MULTIPLE COULOMB SCATTERING

MCS MODEL

"An improved electron multiple-scattering distribution for Monte Carlo transport simulation" - Al.Beteri and D.E. Raeside

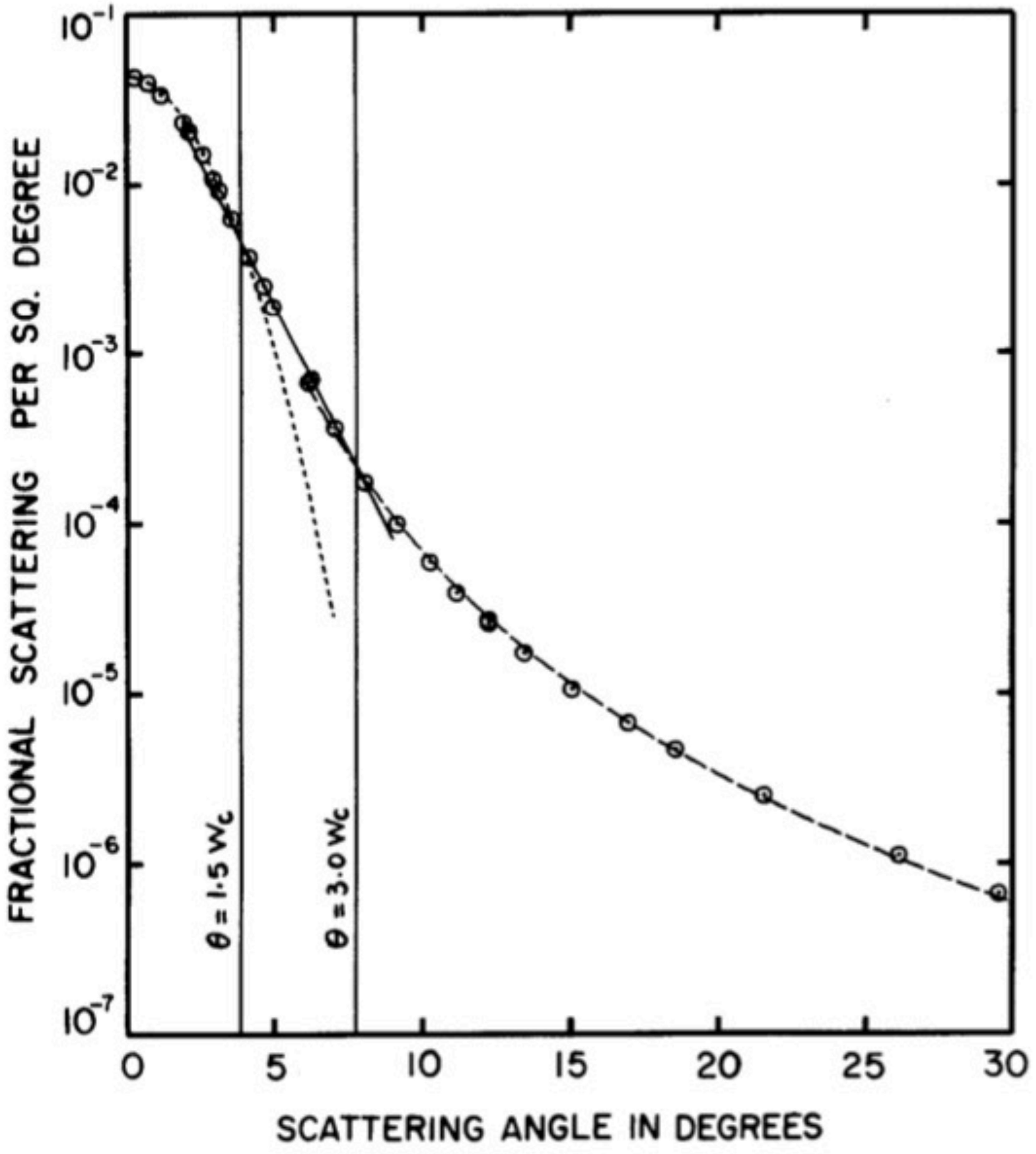
Multiple Coulomb scattering distribution is a composite function:

1. modified Molière Gaussian term for small angle scattering
2. exponential term for the intermediate angle scattering region
3. modified relativistic Mott single-scattering term for large angle scattering

METHODS

The proposed distribution components have the following mathematical forms:

FIG. 1. A comparison of the three components of the composite distribution to the Hanson *et al.* data (Ref. 5) for 15.7-MeV electrons scattered by a 18.66-mg/cm^2 gold foil. The Gaussian component is represented by a dotted line, the exponential component by a solid line, and the single-scattering component by a dashed line. Also shown are the location of the cross over angles at $\theta = 1.5\ W_c$ and $\theta = 3.0\ W_c$.



$$
2\pi\theta f(\theta)\,d\theta
= \begin{cases}
\dfrac{2\theta}{WW_c}\exp\left(-\dfrac{\theta^2}{W_c{}^2}\right)d\theta, & \text{for } 0 \leqslant \theta \leqslant 1.5W_c, \\[2ex]
\dfrac{2\theta P_1}{WW_c}\exp\left(-\dfrac{P_2\theta}{W_c}\right)d\theta, & \text{for } 1.5W_c < \theta \leqslant 3.0W_c, \\[2ex]
\dfrac{2W^2}{B\theta^3}\left(1+\dfrac{\pi W}{\theta}\right)d\theta, & \text{for } 3.0W_c < \theta \leqslant \pi,
\end{cases}
$$

SAMPLING

The random sampling is a two step procedure:

1. generation of random number r1 for the selection of the component of the composite distribution through three integrals:
    - A_G for Gaussian component
    - A_E for exponential component
    - A_S for single-scattering component
2. generation of a second random number r2 for the application of INVERSE TRANSFORM METHOD

INVERSE TRANSFORM METHOD

Each cumulative probability of the three components is set equal to the random number r2:

Gaussian component

$$r_2 = \int_0^{\theta} \frac{2\theta}{WW_c} \exp\left(-\frac{\theta^2}{W_c^2}\right) d\theta \quad \left[\int_0^{1.5W_c} \frac{2\theta}{WW_c} \exp\left(-\frac{\theta^2}{W_c^2}\right) d\theta\right] = \frac{W_c}{W}\left[1 - \exp\left(-\frac{\theta^2}{W_c^2}\right)\right] \Big/ A_G$$

and solving for θ,

$$\theta = W_c[-\ln(1 - 0.8946\, r_2)]^{1/2}.$$

Exponential component

$$r_2 = \int_{1.5W_c}^{\theta} \frac{2\theta P_1}{WW_c} \exp\left(-\frac{P_2\theta}{W_c}\right) d\theta \Big/ \left[\int_{1.5W_c}^{3.0W_c} \frac{2\theta P_1}{WW_c} \exp\left(-\frac{P_2\theta}{W_c}\right) d\theta\right]$$

$$= (C\{(3P_2 + 2)\exp(-1.5P_2) - [(2P_2\theta/W_c) + 2]\exp[-(P_2\theta)/W_c]\})/A_E$$

Single-Scattering component

$$r_2 = \int_{3.0W_c}^{\theta} \frac{2W^2}{B\theta^3}\left(1 + \frac{\pi W}{\theta}\right) d\theta$$

$$\times \left[\int_{3.0W_c}^{\pi} \frac{2W^2}{B\theta^3}\left(1 + \frac{\pi W}{\theta}\right) d\theta\right]^{-1}$$

$$= \left[\frac{W^2}{9BW_c^2}\left(1 + \frac{2\pi W}{9W_c}\right) - \frac{W^2}{B\theta^2}\left(1 + \frac{2\pi W}{3\theta}\right)\right] \Big/ A_S$$

Newton-Raphson Method was used for solving equations relative to exponential and single-scattering components

VALIDATION

- Comparison with experimental results

- 15.7 MeV electrons on Be
- t_material_1 = 0.257 g/cm^2
- t_material_2 = 0.4913 g/cm^2

