



POLITECNICO  
DI TORINO

Dipartimento  
di Scienza Applicata  
e Tecnologia



UNIVERSITÀ  
DI TRENTO

# Exchange enhancement of electron-phonon interactions in rhombohedral graphene

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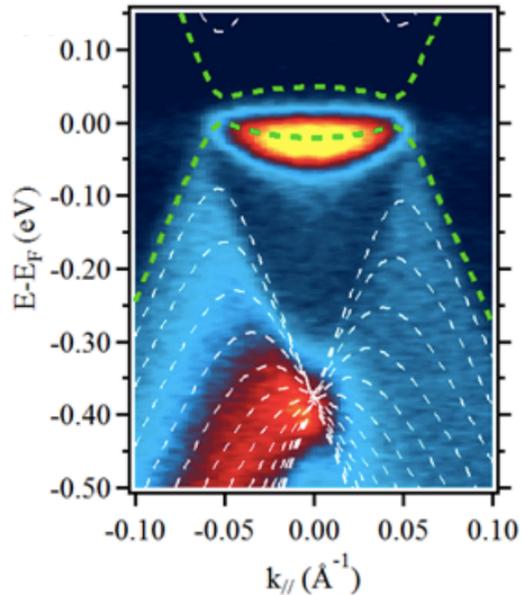
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# Strongly correlated phases in rhombohedral-stacked graphene

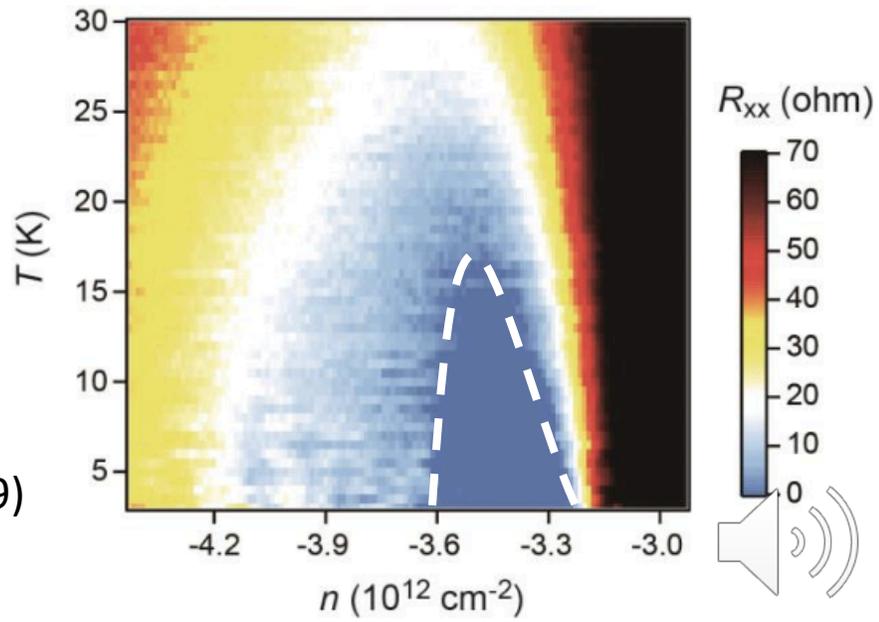
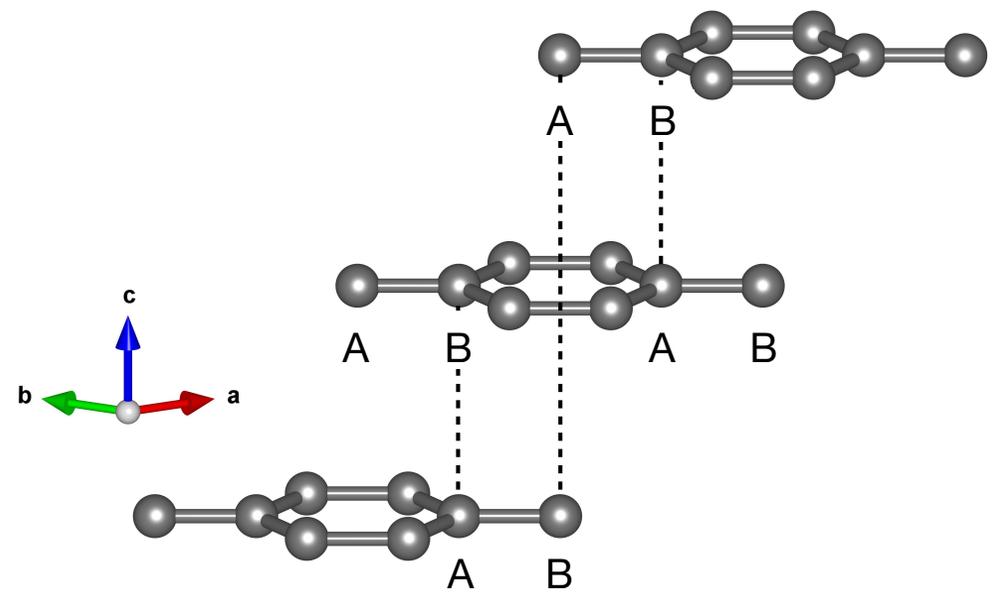


From first-principles calculations it is possible to identify an antiferromagnetic ground state with a band gap of about 40 meV due to electron-electron interaction, compatible with experiments.

