WATERLOO: a Wavelet Coherence Analysis Tool for Glitches in Gravitational Wave Data

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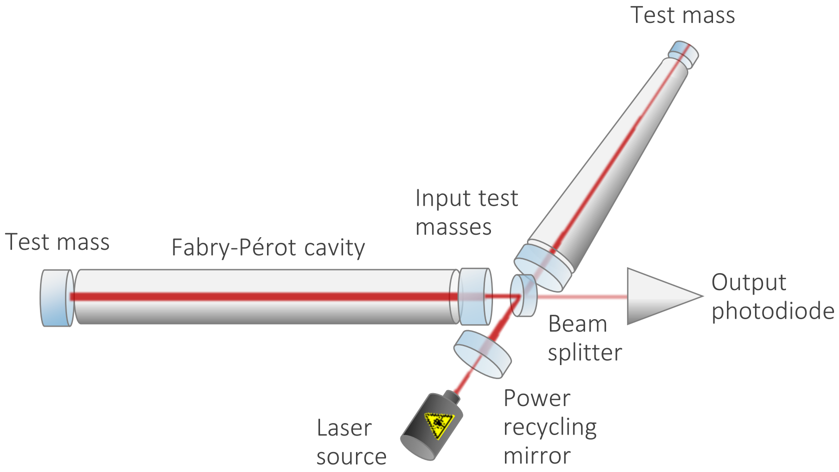
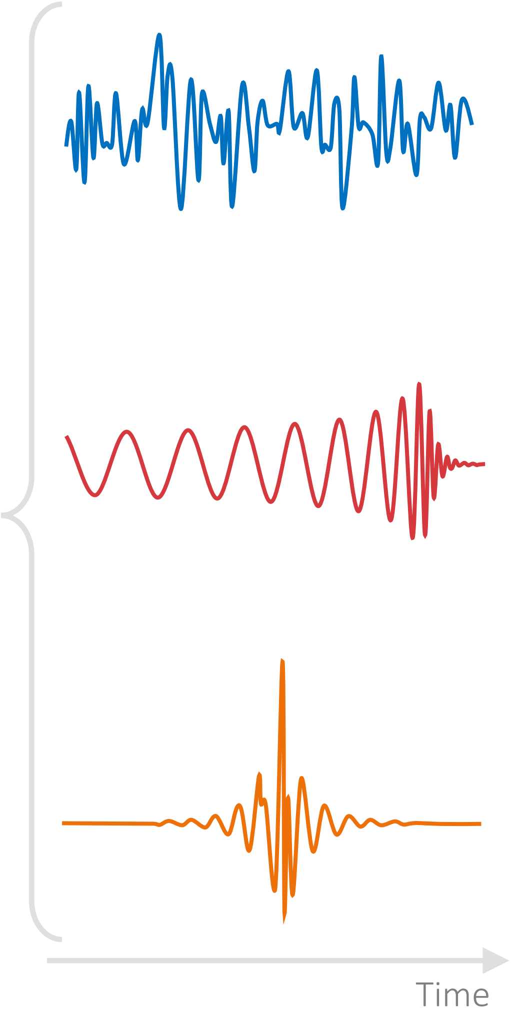
Aerial view of the Virgo interferometer
Credit: Virgo Collab.

Gravitational Wave Detectors Data

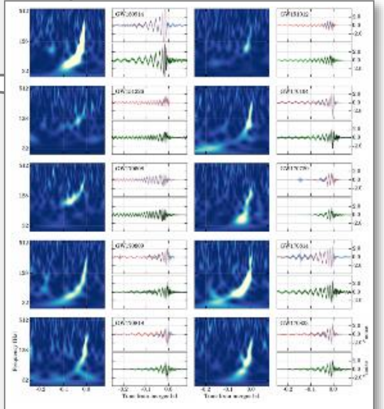
Detector output:

$$x(t) = \underbrace{n(t)}_{\text{Bg. noise}} + \underbrace{h(t)}_{\text{Astro. Sig.}} + \underbrace{g(t)}_{\text{Glitches}}$$





Detector background noise:
- Stationary;
- (mostly) Gaussian.

