# Discrete and higher-form symmetries from wrapped M5-branes

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#### Introduction

- The landscape of QFTs is vast and uncharted
  - Explore the space of QFTs to understand better strong-coupling regimes
  - New formulation of QFT?
- **Geometric engineering** in string/M-/F-theory:
  - Vast classes of strongly coupled fixed points in various dimensions
  - Includes higher-dimensional SCFTs (5d and 6d) that would be very challenging to study with traditional field theory techniques
  - Compactification of higher-dimensional SCFTs yields large families of 4d SCFTs that are strongly coupled (no known Lagrangian)
  - Topology and geometry give a new organizing principle
- Example: class S program and generalizations
  - 4d SCFTs from 6d SCFTs on punctured Riemann surface
  - Many setups in this class are realized with wrapped M5-branes

#### Introduction

- How can we extract physical data of geometrically engineered QFTs directly from their geometric definition?
- A great deal of information about the dynamics is encoded in **global** symmetries and their 't Hooft anomalies
- The concept of global symmetry in QFT has been recently revisited:
  - generalized global symmetries [Gaiotto, Kapustin, Seiberg, Willett 14]
  - new implications for the dynamics (e.g. new 't Hooft anomalies to match)

    [Gaiotto, Kapustin, Komargodski, Seiberg, 17]
- Useful viewpoint on QFTs in geometric engineering for all global symmetries

  Regard QFTs in geometric engineering as "edge modes" in supergravity

#### Introduction

Regard QFTs in geometric engineering as "edge modes" in supergravity

- The QFT is realized by degrees of freedom localized on a "defect" inside a larger ambient space in 10d or 11d
- The "defect" can be a stack of branes or a singularity in the geometry
- At low energies, the ambient space dynamics is described by 10d/11d SUGRA
- The gauge symmetries of SUGRA induce global symmetries in the QFT
- The pullback of the SUGRA fields onto the defect gives non-dynamical background gauge fields for the global symmetries
- In the presence of the "defect", the ambient SUGRA action can acquire a non-zero gauge variation
- This is exactly cancelled by the anomalies of the QFT d.o.f.'s: anomaly inflow

NB: This applies to all global symmetries

In this talk: Apply this viewpoint to generalized global symmetries of QFTs engineered with wrapped M5-branes

### Reminder on generalized symmetries

[Gaiotto, Kapustin, Seiberg, Willett 14]

p-form global symmetry with group G

- Charged operators are p-dimensional  $\mathcal{O}(\mathcal{C}^{(p)})$
- If G is continuous:  $\triangleright$  conserved Noether (p+1)-form current  $d*J_{p+1}$   $\triangleright$  couple to backgr. gauge field: (p+1)-form  $\delta A_{p+1} = d\Lambda_p$
- NB: G is necessarily Abelian for p > 0
- Many aspects of 0-form global symmetries extend to higher-form symmetries
  - > organize the spectrum of operators and give Ward identities
  - > spontaneous symmetry breaking and Goldstone modes
  - > 't Hooft anomalies

### Reminder on generalized symmetries

- 't Hooft anomalies for global symmetries
  - > obstruction to gauging (not inconsistency)
  - > invariant under RG flow
  - > can constrain phases of the QFT
- More familiar situation: perturbative anomalies for ordinary continuous symmetries, computed by triangle diagrams in 4d
- The concept applies to all global symmetries, including discrete and/or generalized symmetries
- To analyze 't Hooft anomalies of discrete and/or generalized symmetries we will use the topological terms in the SUGRA effective action

#### **Outline**

- 1. Introduction
- 2. Discrete higher-form symmetries from BF terms
- 3. 't Hooft anomalies from inflow
- 4. Conclusions and outlook

# Discrete higher-form symmetries from BF terms

## Setup and strategy

- Consider a 4d SCFT with a smooth holographic dual in 11d supergravity
- BPS conditions for  $AdS_5 \times_w M_6$  solutions are analyzed in

[Gauntlett, Martelli, Sparks, Waldram 04]