H. Henck et al., Phys. Rev. B **97**, 245421 (2018)

When stacking rhombohedral graphene on top of hexagonal boron nitride and upon electrostatic doping, a SC dome was observed with a Berezinskii–Kosterlitz–Thouless transition up to  $T_{\text{BKT}} = 14$  K.

S. Moriyama et al., arXiv:1901.09356 [cond-mat.supr-con] (2019)



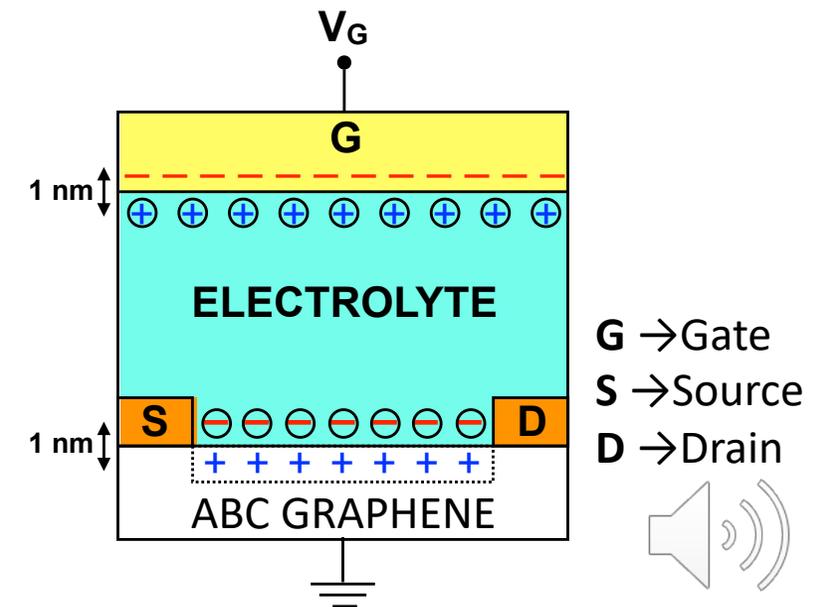
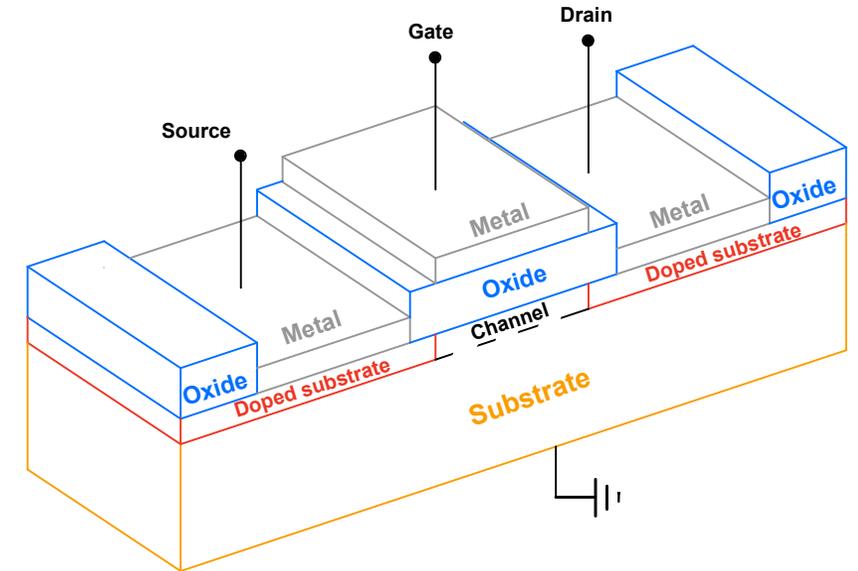
# Field-effect doping

- Effective doping of the first layers of the sample (either electrons or holes);
- Avoids the introduction of impurities and leads to a quasi-2D system;
- The amount of charge induced by ionic liquid ( $10^{14} \div 10^{15} \text{ cm}^{-2}$ ) is one or two orders of magnitude higher than solid state dielectric ( $10^{13} \text{ cm}^{-2}$ );