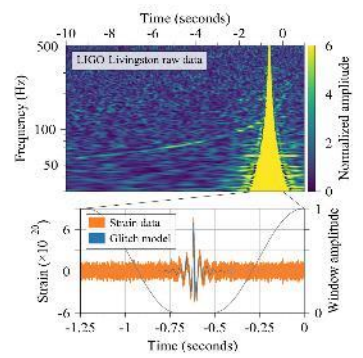


PhysRevX.9.031040

Astrophysical signal

Transient:
- Modelled, e.g. CBCs;
- Unmodelled: bursts.

Continuous:
- Periodic: e.g. pulsars;
- Stochastic background.



PhysRevD.98.084016

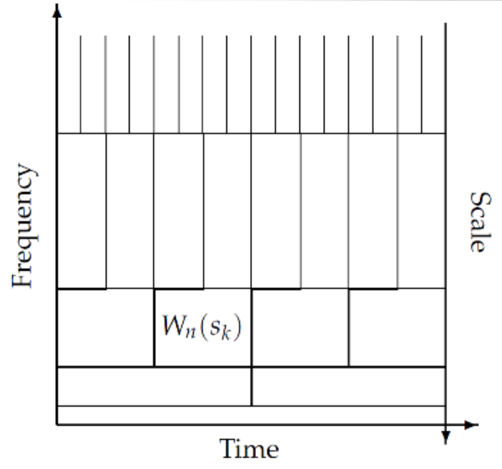
Noise transients:
- Fast ($\lesssim$ 1 sec): "glitches";
- Slow (adiabatic, locally stat.).

Time-frequency Representation

Wavelet transform

- Allows multiple Δt and Δf resolution, suitable for signals (either terrestrial or astrophysical) with peculiar *time-freq.* morphologies (glitches, GW transients);
- ⇨Extension of standard correlation (time) and coherence (analysis).

Implementations: Omega scans (D. Chatterji PhD), Omicron (arXiv:2007.11374), cWB (PRD 93.4, 2016), Bayeswave (CQG 32.13, 2015).

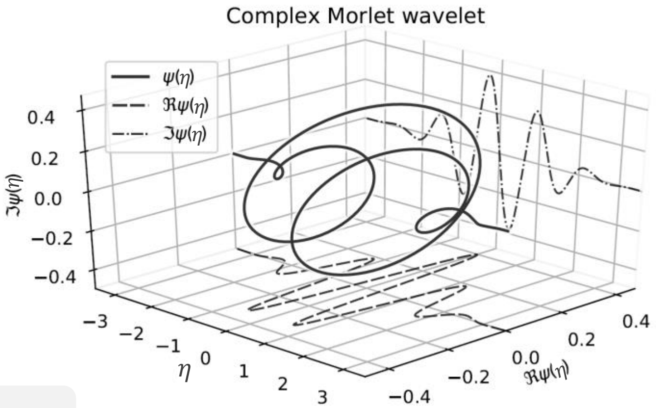


New developments

Exploit complex nature of **Morlet wavelet**:

$$\psi(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\eta^2/2} \rightarrow \boxed{W_n(s) = \frac{f_S}{N} \sum_{k=0}^{N-1} \tilde{x}_k \, \tilde{\psi}^*(s\omega_k) e^{i\omega_k n},}$$

$$\tilde{x}_k = \frac{1}{f_S} \sum_{n=0}^{N-1} x_n \, e^{i\omega_k n/N}, \qquad \omega_k = \frac{2\pi k}{N}, \qquad f^{-1} = \frac{4\pi \, s}{\omega_0 + \sqrt{2+\omega_0^2}}$$



⇨(Complex) **Wavelet coherence** and **instantaneous time delay statistic** for glitch studies.

Wavelet Power Spectrum

Time-frequency representation of the energy of a signal:

$$|W_n(s)|^2.$$

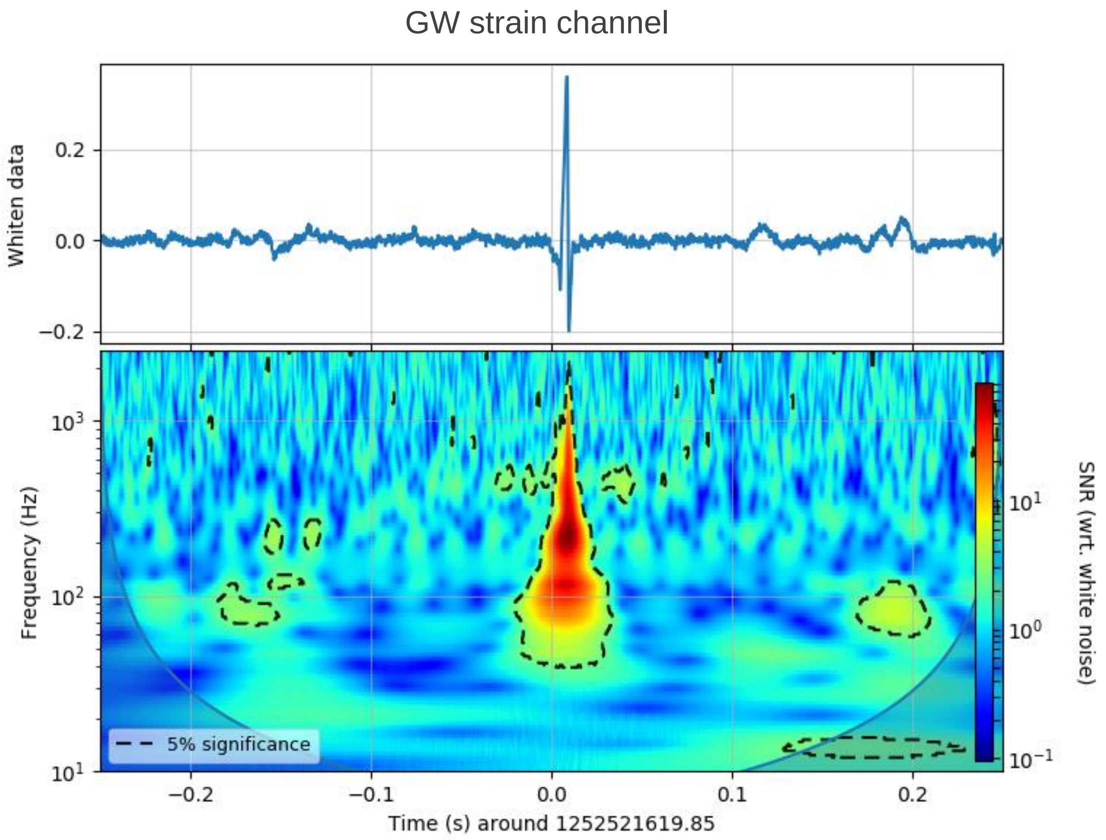
If (the process generating) the data is **stationary** and **Gaussian**:

- Data whitening: $\tilde{x}_k \rightarrow \tilde{x}_k/\sqrt{S_k}$

  S_k: k-th component of the PSD,
- such that: $E[|\tilde{x}_k|^2] = \sigma^2 \quad \forall k$
- and: $\boxed{\frac{|W_n(s)|^2}{N\sigma^2} \sim \frac{1}{2}\chi_2^2}$

We can define a test of stationarity and Gaussianity:

⇨ **detect transients (either of astro. or instrumental origin)**

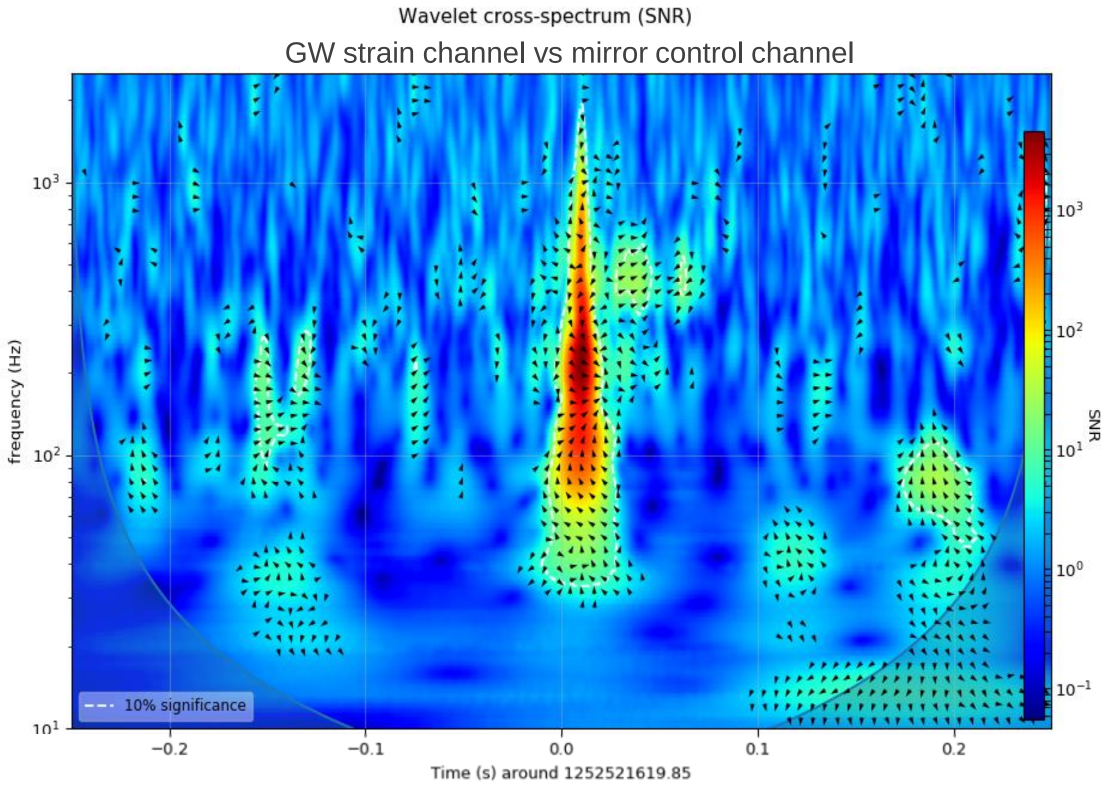


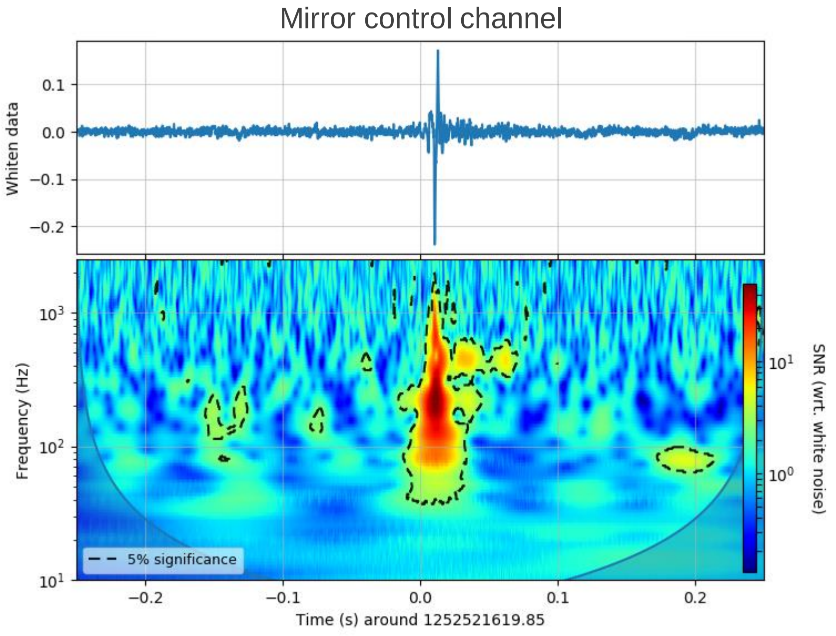
Wavelet Cross-spectrum

Generalises correlation and coherence analyses:

find *similarity* in the time-freq. distribution of energy between a *target* (X) and an *auxiliary* channel (Y):

$$W_n^{XY}(s) = W_n^X(s)^* W_n^Y(s)$$





Wavelet power-spectrum of a Virgo *auxiliary channel*: similar morphology to that of the target strain channel (see [previous page]).

Case study: investigation of some "Mystery glitches" in Virgo during O3a ([VIR-0918A-19])

Complex Wavelet Coherence

- ``Normalised'' wavelet cross-spectrum:

