loop and tree diagram", in J. R. Clauder (Ed.), Magic Without Magic: John Archibald Wheeler. W. H. Freeman, San Francisco (1972).

R. P. Feynman, "Problems in quantizing the gravitational field and the massless Yang-Mills field", in J. R. Clauder (Ed.), Magic Without Magic: John Archibald Wheeler. W. H. Freeman, San Francisco (1972).

Gravity as Quantum Field Theory

According to Feynman, the forces that we observe macroscopically, and that we dub as fundamental, must emerge from quantum field theory in the classical limit, and gravity is no exception.

KEY IDEA: gravity behaves as $1/r^2$ force. It has to be carried by a massless spin-2 quantum, the graviton. Full general relativity should follow from the principles of Lorenzt invariant QFT as applied to a massless spin-2 field and consistency requirements. The graviton has to couple to anything with energy-momentum, including itself: the obtained theory is nonlinear. In whole analogy with electrodynamics, the action is:

$$\int \left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A^\nu}{\partial x^\mu}\right) d^4x + e\int A_\mu j_\mu d^4x + \frac{m}{2}\int \dot{z}_\mu^2 ds + \frac{1}{2}\int T_{\mu\nu} h_{\mu\nu} d^4x + \int (second\ power\ of\ first\ derivatives\ of\ h),$$

$h_{\mu\nu}$ is the new field, satisfying second order equations of the kind:

$$\frac{\partial^2}{\partial x \partial x} h = T.$$

equation of motion for particles

$$g_{\mu\nu} \ddot{z}^\lambda = [\mu\nu, \lambda] \, \dot{z}_\mu \dot{z}_\nu$$

$$g_{\mu\nu} = \epsilon_{\mu\nu} + h_{\mu\nu}$$

PROBLEM: how to find energy-momentum such as to satisfy

$$T_{\mu\nu,\nu} = 0$$

There is coupling with matter, so the linear theory leads to a consistency problem. In order to solve the problem one has to add to the action a non linear term of third order in h.

T satisfies the following equation:

$$g_{\mu\lambda} T_{\lambda\mu,\nu} = [\mu\nu, \lambda]\, T_{\lambda\mu,\lambda}$$

How to find the general solution? By finding an expression which is invariant under the following transformation:

$$g'_{\mu\nu} = g_{\mu\nu} + g_{\mu\lambda}\frac{\partial A_\lambda}{\partial x^\nu} + g_{\nu\lambda}\frac{\partial A_\lambda}{\partial x^\mu} + A_\lambda \frac{\partial g_{\mu\nu}}{\partial x^\lambda}$$

This procedure, sketched at Chapel Hill (1957) and fully developed in the Caltech Lectures on Gravitation (1962-63), leads (in the classical limit) to Einstein gravitational field equations.

Feynman approach to gravity would have been pursued also by other people such as Weinberg and Deser.

S. Weinberg, Phys. Rev. **138** (1965) B988.

S. Deser, Gen. Rel. Grav. 1 (1970) 9

D. G. Boulware and S. Deser, Annals Phys. **89** (1975) 193.

Fighting with loops: the renormalization of gravity

I suggested that he try the analogous problem in Yang–Mills theory, a much simpler nonlinear gauge theory than Einsteinian gravitation. Richard asked what Yang–Mills theory was.

Murray Gell-Mann

AN International Conference on the Theory of Weak and Strong Interactions took place June 14–16, 1961, at the Department of Physics of the University of California, San Diego, in La Jolla. The theory of gravitation was discussed in the afternoon. R. P. Feynman, in reviewing the current status of his work on renormalization of the gravitational field, reported having encountered the difficulty that the theory appeared to be nonunitary, and he discussed this same difficulty in the Yang-Mills theory.

W. R. Frazer (1961)

I started with the Lagrangian of Einstein for the interacting field of gravity and I had to make some definition for the matter since Im dealing with real bodies and make up my mind what the matter was made of; and then later I would check whether the results that I have depend on the specific choice or they are more powerful R. P. Feynman, Acta Phys. Polon. 24 (1963) 697.

approximation for the metric

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$L = \int \left(h_{\mu\nu,\sigma}\bar{h}_{\mu\nu,\sigma} - 2\bar{h}_{\mu\sigma,\sigma}\bar{h}_{\mu\sigma,\sigma}\right) + \frac{1}{2}\int \left(\phi_{,\mu}^2 - m^2\phi^2\right) d\tau$$
$$+\kappa \int \left(\bar{h}_{\mu\nu}\phi_{,\mu}\phi_{,\nu} - m^2\frac{1}{2}h_{\sigma\sigma}\phi^2\right) + \kappa\int hhh + \kappa^2\int hh\phi\phi + \ldots$$