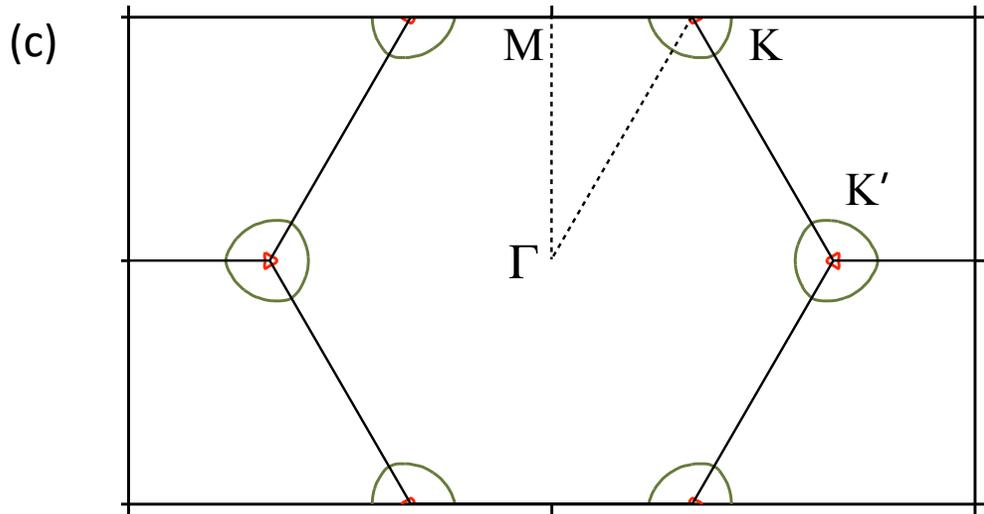
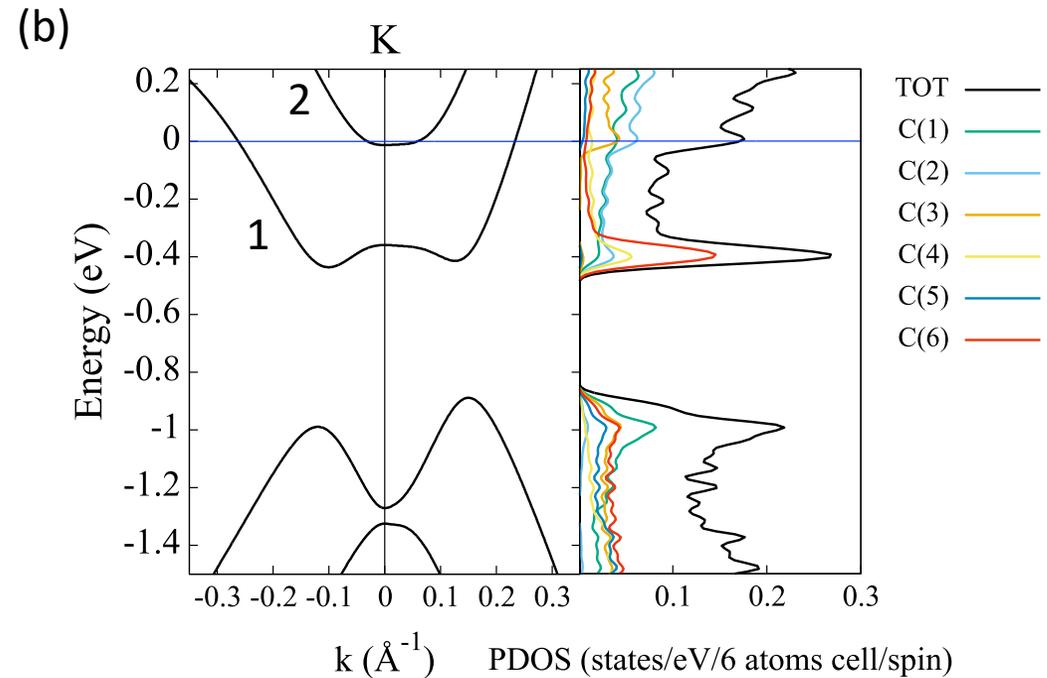
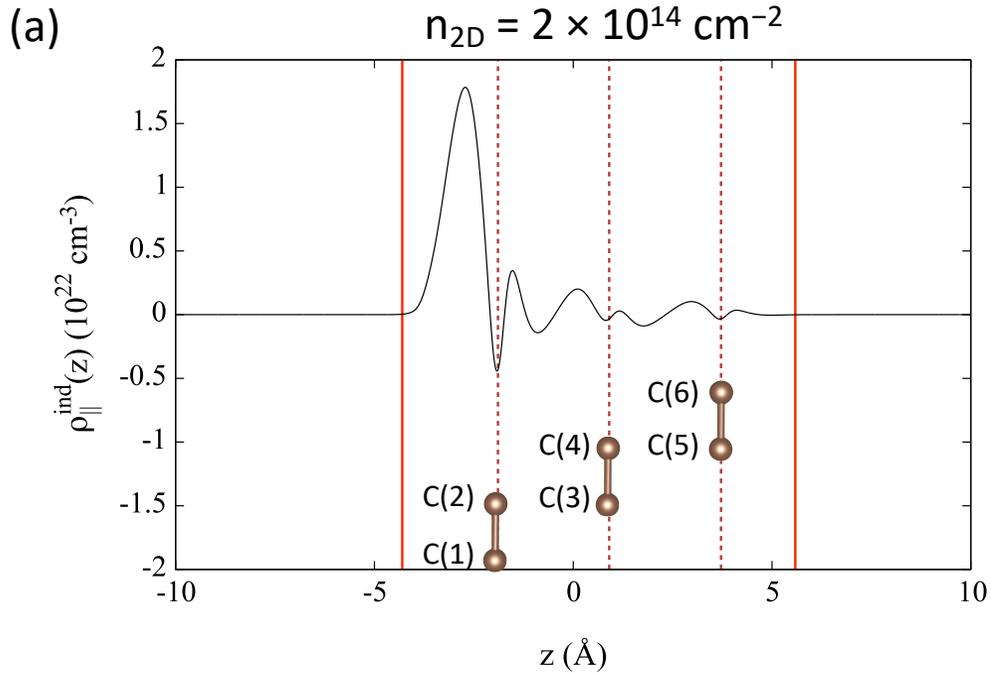
## Key features of the DFT-implemented FET model