- Reduction of M-theory on  $M_6$  gives a 5d low-energy effective action
  - > p-form gauge fields in 5d bulk correspond to global symmetries on the boundary field theory
  - $\triangleright$  't Hooft anomalies are encoded in topological terms in the 5d action
- Input data about the setup
  - $\triangleright$  topology of the internal space  $M_6$  (cohomology groups)
  - $\triangleright$  isometries of the internal space  $M_6$
  - $\triangleright$   $G_4$ -flux configuration on  $M_6$

## p-form gauge fields in 5d

- p-form gauge fields in the 5d effective action have two sources:
  - 1) 1-form gauge fields from isometries of  $M_6$ 
    - (possibly non-Abelian) 0-form global symmetry
  - 2) p-form gauge fields from expansion of  $C_3$  onto cohomology classes of  $M_6$

$$\delta C_3 = c_3 + B_2^u \wedge \lambda_{1u} + A_1^\alpha \wedge \omega_{2\alpha} + a_0^x \wedge \Lambda_{3x}$$

 $\lambda_{1u}, \omega_{2\alpha}, \Lambda_{3x}$ : closed forms on  $M_6$  with integral periods; their cohomology classes are a basis of  $H_p(M_6, \mathbb{Z})_{\text{free}}$  for p=1,2,3

$$u = 1, \dots, b_1(M_6)$$
  $\alpha = 1, \dots, b_2(M_6)$   $x = 1, \dots, b_3(M_6)$ 

 $c_3, B_2^u, A_1^\alpha, a_0^x$ : external gauge fields in 5d; their field strengths have periods quantized in units of  $2\pi$ 

• NB: 0-form gauge field  $a_0^x = \text{compact scalar with period } 2\pi$ 

## p-form gauge fields in 5d

$$\delta C_3 = c_3 + B_2^u \wedge \lambda_{1u} + A_1^\alpha \wedge \omega_{2\alpha} + a_0^x \wedge \Lambda_{3x}$$

• Naïve expectation for global symmetries of the boundary QFT:

$$U(1)$$
 2-form symmetry  $U(1)^{b_1(M_6)}$  1-form symmetry  $U(1)^{b_2(M_6)}$  0-form symmetry



• Some 5d *p*-form gauge fields enter **topological mass couplings** 

$$S_{\text{top}} = -\frac{1}{2\pi} \int_{\mathcal{M}_5} \left[ N_{\alpha} c_3 \wedge dA_1^{\alpha} + \frac{1}{2} k_{uv} B_2^u \wedge dB_2^v \right]$$
$$N_{\alpha} = \int_{M_6} \frac{G_4}{2\pi} \wedge \omega_{2\alpha} \qquad k_{uv} = \int_{M_6} \frac{G_4}{2\pi} \wedge \lambda_{1u} \wedge \lambda_{1v}$$

- Some 5d U(1) p-form gauge symmetries are Higgsed to a cyclic subgroup
- The global symmetries on the field theory side depend on a choice of boundary conditions for the topological couplings

#### BF-like terms in five dimensions

- In the deep IR the topological mass couplings dominate
- We consider a topological 5d theory with action

$$S = \frac{k}{2\pi} \int_{\mathcal{M}_5} c_3 \wedge d\mathcal{A}_1 + \frac{M}{2\pi} \int_{\mathcal{M}_5} \widetilde{B}_2 \wedge dB_2$$

analog of 4d BF theory see e.g. the review [Banks, Seiberg 10]

- Let us first take 5d (Euclidean) spacetime  $\mathcal{M}_5$  to be a closed manifold
  - $\triangleright$  Invariance under large gauge transformations requires  $k, M \in \mathbb{Z}$
  - $\triangleright$  On shell:  $dc_3 = dA_1 = d\widetilde{B}_2 = dB_2 = 0$  (no local operators)
  - ➤ Natural observables: holonomies ("Wilson lines")

$$W_c(\mathcal{C}_3, n) = e^{in \int_{\mathcal{C}_3} c_3} \qquad W_{\mathcal{A}}(\mathcal{C}_1, n) = e^{in \int_{\mathcal{C}_1} \mathcal{A}_1}$$
$$W_{B, \widetilde{B}}(\mathcal{C}_2, n, \widetilde{n}) = e^{i \int_{\mathcal{C}_2} (\widetilde{n} B_2 - n \widetilde{B}_2)}$$

