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# JET CLUSTERING

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# ANTI $k_T$ ALGORITHM

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- M. Cacciari, G. P. Salam and G. Soyez, The anti- $k_t$  jet clustering algorithm, JHEP 04 (2008) 063, arXiv: 0802.1189 [hep-ph].
- M. Cacciari, G. P. Salam and G. Soyez, FastJet user manual, Eur. Phys. J. C 72 (2012) 1896, arXiv: 1111.6097 [hep-ph].

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- M. Cacciari, G. P. Salam and G. Soyez, The anti- $k_t$  jet clustering algorithm, JHEP 04 (2008) 063, arXiv: 0802.1189 [hep-ph].

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## The anti- $k_t$ jet clustering algorithm

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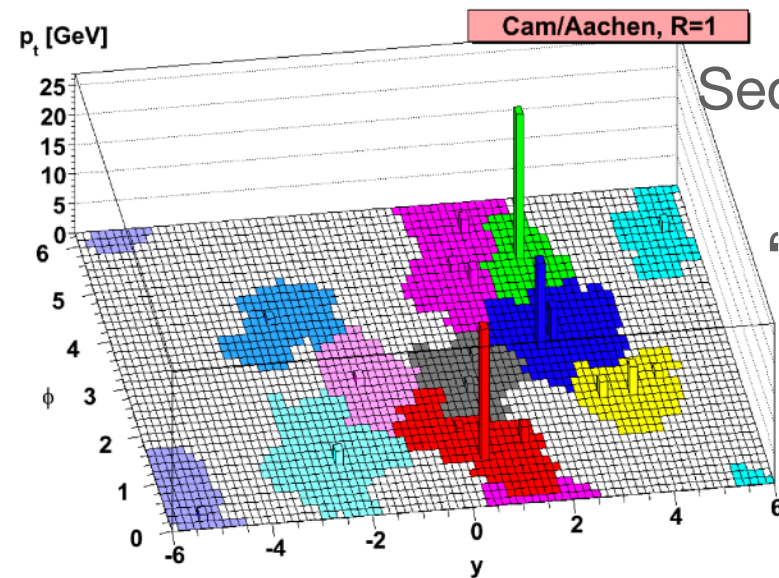
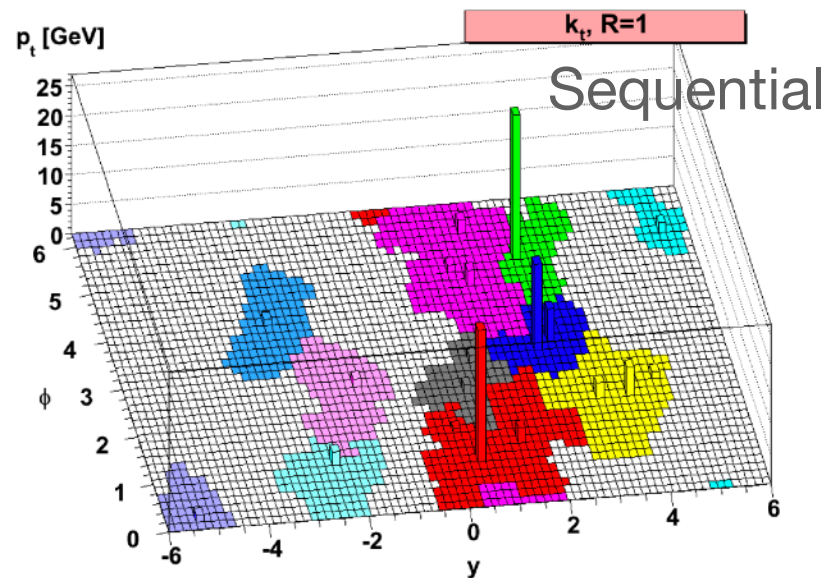
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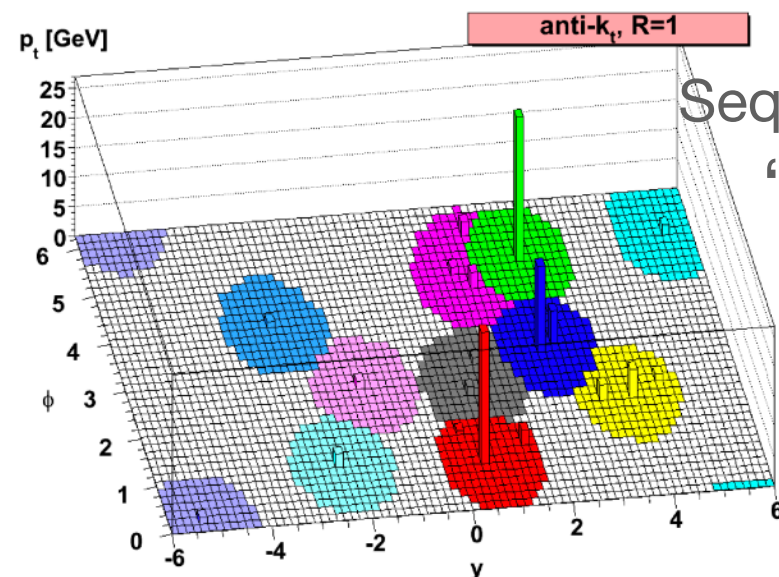
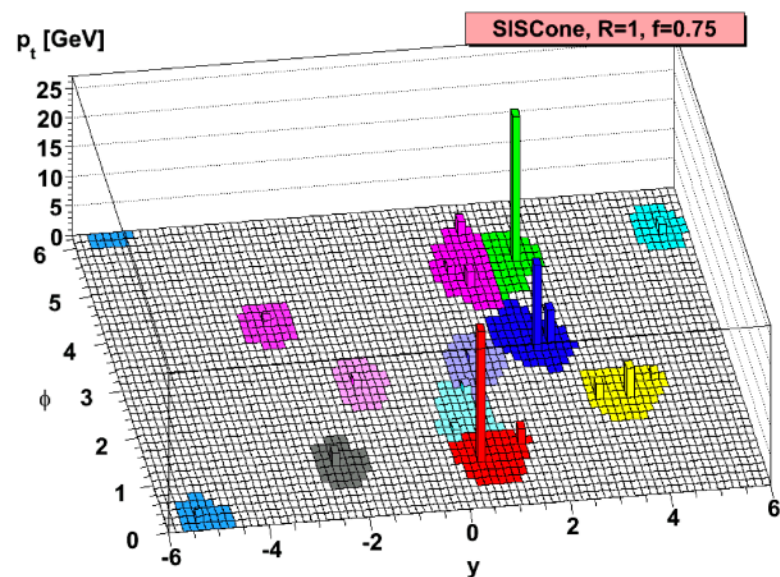
**Abstract:** The  $k_t$  and Cambridge/Aachen inclusive jet finding algorithms for hadron-hadron collisions can be seen as belonging to a broader class of sequential recombination jet algorithms, parametrised by the power of the energy scale in the distance measure. We examine some properties of a new member of this class, for which the power is negative. This “anti- $k_t$ ” algorithm essentially behaves like an idealised cone algorithm, in that jets with only soft fragmentation are conical, active and passive areas are equal, the area anomalous dimensions are zero, the non-global logarithms are those of a rigid boundary and the Milan factor is universal. None of these properties hold for existing sequential recombination algorithms, nor for cone algorithms with split-merge steps, such as SISCone. They are however the identifying characteristics of the collinear unsafe plain “iterative cone” algorithm, for which the anti- $k_t$  algorithm provides a natural, fast, infrared and collinear safe replacement.

