Multifield Quintessence Models and the Swampland

Based on:

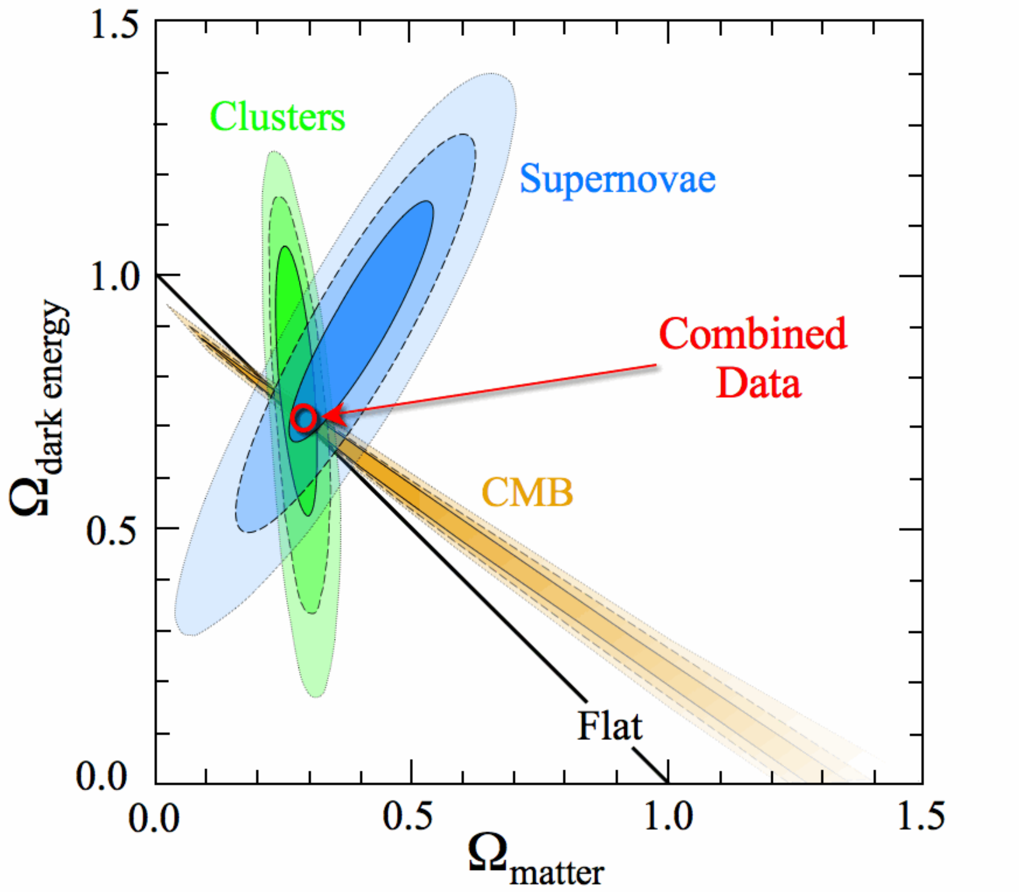
2002.02695 [gr-qc] & 2007.11011 [hep-th]

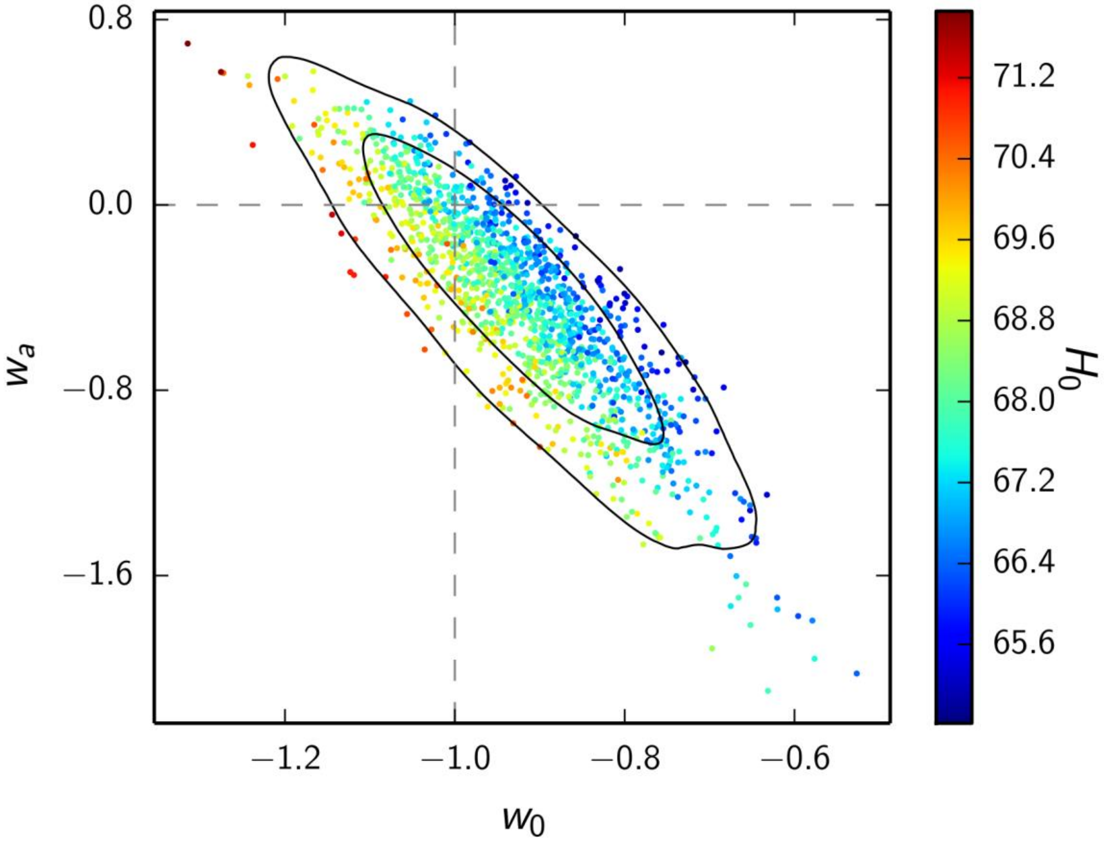
with M. Cicoli & Francisco Gil Pedro

Outline

- Introduction & Motivation
- Single field quintessence and Swampland bounds
- Multifield models: an opportunity!
- Dynamics of 2-field models
- Non-geodesic trajectories and Swampland bounds
- Different scenarios
- Discussion