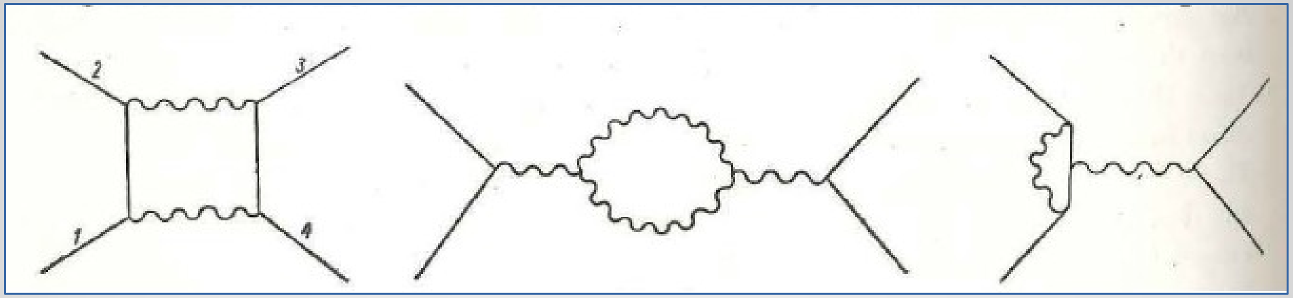
$$\bar{h}_{\mu\nu} = \frac{1}{2}\left(h_{\mu\nu} + h_{\nu\mu} - \delta_{\mu\nu}h_{\sigma\sigma}\right)$$

By varying the Lagrangian with respect to h and ϕ, he obtains the equations of motion with a source term:

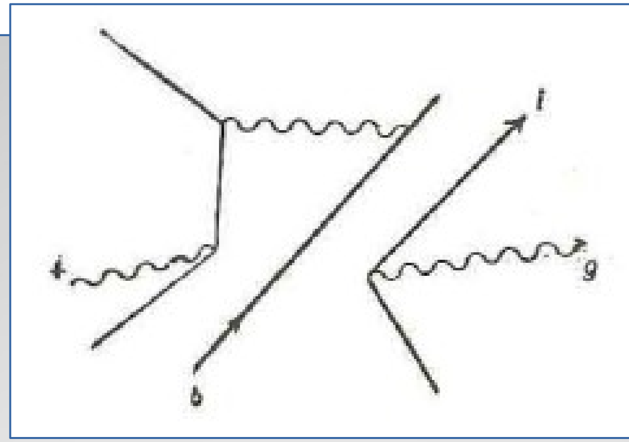
$$h_{\mu\nu,\sigma\sigma} - \bar{h}_{\sigma\nu,\sigma\mu} - \bar{h}_{\sigma\mu,\sigma\nu} = \bar{S}_{\mu\nu}(h,\phi)\,,$$

$$\phi_{,\sigma\sigma} - m^2\phi = \chi(\phi, h)\,.$$

The first equation is singular, but Feynman worked out the problem by resorting to the coordinate invariance of general relativity and came up with a description of two bodies interacting by the exchange of a virtual graviton. But the problem was to obtain high-order radiative corrections, which required to go beyond tree-level approximation:



rather interesting. As a matter of fact, I proved that if you have a diagram with rings in it there are enough theorems altogether, so that you can express any diagram with circuits completely in terms of diagrams with trees and with all momenta for tree diagrams in physically attainable regions and on the mass shell.



By working out one-loop calculations Feynman realized that unitarity gets lost because some contributions arise from longitudinal polarization states of the graviton, which don't cancel. Indeed one has to sum the whole set of tree diagrams corresponding to a definite process in order to guarantee gauge invariance.

But how to get the same result by integrating the closed loop directly? On one hand, a mass term has to be added to the lagrangian in order to make it non singular but it breaks gauge invariance; on the other hand a contribution has to be subtracted, obtained by taking a ghost particle going around the ring and artificially coupled to the external field. In this way unitarity and gauge invariance would be recovered.

R. P. Feynman, Acta Phys. Polon. 24 (1963) 697

Conclusions

I do not know whether it will be possible to develop a cure for treating the multi-ring diagrams. I suspect not - in other words, I suspect that the theory is not renormalizable. Whether it is a truly significant objection to a theory, to say that it is not renormalizable, I don't know (| Lecture 16, pp. 211-212).

R. P. Feynman, F. B. Morinigo, W. G. Wagner and B. Hatfield, "Feynman lectures on gravitation," Reading, USA: Addison-Wesley (1995).

The search for quantum gravity is still open !!