Phenomenology of Combined Resummation for Transverse Momentum Distributions: Higgs and Drell-Yan

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UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

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Theoretical Predictions

QCD factorization theorem as a main guiding principle for making theoretical predictions:

$$\frac{d\sigma}{d\xi_p}(\xi_p,\alpha_s) = \tau' \sum_{ab} \int_{\tau'}^{1} \frac{dx}{x} \int_{x}^{1} \frac{dy}{y} f_a(y)\, f_b\left(\frac{x}{y}\right) \times \frac{d\hat{\sigma}_{ab}}{d\xi_p}(x,\xi_p,\alpha_s)$$

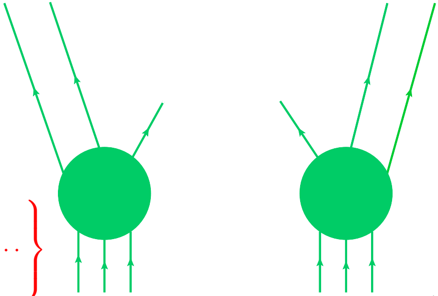
with $\xi_p = p_T^2/Q^2$ and

$$x = \frac{Q^2}{\hat{s}}\left(\sqrt{1+\xi_p}+\sqrt{\xi_p}\right)^2$$

$$\tau' = \frac{Q^2}{s}\left(\sqrt{1+\xi_p}+\sqrt{\xi_p}\right)^2$$

Fixed-order (FO) expansion:

$$\frac{d\hat{\sigma}_{ab}}{d\xi_p} = \sigma^{(0)}\left\{\underbrace{1}_{\text{LO}} + \underbrace{\alpha_s\Sigma_{ab}^{(1)}}_{\text{NLO}} + \underbrace{\alpha_s^2\Sigma_{ab}^{(2)}}_{\text{NNLO}} + \cdots\right\}$$



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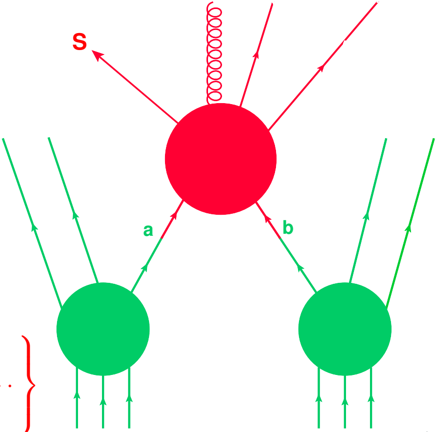

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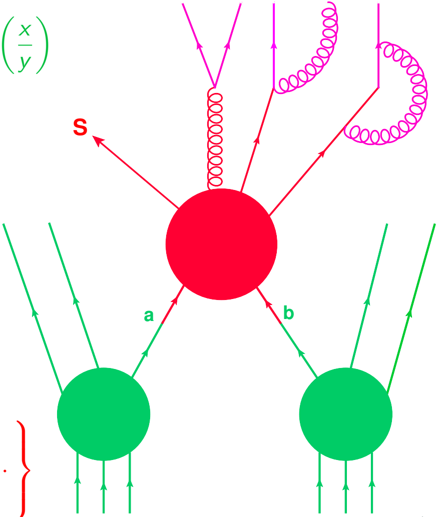

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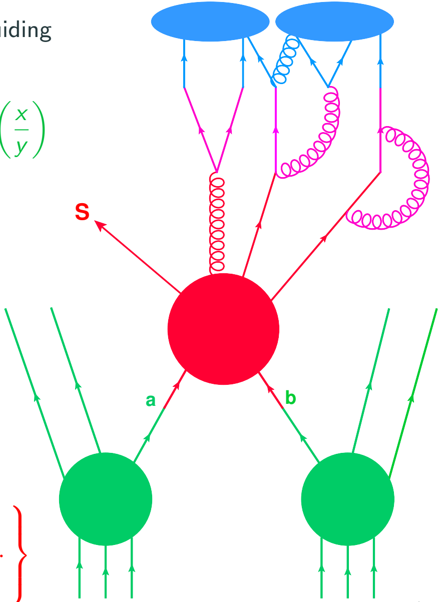

QCD beyond FO

Fixed-order calculations assume that $\Sigma_{ij}^{(n)}$ are **well-behaved** which allows for the perturbative expansion of the partonic cross-section.