#### BF-like terms in five dimensions

• Correlators of Wilson lines

$$\langle W_c(\mathcal{C}_3, n) W_{\mathcal{A}}(\mathcal{C}_1, n') \rangle = \exp \left[ 2\pi i \frac{nn'}{k} L(\mathcal{C}_3, \mathcal{C}_1) \right] \qquad (L = \text{linking number})$$

$$\langle W_{B, \widetilde{B}}(\mathcal{C}_2, n, \widetilde{n}) W_{B, \widetilde{B}}(\mathcal{C}'_2, n', \widetilde{n}') \rangle = \exp \left[ 2\pi i \frac{n\widetilde{n}' - \widetilde{n}n'}{M} L(\mathcal{C}_2, \mathcal{C}'_2) \right]$$

- The 5d action describes p-form gauge fields with **discrete** gauge group
  - $ightharpoonup c_3, \mathcal{A}_1$  are flat gauge fields with holonomies in  $\mathbb{Z}_k \subset U(1)$
  - $\triangleright \widetilde{B}_2, B_2$  are flat gauge fields with holonomies in  $\mathbb{Z}_M \subset U(1)$
- Equivalent viewpoint: **Stückelberg** action

$$S = -\frac{1}{2} \int_{\mathcal{M}_5} g_{\phi\phi} (d\phi - k \mathcal{A}_1) \wedge *(d\phi - k \mathcal{A}_1) \qquad \begin{array}{l} d\phi = *dc_3 \\ \phi : \text{ compact scalar of period } 2\pi \end{array}$$
 in the IR: 
$$\mathcal{A}_1 = \frac{1}{k} d\phi$$

## Spacetimes with a boundary

- For holographic applications we have to consider 5d spacetimes with boundary
- Choice of topological boundary conditions for the pairs  $(c_3, A_1)$ ,  $(\widetilde{B}_2, B_2)$

case	boundary conditions	global symm. of boundary theory
(a)	$c_3$ : free $\mathcal{A}_1$ : Dirichlet	$\mathbb{Z}_k$ 0-form symm.
(b)	$c_3$ : Dirichlet $\mathcal{A}_1$ : free	$\mathbb{Z}_k$ 2-form symm.
(c) $k = mm'$ $(m \neq 1, k)$	$c_3$ : free modulo $\mathbb{Z}_m$ $\mathcal{A}_1$ : free modulo $\mathbb{Z}_{m'}$	$\mathbb{Z}_{m'}$ 0-form symm. $\mathbb{Z}_m$ 2-form symm.

"free modulo  $\mathbb{Z}_m$ ": for  $\mathcal{C}_3 \subset \partial \mathcal{M}_5$  consider the holonomy  $e^{i \int_{\mathcal{C}_3} c_3} = (e^{2\pi i/k})^x$ x must satisfy  $x = p \mod m$  for a fixed p, but is otherwise free

## Spacetimes with a boundary

• Depending on b.c. different bulk Wilson lines are or are not allowed to end on the boundary

case	boundary conditions	can end on boundary?
(a)	$c_3:$ free $\mathcal{A}_1:$ Dirichlet	$e^{i\int_{\mathcal{C}_3} c_3}$ no local op. charged under $e^{i\int_{\mathcal{C}_1} A_1}$ yes ———————————————————————————————————
(b)	$c_3:$ Dirichlet $\mathcal{A}_1:$ free	$e^{i\int_{C_3} c_3}$ yes $\longrightarrow$ surface op. charged under $e^{i\int_{C_1} A_1}$ no 2-form glob. symm.
(c)	$c_3$ : free modulo $\mathbb{Z}_m$ $\mathcal{A}_1$ : free modulo $\mathbb{Z}_{m'}$	$e^{i\int_{\mathcal{C}_3} c_3}  m' \text{ at a time}$ both local op. and surface op.