The ΛCDM model

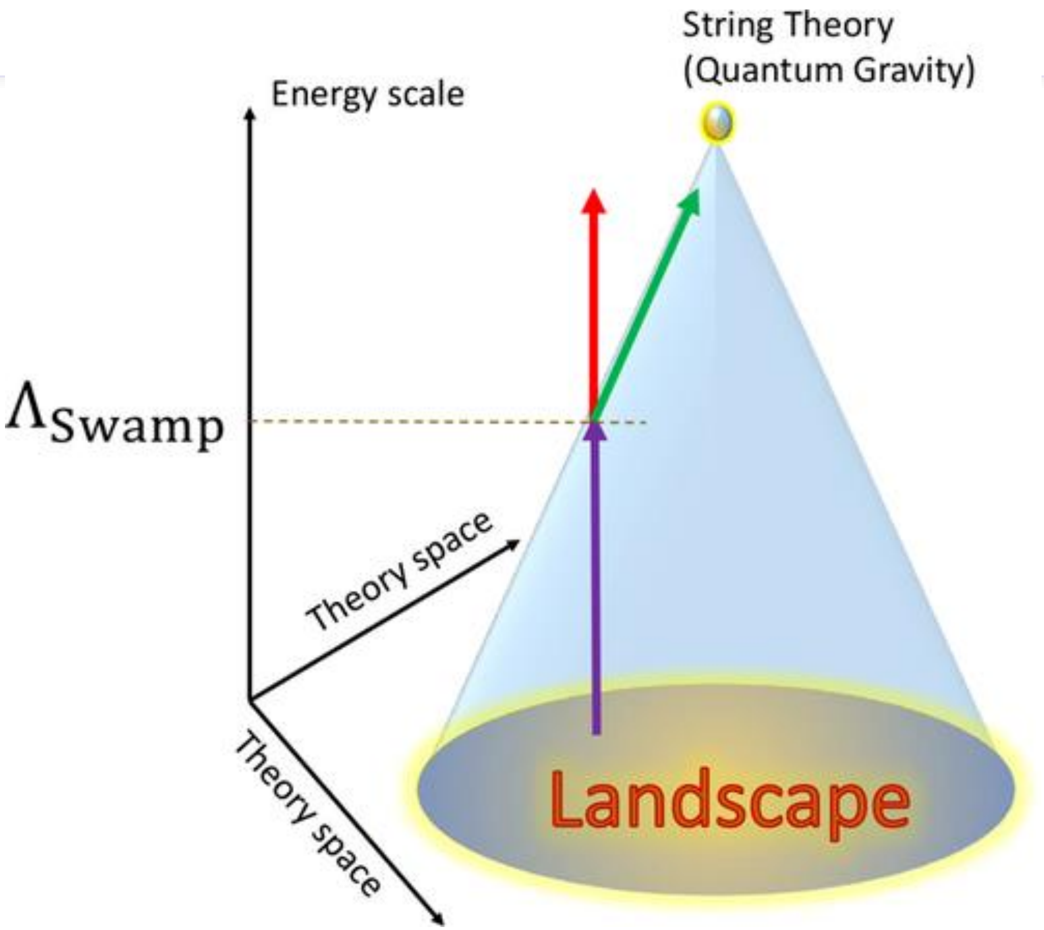




$\Omega_\phi = 0.7 \quad \omega_\phi = -1 \pm 0.09$ at 95% CL

[Credits: Planck coll. 1502,01589]

Swampland Conjectures



[Credits: Review by E. Palti, 2019]

The dS Swampland Conjecture

$$\frac{V_\phi}{V} \geq \frac{c}{M_{\mathrm{Pl}}}$$

Only models with $c \approx 0{,}6$ remain compatible with the latest observations!

→ Tension between theory & observations…

[Obied, Ooguri, Spodyneiko, Vafa, 2018]

Single-field Quintessence Models

$$S[g_{\mu,\nu}, \phi] = \int d^4x \sqrt{-g} \left(\frac{M_{\text{Pl}}^2}{2} \mathcal{R} - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right)$$

$$ds_4^2 = -dt^2 + a(t)^2 \, ds_{\mathbb{R}^3}^2$$ and $$\phi = \phi(t)$$

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} < 1$$

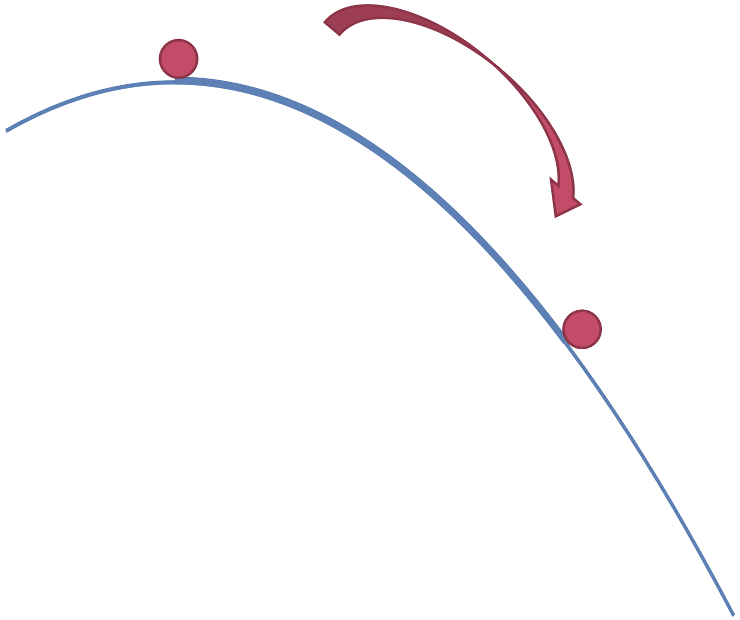
⇄

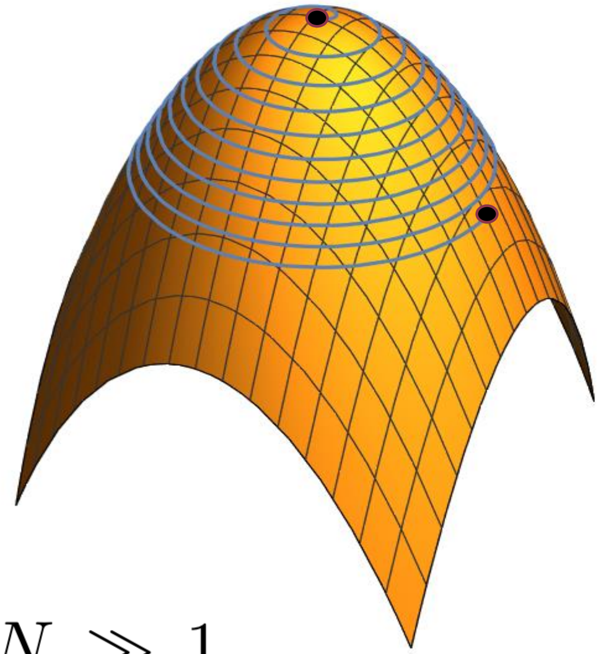
$$\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_\phi}{V} \right)^2 \approx \epsilon_H < 1$$

Cosmic acceleration

Flatness of scalar potential *V*

Multifield Models: an opportunity?