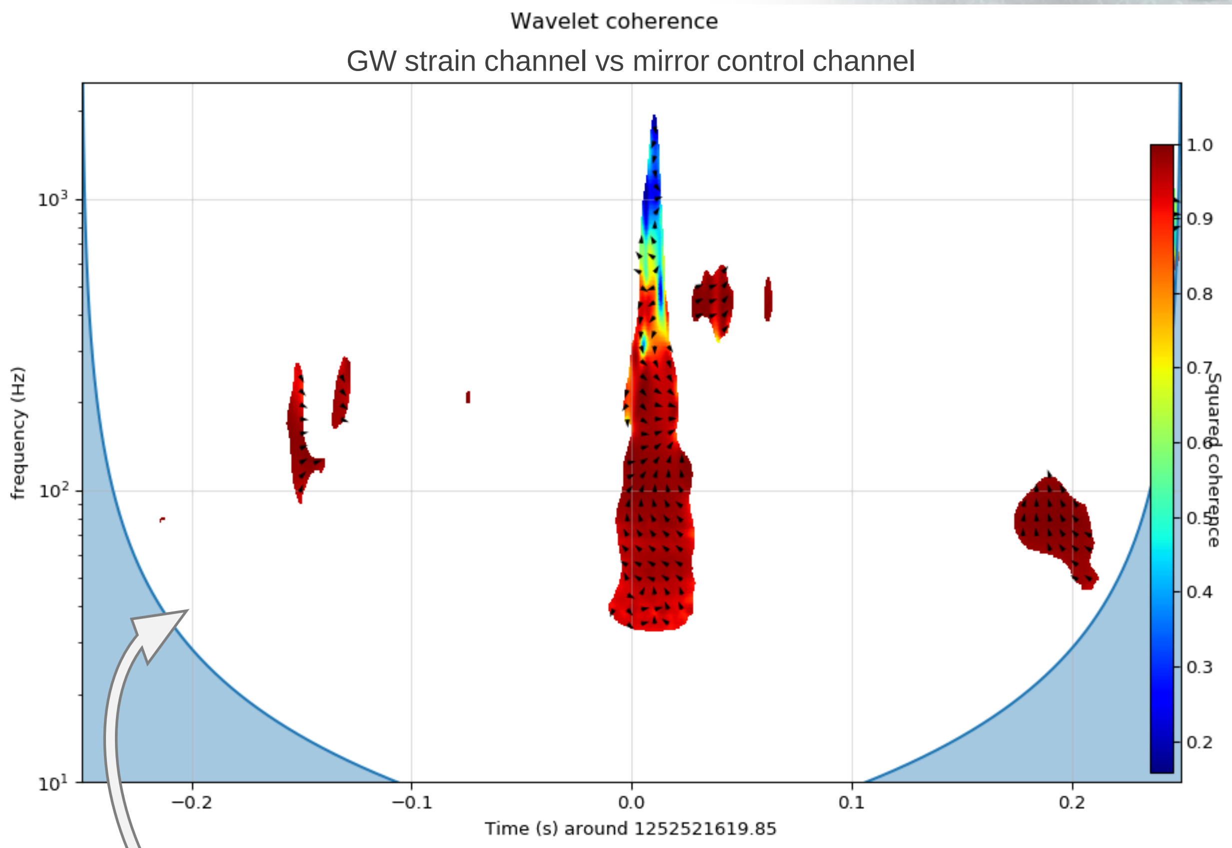
$$|C_n^{XY}(s)|^2 \in [0,1]$$

- Locally averaged over both time and scale (angle braces "$\langle \dots \rangle$", Torrence 1999) to reduce variance (akin to Welch's method for PSD estimate, Welch1967):

$$\boxed{C_n^{XY}(s) = \frac{|\langle s^{-1} W_n^{XY}(s) \rangle| \, e^{i\theta_n^{XY}(s)}}{\sqrt{\left\langle s^{-1} |W_n^X(s)|^2 \right\rangle \left\langle s^{-1} |W_n^Y(s)|^2 \right\rangle}}}$$

with $\theta_n^{XY}(s) = \tan^{-1}(\Im W_n^{XY}(s) / \Re W_n^{XY}(s))$;

- **Complex quantity (notice the arrows ◀)**: to be exploited in the next page…



Only the $\alpha = 10\%$ critical region for $W_n^{XY}(s)$, under the *null hypothesis* of stationarity and Gaussainty, is shown.