PRELIMINARY

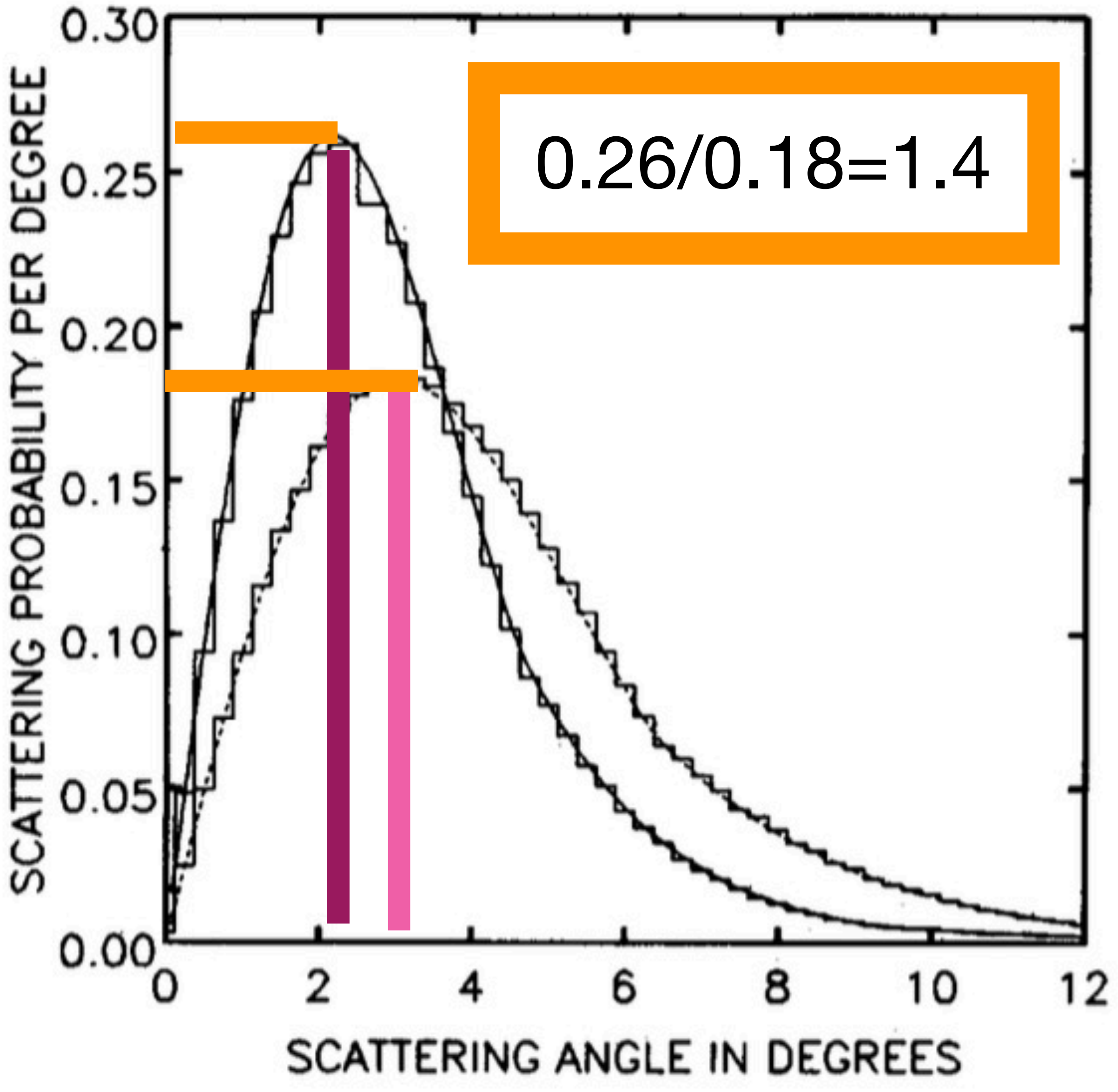