$\Delta N < 1$

$\Delta N \gg 1$

[Achucarro, Palma, 2018], [Christodoulidis, Roest, Sfakianakis, 2019]

A 2-field Model

$$S[g_{\mu,\nu},\phi] = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}\mathcal{R} - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}f(\phi_1)^2(\partial\phi_2)^2 - V(\phi_1)\right)$$

$$ds_4^2 = -dt^2 + a(t)^2\, ds_{\mathbb{R}^3}^2 \qquad \text{and} \qquad \phi_i = \phi_i(t)$$

The EOM's read

$$\begin{cases} \ddot\phi_1 + 3H\dot\phi_1 - f\, f_1\dot\phi_2^2 + V_1 = 0 \ , \\ \ddot\phi_2 + 3H\dot\phi_2 + 2\frac{f_1}{f}\dot\phi_2\dot\phi_1 = 0 \ , \end{cases}$$

Together with the Friedman equation

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{\dot\phi_1^2}{2} + \frac{f^2}{2}\dot\phi_2^2 + V + \rho_b\right)$$

Now introduce $k_1(\phi_1) \equiv -M_{\rm Pl}\,\frac{f_1}{f}$ and $k_2(\phi_1) \equiv -M_{\rm Pl}\,\frac{V_1}{V}$

as well as: $x_1 \equiv \frac{\dot{\phi}_1}{\sqrt{6}HM_{\rm Pl}}\,, \quad x_2 \equiv \frac{f\dot{\phi}_2}{\sqrt{6}HM_{\rm Pl}}\,, \quad y_1 \equiv \frac{\sqrt{V}}{\sqrt{3}HM_{\rm Pl}}$

$$x_1' = 3x_1(x_1^2 + x_2^2 - 1) + \sqrt{\frac{3}{2}}(-2k_1x_2^2 + k_2y_1^2) - \frac{3}{2}\gamma x_1(x_1^2 + x_2^2 + y_1^2 - 1)\,,$$

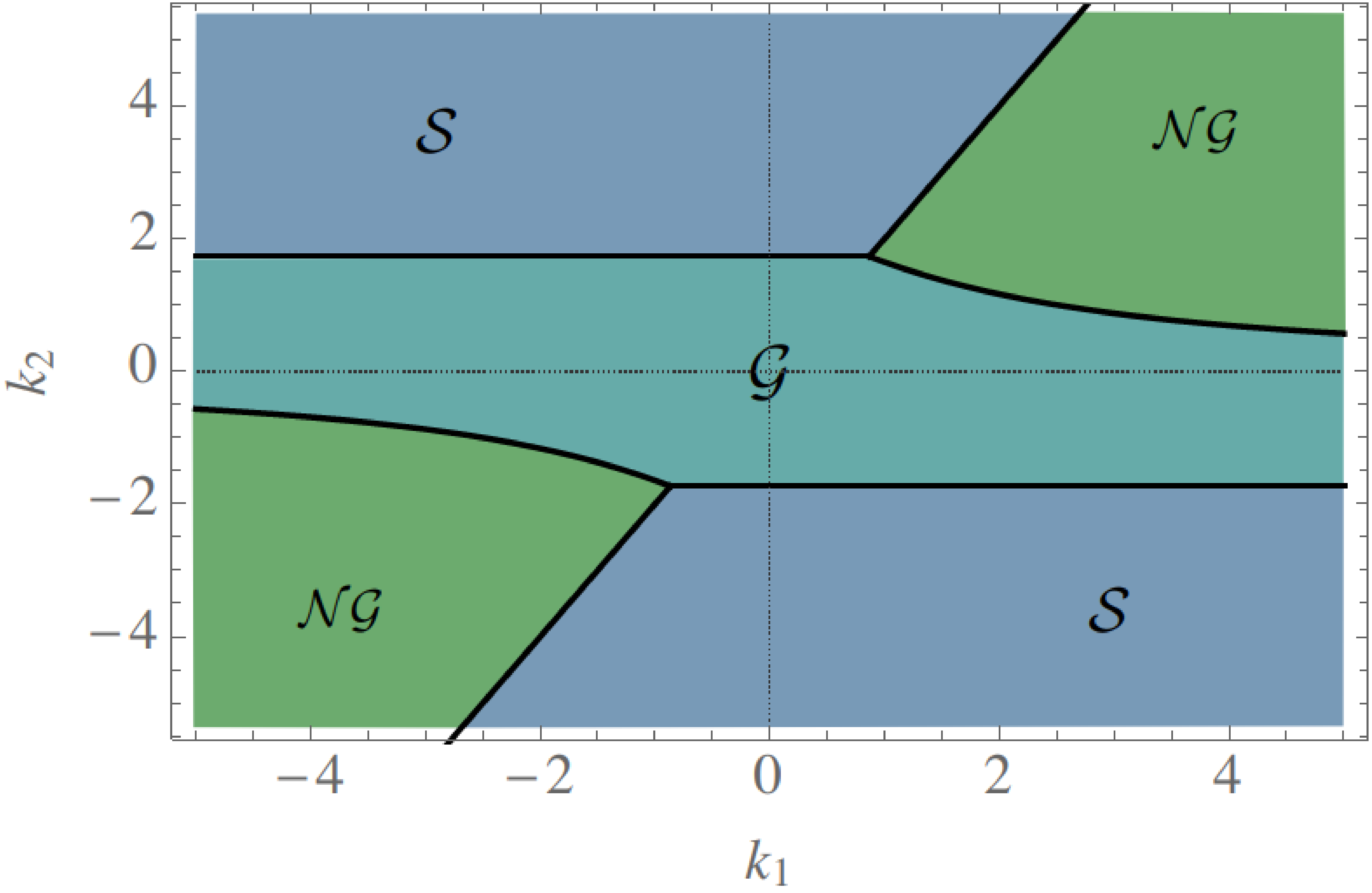
$$x_2' = 3x_2\left(x_1^2 + x_2^2 - 1\right) + \sqrt{6}k_1x_1x_2 - \frac{3}{2}\gamma x_2\left(x_1^2 + x_2^2 + y_1^2 - 1\right)\,,$$

$$y_1' = -\sqrt{\frac{3}{2}}k_2x_1y_1 - \frac{3}{2}\gamma y_1\left(x_1^2 + x_2^2 + y_1^2 - 1\right) + 3y_1\left(x_1^2 + x_2^2\right)\,,$$

The Fixed Points Analysis

$$x_1' = x_2' = y_1' = 0$$

	x_1	x_2	y_1	Ω_ϕ	ω_ϕ	existence
$\mathcal{K}_+$	1	0	0	1	1	all k_1, k_2, γ
$\mathcal{K}_-$	-1	0	0	1	1	all k_1, k_2, γ
$\mathcal{F}$	0	0	0	0	undefined	all k_1, k_2, γ
$\mathcal{S}$	$\frac{\sqrt{3/2}\gamma}{k_2}$	0	$\frac{\sqrt{3/2\gamma(2-\gamma)}}{k_2}$	$\frac{3\gamma}{k_2^2}$	$\gamma - 1$	$0 < \gamma < 2 \wedge k_2^2 \geq 3\gamma$
$\mathcal{G}$	$\frac{k_2}{\sqrt{6}}$	0	$\sqrt{1 - \frac{k_2^2}{6}}$	1	$-1 + \frac{k_2^2}{3}$	$k_2 < \sqrt{6}$
$\mathcal{NG}$	$\frac{\sqrt{6}}{(2k_1+k_2)}$	$\frac{\pm\sqrt{k_2^2+2k_2k_1-6}}{2k_1+k_2}$	$\sqrt{\frac{2k_1}{2k_1+k_2}}$	1	$\frac{k_2-2k_1}{k_2+2k_1}$	$k_2 \geq \sqrt{6+k_1^2} - k_1$