What happens when the smallness of the running of the coupling α_s is compensated by large logarithms?

$$\alpha_s^n \log^m(\mu) \sim 1$$

Indeed, typical behavior of $\Sigma^{(n)}$ in a **multi-scale** problem:

$$\Sigma^{(1)} = c_{21}\mathrm{L}^2 + c_{11}\mathrm{L}, \quad \Sigma^{(2)} = c_{42}\mathrm{L}^4 + c_{32}\mathrm{L}^3 + c_{22}\mathrm{L}^2 + c_{21}\mathrm{L} \quad (\mathrm{L} = \log(\mu))$$

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$\Longrightarrow$ **Performing all-order resummation of the logarithms.**

Resummation in a Nutshell

$$\frac{d\hat{\sigma}_{ab}}{d\xi_p} = 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{m} c_{n,k} \mathrm{L}^m \quad \text{(Factoring out } \sigma^{(0)}\text{)}$$

Fixed-coupling α_s with $m = 2n$ (soft resummation) at LL:

$$\frac{d\hat{\sigma}_{ab}^{\mathrm{LL}}}{d\xi_p} = 1 + \sum_{n=1}^{\infty} c_{n,2n} \left(\alpha_s \mathrm{L}^2\right)^n$$

On the other hand, multiple emissions factorize (eikonal approximation):

$$c_{n,2n} = \frac{(c_{1,2})^n}{n!}$$

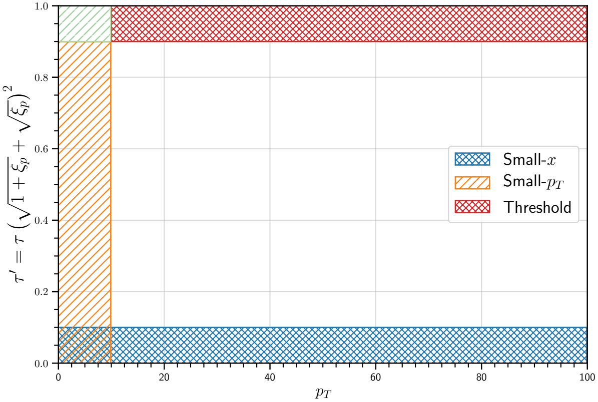
Exponentiation:

$$\frac{d\hat{\sigma}_{ab}^{\mathrm{LL}}}{d\xi_p} = \exp\left(\alpha_s c_{1,2} \mathrm{L}^2\right)$$

Beyond LL, it can be shown that

$$\frac{d\hat{\sigma}_{ab}^{\mathrm{LL}}}{d\xi_p} = \mathcal{H} \exp\left(\underbrace{\alpha_s^{-1} f_1(\alpha_s \mathrm{L})}_{\mathrm{LL}} + \underbrace{f_2(\alpha_s \mathrm{L})}_{\mathrm{NLL}} + \underbrace{\alpha_s f_3(\alpha_s \mathrm{L})}_{\mathrm{NNLL}} + \cdots\right)$$

Which regions to resum?



- **Small-pT Limit** : $\hat{\tau} \approx x$ or $\xi_p \to 0 \iff \ln \xi_p$.
- **High-energy Limit**: $s \to \infty$ or $\hat{\tau} \to 0 \iff \ln s$.
- **Threshold Limit**: $\hat{\tau} \to \hat{\tau}_{max} = \left(\sqrt{1+\xi_p} - \sqrt{\xi_p}\right)^2$ or $x \to 1$ $\iff \ln(1-x)$.

Combining Resummations?

SMALL-p_t/TRANSVERSE MOMENTUM RESUMMATION

SMALL-x/HIGH ENERGY RE-SUMMATION

SOFT-x/THRESHOLD RESUMMATION

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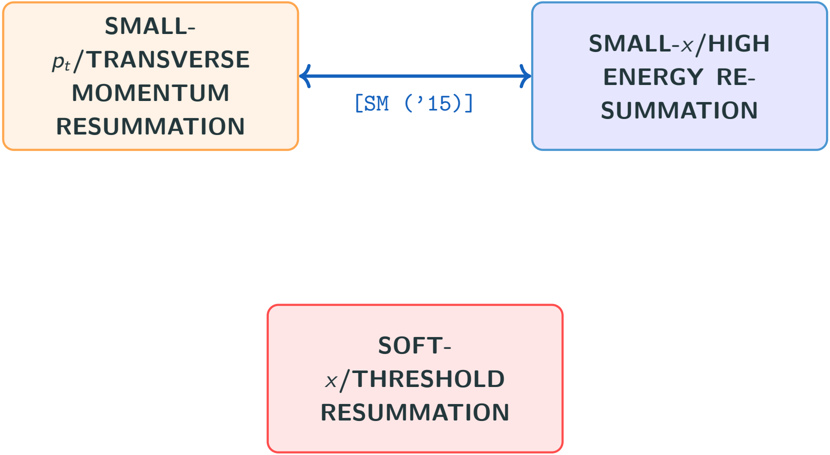

