

---

# COBORDISMS AND THE SWAMPLAND

---

Miguel Montero

Harvard

Tor Vergata Theory Seminar

March 29 2021



---

String theory is good for **many things**.

---

---

String theory is good for **many things**.

Understand field theory

---

---

String theory is good for **many things**.

Mathematical connections

Understand field theory

---

String theory is good for **many things**.

Mathematical connections

Understand field theory

Quantum gravity

Real world

---

---

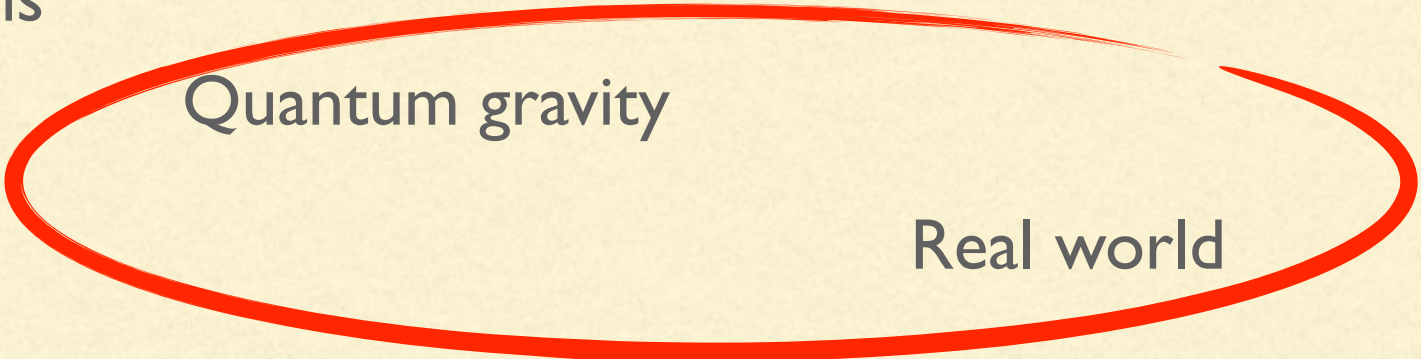
String theory is good for **many things**.

Mathematical connections

Understand field theory

Quantum gravity

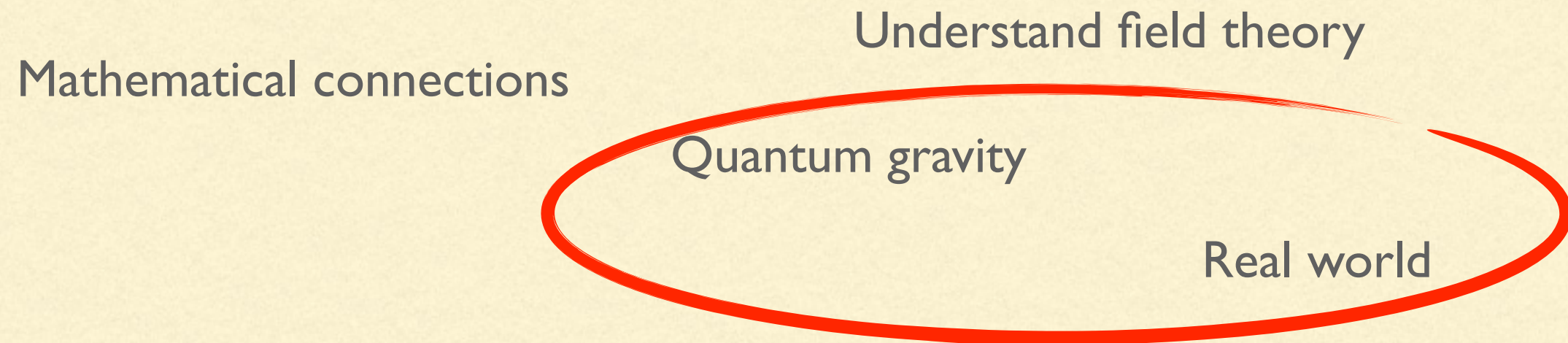
Real world



---

---

String theory is good for **many things**.

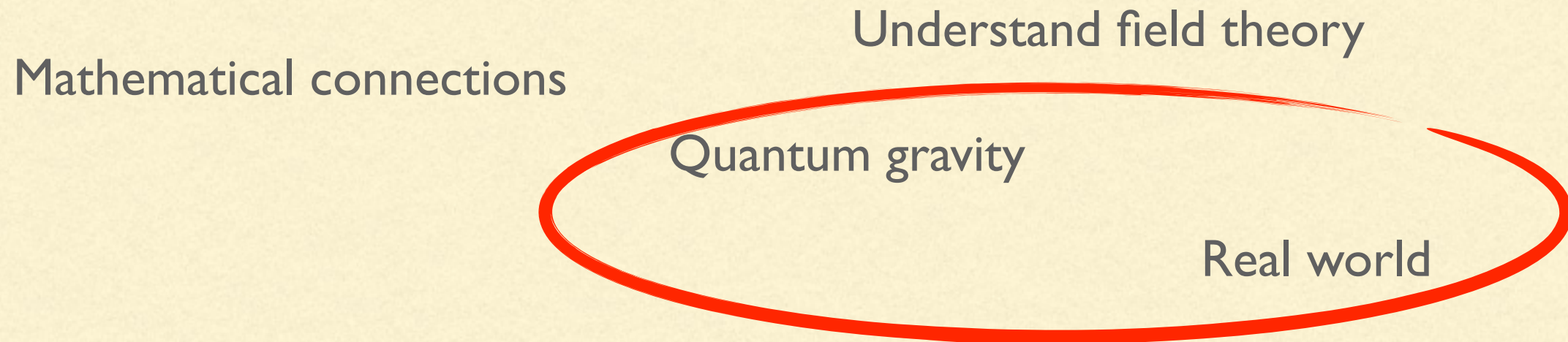


“Traditional” approach

---

---

String theory is good for **many things**.



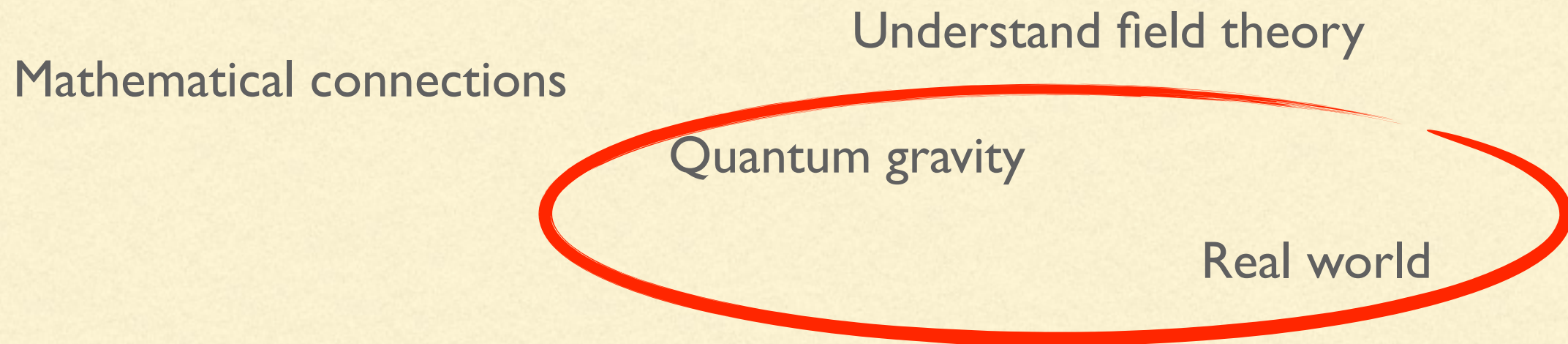
“Traditional” approach

Concrete string model

---

---

String theory is good for **many things**.



“Traditional” approach

Concrete string model



Concrete low-energy EFT

(SM, dark matter, inflation...)

---

---

String theory is good for **many things**.

Mathematical connections

Understand field theory

Quantum gravity

Real world

“Traditional” approach

Swampland approach

Concrete string model



Concrete low-energy EFT

(SM, dark matter, inflation...)

---

---

String theory is good for **many things**.

Mathematical connections

Understand field theory

Quantum gravity

Real world

“Traditional” approach

Swampland approach

Concrete string model

**General** property of gravity



Concrete low-energy EFT

(SM, dark matter, inflation...)

---

---

String theory is good for **many things**.

Mathematical connections

Understand field theory

Quantum gravity

Real world

“Traditional” approach

Swampland approach

Concrete string model

**General** property of gravity



Concrete low-energy EFT

**General** low-energy EFT

(SM, dark matter, inflation...)