## Spacetimes with a boundary

#### Remarks:

• In case (a), specifying the b.c. for  $\mathcal{A}_1$  is the same as fixing a **background** 1-form gauge field for the global  $\mathbb{Z}_k$  0-form symmetry of the theory on  $\mathcal{M}_4 = \partial \mathcal{M}_5$ 

if there is no torsion in homology,  $\operatorname{Hom}(\pi_1(\mathcal{M}_4), \mathbb{Z}_k) \cong H^1(\mathcal{M}_4, \mathbb{Z}_k)$ 

- Gauging the global  $\mathbb{Z}_k$  0-form symmetry of case (a) gives case (b); gauging subgroups gives case (c)
- In case (c) there is a **mixed 't Hooft anomaly** between the  $\mathbb{Z}_{m'}$  0-form symm. and the  $\mathbb{Z}_m$  2-form symm.
- Formally we detect it from the 6-form that encodes the 5d top. mass terms

$$\frac{S}{2\pi} = \int_{\mathcal{M}_5} I_5^{(0)} \qquad I_6 = dI_5^{(0)} \qquad I_6 = k \frac{dc_3}{2\pi} \wedge \frac{d\mathcal{A}_1}{2\pi} + M \frac{d\tilde{B}_2}{2\pi} \wedge \frac{dB_2}{2\pi}$$

• Analogous remarks hold for the  $(\widetilde{B}_2, B_2)$  pair

't Hooft anomalies from inflow

## Embedding in the full gravity theory

- The BF-like terms embed in the full gravity theory obtained from reduction of M-theory on  $M_6$
- The 5d effective action includes kinetic terms for all p-form fields
- The system admits **singleton** modes
  - > Pure gauge in the bulk, dynamical on the conformal boundary
  - ➤ Hamiltonian determined combining kinetic terms and BF terms

[Maldacena, Moore, Seiberg 01; Belov, Moore 04]

- $\triangleright$  Singleton 0-form gauge field from  $(c_3, \mathcal{A}_1)$
- $\triangleright$  Singleton 1-form gauge field from  $(\tilde{B}_2, B_2)$
- Singleton modes do not gravitate
  - > They are holographically dual to **decoupled** sectors

(terminology "singleton" from representation theory of superconformal algebra)

## Singletons and decoupling

• Decoupling singleton: sum over topological sectors

$$Z = \sum_{eta} Z_{eta} Z_{ ext{singleton}}^{eta}$$

 $Z_{\text{singleton}}^{\beta}$ : conformal blocks of singleton sector

 $Z_{\beta}$ : partition function of gravitating modes

 $\beta$ : label for topological sectors

- On field theory side:  $Z_{\text{singleton}}^{\beta} \rightarrow \text{free modes}; Z_{\beta} \rightarrow \text{interacting SCFT}$
- E.g.:  $(c_3, \mathcal{A}_1)$  system

start from case (a):

- Dirichlet b.c. for  $A_1$
- Interacting SCFT with  $\mathbb{Z}_k$  0-form glob. symm.
- Choice of b.c. = choice of backgr. gauge field  $\beta \in H^1(\mathcal{M}_4, \mathbb{Z}_k)$
- $Z_{\beta}$  = partition function of SCFT with  $\beta$  backgr. gauge field

to get case (b):

- Gauge 0-form symm.
- $\widehat{Z} \propto \sum_{\beta} Z_{\beta}$
- Analogous remarks for  $(\widetilde{B}_2, B_2)$

cfr. AdS<sub>5</sub> x S<sup>5</sup> type IIB analysis of [Witten 98; Belov, Moore 04; Gaiotto, Kapustin, Seiberg, Willett 14; Hofman, Igbal 17]