[Th. Sohler, M. Calandra and F. Mauri, *Phys. Rev. B* **96**, 075448 (2017)]

- Ionic layer  $\rightarrow$  sheet of uniformly distributed charges;
- Potential barrier which prevents charge spilling;
- Coulomb interaction truncated along the direction perpendicular to the sample surface.



# Electronic structure

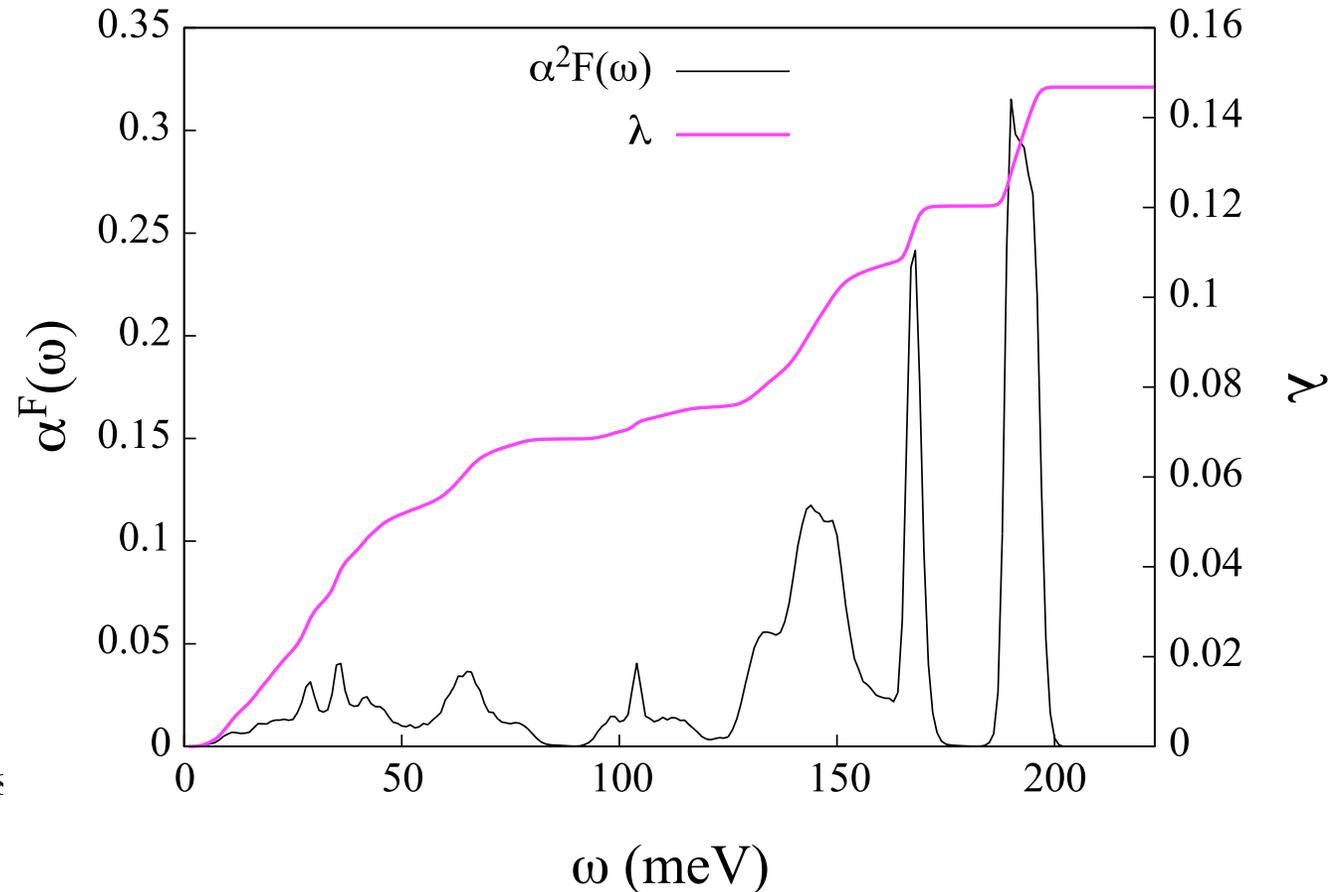
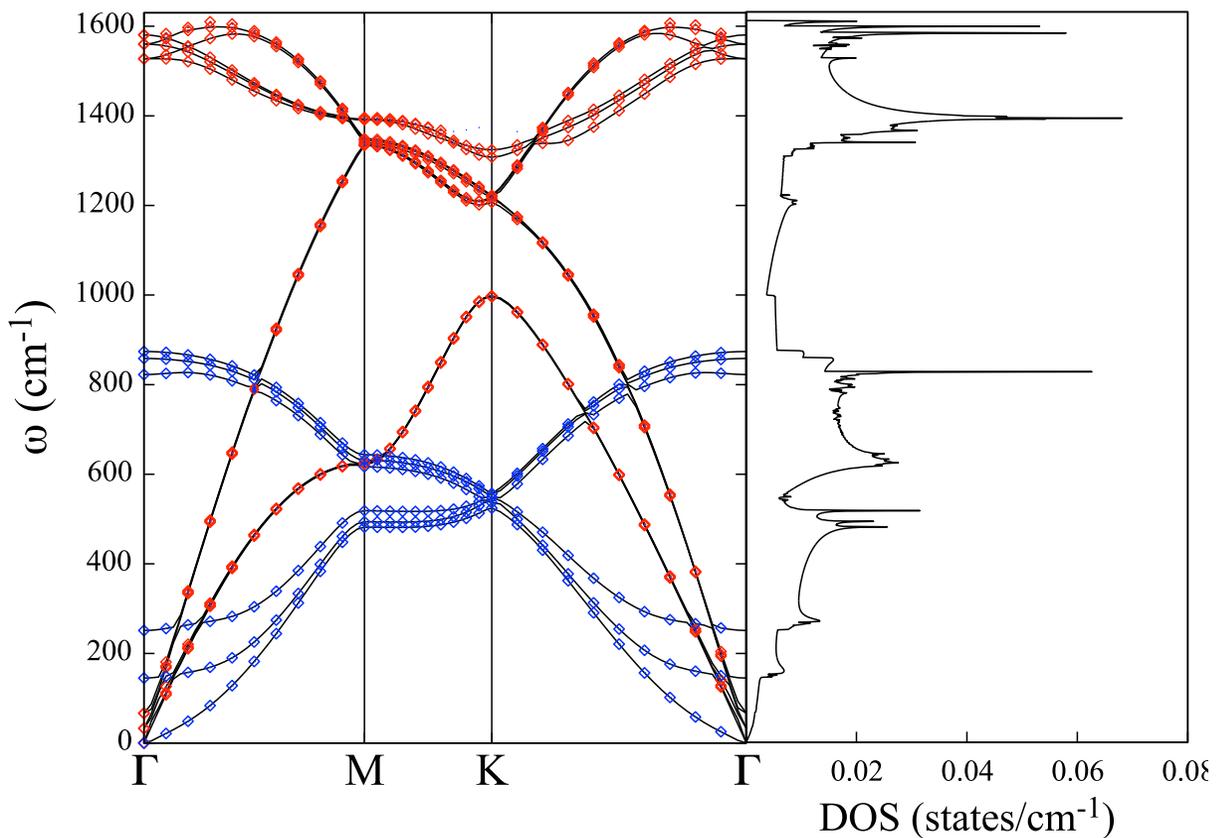


- The induced charge density is concentrated on the first layer;
- The first two carbon atoms (C(1) and C(2)) contribute to the van Hove singularity at the Fermi level;
- The Fermi surface consists of two concentric electron pockets centered around **K** and **K'**.



# Vibrational properties and superconductivity

Wannier interpolation of e-ph matrix elements:  
M. Calandra, G. Profeta, and F. Mauri, Phys. Rev. B **82**,  
165111 (2010)