---

---

**Main Swampland Idea:** Ascertain **universal** features  
of quantum gravity

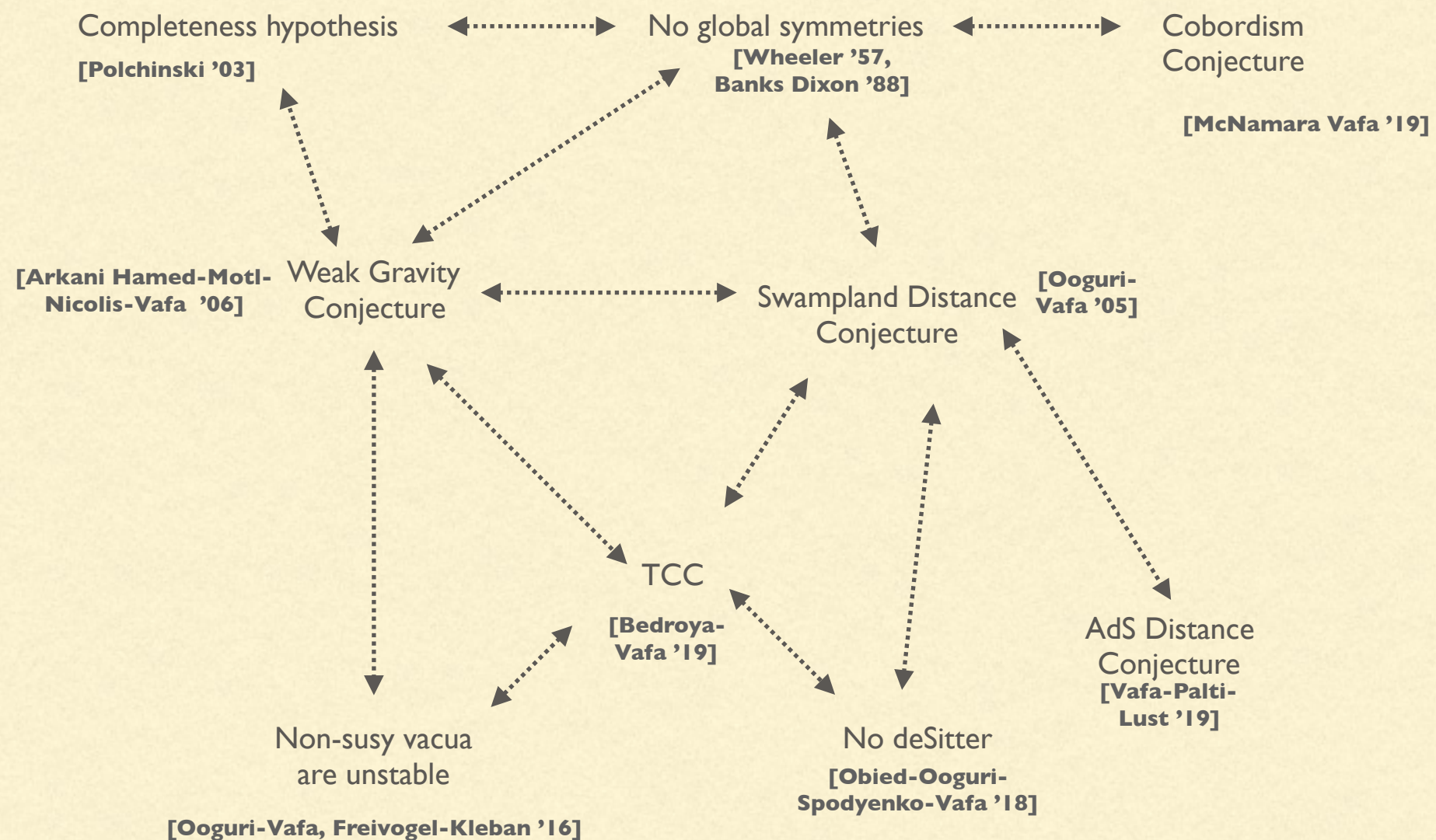
---

# **Main Swampland Idea:** Ascertain **universal** features of quantum gravity

We organize this in a web of interconnected **conjectures**

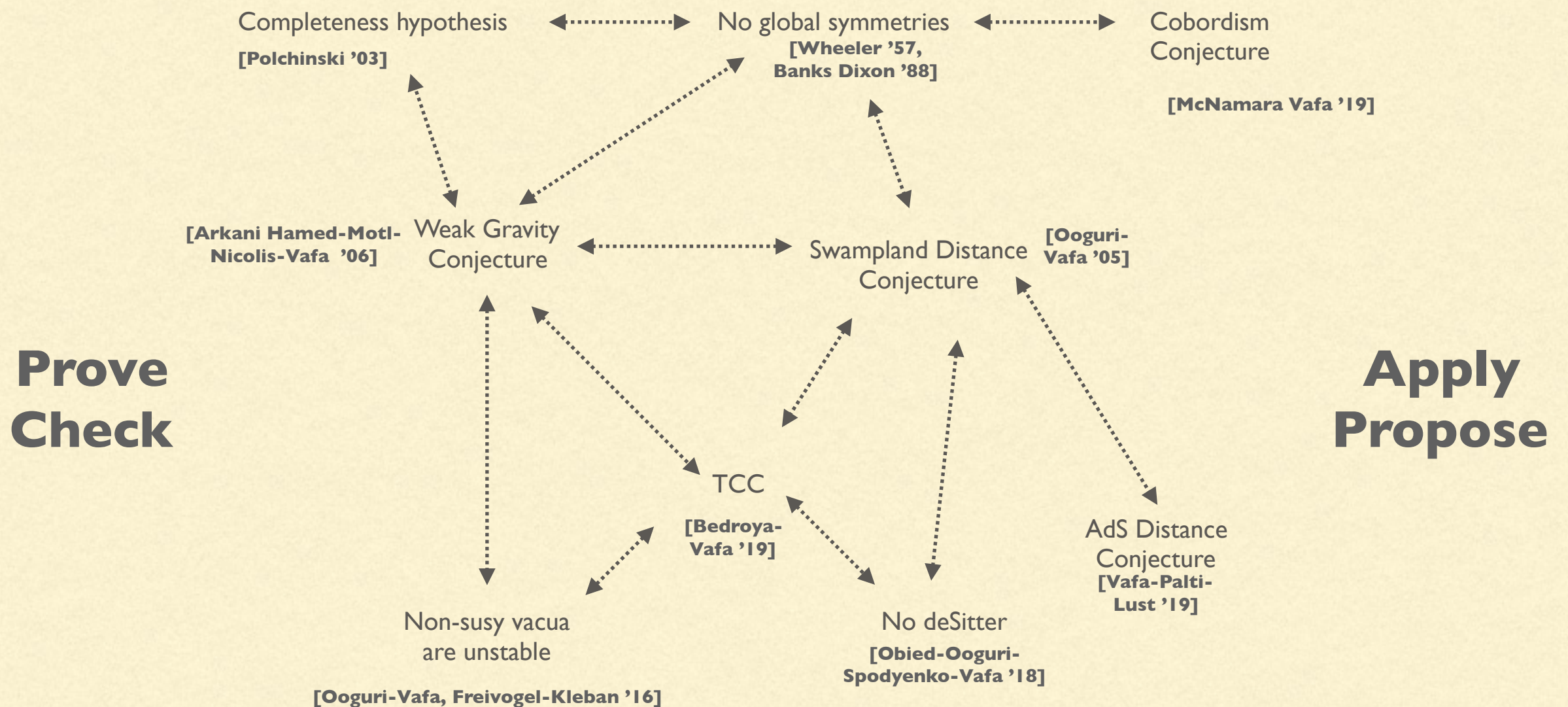
# Main Swampland Idea: Ascertain **universal** features of quantum gravity

We organize this in a web of interconnected **conjectures**



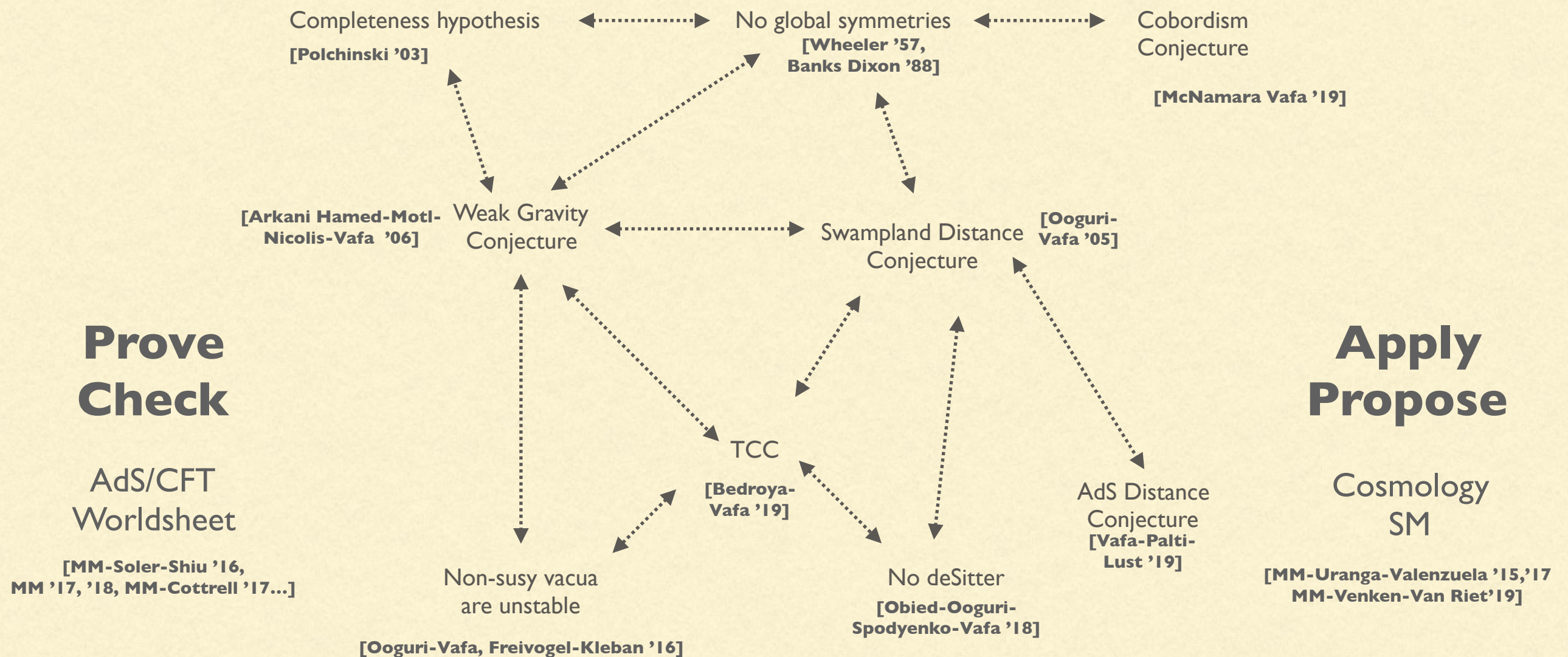
# Main Swampland Idea: Ascertain **universal** features of quantum gravity

We organize this in a web of interconnected **conjectures**



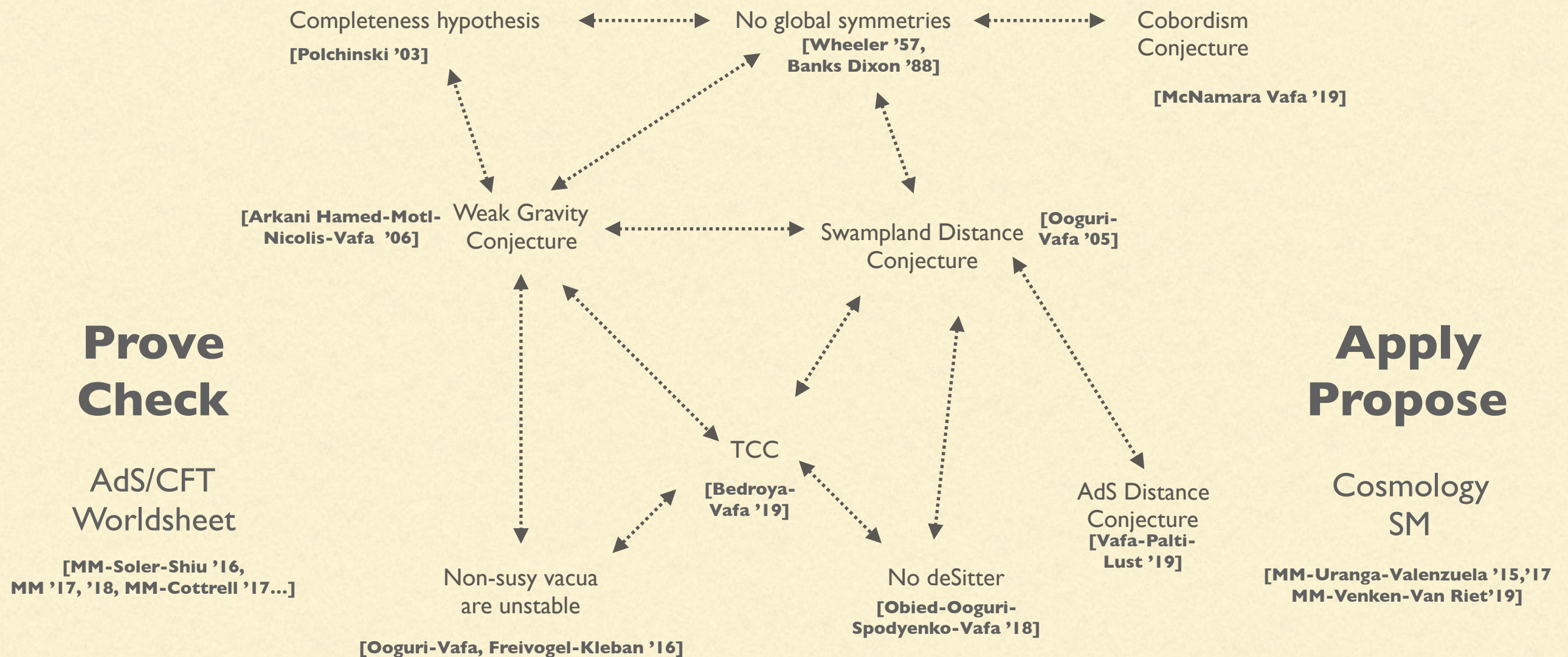
# Main Swampland Idea: Ascertain **universal** features of quantum gravity