#### 't Hooft anomalies and inflow

- In the full gravity theory (gravitating + singletons)
  - ➤ Kinetic terms & BF terms in the 5d action
  - **2nd order action**: impose Dirichlet b.c. for  $c_3$ ,  $A_1$ ,  $\widetilde{B}_2$ ,  $B_2$

cfr. [Hofman, Iqbal 17] and 3d Abelian CS analysis of [Gukov, Martinec, Moore, Strominger 04]

- $\triangleright$  The full boundary theory has continuous U(1) p-form symmetries
- Expectation: the anomalies of the full boundary theory (interacting + decoupling) are balanced by anomaly inflow from M-theory bulk
- Since we deal with continuous p-form symmetries, we can describe anomalies using a 6-form anomaly polynomial  $I_6$
- A single 6-form  $I_6$  contains information about 't Hooft anomalies of different SCFTs (corresponding to different top. b.c. in 5d)

### Tools for anomaly inflow

- Systematic tools for anomaly inflow based on [Freed, Harvey, Minasian, Moore 98] developed in [Bah, FB, Minasian, Nardoni 19,20]
- Input: (1) topology and isometries of  $M_6$ 
  - (2)  $G_4$ -flux quantum numbers
- Auxiliary geometry

 $M_6 \hookrightarrow M_{12} \to \mathcal{M}_6$  external spacetime (Euclidean; descent formalism)

fibration includes

connections for isometries of  $M_6$ 

- Cohomology classes of  $M_6$  are represented by closed forms:  $\Omega_4^{\alpha}, \Lambda_{3x}, \omega_{2\alpha}, \lambda_{1u}$
- Upgrade to globally-def. closed forms on  $M_{12}$ :  $(\Omega_4^{\alpha})', (\Lambda_{3x})', (\omega_{2\alpha})', (\lambda_{1u})'$

they contain connections for isometries of  ${\cal M}_6$ 

## Tools for anomaly inflow

• Construct:

$$E_4 = N_{\alpha} \left(\Omega_4^{\alpha}\right)' + \frac{da_0^x}{2\pi} \left(\Lambda_{3x}\right)' + \frac{dA_1^{\alpha}}{2\pi} \left(\omega_{2\alpha}\right)' + \frac{dB_2^u}{2\pi} (\lambda_{1u})' + \frac{dc_3}{2\pi}$$
input flux quanta
$$U(1) \text{ backgr. } p\text{-form gauge fields}$$
(forms on  $\mathcal{M}_6$  pulled back to  $M_{12}$ )

- Interpretation:  $E_4$  encodes the 11d boundary condition for  $G_4$  near M5-branes
- Compute

$$I_6^{\rm inflow} = \int_{M_6} \left[ -\frac{1}{6} E_4^3 - E_4 \wedge X_8 \right]$$

$$\int_{M_6} = \text{fiber integration in } M_6 \hookrightarrow M_{12} \to M_6$$

$$X_8 \propto p_1^2 - 4 p_2 \text{ (Pontryagin classes of } TM_{12} \text{)}$$

#### Example I: BBBW

- We consider a stack of N M5-branes wrapped on a Riemann surface without punctures, preserving 4d  $\mathcal{N}=1$  supersymmetry

  [Bah, Beem, Bobev, Wecht 12]
- 11d background:  $\mathbb{R}^{1,3} \times \mathbb{R} \times Y_3$  where  $Y_3$ : total space of  $\mathcal{L}_1 \oplus \mathcal{L}_2 \to \Sigma_g$   $c_1(\mathcal{L}_1) = p \qquad c_1(\mathcal{L}_2) = q \qquad \text{CY condition:} \quad p + q = -\chi(\Sigma_g) = 2g 2$
- M5-brane stack sits at a point on  $\mathbb{R}$ , at the zero section of  $Y_3$ , and is extended along  $\mathbb{R}^{1,3}$  and the Riemann surface
- For suitable values of p, q, g the system flows to an interacting SCFT which has a **smooth** gravity dual solution in 11d supergravity

$$AdS_5 \times_w M_6$$
 
$$S^4 \hookrightarrow M_6 \to \Sigma_g$$
 struct. group:  $U(1) \times U(1) \subset SO(5)$ 