$\mathcal{S}$
$\mathcal{NG}$
$\mathcal{G}$
$\mathcal{NG}$
$\mathcal{S}$
4
2
0
−2
−4
k_2
−4
−2
0
2
4
k_1

To better connect with observations, the scalar dynamics can be formulated in terms of the density parameter and the equation of state parameter, respectively:

$$\Omega_\phi = x_1^2 + x_2^2 + y_1^2 \qquad \text{and} \qquad \omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{x_1^2 + x_2^2 - y_1^2}{x_1^2 + x_2^2 + y_1^2}$$

The time evolution within the $(\omega_\phi, \Omega_\phi)$ plane is determined by the following ODE's

$$\Omega_\phi' = -3(\Omega_\phi - 1)\Omega_\phi(\omega_b - \omega_\phi)$$

$$\omega_\phi' = (\omega_\phi - 1)\left(-k_2\sqrt{3(\omega_\phi + 1)\Omega_\phi - 6x_2^2} + 3(1 + \omega_\phi)\right)$$

NG Dynamics & Swampland Bounds

The displacement in field space can be expressed as

$$\Delta\phi \equiv \int dt \sqrt{\gamma_{ij}\dot{\phi}^i\dot{\phi}^j}$$

Which in our case, evaluates to:

$$\frac{\Delta\phi}{M_{Pl}} = \int dN \sqrt{-\left(\frac{3}{2k_1}\right)^2 + 3\,\frac{k_2}{k_1}}$$

$$\frac{\Delta\phi}{M_{\rm Pl}} \sim \frac{\Delta N}{\sqrt{k_1}} < 1$$

→

This means that we can realize even O(10) e-folds of accelearation **without violating the Distance Swampland Conjecture**!!!!

The simplest scenario: Exp-type *f* & *V*

$$f = f_0 \, e^{-k_1 \phi_1}$$

$$V = V_0 \, e^{-k_2 \phi_1}$$

→ k_1 and k_2 are constant!

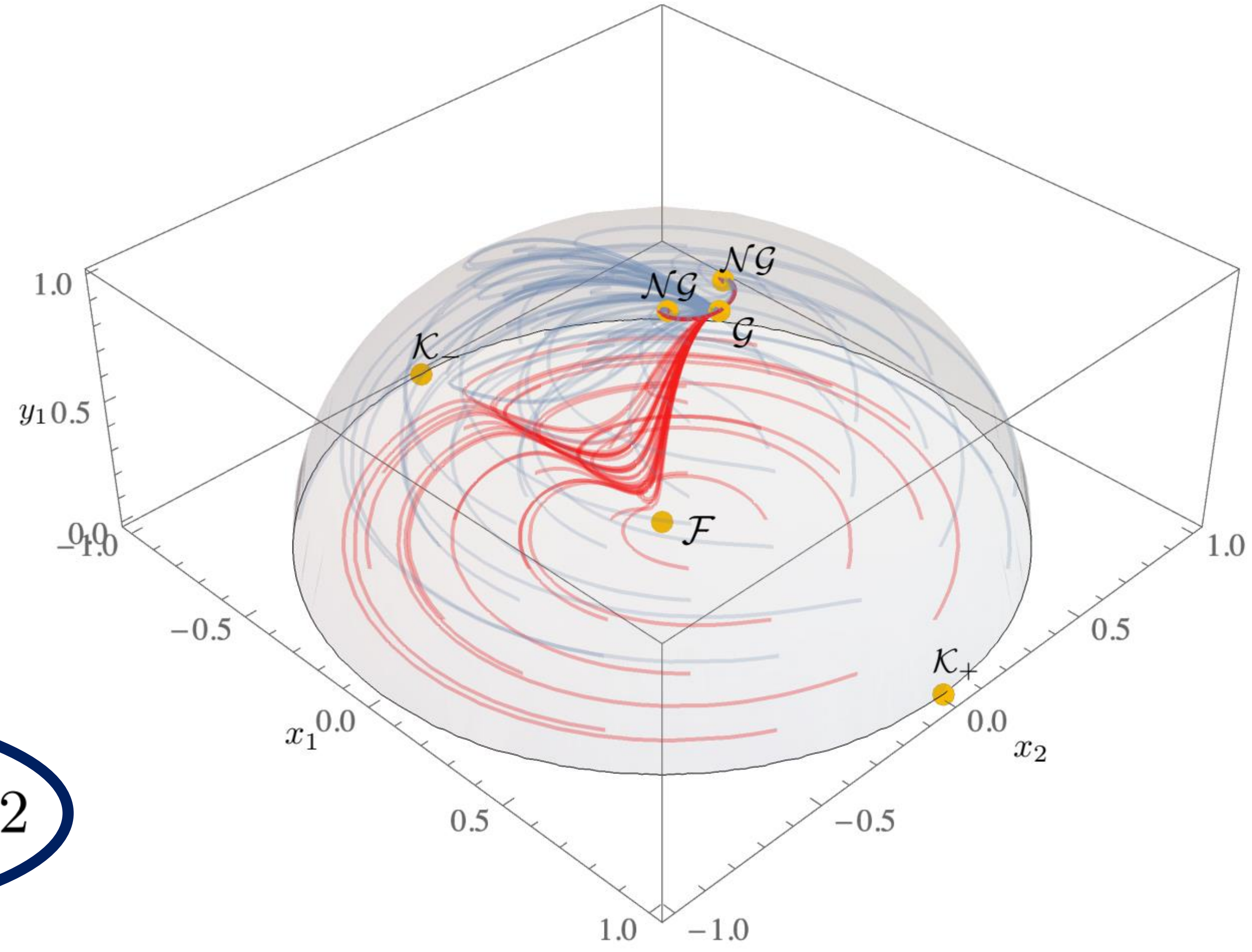
$$\omega_\phi = \frac{k_2 - 2k_1}{k_2 + 2k_1} = -1 + \frac{2}{1 + 2\frac{k_1}{k_2}}$$

Should be small…

→ Need a little «hierarchy» between them!

$$k_1 \gg k_2$$

The system evolves from matter domination towards $\mathcal{G}$ (saddle), before settling into $\mathcal{NG}$ (stable).