Instantaneous Time Delay Statistic

Computed from the phase θ_n^{XY} of the previous complex wavelet coherence, in the α-critical region only:

$$\Theta_n^{XY} = \frac{1}{2\pi}\sum_k \frac{\theta_n^{XY}\left(\frac{1}{f_k+\delta f_k}\right) - \theta_n^{XY}\left(\frac{1}{f_k}\right)}{\delta f_k}$$

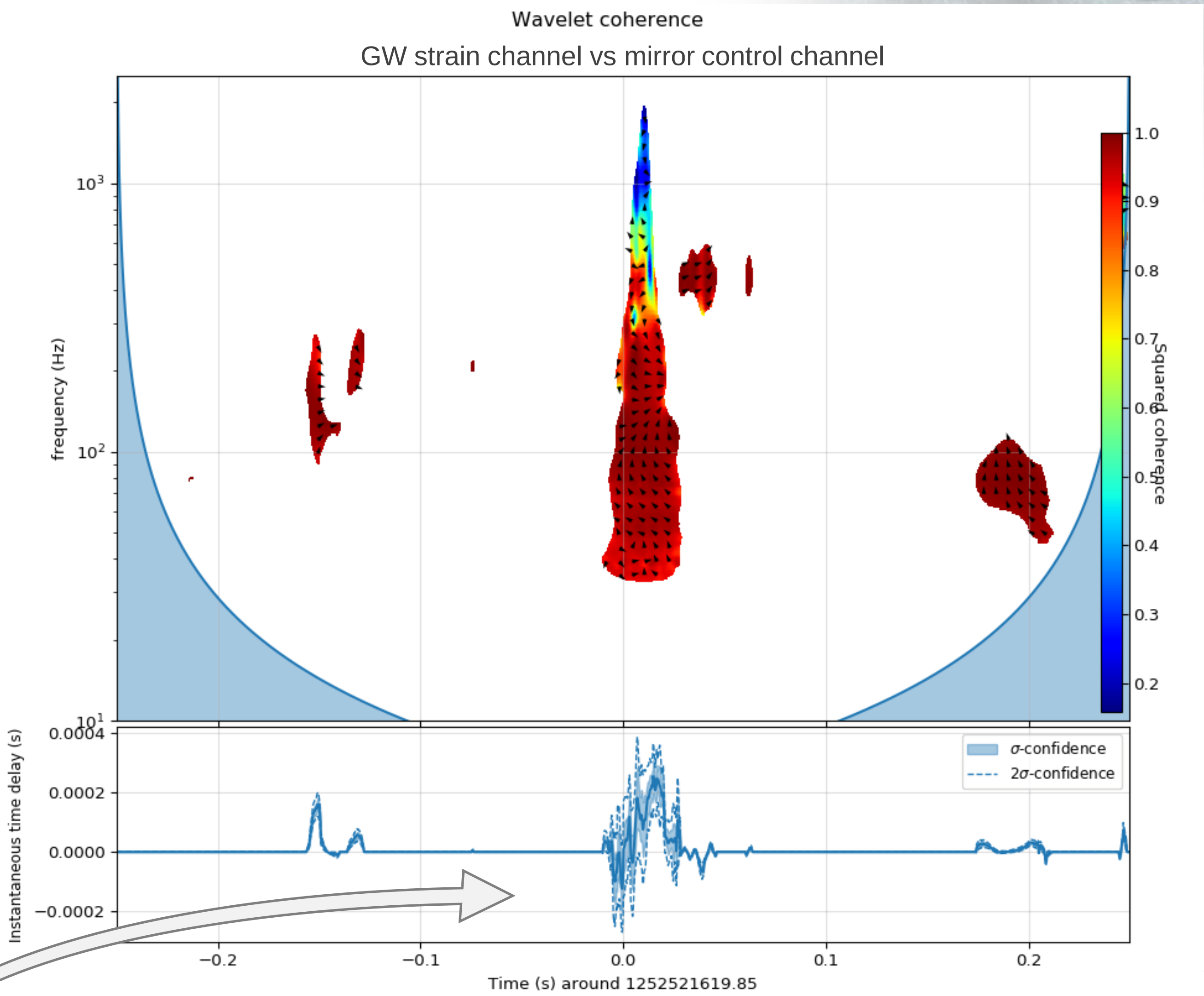
Property: if the same (fast) noise transient appears in two different channels (X and Y) with a time delay τ, that is

$$y_t = x_{t+\tau}, \qquad (y \text{ leads } x)$$

then:

$$\boldsymbol{\Theta_n^{XY} \approx \tau}$$

Case study: Virgo O3a "mystery glitches".