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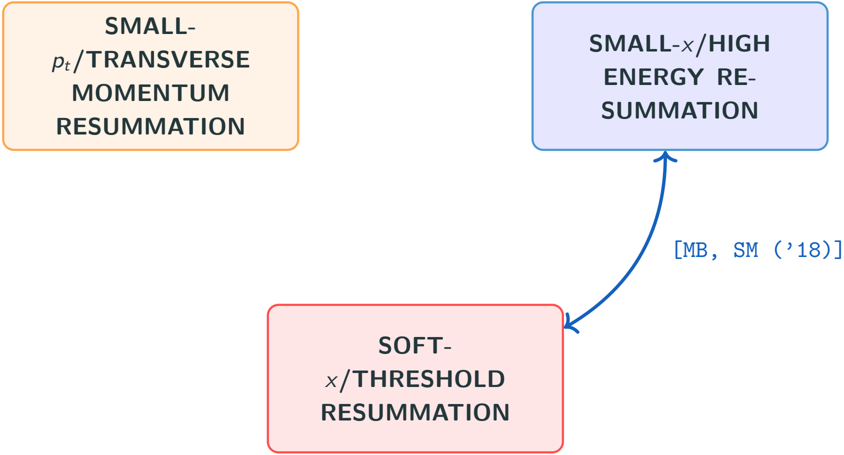

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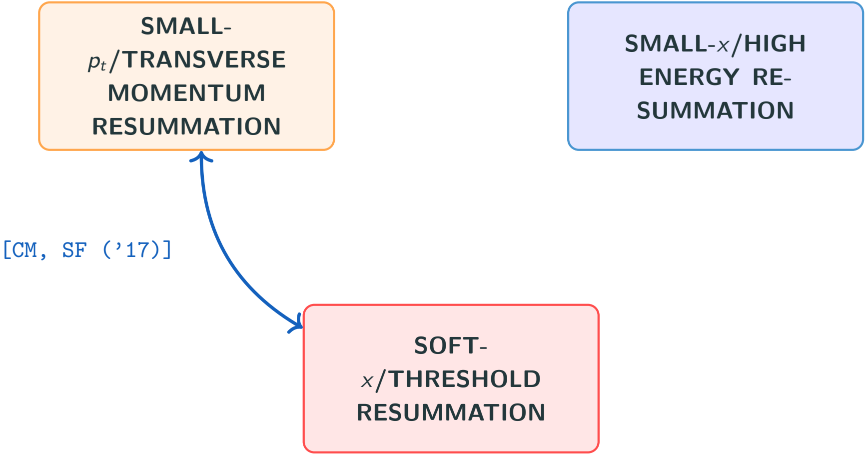

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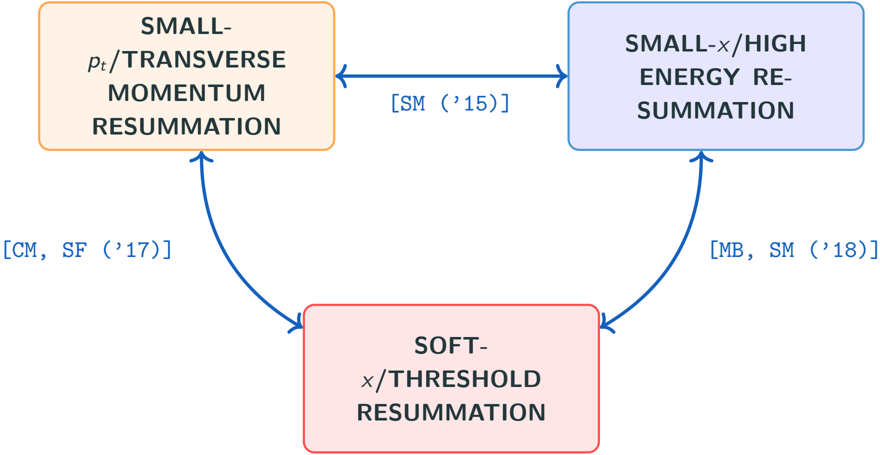


Threshold-improved Small-p_T (TIpT) resummation

Motivation?

Large logarithms present in threshold and transverse momentum **both originate from soft-gluon emission**.

Which requirements should such a resummation satisfy?

(1) **Reproduces standard small-p_T** (SSpT) resummation to some fixed logarithmic accuracy in the limit p_T is small $(p_T \to 0)$.

(2) **Reproduces threshold resummation** to some given logarithmic accuracy in the soft limit $(x \to 1)$.

(3) **Leads to the total cross section upon integration over p_T:**

$$\exp\left\{S(\alpha_s^n \mathrm{L}^m)\right\}_{\mathrm{L}\to 0} = 1 \longrightarrow \int_0^\infty d\xi_p \left(\frac{d\hat{\sigma}}{d\xi_p}\right) = \hat{\sigma}^{\mathrm{TOT}}$$

(In SSpT, this is enforced by the **Unitarity Constraint**.)