Plot:

$$k_1 = 10, \; k_2 = 1/2$$

Stringy Examples…

Stabilize everything but **ONE** complex modulus

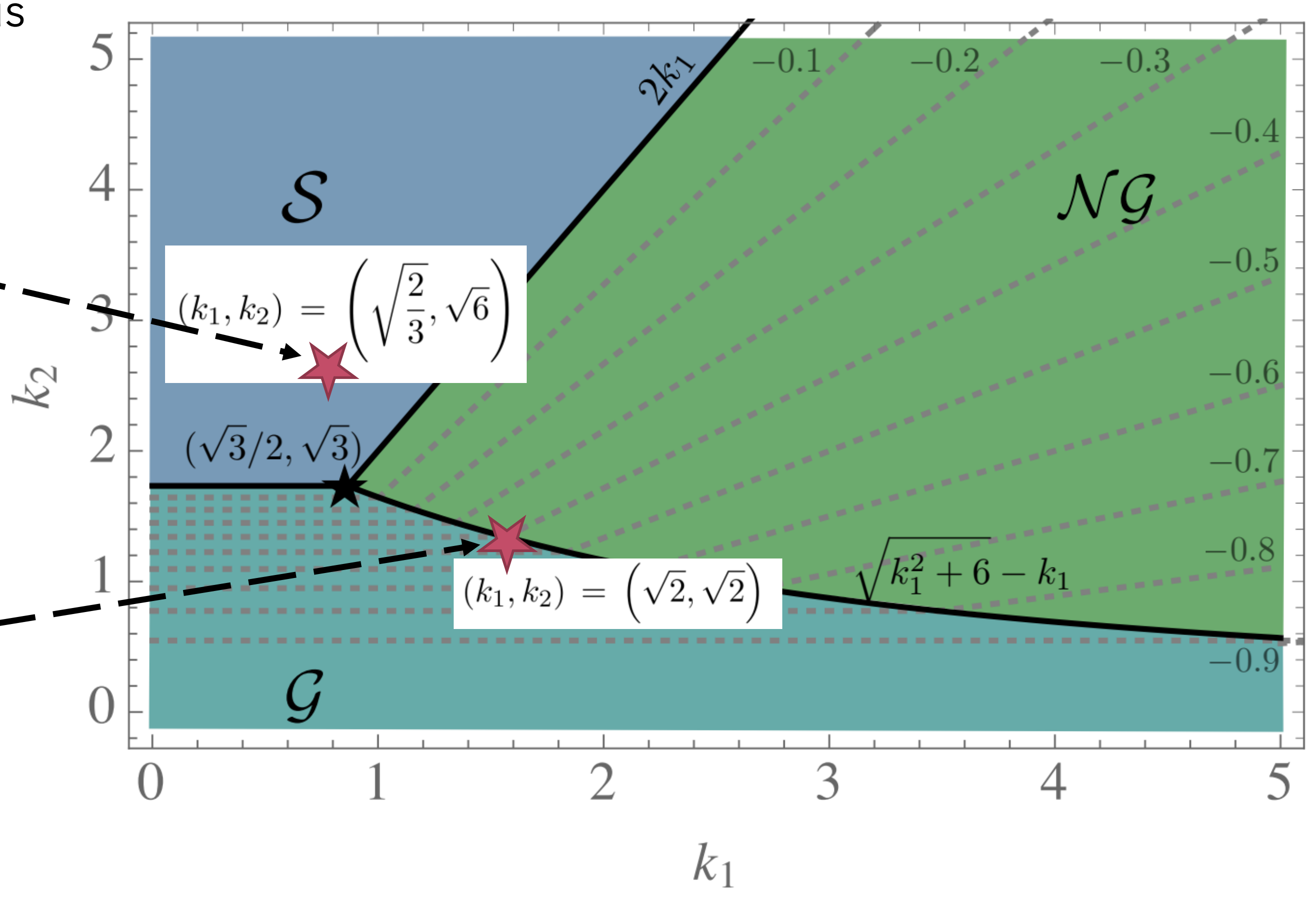
- Type IIB on a CY 3-fold with O3 planes

$$T = C_{(4)} + i\,\mathrm{vol}_6$$

[Saltman, Silverstein, 2002]

- Heterotic on CY 3-fold

$$S = B_{(6)} + i\,e^{-\Phi}$$



The case of non-Exp *f*'s

k_1 and k_2 are no longer constant!

Their time evolution is given by

$$k_i' = \sqrt{6} k_i^2 x_1 \left(1 - \Gamma_i\right)$$

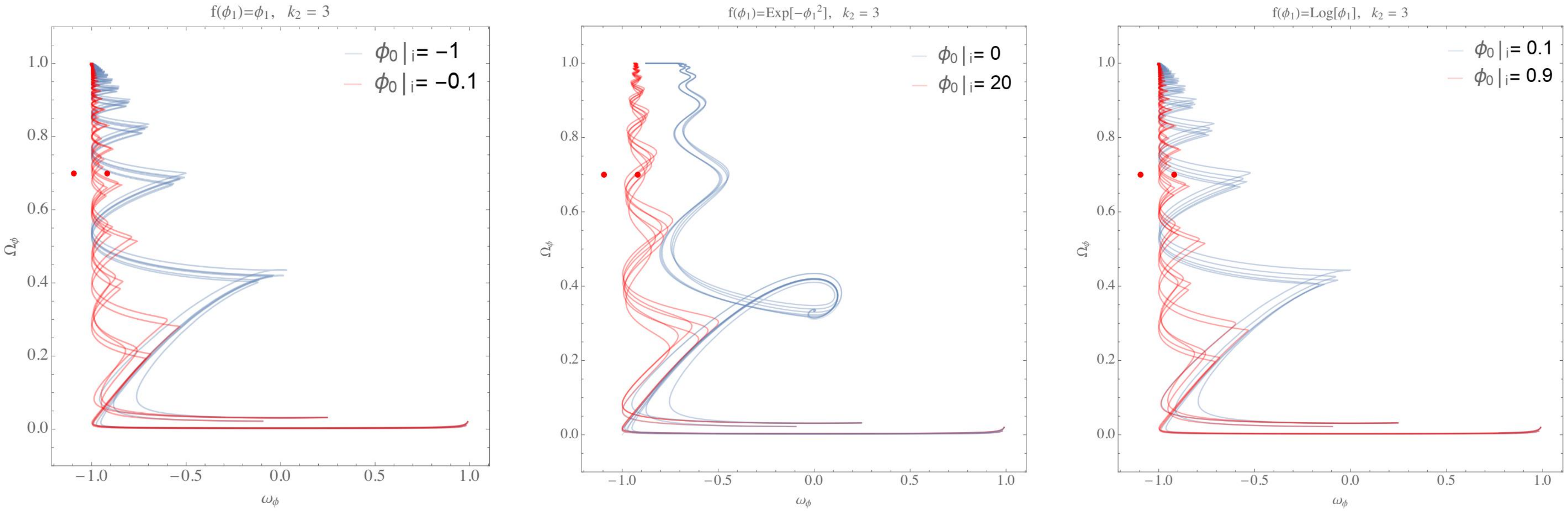
Where:

$$\Gamma_1 \equiv \frac{f_{11} f}{f_1^2}$$

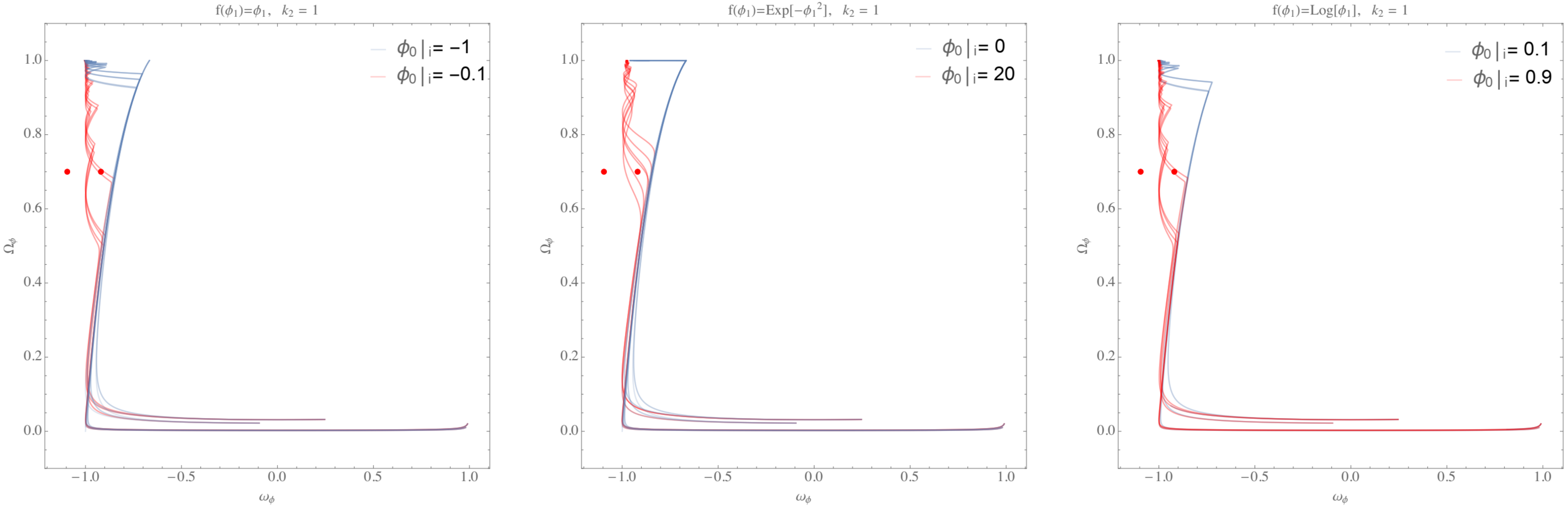
and

$$\Gamma_2 \equiv \frac{V_{11} V}{V_1^2}$$

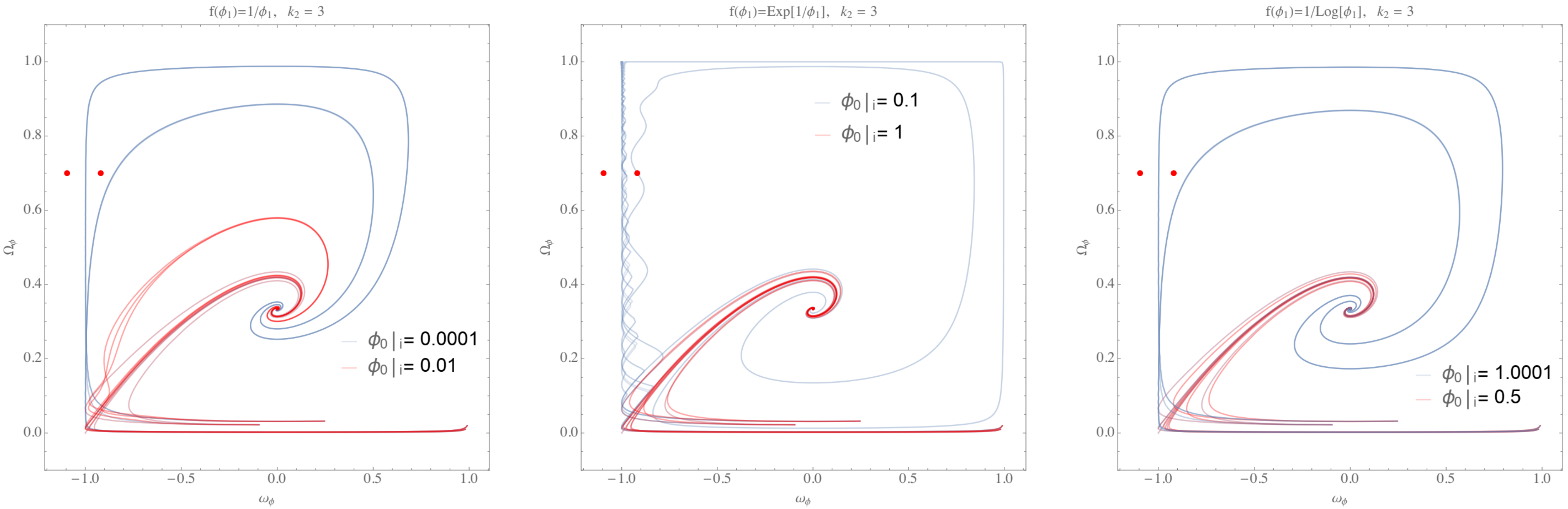
Growing kinetic coupling, steep potential



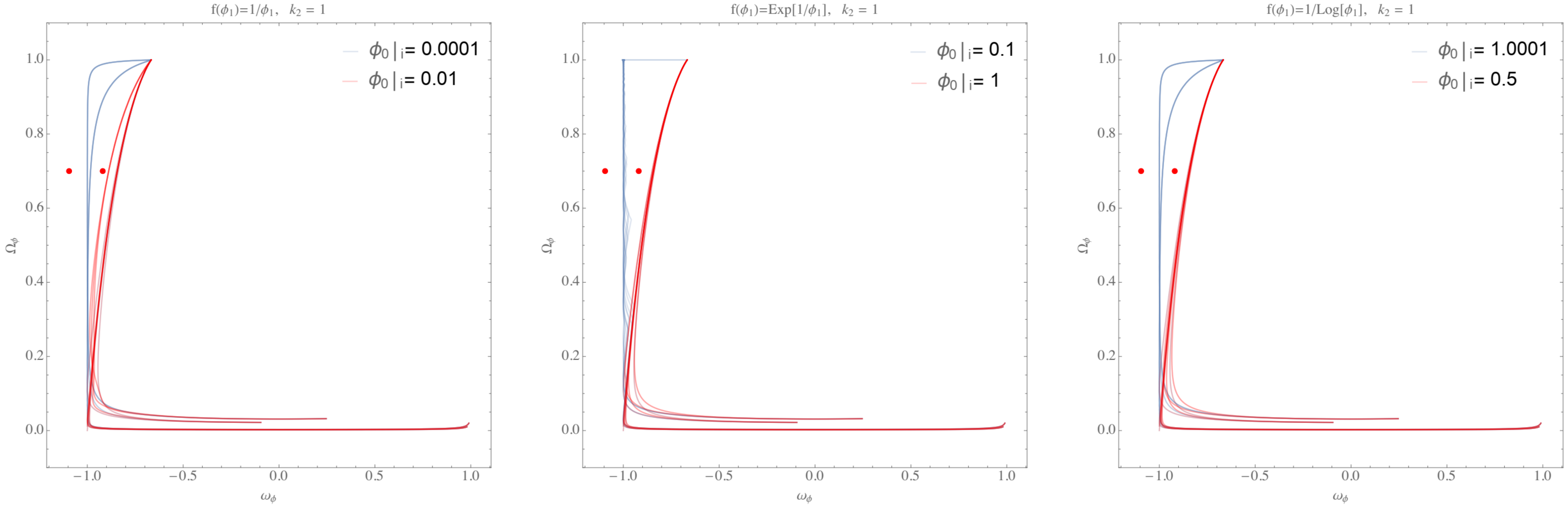
Growing kinetic coupling, shallow potential