Notes and Examples (I)

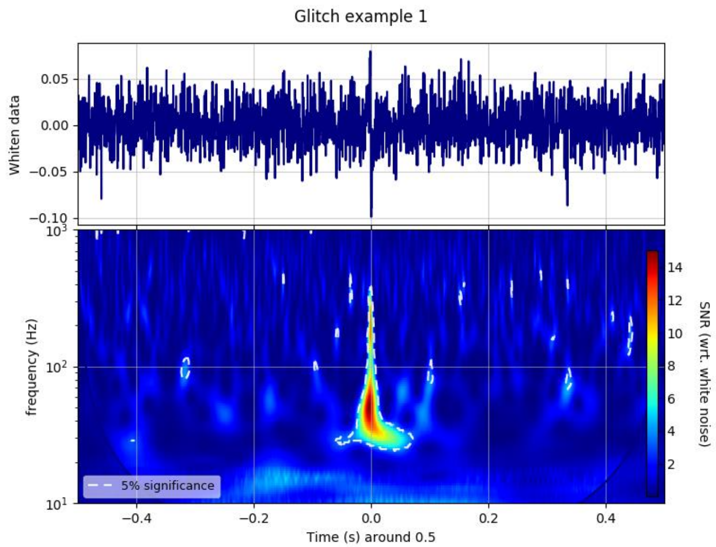
Example: (Dirac) impulse signals, $y_n = \delta_{n0}$, $x_n = \delta_{nm}$:

$$W_n^Y(s) = \psi^*\left(-\frac{n}{s\,f_s}\right), \qquad W_n^X(s) = \psi^*\left(\frac{m-n}{s\,f_s}\right)$$

$$W_n^X(s)^*\, W_n^Y(s) = \underbrace{\left(\frac{f_s}{\sqrt{\pi} s}\right) e^{-\frac{(m-n)^2+n^2}{2s^2 f_s^2}}}_{\text{Amplitude}}\ \underbrace{e^{\frac{2\pi i\, m}{s\, f_s}}}_{\text{Phase}} \qquad \Rightarrow \qquad \boldsymbol{\Theta_n^{XY} = m}$$

Example: simulated glitch $g(t) = A \cdot \text{sign}(t_0 - t)\, e^{-|t-t_0|/d}$

- A: amplitude,
- t_0: glitch time,
- d: decay time.
- Background: white gaussian noise N$(0, \sigma^2)$
- SNR $= A^2/\sigma^2 = 10$
- $\boldsymbol{\tau_0 = 1}$ ms



Notes and Examples (II)

Example: simulated glitch $g(t) = A \cdot \mathrm{sign}(t_0 - t)\, e^{-|t-t_0|/d}$

A: amplitude,

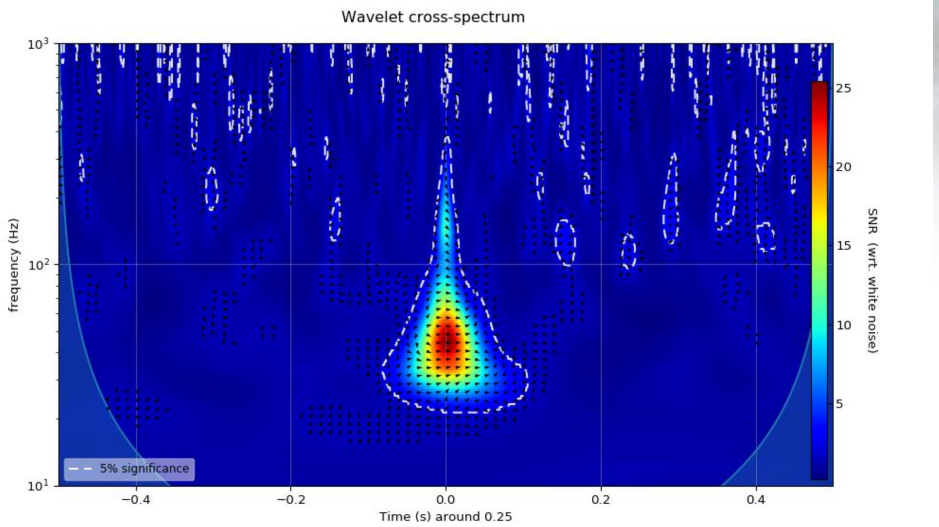
t_0: glitch time,

d: decay time.

