Hamiltonian truncation in AdS

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Outline

1. Motivation, setup, expectations
2. Trouble from the end of the world (a.k.a cutoff effects)
3. Examples:
   - free boson & fermion,
   - Ising w/ $(T-T_c)$ or w H
   - ϕ^4
4. Outlook

1. Motivation & setup

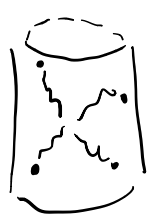
QFT w/ a M scale in AdS w/ radius R $\longleftrightarrow$ golden standard CFT + λ relevant

- $MR \ll 1$ perturbation theory
- $MR \gg 1$ flat space physics

conformal symmetry

- Massive QFTs w/ conformal bootrap?

S-Matrix bootstrap [Paulos, Penedones, ...]



spectrum?

$\downarrow$

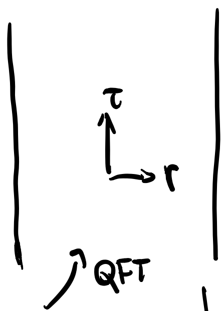
1 parameter family of CFT correlators

HARD

Hamiltonian truncation:

- H breaks conformal sym (regulator)
- Straightforward to set up for any RG flow
- Window into strongly coupled AdS physics
- Example of HT in ∞ vol. (UV/IR connection)

• Setup: 2d AdS_2



$$ds^2 = \frac{R^2}{\cos^2 r}\left(d\tau^2 + dr^2\right)$$

$$r \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$H = H_0 + \bar{\lambda} \int \frac{dr}{\cos^2 r} R^{\Delta_V} V(\tau = 0, r) = H_0 + V$$

dimensionless H $\longleftrightarrow$ ∂_τ

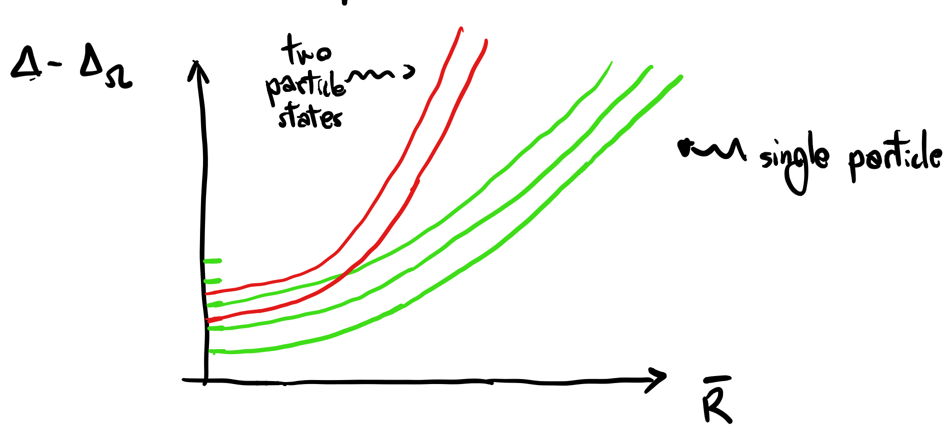
solvable (2d CFT or free massive th)

local op dim $\Delta_V < 1$

$$\bar{\lambda} = \lambda R^{2-\Delta_V} \quad \text{dimensionless}$$

$$M = \lambda^{\frac{1}{2-\Delta_V}}$$

$$\bar{R} = \bar{\lambda}^{\frac{1}{2-\Delta_V}} = MR \quad \text{dimensionless AdS radius}$$



$$ds^2 \sim R^2 d\tau^2 = d\tau^2_{flat}$$

$$\Delta - \Delta_{\cancel{min}\,\Omega} \longrightarrow E_{flat} R = \frac{\epsilon}{M} \bar{R}$$

$$\frac{\Delta_2 - \Delta_\Omega}{\Delta_1 - \Delta_\Omega} \rightarrow \frac{m_2}{m_1}$$

- HT:

1) Truncate $\mathcal{H}$ of H_0 to $\Delta < \Lambda$
2) diagonalize $H = H_0 + \bar{\lambda} V$ in the subspace:

$$\mathcal{E} = \Delta(\bar{\lambda}, \Lambda)$$

3) Extrapolate $\cancel{\Delta(\bar{\lambda}, \infty)}$

Vacuum energy is ∞

$$\langle T_\tau{}^\tau \rangle \neq 0 \quad = \text{const.}$$

$$\Delta(\bar{\lambda}) - \Delta_\Omega(\bar{\lambda}) \text{ are finite}$$

$$\left[\Delta(\bar{\lambda}, \Lambda) - \Delta_\Omega(\,, \Lambda) \xrightarrow{\Lambda \to \infty} \text{oscillates} \right]$$