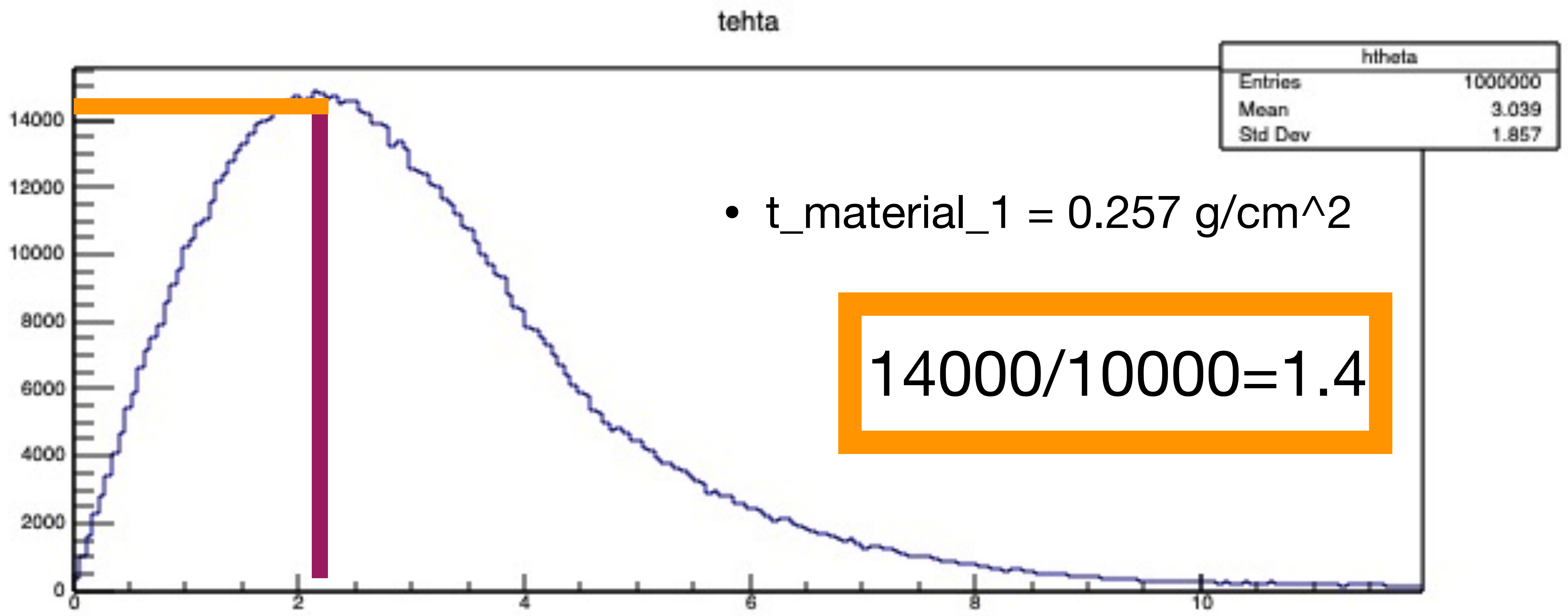
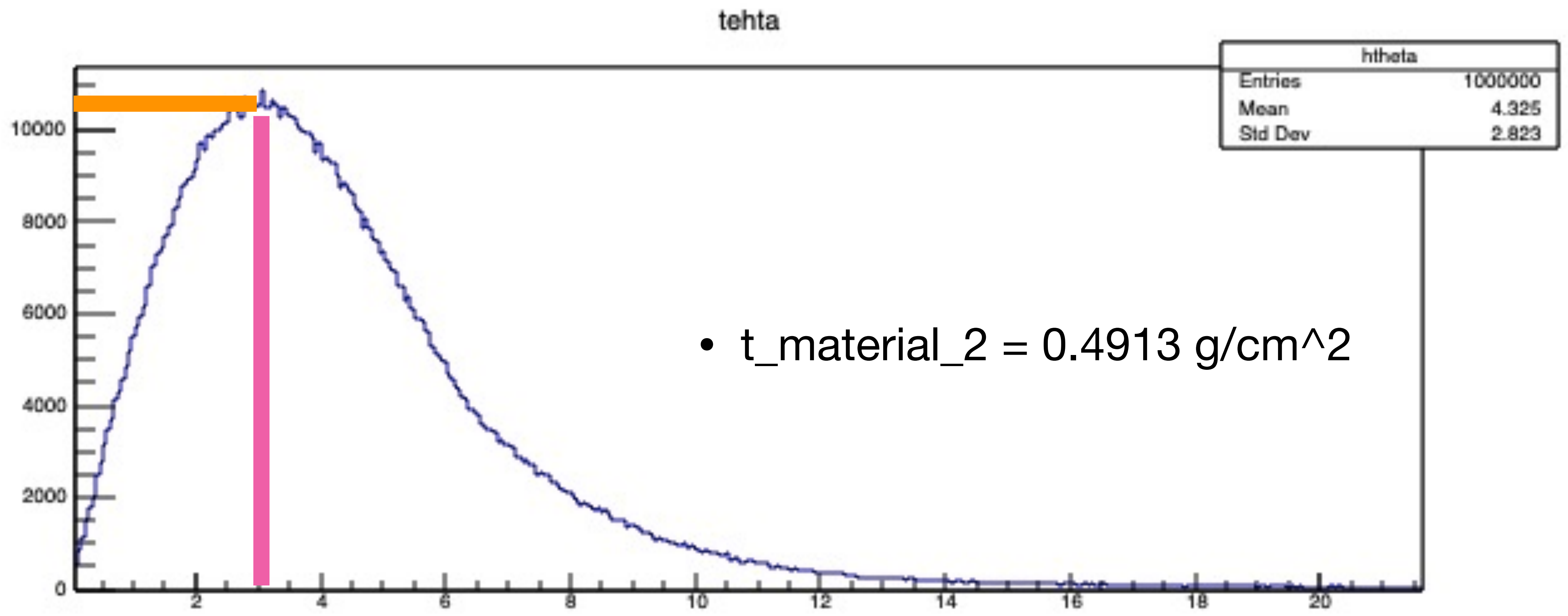