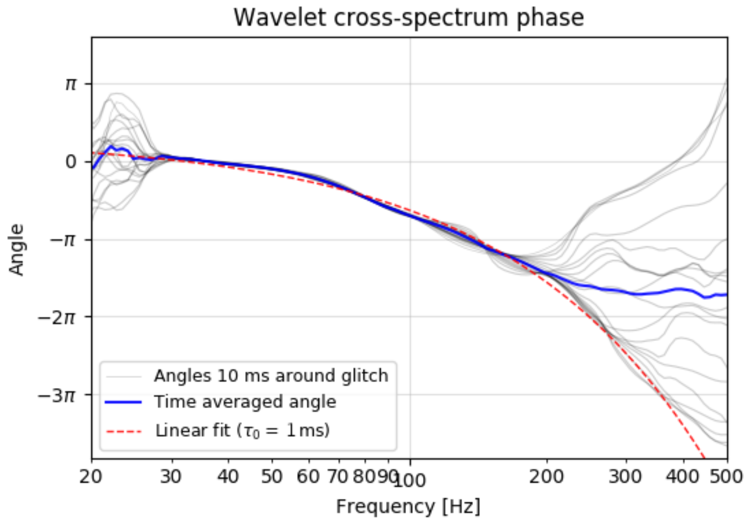
Background: white gaussian noise $\mathrm{N}(0, \sigma^2)$

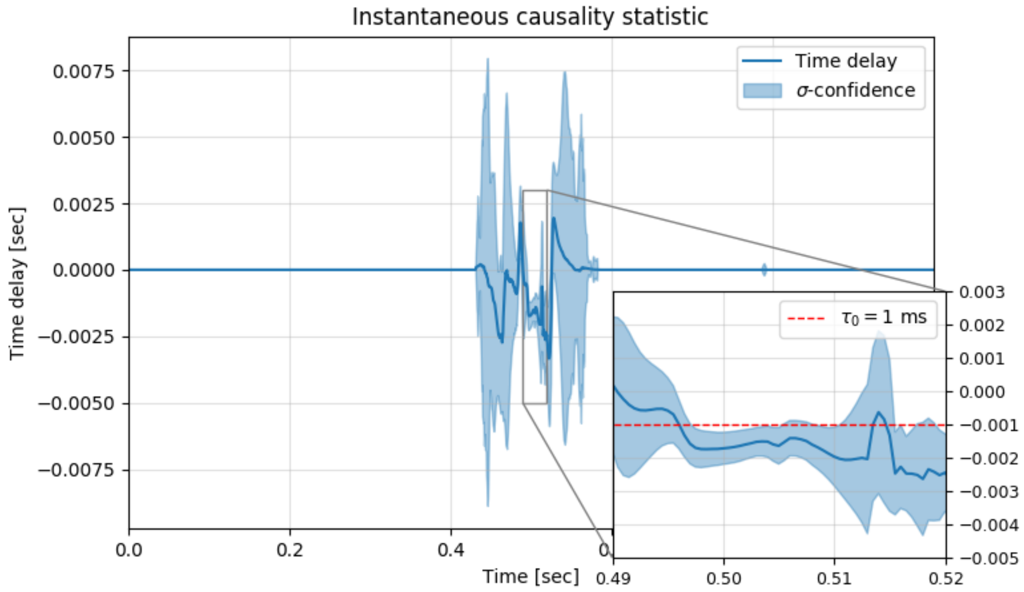
$\mathrm{SNR} = A^2/\sigma^2 = 10$

$\boldsymbol{\tau_0} = \mathbf{1}$ **ms**



- Characteristic linear behaviour of the phase angle spectrum;
- $\boldsymbol{\tau_0}$ recovered at the time of the glitches.





Conclusions

The **complex wavelet coherence** has been used for the study of glitches in gravitational wave detectors data:

- Time-frequency generalisation of standard techniques based on “correlation” (time) and “coherence” (freq.);
- Suitable for the study of glitches in the main strain channel and in auxiliary ones;
- Test of the significance of the similarity of their time-freq. morphologies.

New “**instantaneous time delay** statistic”:

- Exploit the complex nature of the wavelet coherence;
- If the same transient appears in two channels with a delay τ, the statistic equals this value;
- Frequency averaging (sum) makes it robust with respect to the uncertainties we have with only one realization of the processes, and allows to define “confidence regions” based on its standard deviation;
- Its evaluation only on the critical region of the wavelet cross-spectrum is an additional aid to get rid of spurious effects due to noise;
- Being the temporal precedence a necessary condition for one signal to constitute the “cause” of the other, it is tempting to interpret the previous instantaneous time delay, in fact, as a hint for a **causality relation**.