2. Cutoff effects : a cure

2.1. Free boson: the example.

$$H_0 = \int (\partial\phi)^2 + m^2\phi^2$$

$$V = \underbrace{\lambda R^2}_{\bar\lambda} \int \phi^2$$

$$m^2R^2 = \Delta(\Delta - d) \iff \Delta = \frac{1}{2} + \sqrt{m^2R^2 + \frac{1}{4}}$$

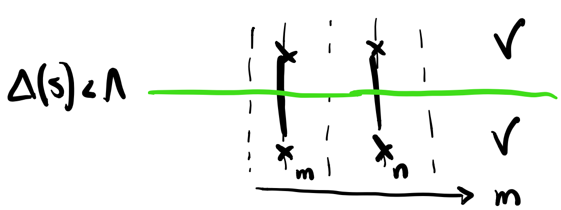
$$\Delta(\lambda) = \Delta + \frac{\bar\lambda}{\cdots} - \frac{\bar\lambda^2}{2(\Delta-\frac{1}{2})^3} + \ldots$$

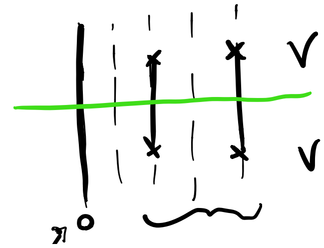
$$V = \underset{\xi_?}{a^+_m a^+_n} + a^+_m a_n + a_m a_n$$

RSPT :

$$\mathcal{E}_\chi = \ldots + \sum^{\Delta(s)<\Lambda} \frac{|\langle\chi|V|s\rangle|^2}{\Delta(s) - \Delta(\chi)} \sim \Delta(s)$$

$|\chi\rangle = |\Omega\rangle$ $\qquad$ $|\chi\rangle = a^+_0|\Omega\rangle$





e_1

$\Delta_{max} = \Lambda$ $\qquad\qquad$ $\Delta_{max} = \Lambda - e_1$

$$|\langle\chi|V|S(\Lambda)\rangle|^2 \xrightarrow[\Lambda\to\infty]{} 0 \quad \times$$

Prescription:

$$\mathcal{E}_i(\Lambda + e_i) - \mathcal{E}_\Omega(\Lambda) \xrightarrow[\Lambda\to\infty]{} \text{physical gaps}$$

A sketch of the genezl argument (PT)

Intermezzo:

$$\langle i| V(\tau_1 + \dots \tau_{n-1}) \dots V(\tau_1+\tau_2)\, \overset{|\alpha\rangle\langle\alpha|}{\downarrow} V(\tau_1)\, \overset{\{}{\downarrow} V(0)|i\rangle_{\text{conn}}$$

wrt $|i\rangle$

$$= \prod_{\ell=1}^{n-1} \int_0^\infty d\alpha_\ell\, e^{-(\alpha_\ell - e_i)\tau_\ell}\, \rho_{i,n}(\vec{\alpha})$$

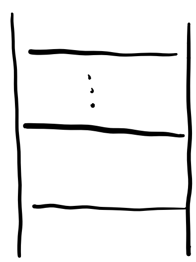
$$\mathcal{E}_i = e_i + \sum_{n=1}^{\infty} (-1)^{n+1} c_{i,n} \bar{\lambda}^n$$

$$\left[\, c_{i,n} = \prod_{\ell=1}^{n-1} \int_0^\Lambda \frac{d\alpha_\ell}{\alpha_\ell - e_i}\, \rho_{i,n}(\vec{\alpha}) \,\right]$$

$$c_{i,n}(\Lambda + e_i) - c_{0,n}(\Lambda) \sim \Delta\rho_{i,n}(\alpha)$$

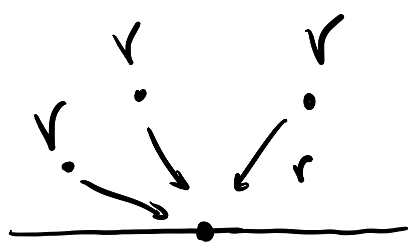
$$\Delta\rho_{i,n} \longleftrightarrow \langle i | V \dots V | i \rangle - \langle \Omega | V \dots V | \Omega \rangle$$

$V = \int r$

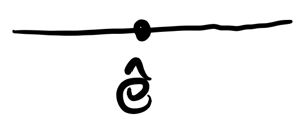


- $r \to$ boundary non singular (fine tune)
- $\boxed{(r \times r \times r) \longrightarrow 1 + r^{\Delta_*} \mathcal{O}_{\Delta_*}}$

Poincaré



$= \sum_{\hat{e}}$



- $1 \rightsquigarrow \mathcal{E}_{\Omega} \sim \Lambda$
- $r^{\Delta_*} \mathcal{O}_{\Delta_*} \rightsquigarrow |\Delta\rho| < \alpha^{1-\Delta_*}$

$$\rightsquigarrow$$

$$\left[\mathcal{E}_{gap}(\infty) - \mathcal{E}_{gap}(\Lambda) \sim \Lambda^{1-\Delta_*} \qquad \Delta_* > 1 \right]$$