FIG. 4. The calculated composite probability distributions $2\pi\theta f(\theta)$ for 15.7-MeV electrons scattered by a 257.0-mg/cm^2 beryllium foil, represented by a solid line, and a 491.3-mg/cm^2 beryllium foil, represented by a dotted line, are compared to the distribution of 0.5×10^6 angles randomly sampled from the composite distribution corresponding to each thickness, shown as histograms. The excellent agreement verifies the correctness of the sampling algorithm.





VALIDATION

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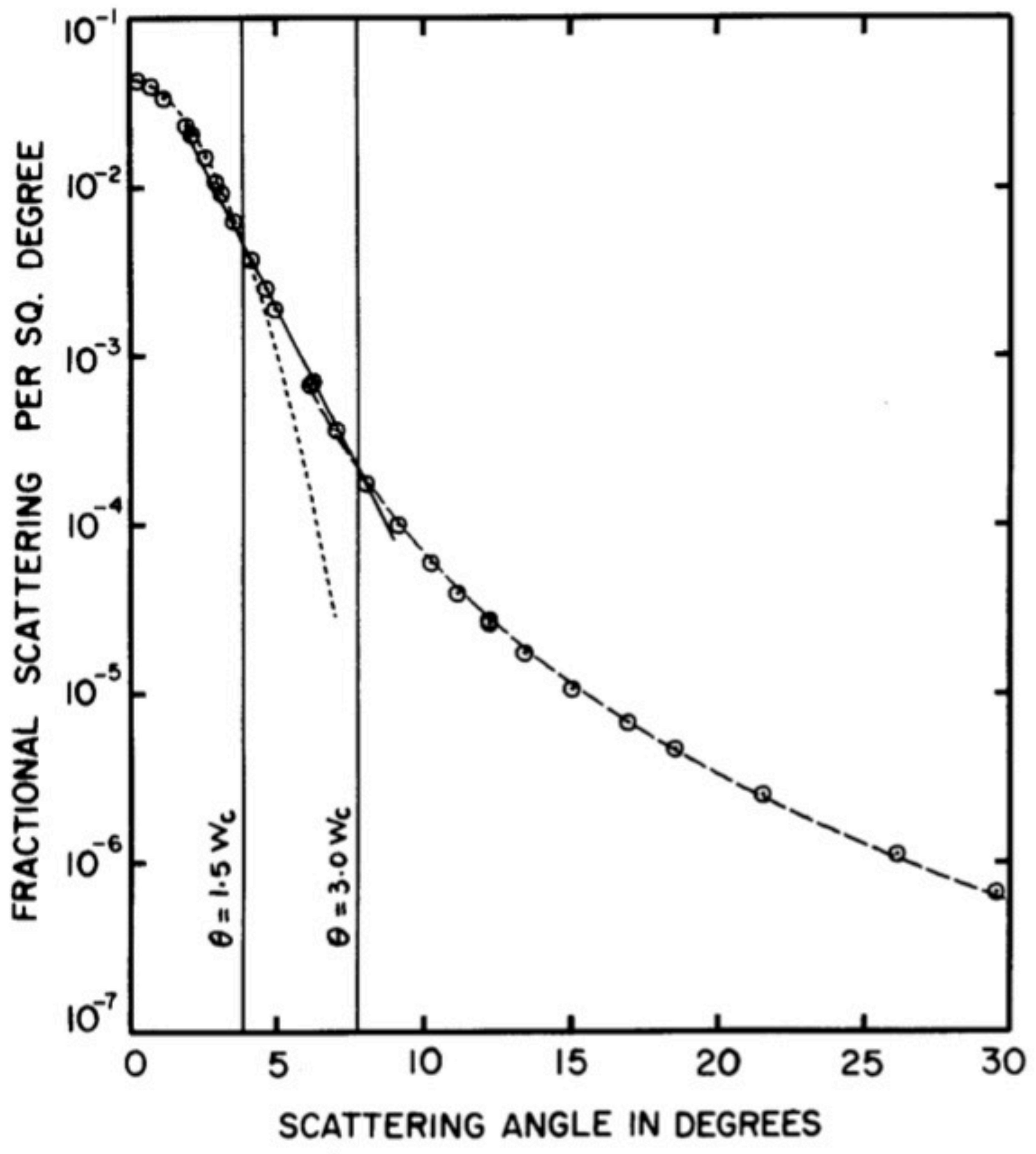