We organize this in a web of interconnected **conjectures**

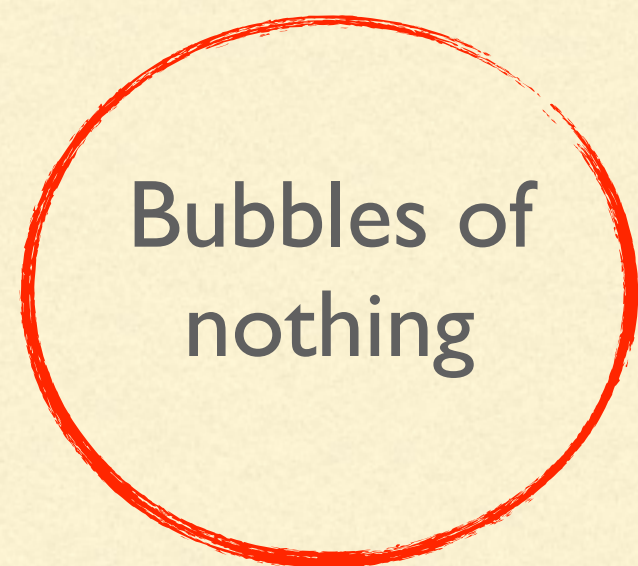


# Main Swampland Idea: Ascertain **universal** features of quantum gravity

We organize this in a web of interconnected **conjectures**



**Today's menu:** One of each



---

Bubbles of  
nothing

String  
universality in  
 $d > 6$

---

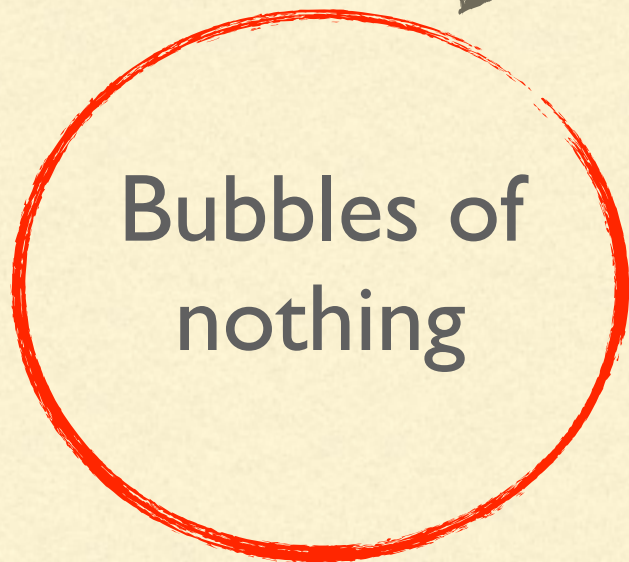
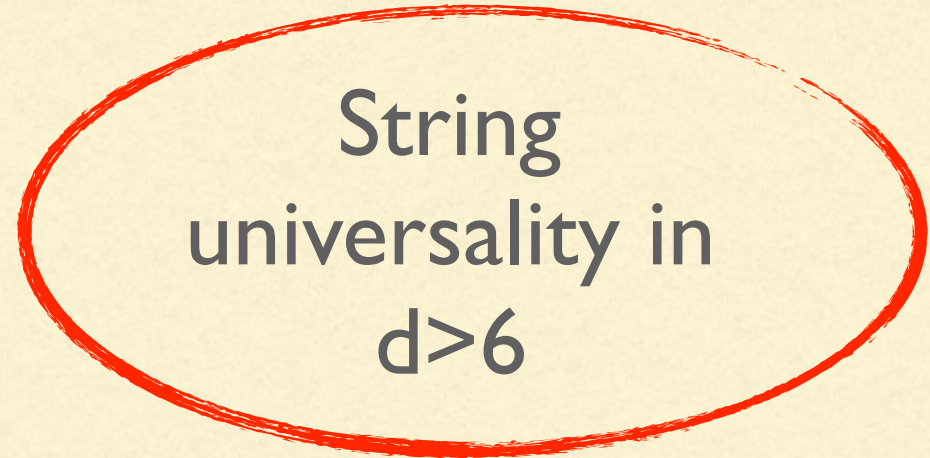
Cobordisms



Bubbles of  
nothing



String  
universality in  
 $d > 6$



---

Cobordisms

Bubbles of  
nothing

String  
universality in  
 $d > 6$

**[(Garcia-Extebarria)-MM-  
Sousa-Valenzuela '20]**

**[MM-Vafa'20]**

[Witten '82]  
[Blanco-Pillado-Shlaer-Sousa-Urrestilla '16]  
[Dibitetto-Petri-Schillo '20]

[Kim, Shiu, Vafa '19]  
[McNamara, Vafa '19]  
[Kim, Hourii Tarazi, Vafa '19]