Caveat: p=q=0 gives 4d  $\mathcal{N}=4$  SYM, which does not have a smooth gravity dual in 11d supergravity (dualize to standard type IIB frame)

### Example I: BBBW

- Isometries of  $M_6$ :  $U(1)_{\phi_1} \times U(1)_{\phi_2}$
- (Co)homology of  $M_6$ : one 4-cycle  $\rightarrow$  one flux quantum N one 2-cycle  $\rightarrow$  one 1-form gauge field  $A_1$  2g 1-cycles  $\rightarrow$  g pairs of 2-form gauge fields  $(\widetilde{B}_2^i, B_{2i})$  one 0-cycle  $\rightarrow$  one 3-form gauge field  $c_3$  no 3-cycles  $\rightarrow$  no 0-form gauge fields (axions)

• Inflow result:

$$I_6^{\text{inflow}} = -\frac{2}{3} \left( N^3 - \frac{1}{4} N \right) \left[ p c_1^{\phi_1} (c_1^{\phi_2})^2 + q c_1^{\phi_2} (c_1^{\phi_1})^2 \right] - \frac{1}{6} N \left[ p (c_1^{\phi_1})^3 + q (c_1^{\phi_2})^3 \right]$$

$$+ \frac{1}{24} N \left[ p c_1^{\phi_1} + q c_1^{\phi_2} \right] p_1(T) + \frac{1}{(2\pi)^2} \left[ -N dc_3 dA_1 - N d\widetilde{B}_2^i dB_{2i} \right]$$

- $\triangleright$  no interplay between isometries and  $c_3$ ,  $A_1$ ,  $\widetilde{B}_2^i$ ,  $B_{2i}$
- $\triangleright$  BF terms  $dc_3 dA_1, d\widetilde{B}_2^i dB_{2i}$ :
  - choice of b.c. gives various SCFTs with different global symm.
  - they encode mixed 't Hooft anomalies if we select b.c. of type (c)

- Smooth supersymmetric  $AdS_5$  solutions first discovered in [Gauntlett, Martelli, Sparks, Waldram 04]
- Internal space  $M_6$ :  $\mathbb{CP}^1$  bundle over product of two Riemann surfaces
- If one of the Riemann surfaces is a torus it is best to dualize to type IIB cfr.  $Y^{p,q}$  [Gauntlett, Martelli, Sparks, Waldram 04] and [Gauntlett, O Colgain, Varela 06]
- We consider one 2-sphere and a genus-g surface  $\Sigma_g$  with g=0 or g>2
- Interpretation: near-horizon limit of M5-branes probing a flux background  $\triangleright$  Resolved  $\mathbb{C}^2/\mathbb{Z}_2$  fibered over  $\Sigma_g$  and stabilized by  $G_4$ -flux [Bah, FB 19]
- Strategy here: read off symmetries and anomalies from holographic data

- Isometries of  $M_6$ :  $U(1)_{\psi} \times SU(2)_{\varphi}$
- (Co)homology of  $M_6$ :

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three 4-cycles \rightarrow three flux quanta N, N_+, N_-

three 2-cycles \rightarrow three 1-form gauge fields A_1, A_1^+, A_1^-

2g 1-cycles \rightarrow g pairs of 2-form gauge fields B_{2i}, \widetilde{B}_2^i

one 0-cycle \rightarrow one 3-form gauge field c_3

4g 3-cycles \rightarrow 4g 0-form gauge fields a_{0i}^{\pm}, \widetilde{a}_0^{i\pm}
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- BF terms:  $\frac{1}{2\pi}c_3 \wedge \left(N dA_1 + N_+ dA_1^+ + N_- dA_1^-\right) + \frac{N}{2\pi}\widetilde{B}_2^i \wedge dB_{2i}$ 
  - $\triangleright$  One linear comb. of A's gives a  $\mathbb{Z}_k$  5d gauge field,  $k = \gcd(N, N_+, N_-)$

$$N A_1 + N_+ A_1^+ + N_- A_1^- \equiv k A_1$$

- $\triangleright$  The other two vectors remain standard U(1) fields
- ➤ Discrete 2-forms gauge fields in 5d