- M. Cacciari, G. P. Salam and G. Soyez, The anti-kt jet clustering algorithm, JHEP 04 (2008) 063, arXiv: 0802.1189 [hep-ph].

Infrared and  
collinear  
(IRC)



‘soft-adaptable’



‘soft-resilient’

Figure 1: A sample parton-level event (generated with Herwig [8]), together with many random soft “ghosts”, clustered with four different jets algorithms, illustrating the “active” catchment areas of the resulting hard jets. For  $k_t$  and Cam/Aachen the detailed shapes are in part determined by the specific set of ghosts used, and change when the ghosts are modified.



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In this paper it is not our intention to advocate one or other type of algorithm in the debate concerning soft-resilient versus soft-adaptable algorithms. Rather, we feel that this debate can be more fruitfully served by proposing a simple, IRC safe, soft-resilient jet algorithm, one that leads to jets whose shape is not influenced by soft radiation. To do so, we take a quite non-obvious route, because instead of making use of the concept of a stable cone, we start by generalising the existing sequential recombination algorithms,  $k_t$  [1] and Cambridge/Aachen [2].

As usual, one introduces distances  $d_{ij}$  between entities (particles, pseudojets)  $i$  and  $j$  and  $d_{iB}$  between entity  $i$  and the beam (B). The (inclusive) clustering proceeds by identifying the smallest of the distances and if it is a  $d_{ij}$  recombining entities  $i$  and  $j$ , while if it is  $d_{iB}$  calling  $i$  a jet and removing it from the list of entities. The distances are recalculated and the procedure repeated until no entities are left.

The extension relative to the  $k_t$  and Cambridge/Aachen algorithms lies in our definition of the distance measures:

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}^2}{R^2}, \quad \begin{array}{l} p=1, kT \\ p=0, Achen \end{array} \quad (1a)$$

$$d_{iB} = k_{ti}^{2p}, \quad p=-1 \text{ Anti-}kT \quad (1b)$$

where  $\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$  and  $k_{ti}$ ,  $y_i$  and  $\phi_i$  are respectively the transverse momentum, rapidity and azimuth of particle  $i$ . In addition to the usual radius parameter  $R$ , we have added a parameter  $p$  to govern the relative power of the energy versus geometrical ( $\Delta_{ij}$ ) scales.

For  $p = 1$  one recovers the inclusive  $k_t$  algorithm. It can be shown in general that for  $p > 0$  the behaviour of the jet algorithm with respect to soft radiation is rather similar to that observed for the  $k_t$  algorithm, because what matters is the ordering between particles and for finite  $\Delta$  this is maintained for all positive values of  $p$ . The case of  $p = 0$  is special and it corresponds to the inclusive Cambridge/Aachen algorithm.

# $K_T$ splitting scale in $Z \rightarrow \ell\ell$ events at 8 TeV

- Test of pQCD with jet production rate as a function of the **jet-resolution scale** of the anti- $K_T$  jet-clustering algorithm:

► initial condition:  $N$  input momenta, distances:  $d_{ij}$  and  $d_{ib}$  for all  $i$  and  $j > i$   
 $d_K = \min(d_{ij}, d_{ib})$  for all  $i, j$

► at any iteration, the number of momenta available for clustering decreases:  $k+1 \rightarrow k$

## Observables

Differential cross sections of  $Z \rightarrow ee$  and  $Z \rightarrow \mu\mu$  as a function of  $\sqrt{d_K}$  for  $k=0-7$  corrected at **charged** (and at **charged + neutral**) particle level via Bayesian unfolding