---

---

# **Plan** of the talk

---

# **Plan** of the talk

Review of  
BoN's

---

## **Plan** of the talk

Review of  
BoN's

Cobordisms



---

## Plan of the talk

Review of  
BoN's

Cobordisms

New BoN's



---

## Plan of the talk

Review of  
BoN's

Cobordisms

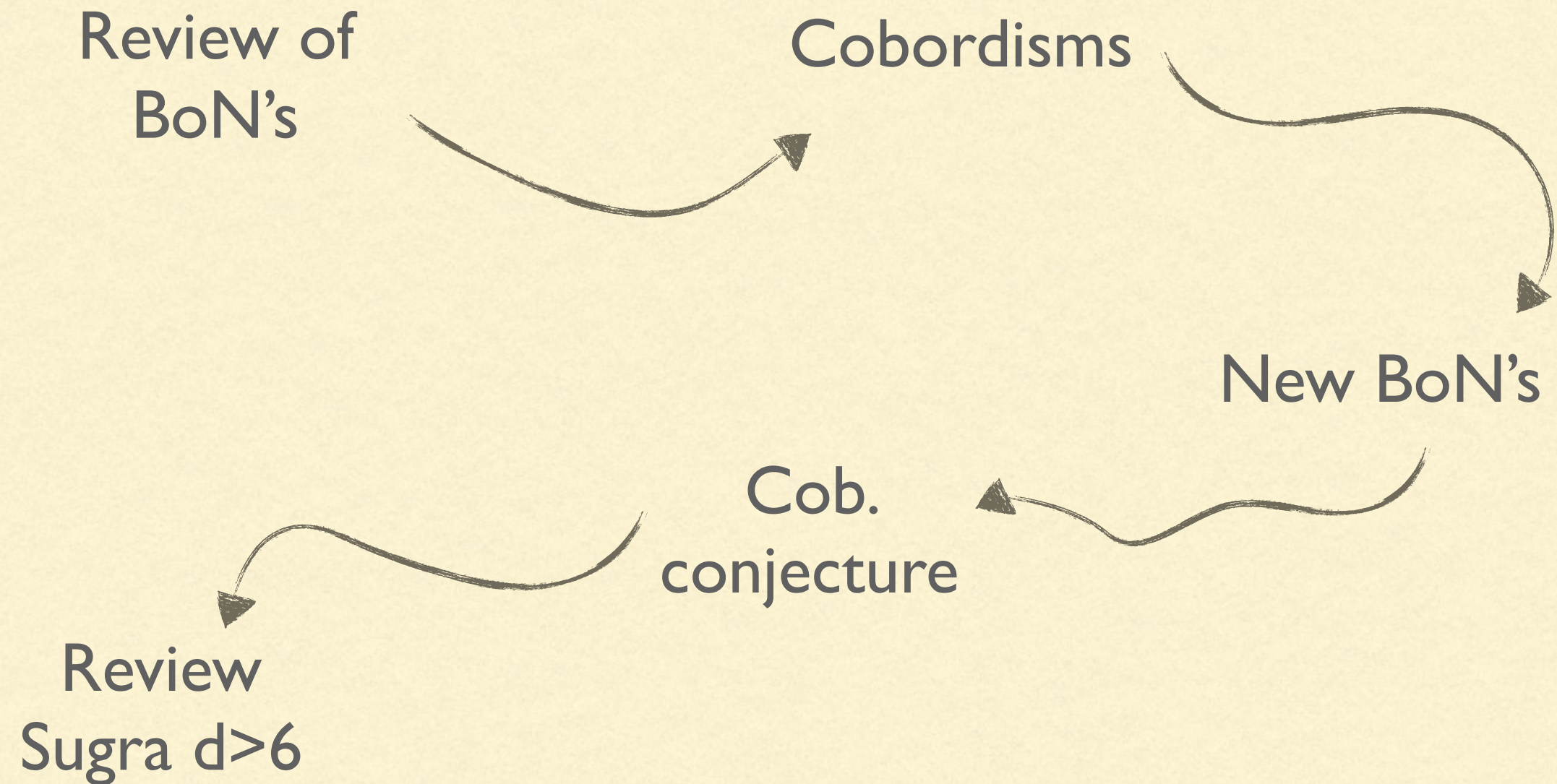
New BoN's

Cob.  
conjecture



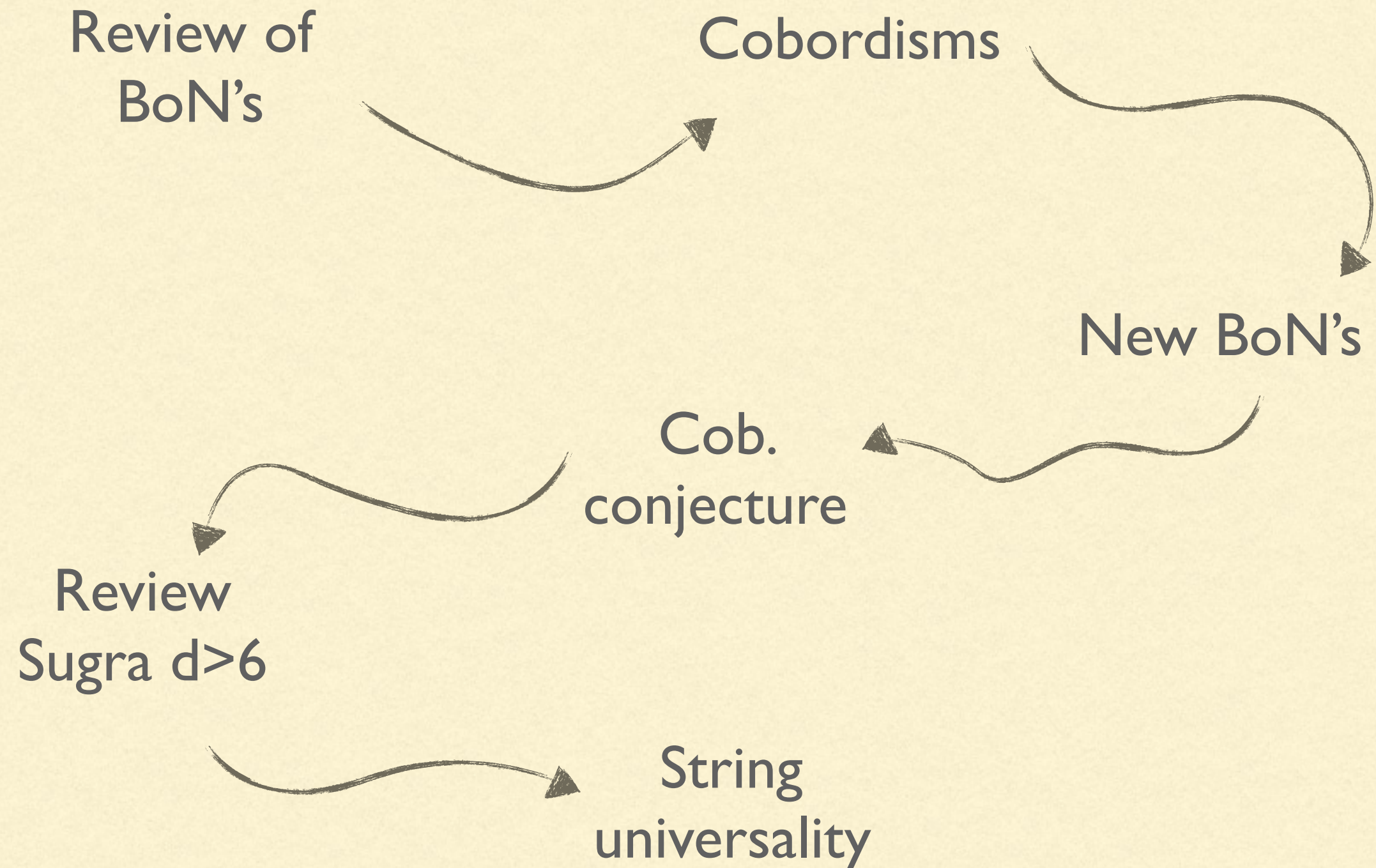
---

## Plan of the talk



---

## Plan of the talk









**Please interrupt me!!**

(at any point with questions, comments, whatever!)

---

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

**No examples! (that I know of)**

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

**No examples! (that I know of)**

Einstein  
gravity

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

**No examples! (that I know of)**

Einstein  
gravity

- Far-reaching **Swampland conjectures:**

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

**No examples! (that I know of)**

Einstein  
gravity

- Far-reaching **Swampland conjectures:**

No stable nonsusy AdS [Ooguri-Vafa '16]

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

**No examples! (that I know of)**

Einstein  
gravity

- Far-reaching **Swampland conjectures:**

No stable nonsusy AdS [Ooguri-Vafa '16]

Vacua morghulis [Freivogel-Kleban '16]

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

**No examples! (that I know of)**

Einstein  
gravity

- Far-reaching **Swampland conjectures:**

No stable nonsusy AdS [Ooguri-Vafa '16]

Vacua morghulis [Freivogel-Kleban '16]

- If instability is universal QG property, maybe it is a good idea to look for a **universal vacuum decay mechanism.**
-

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

**No examples! (that I know of)**

Einstein  
gravity

- Far-reaching **Swampland conjectures:**

No stable nonsusy AdS [Ooguri-Vafa '16]

Vacua morghulis [Freivogel-Kleban '16]

- If instability is universal QG property, maybe it is a good idea to look for a **universal vacuum decay mechanism.**

**Bubble of Nothing**

---

- 
- **Motivation:** Constructing nonsusy stable vacua in gravity is hard!

**No examples! (that I know of)**

Einstein  
gravity

- Far-reaching **Swampland conjectures:**

No stable nonsusy AdS [Ooguri-Vafa '16]

Vacua morghulis [Freivogel-Kleban '16]

- If instability is universal QG property, maybe it is a good idea to look for a **universal vacuum decay mechanism.**

**Bubble of Nothing**

(Not completely universal, but  
makes sense whenever we have  
internal geometry)

---

---

Consider the metric [Witten '81]

$$ds_5^2 = -r^2 d\psi^2 + \frac{dr^2}{1 - R^2/r^2} + r^2 \cosh^2 \psi d\Omega_2^2 + \left(1 - \frac{R^2}{r^2}\right) d\theta^2.$$

---

Consider the metric [Witten '81]

$$ds_5^2 = -r^2 d\psi^2 + \frac{dr^2}{1 - R^2/r^2} + r^2 \cosh^2 \psi d\Omega_2^2 + \left(1 - \frac{R^2}{r^2}\right) d\theta^2.$$

its an unusual **Wick rotation** of the 5d Schwarzschild BH.

Spacetime **ends** at  $r=R$ .

---

---

Consider the metric [Witten '81]

$$ds_5^2 = -r^2 d\psi^2 + \frac{dr^2}{1 - R^2/r^2} + r^2 \cosh^2 \psi d\Omega_2^2 + \left(1 - \frac{R^2}{r^2}\right) d\theta^2.$$

its an unusual **Wick rotation** of the 5d Schwarzschild BH.

Spacetime **ends** at  $r=R$ .

If one changes coordinates

$$x = r \cosh \psi, t = r \sinh \psi$$

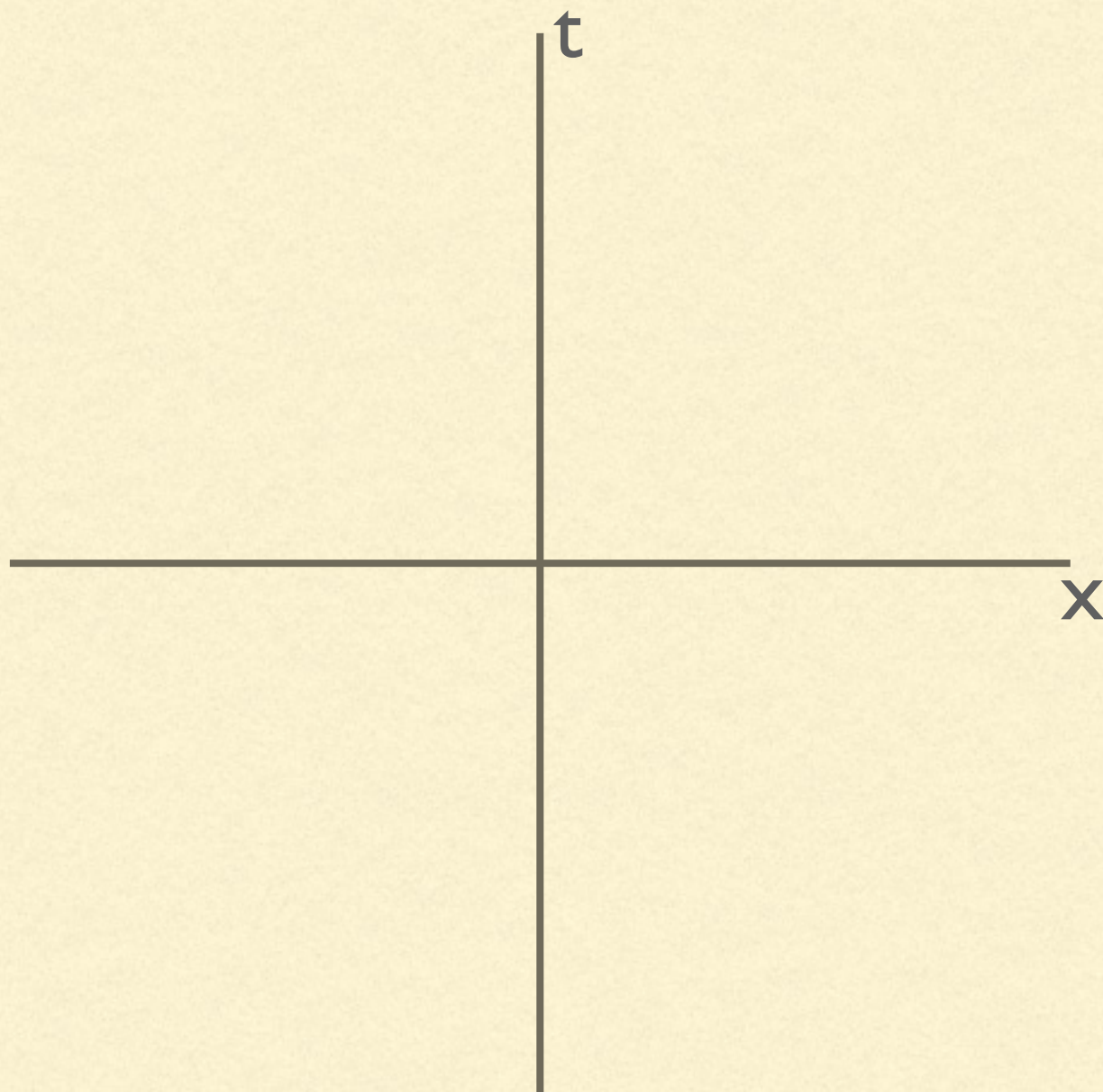
It looks like **Rindler (i.e. Minkowski) space** for  $r \gg R$ .