This system has a rich variety of terms in the inflow anomaly polynomial:

➤ Mixed terms between isometries/gravity and discrete symmetries, e.g.

$$c_2(SU(2)_{\varphi}) \frac{d\mathcal{A}_1}{2\pi}$$
  $p_1(T) \frac{d\mathcal{A}_1}{2\pi}$ 

➤ Mixed terms between discrete symmetries and axions, e.g.

$$\frac{d\mathcal{A}_1}{2\pi} \left( \frac{da_0}{2\pi} \frac{d\widetilde{a}_0}{2\pi} \right)^2 \qquad \left( \frac{d\widetilde{a}_0}{2\pi} \frac{dB_2}{2\pi} \right) \left( \frac{da_0}{2\pi} \frac{d\widetilde{a}_0}{2\pi} \right) \qquad \text{(indices are suppressed)}$$

➤ All three kinds of symmetries at once, e.g.

$$c_1(U(1)_{\psi}) \frac{d\mathcal{A}_1}{2\pi} \left( \frac{da_0}{2\pi} \frac{d\widetilde{a}_0}{2\pi} \right) \qquad c_1(U(1)_{\psi}) \left( \frac{d\widetilde{a}_0}{2\pi} \frac{dB_2}{2\pi} \right)$$

• Terms with the 0-form gauge fields  $a_{0i}^{\pm}$ ,  $\widetilde{a}_{0}^{i\pm}$  are interpreted as anomalies in the space of coupling constants

[Cordova, Freed, Ho Tat Lam, Seiberg 19]

• Exactly marginal couplings associated to 4d operators

$$\Delta(\mathcal{O}) = 4$$
  $\Delta \mathcal{L} \sim a_0 \mathcal{O}$   $a_0$  has period  $2\pi$   $\Rightarrow$   $\int_{\mathcal{M}_4} \mathcal{O} *_4 1 \in \mathbb{Z}$  (analogous to  $\Delta \mathcal{L} \sim \theta \, \epsilon_{\mu\nu\rho\sigma} \mathrm{tr} \, (F^{\mu\nu} \, F^{\rho\sigma})$  in gauge theory)

• We expect that this rich structure of 't Hooft anomalies has interesting implications for the dynamics of the theory

#### Conclusions and outlook

#### Conclusions

- Topological mass terms in 5d AdS supergravity encode discrete global symmetries of the dual field theory
  - ➤ The same bulk theory with different topological boundary conditions gives field theories with different discrete global symmetries
- We can capture 't Hooft anomalies with a 6-form inflow anomaly polynomial
  - > Systematic geometric tools using data from holographic solution
- Rich interplay between all p-forms fields from expansion of  $C_3$  (p=0,1,2,3)
  - ➤ Higher-form symmetries
  - > Discrete symmetries
  - $\triangleright$  Anomalies in the space of coupling constants, or "(-1)-form" symmetries

#### Outlook

- Dynamical implications of anomalies
- Analysis of singleton sector and decoupling modes
  - $\triangleright$  potential avenue for exact anomalies, including  $\mathcal{O}(N^0)$  terms
- Complement the holographic/inflow approach with field theory analysis
  - $\triangleright$  e.g. theories engineered with N=2 M5-branes
- Use topological mass terms in 5d to organize/classify extended operators
  - $\triangleright$  line operators in class S from  $(\widetilde{B}_2^i, B_{2i})$  system?
- Explore other sources of discrete symmetries
  - > torsion cycles in the internal geometry
  - > discrete isometries
  - **>** ...

[Camara, Ibanez, Marchesano 11; Bersaluce-Gonzales, Camara, Marchesano, Regalado, Uranga 12]

#### Thank you!