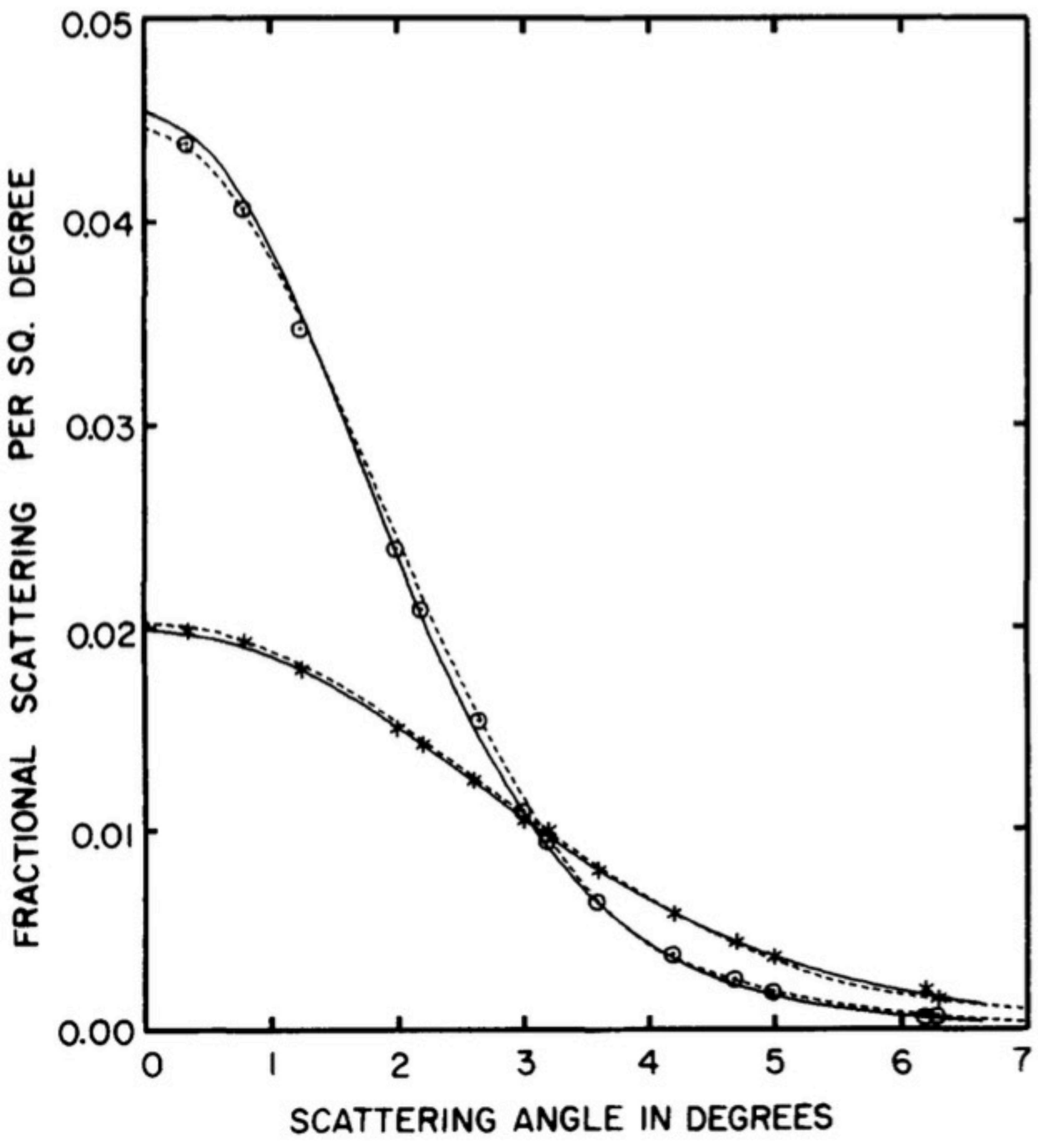

PROBLEMS

- W_c parameter is not well defined at low energies (by the authors of the article)