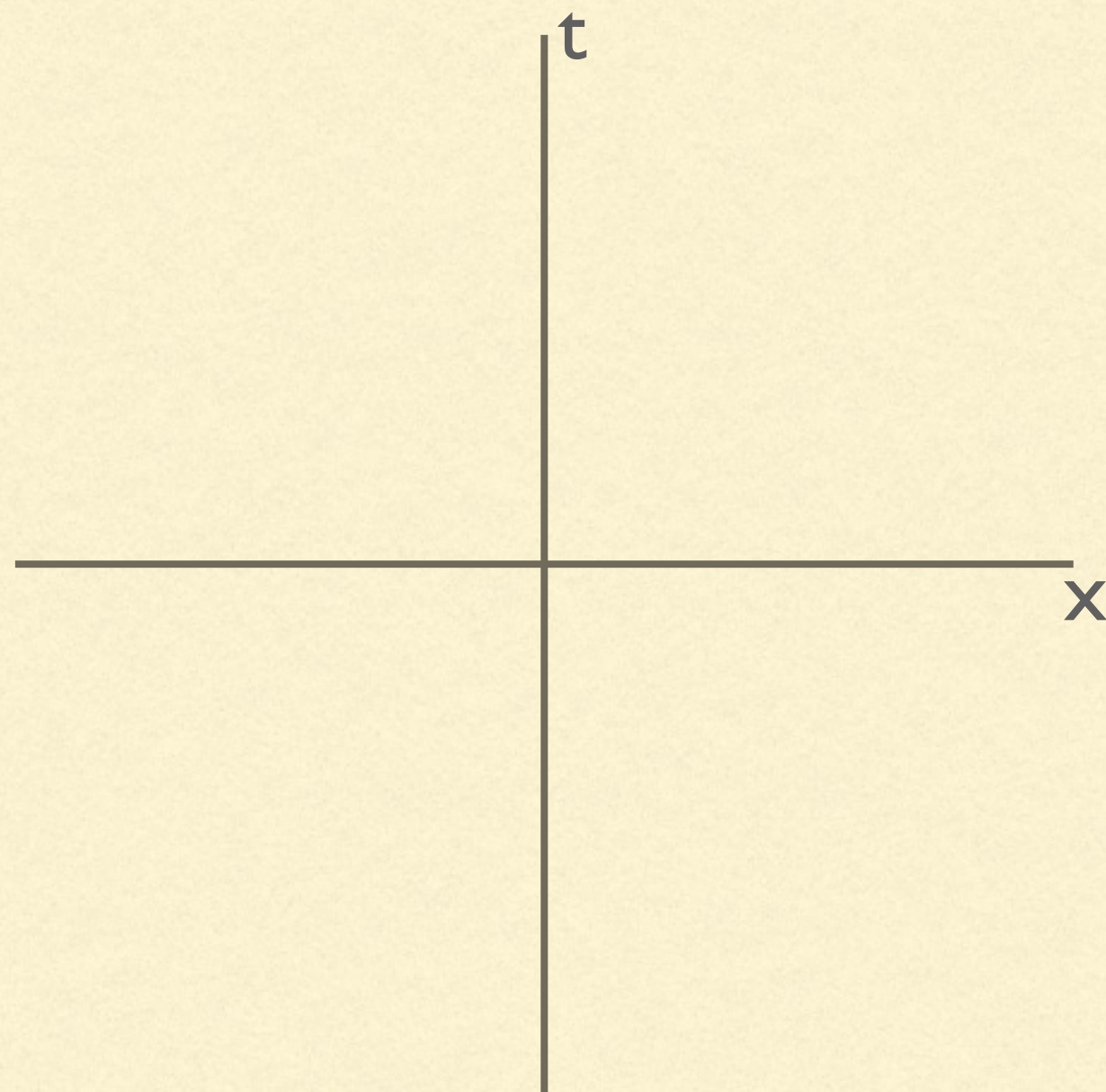
$$ds_5^2 \underset{r \rightarrow \infty}{\approx} -dt^2 + dx^2 + x^2 d\Omega_2^2 + R^2 d\theta^2 \quad \text{(times a circle)}$$

---

---

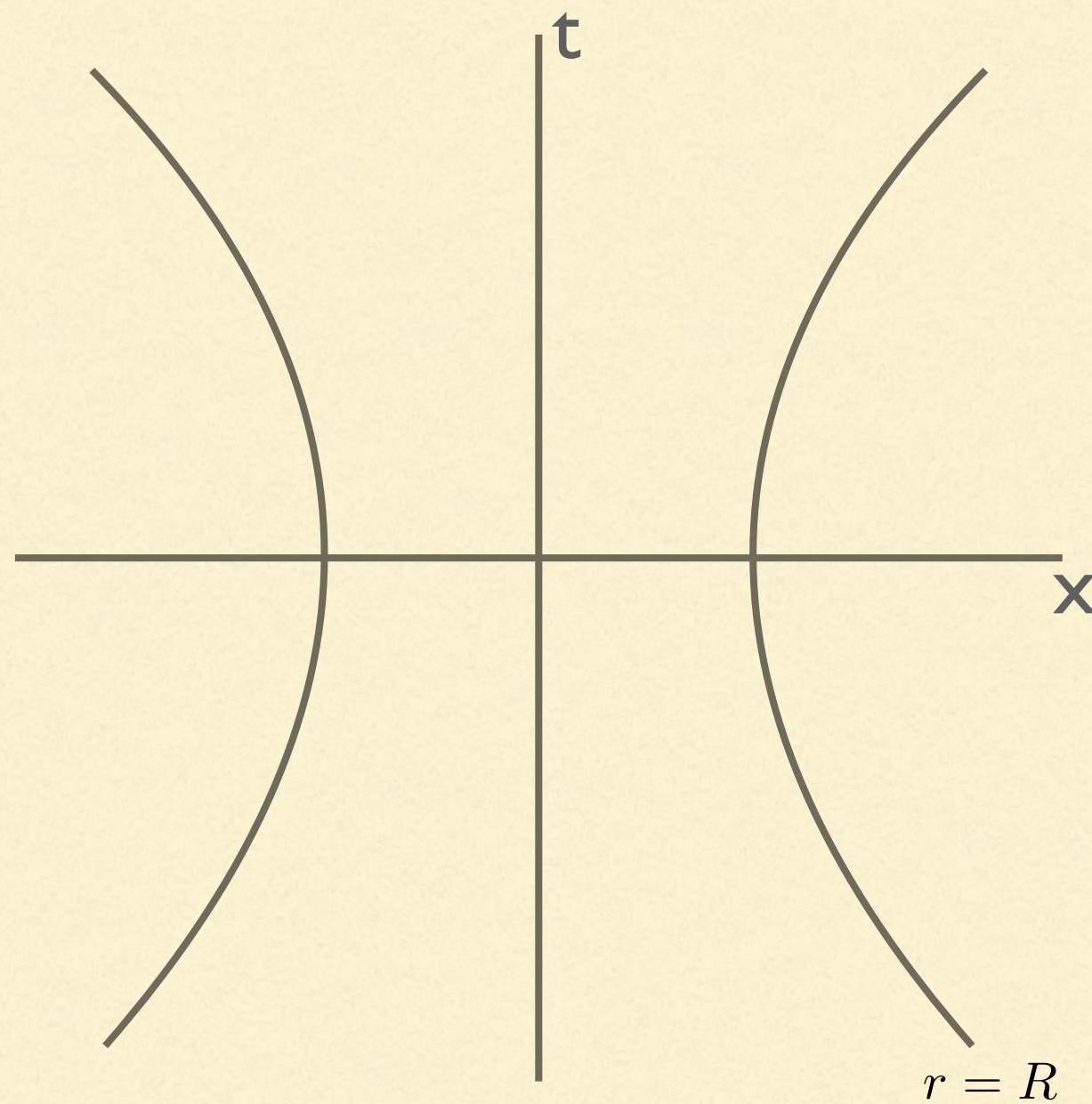
So it looks like Minkowski...





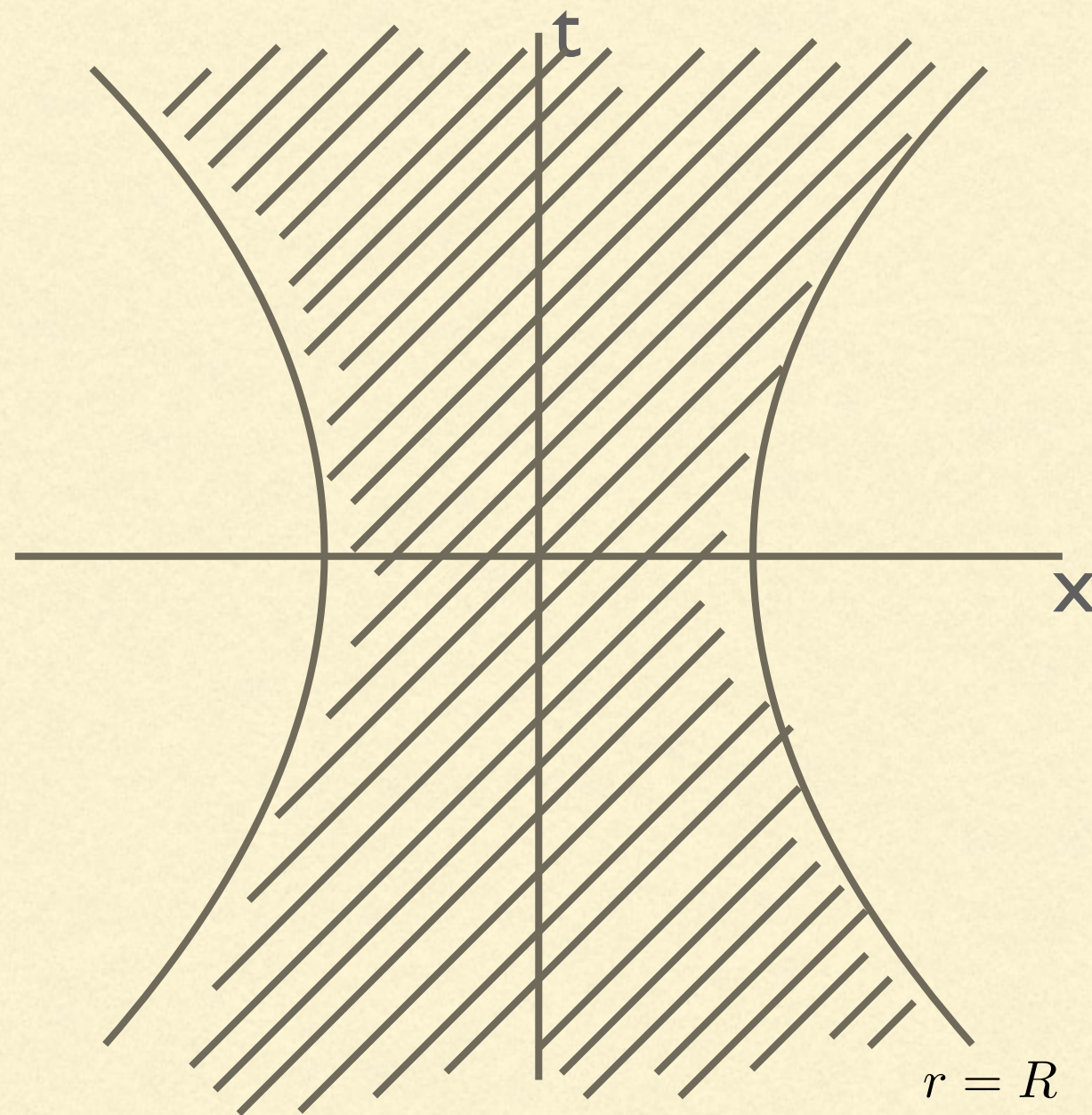
So it looks like Minkowski...