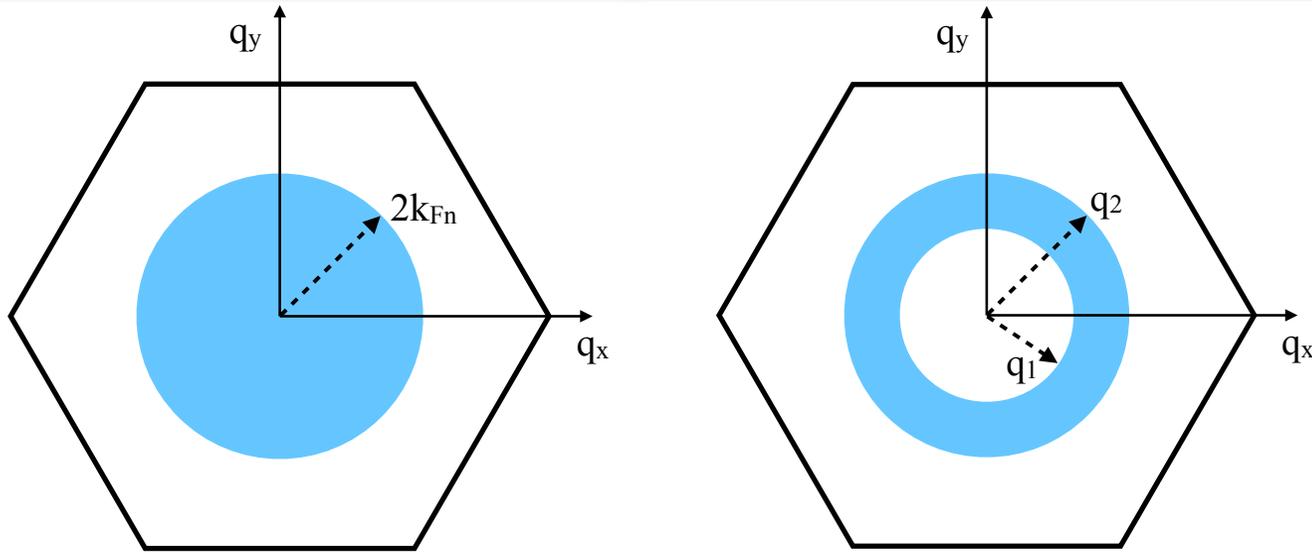


	$\lambda_{\Gamma}$	$\lambda_K$	$\lambda_{\text{TOT}}$	$\omega_{\text{log}}$ (meV)
Wannier (PBE)	0.095	0.053	0.148	74.27



# Simplified superconductive model

Appl. Surf. Sci. 496, 143709 (2019)  
arXiv:2001.08952 [cond-mat.supr-con]



$$|\mathbf{q}| = \sqrt{k_{Fn}^2 + k_{Fm}^2 - 2k_{Fn}k_{Fm} \cos \theta}$$

$$\lambda = \sum_{\mathbf{q}, \nu} \frac{N_{tot}(0) \langle g_{\nu}^2 \rangle_{\mathbf{q}}}{\omega_{\mathbf{q}\nu}}$$

$$\log\{\omega_{log}\} = \frac{N_{tot}(0)}{2\lambda} \sum_{\mathbf{q}, \nu} \frac{\langle g_{\nu}^2 \rangle_{\mathbf{q}}}{\omega_{\mathbf{q}\nu}} \log\{\omega_{\mathbf{q}\nu}\}$$

McMillan/Allen-Dynes:

$$T_c = \frac{\omega_{log}}{1.2} \exp\left\{-\frac{1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)}\right\}$$

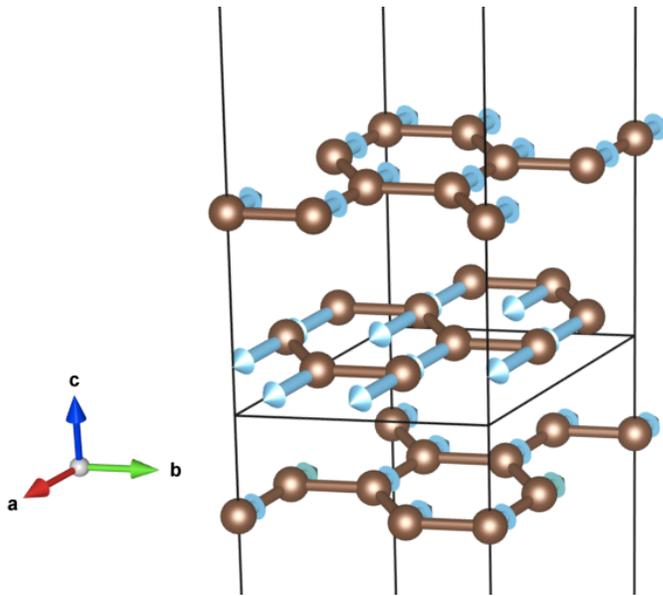
	$\lambda_{\Gamma}$	$\lambda_{K}$	$\lambda_{TOT}$	$\omega_{log}$ (meV)
Wannier (PBE)	0.095	0.053	0.148	74.27
Simple PBE	0.096	0.012	0.108	113

- The simplified model slightly underestimates the Wannier-interpolated values;
- In both cases, no superconductivity is found ( $\mu^*=0.1$ ).