Same lepton definition as in  $Z$ +jets analysis

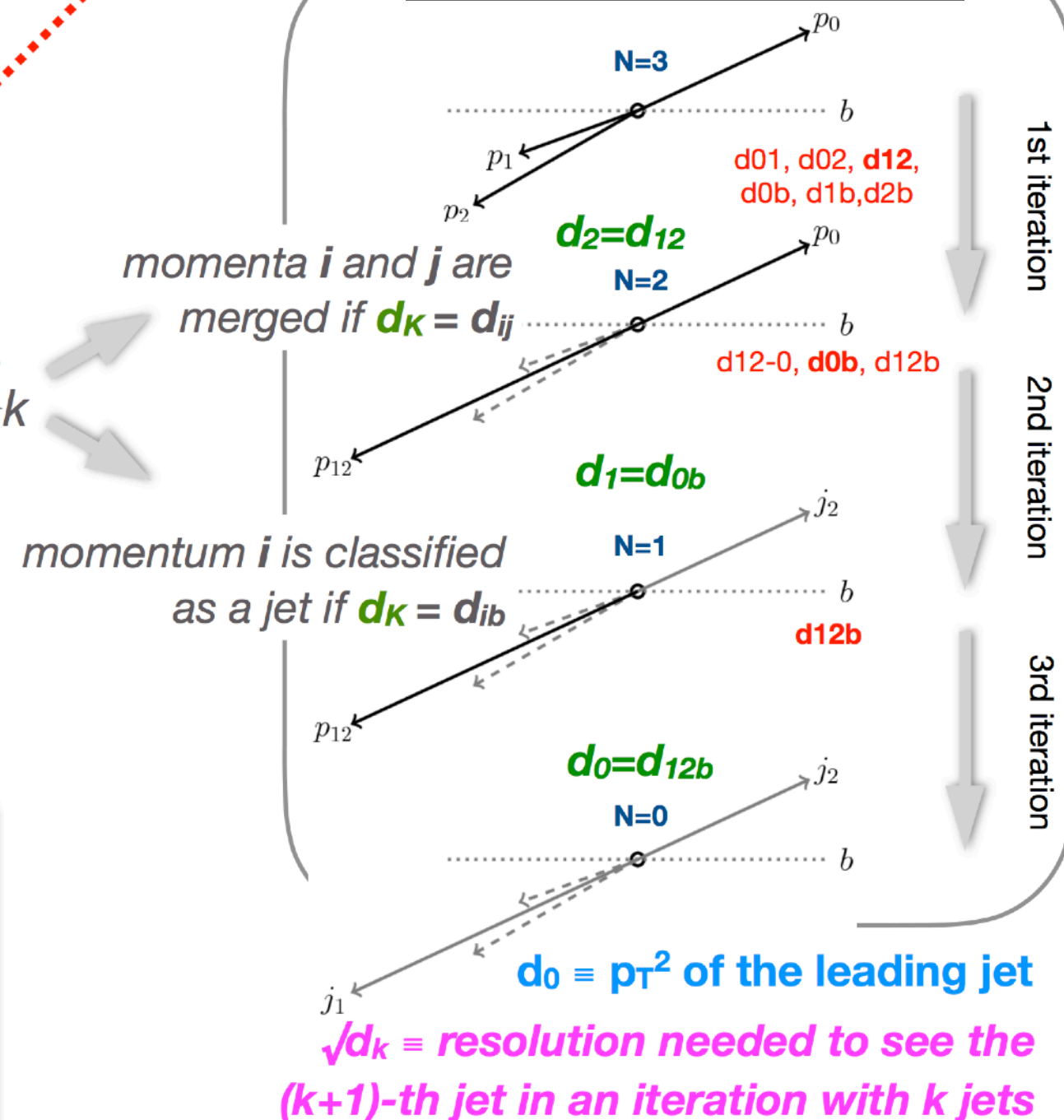
$71 < M_{\ell\ell} < 111$  GeV

**Hadrons**: charged tracks with  $p_T > 400$  MeV and  $|\eta| < 2.4$  used to measure  $\sqrt{d_K}$  with  **$R=0.4$**  and  **$R=1.0$**

$$d_{ij} = \min(p_{T,i}^2, p_{T,j}^2) \times \frac{\Delta R_{ij}^2}{R^2},$$

$$d_{ib} = p_{T,i}^2,$$

## The $k_T$ algorithm flow





## 2.1 General behaviour

The functionality of the anti- $k_t$  algorithm can be understood by considering an event with a few well-separated hard particles with transverse momenta  $k_{t1}, k_{t2}, \dots$  and many soft particles. The  $d_{1i} = \min(1/k_{t1}^2, 1/k_{ti}^2)\Delta_{1i}^2/R^2$  between a hard particle 1 and a soft particle  $i$  is exclusively determined by the transverse momentum of the hard particle and the  $\Delta_{1i}$  separation. The  $d_{ij}$  between similarly separated soft particles will instead be much larger. Therefore soft particles will tend to cluster with hard ones long before they cluster among themselves. If a hard particle has no hard neighbours within a distance  $2R$ , then it will simply accumulate all the soft particles within a circle of radius  $R$ , resulting in a perfectly conical jet.

If another hard particle 2 is present such that  $R < \Delta_{12} < 2R$  then there will be two hard jets. It is not possible for both to be perfectly conical. If  $k_{t1} \gg k_{t2}$  then jet 1 will be conical and jet 2 will be partly conical, since it will miss the part overlapping with jet 1. Instead if  $k_{t1} = k_{t2}$  neither jet will be conical and the overlapping part will simply be divided by a straight line equally between the two. For a general situation,  $k_{t1} \sim k_{t2}$ , both cones will be clipped, with the boundary  $b$  between them defined by  $\Delta R_{1b}/k_{t1} = \Delta_{2b}/k_{t2}$ .

Similarly one can work out what happens with  $\Delta_{12} < R$ . Here particles 1 and 2 will cluster to form a single jet. If  $k_{t1} \gg k_{t2}$  then it will be a conical jet centred on  $k_1$ . For  $k_{t1} \sim k_{t2}$  the shape will instead be more complex, being the union of cones (radius  $< R$ ) around each hard particle plus a cone (of radius  $R$ ) centred on the final jet.

The key feature above is that the soft particles do not modify the shape of the jet, while hard particles do. I.e. the jet boundary in this algorithm is resilient with respect to soft radiation, but flexible with respect to hard radiation.<sup>3</sup>