...but remember to  
**cut out** space at  
 $r=R$ !



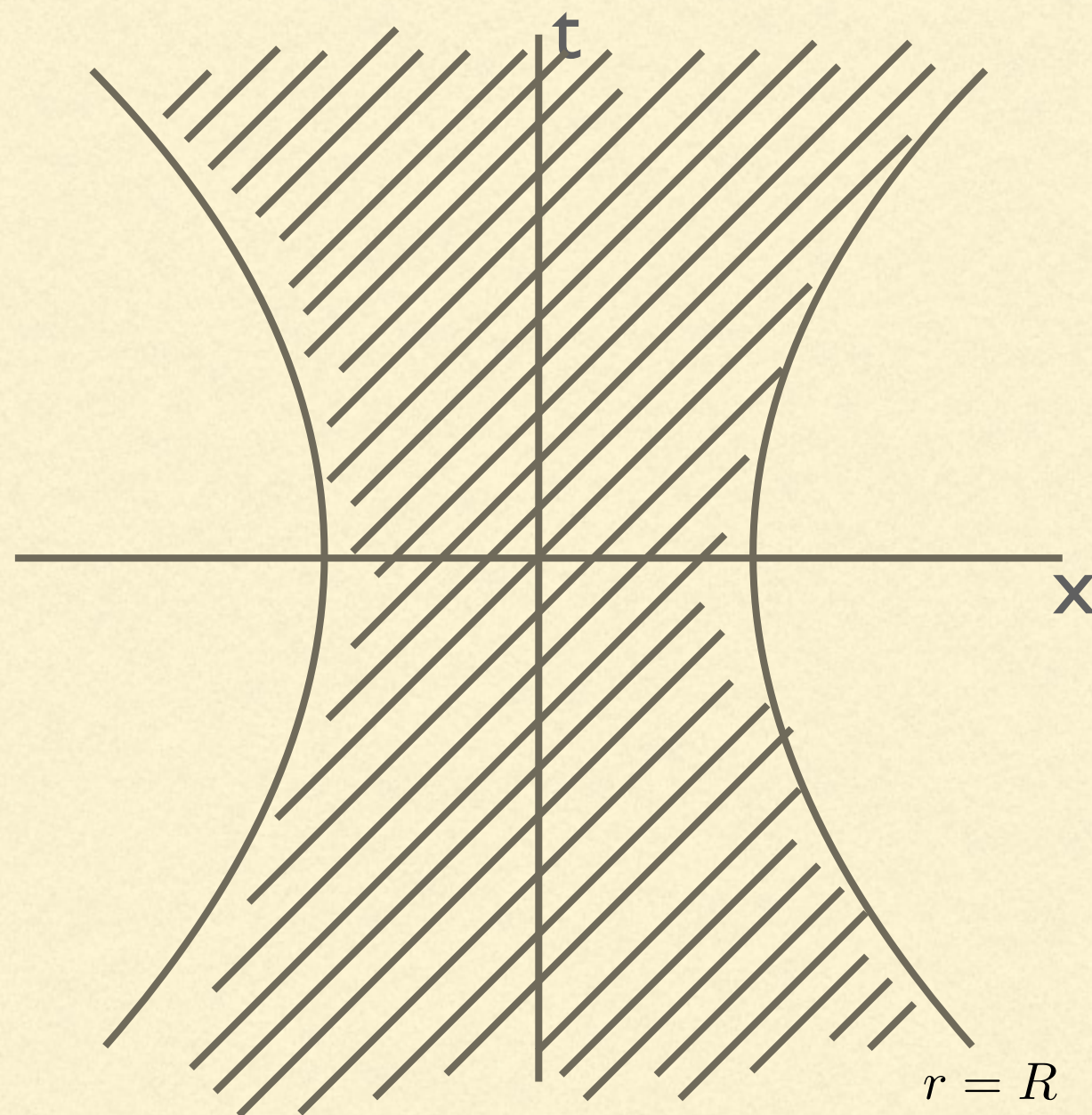
So it looks like Minkowski...

...but remember to  
**cut out** space at  
 $r=R$ !



So it looks like Minkowski...

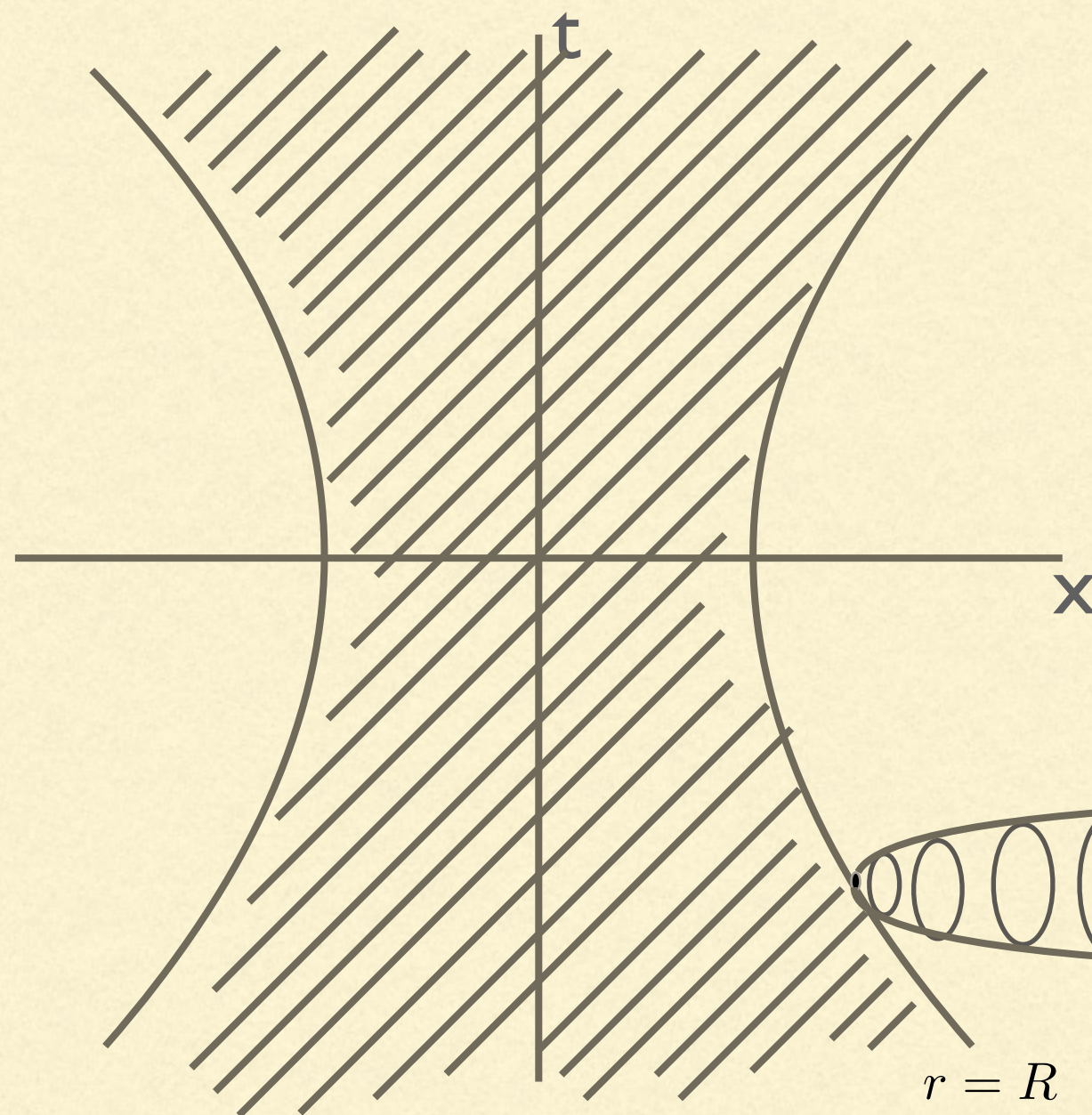
...but remember to  
**cut out** space at  
 $r=R$ !



So it looks like Minkowski...

...but remember to  
**cut out** space at  
 $r=R$ !

There is **Nothing**  
inside!



So it looks like Minkowski...

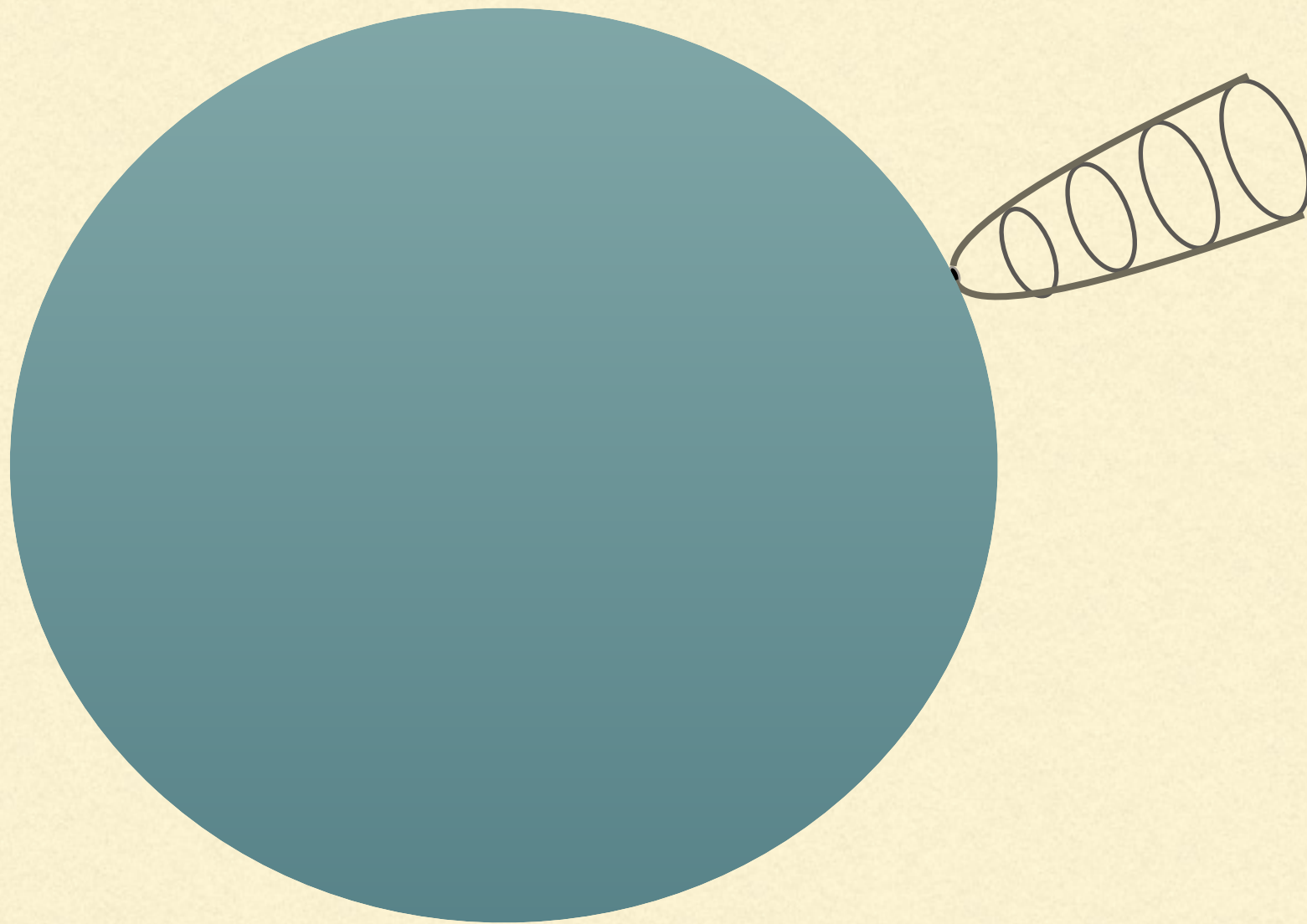
...but remember to  
**cut out** space at  
 $r=R$ !