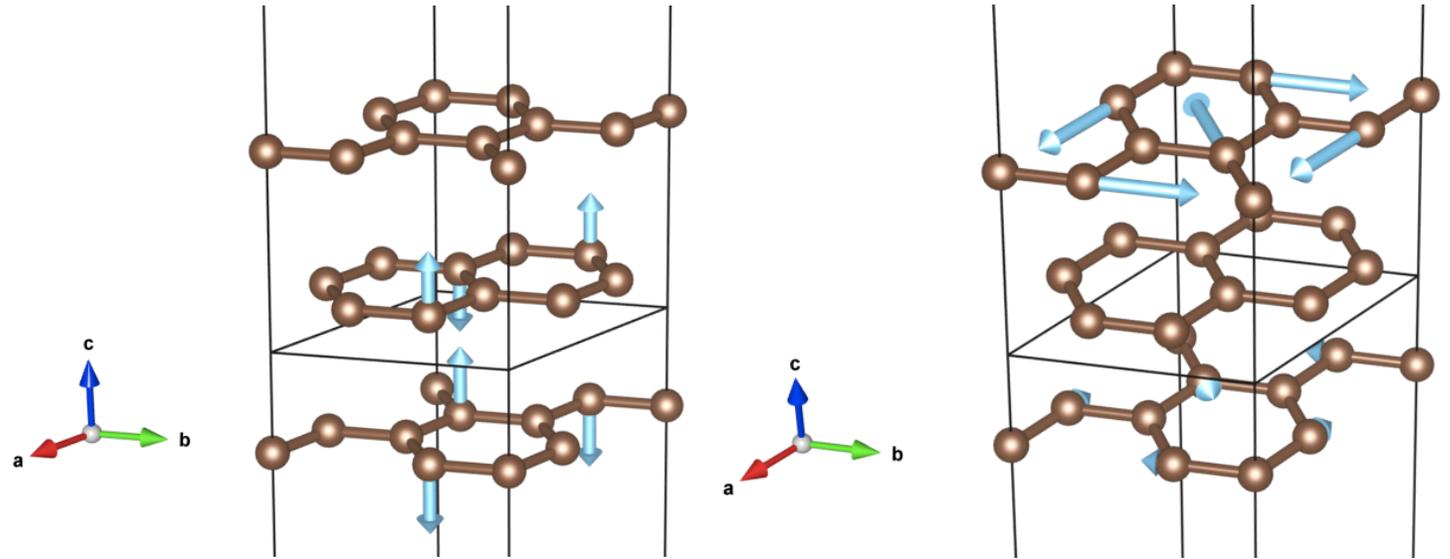


# Simplified superconductive model

$q=\Gamma$



$q=K$

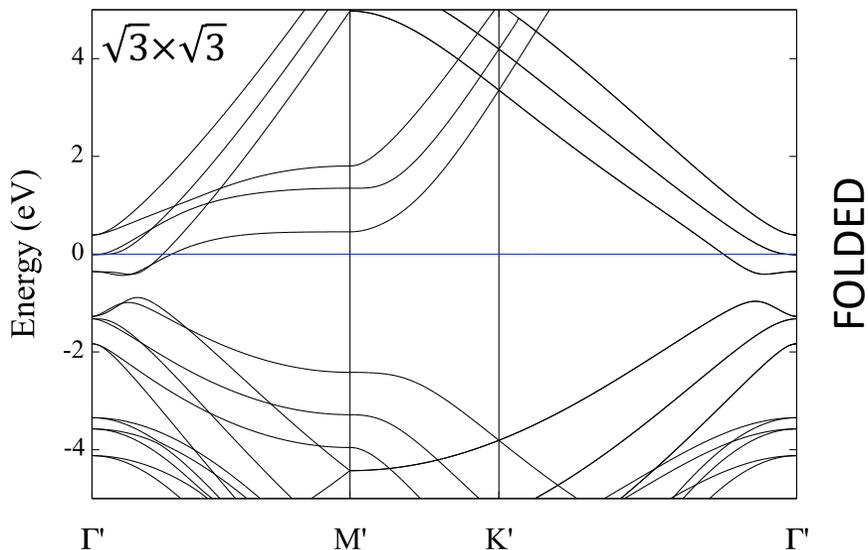
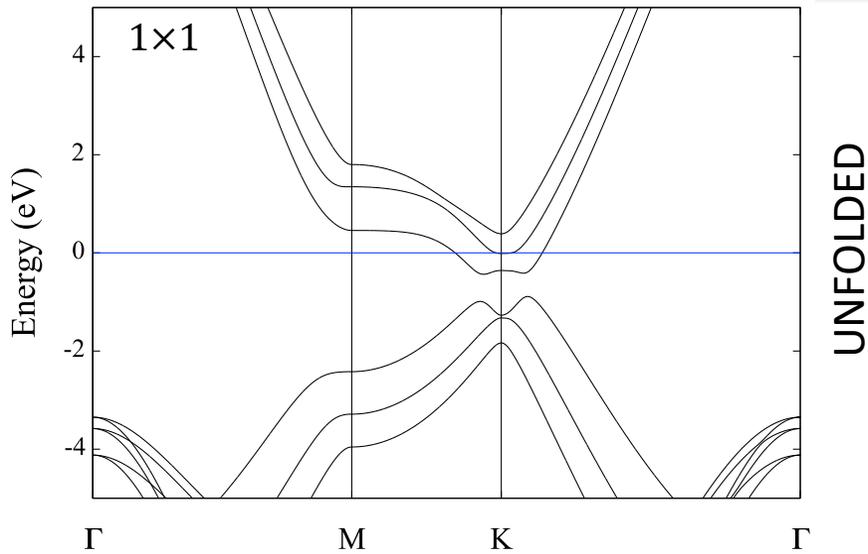