Combined Resummation [Forte, Muselli, Ridofli ('17)]

Constructing TIpT resummation

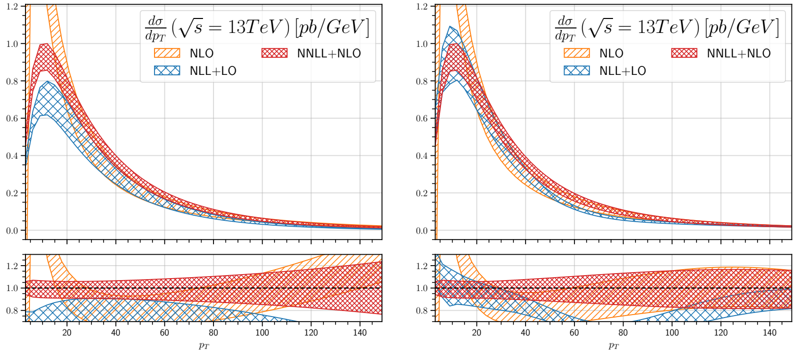
- Switch to Conjugate Space ($\xi_p \to b, \tau' \to N$)
- Modified Phase Space factorization: take small-p_T limit at fixed N/b
- Modified argument in the logarithms: $\chi = \bar{N}^2 + (b^2 Q^2)/b_0^2$
- Different treatment of DGLAP evolution: in order to compute N^{k}LL threshold-improved small-p_T resummation, the large-N behavior of the evolution has to be included up to N^{k}LO.

Combined Expression

$$\frac{\mathrm{d}\hat{\sigma}_{ab}}{\mathrm{d}\xi_p}(N, \xi_p, \alpha_s) = \left(1 - \mathrm{T}(N, \xi_p)\right) \frac{\mathrm{d}\hat{\sigma}_{ab}^{\text{TIpT}}}{\mathrm{d}\xi_p}(N, \xi_p, \alpha_s) + \mathrm{T}(N, \xi_p) \frac{\mathrm{d}\hat{\sigma}_{ab}^{\text{thrs}}}{\mathrm{d}\xi_p}(N, \xi_p, \alpha_s)$$

- For small-p_T, T gets rid of the $\xi_p \to 0$ **singularity** and only keeps $\mathrm{d}\hat{\sigma}_{ab}^{\text{TIpT}}$ which contributes to the total cross-section.
- For finite p_T and large-N, T gets rid of the $N \to \infty$ **singularity** and only keeps $\mathrm{d}\hat{\sigma}_{ab}^{\text{thrs}}$ which reproduces the soft behavior.

Higgs boson production [T.R. ('20)]

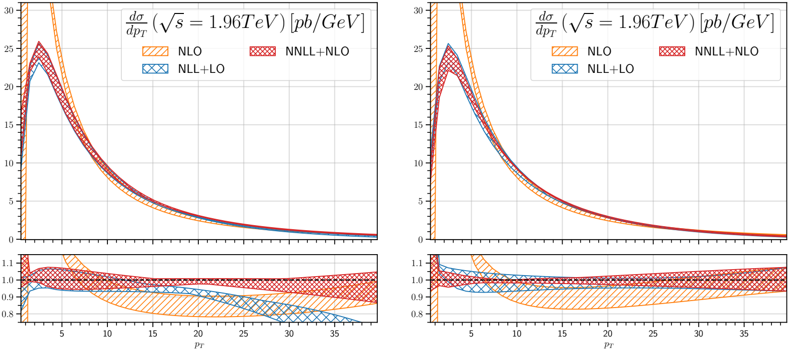


(a) SSpT resummation **(b)** combined resummation (TIpT + Threshold)

Combined TIpT and Threshold resummation yields:

- Faster convergence
- Reduced uncertainties
- Better agreement with FO results for moderate and large values of P_T

Drell–Yan (through Z boson production) [T.R. ('20)]



(a) SSpT resummation **(b)** combined resummation (TIpT + Threshold)

Combined TIpT and Threshold resummation yields:

- Moderate improvements in the small-p_T region
- Reduced uncertainties in medium-large p_T region
- Better agreement with FO results for moderate and large values of P_T

Conclusions and & Outlooks

Summary

- Resummed calculations needed for accurate predictions
- TIpT elucidates the relation between collinear and soft logarithms
- TIpT accelerates convergences and is more reliable beyond small-p_T

Outlook

- Extend accuracy of the resummation to N3LL in order to produce N3LL+NNLO predictions
- Combined resummation without relying on a profile matching function
- Include high-energy (or small-x) resummation

THANK YOU