The KK circle  
**shrinks to zero**  
**size** at the bubble

There is **Nothing**  
inside!

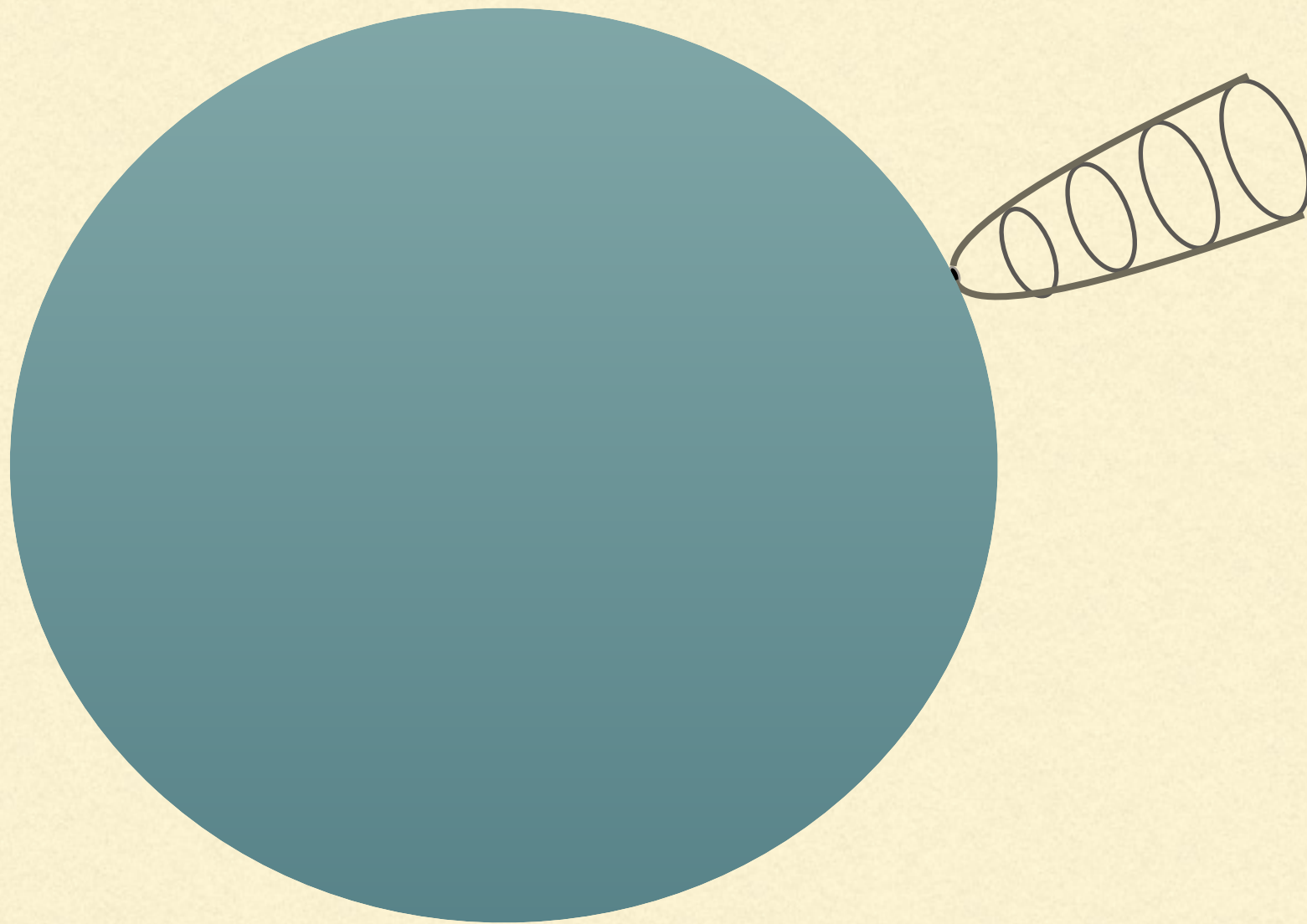
---

Snapshot at constant time:



---

Snapshot at constant time:



- Finite **probability of nucleation** from Euclidean methods

$$\Gamma \sim \exp \left( -\frac{\pi R^2}{8G} \right)$$



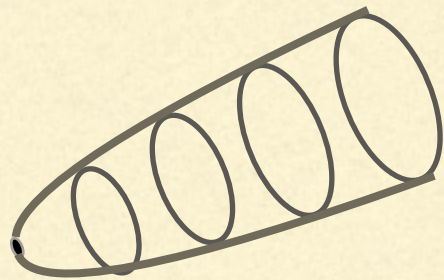
Vacuum seems unstable! We  
should PANIC



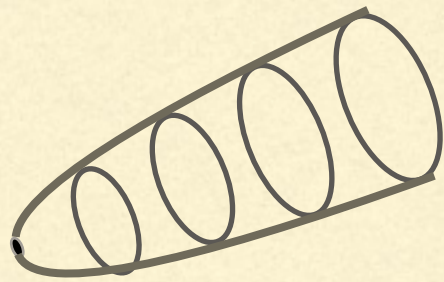
Vacuum seems unstable! We  
should PANIC

But wait! **Fermions** can  
protect us....





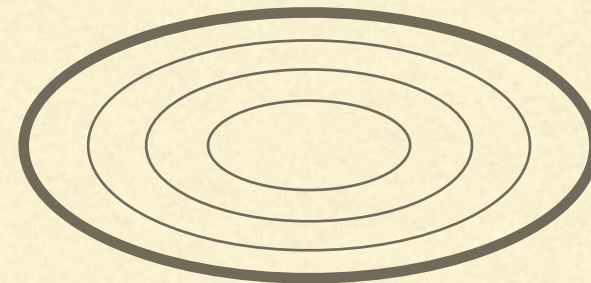
If we have fermions, they must be  
**well defined** on the cigar....

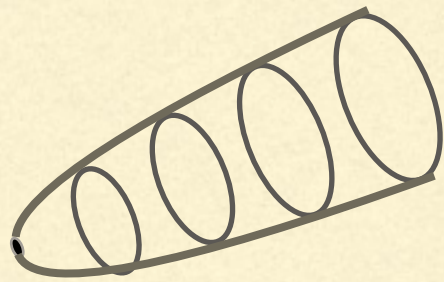


If we have fermions, they must be  
**well defined** on the cigar....



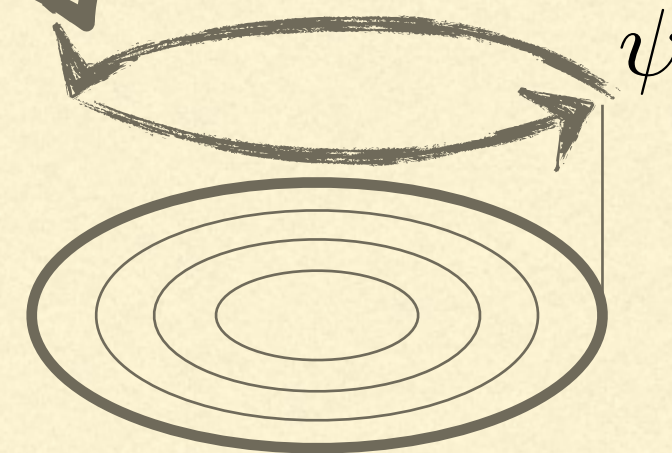
i.e. they must be **well-defined**  
on the disk...



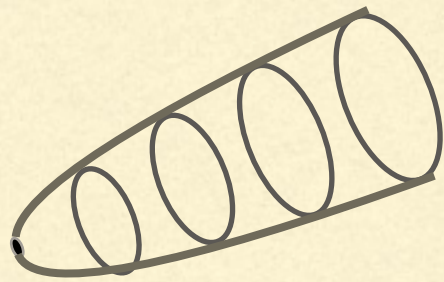


If we have fermions, they must be **well defined** on the cigar....

i.e. they must be **well-defined** on the disk...

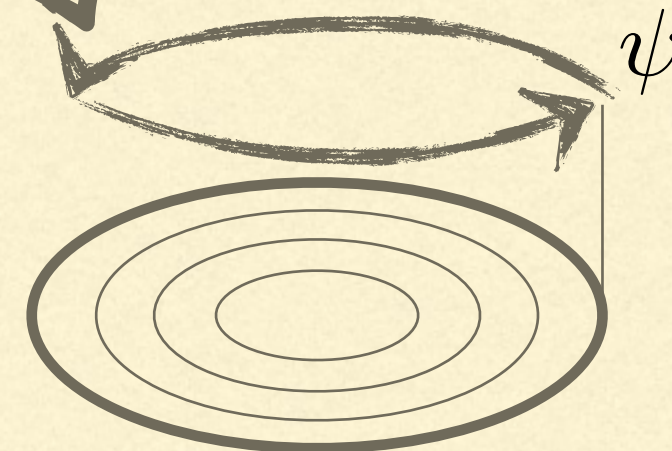


But fermions pick a - sign when rotated by  $2\pi$ !



If we have fermions, they must be **well defined** on the cigar....

i.e. they must be **well-defined** on the disk...



**Bubble does  
not exist for  
periodic (SUSY)  
fermions**

But fermions pick a - sign when  
rotated by  $2\pi$ !