	$\lambda_{\Gamma}$	$\lambda_K$	$\lambda_{TOT}$	$\omega_{log}$ (meV)
Wannier (PBE)	0.095	0.053	0.148	74.27
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# Finite differences computation of e-ph matrix elements



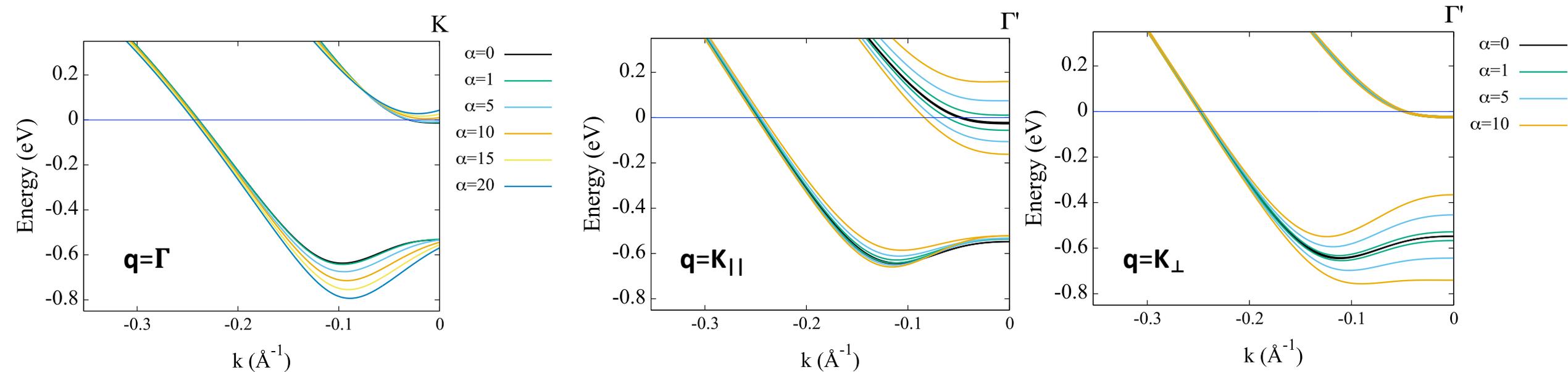
- The electron-phonon matrix elements are proportional to the variations of the KS Hamiltonian with respect to atom displacement:

$$g_{\mathbf{K}n,\mathbf{K}+q}^{\nu} \propto \left\langle \phi_n \left| \frac{\delta H_{KS}}{\delta \mathbf{u}_q^{\nu}} \right| \phi_m \right\rangle = \left\langle \phi_n \left| \frac{H_{KS}(\alpha \mathbf{u}_q^{\nu}) - H_{KS}(0)}{|\mathbf{u}_q^{\nu}|} \right| \phi_m \right\rangle$$

- We need to calculate the variation of the eigenvalue (i.e. the electronic band) as a function of an atomic displacement.
- This technique requires the knowledge of the phonon displacement for the modes giving rise to strong electron-phonon interactions;
- Notice that, in order to obtain the correct phonon displacement, the  $\mathbf{q}=\mathbf{K}$  mode will be computed in a  $\sqrt{3} \times \sqrt{3}$  supercell, where  $\mathbf{K}$  is folded into  $\mathbf{\Gamma}$



# Electron-phonon boost via hybrid functionals



	$\lambda_{\Gamma}$	$\lambda_{K}$	$\lambda_{TOT}$
Simple PBE	0.096	0.012	0.108
Simple PBE0	0.218	0.028	0.246
Simple B3LYP	0.197	0.035	0.232



# Conclusions

- A simplified superconductive model correctly describes the system under study from a qualitative point of view, slightly underestimating  $\lambda$ ;
- The inclusion of the exact exchange interaction among electrons via hybrid functionals boost the electron-phonon coupling constant  $\lambda$  by approximately a factor 2;
- The McMillan/Allen-Dynes formula ( $\mu^*=0.1$ ), together with the computed values of  $\lambda$  and  $\omega_{\log}$ , shows that phonon mediated superconductivity is not induced in such system for  $n_{2D} < 2 \cdot 10^{14} \text{ cm}^{-2}$ .

Thank you for your attention!

## Acknowledgments

Computational resources were provided by the **CINECA** award «**ISCRA C**» (HP10CY4S13, 2020) and by **HPC@POLITO**, which is a project of Academic Computing within the Department of Control and Computer Engineering at the Politecnico di Torino (<http://hpc.polito.it>)