Decaying kinetic coupling, steep potential



Decaying kinetic coupling, shallow potential



Towards more general scenarios…

It's **hard to be exhaustive** and the equations get much more complicated…

However, an **interesting** case is given by the following setup

$$V = V_0 \ \phi_1^{p_2}$$

and

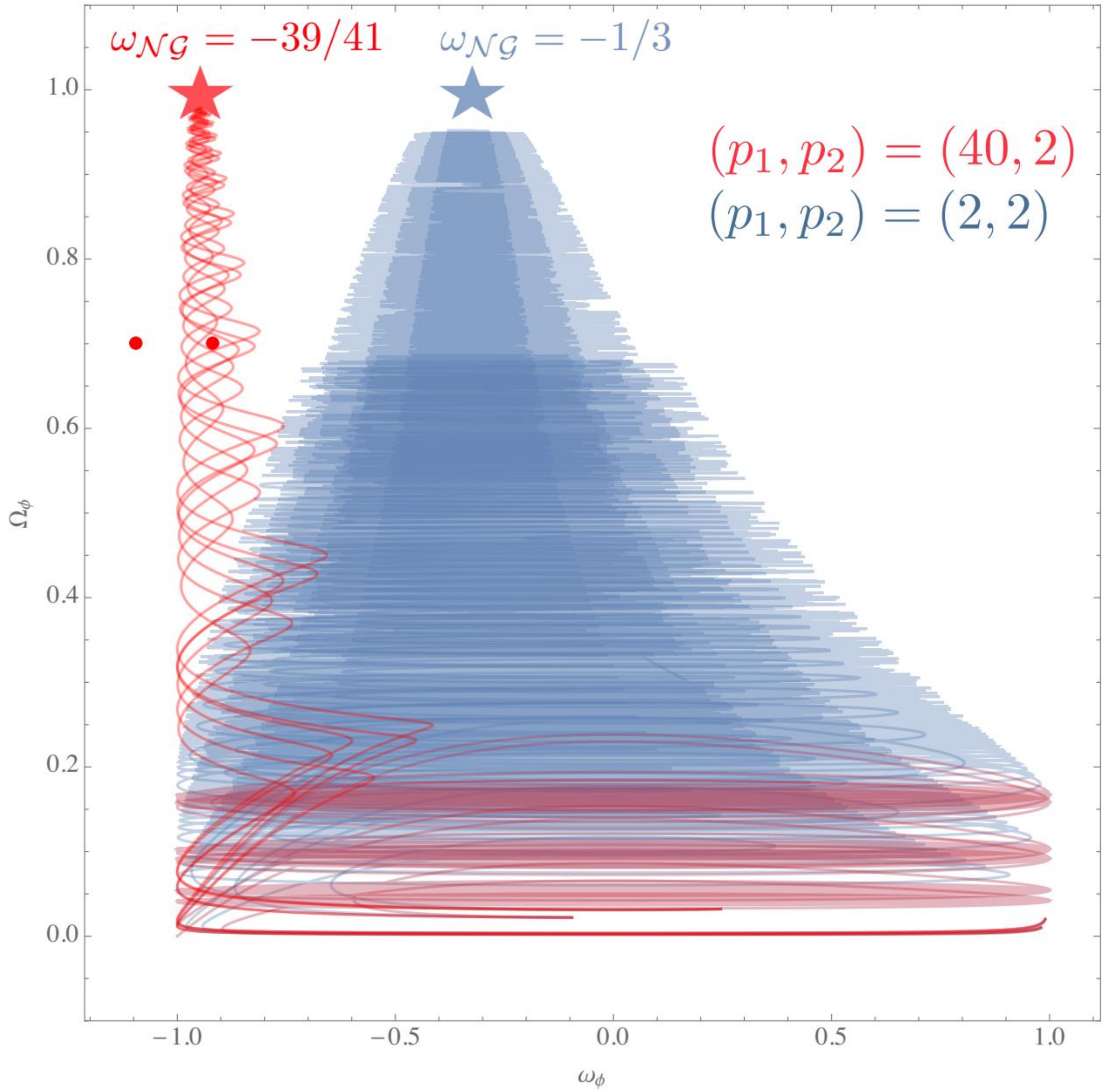
$$f = f_0 \ \phi_1^{p_1}$$

Even though the k's are not constant, their ratio still is! →

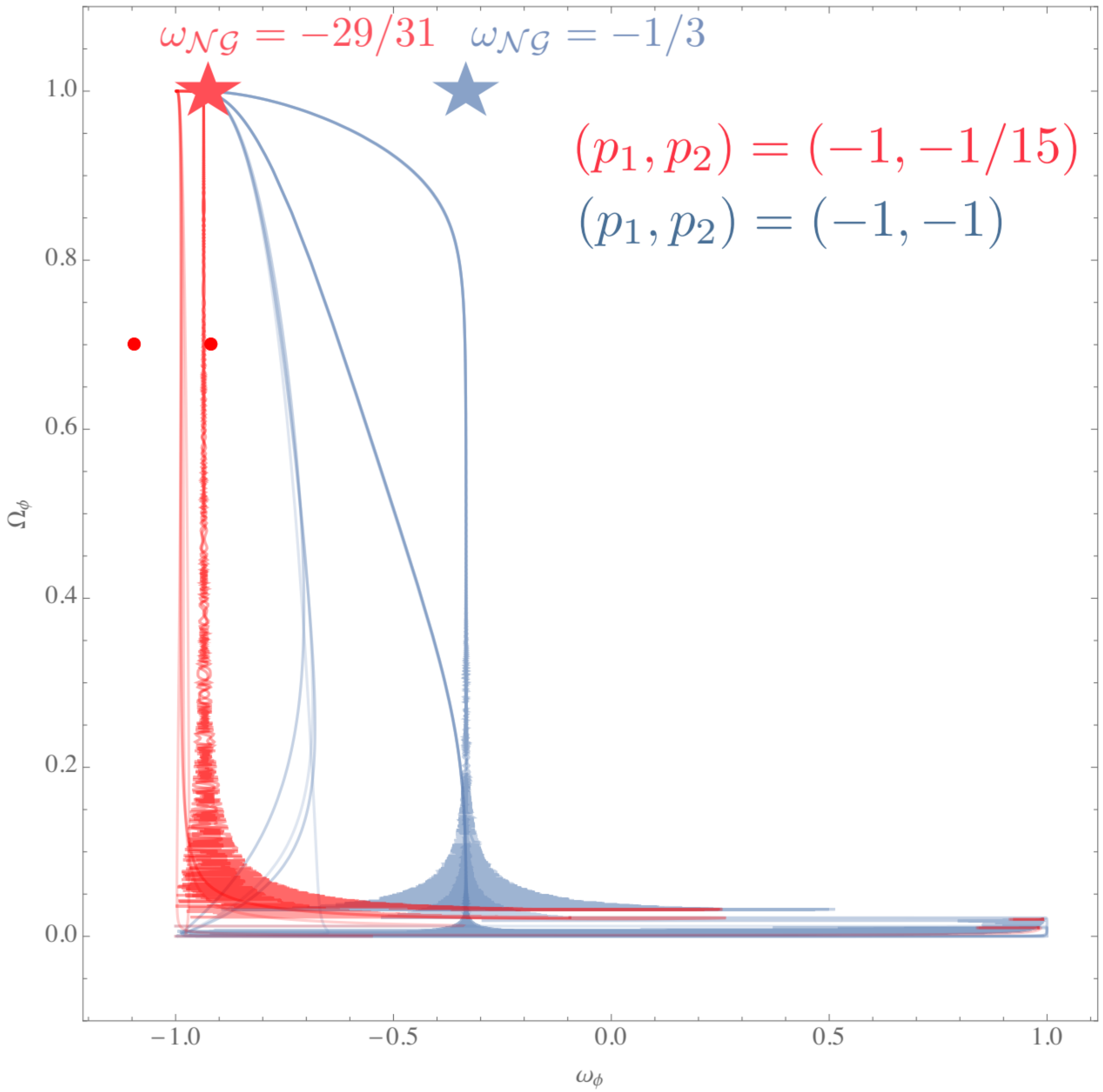
$$\frac{k_1}{k_2} = \frac{p_1}{p_2}$$

$$\omega_{\mathcal{NG}} = \frac{p_2 - 2p_1}{2p_1 + p_2}$$

← Need the same hierarchy but this time for the p's!



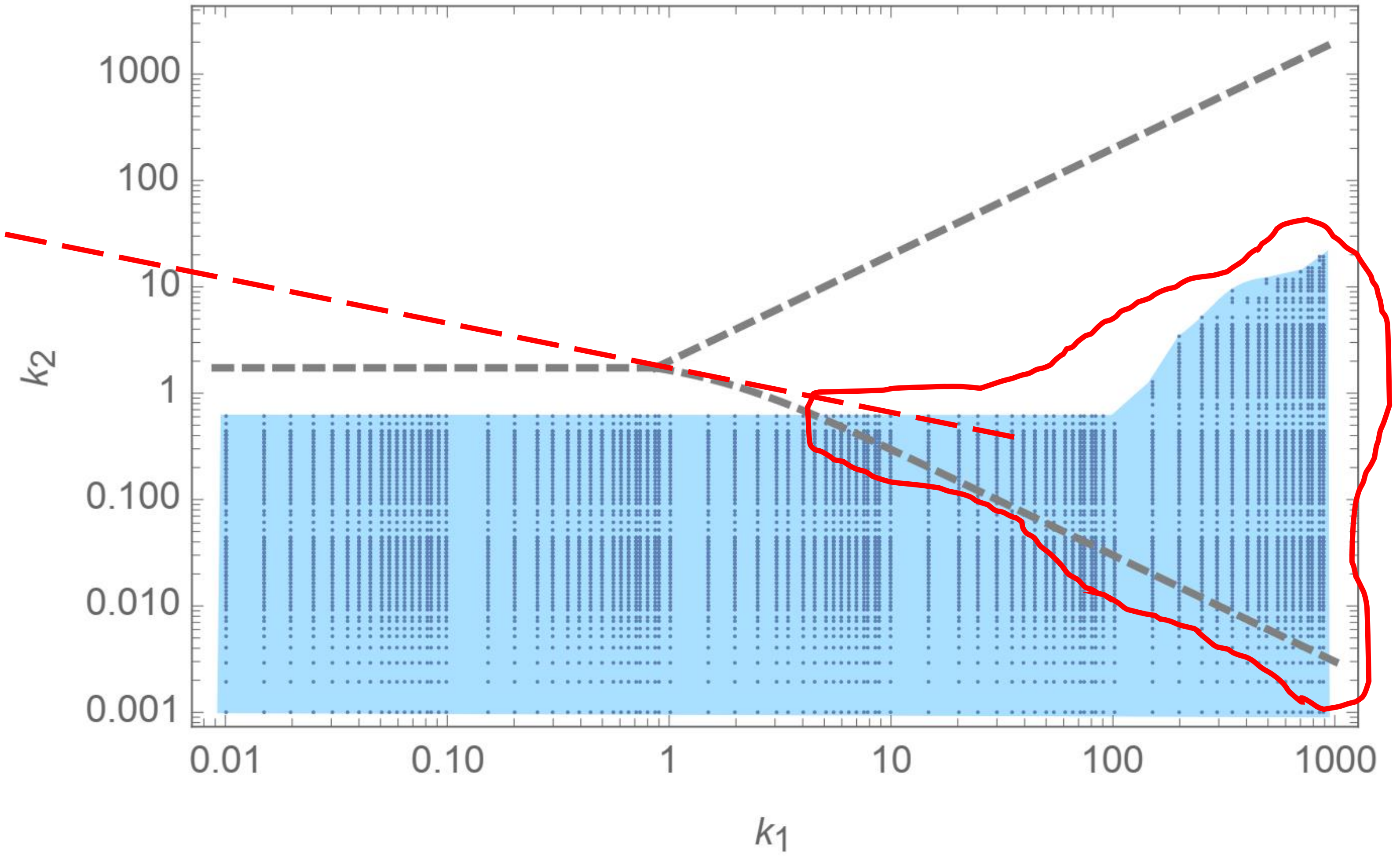
$\omega_{\mathcal{NG}} = -39/41$
$\omega_{\mathcal{NG}} = -1/3$
$(p_1, p_2) = (40, 2)$
$(p_1, p_2) = (2, 2)$
Ω_ϕ
ω_ϕ



$\omega_{\mathcal{NG}} = -29/31$
$\omega_{\mathcal{NG}} = -1/3$
$(p_1, p_2) = (-1, -1/15)$
$(p_1, p_2) = (-1, -1)$
Ω_ϕ
ω_ϕ

What about string theory?

Here's where you would like your model to be!



BUT…

Conclusions & Outlook

- Cosmic acceleration at late times is a challenge for QG
- A dS phase has been conjectured to be in the Swampland
- What about Quintessence? Single-field needs flat potentials and hence has the same fate as dS
- Multifield Quintessence offers a way out cause it may realize a quasi-dS phase with NO need for flat potentials
- A simple setup though, still needs some hierarchy which is not very natural for stringy constructions

→ We are working on this, so stay tuned…

Thank you for your attention!