---

# TWO WRONG LESSONS

---

---

# TWO WRONG LESSONS

---

- Only constructed bubble for a circle — few other examples.  
Seems nongeneric

---

# TWO WRONG LESSONS

---

- Only constructed bubble for a circle — few other examples.  
Seems nongeneric
  - They won't arise for SUSY theories because as we have seen the b.c's for the fermions lead to a **topological protection**.
-

---

# TWO WRONG LESSONS

---

- Only constructed bubble for a circle — few other examples.  
Seems nongeneric
- They won't arise for SUSY theories because as we have seen the b.c's for the fermions lead to a **topological protection**.

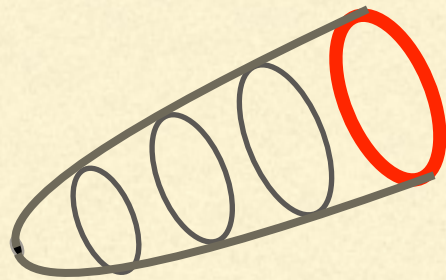
Both statements are **good for nothing**  
(they are just wrong)

---

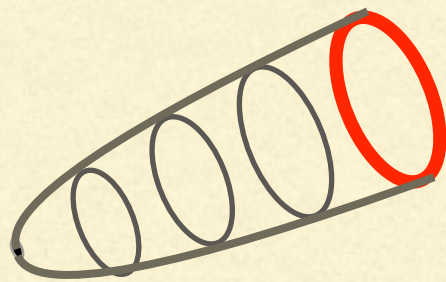
- 
- Suppose we want to construct a bubble for some  $d$ -dimensional manifold  $C_d$ .

- 
- Suppose we want to construct a bubble for some  $d$ -dimensional manifold  $C_d$ .
  - Intuitively,  $C_d$  must “shrink to a point”

- 
- Suppose we want to construct a bubble for some  $d$ -dimensional manifold  $C_d$ .
  - Intuitively,  $C_d$  must “shrink to a point”

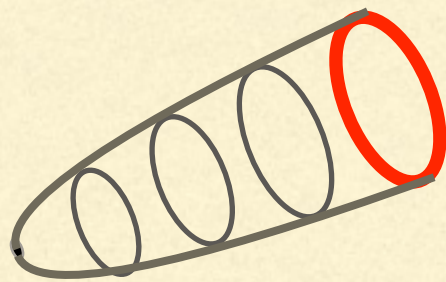


- 
- Suppose we want to construct a bubble for some  $d$ -dimensional manifold  $C_d$ .
  - Intuitively,  $C_d$  must “shrink to a point”

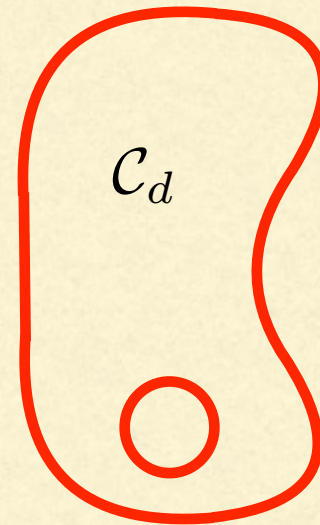


The circle is the  
**boundary** of the disk

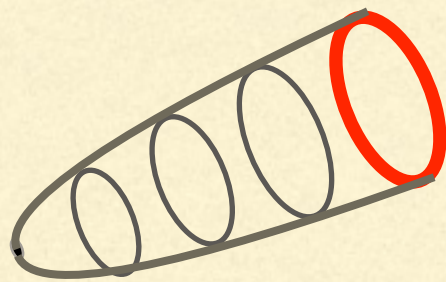
- Suppose we want to construct a bubble for some  $d$ -dimensional manifold  $C_d$ .
- Intuitively,  $C_d$  must “shrink to a point”



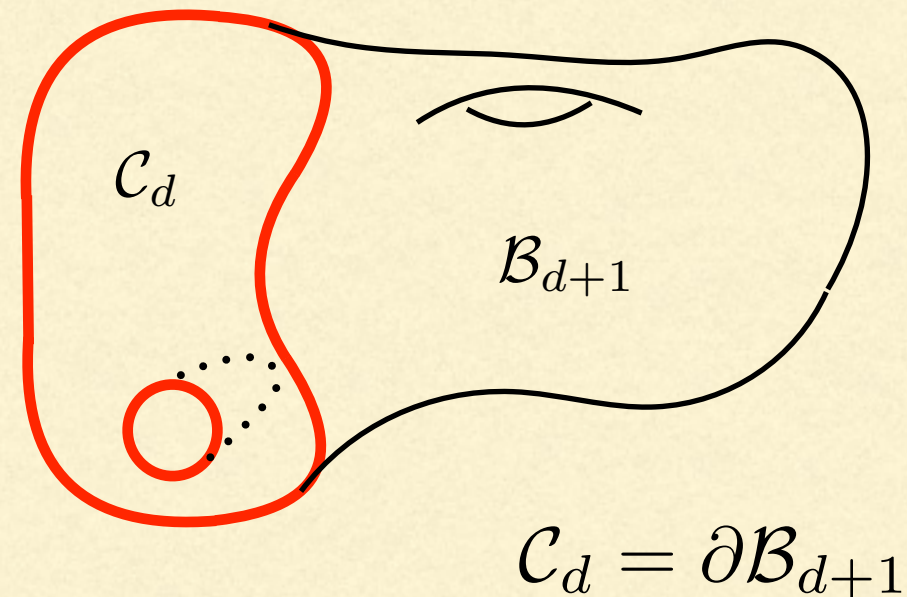
The circle is the  
**boundary** of the disk



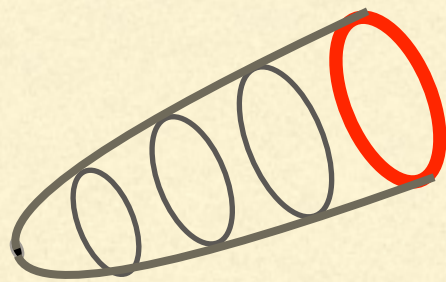
- Suppose we want to construct a bubble for some  $d$ -dimensional manifold  $\mathcal{C}_d$ .
- Intuitively,  $\mathcal{C}_d$  must “shrink to a point”



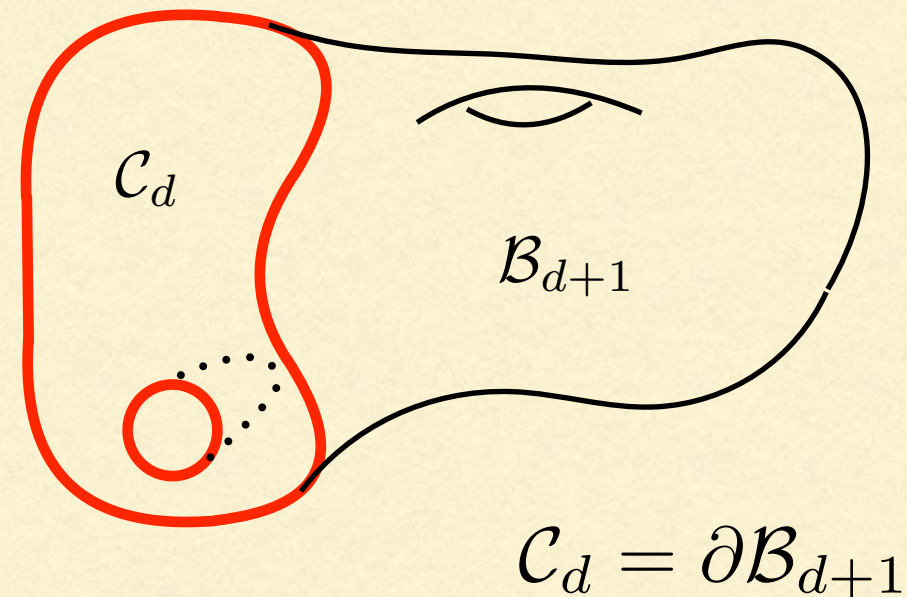
The circle is the  
**boundary** of the disk



- Suppose we want to construct a bubble for some  $d$ -dimensional manifold  $C_d$ .
- Intuitively,  $C_d$  must “shrink to a point”



The circle is the  
**boundary** of the disk

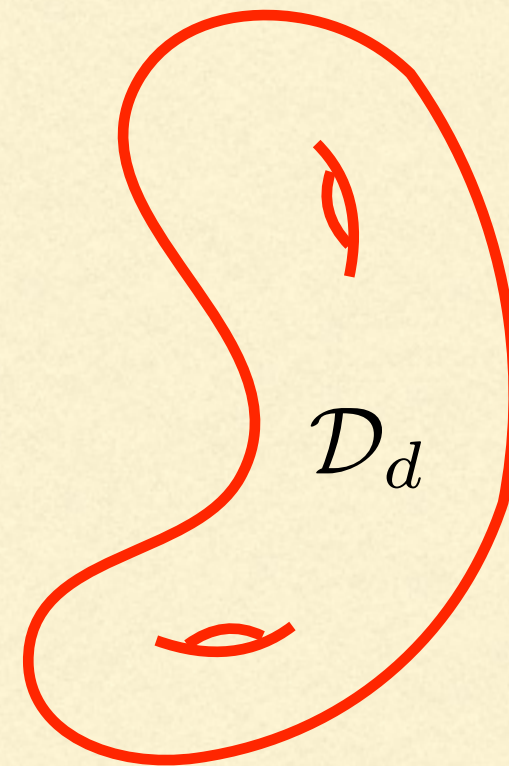
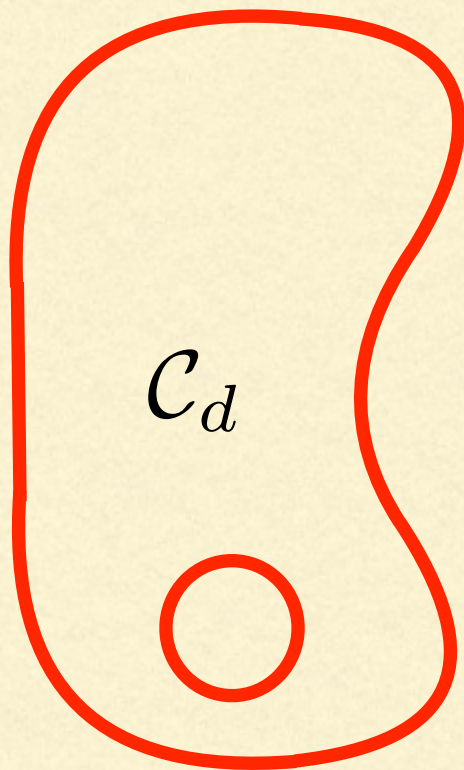


- $C_d$  must be a boundary; which manifolds are boundaries has been studied by mathematicians

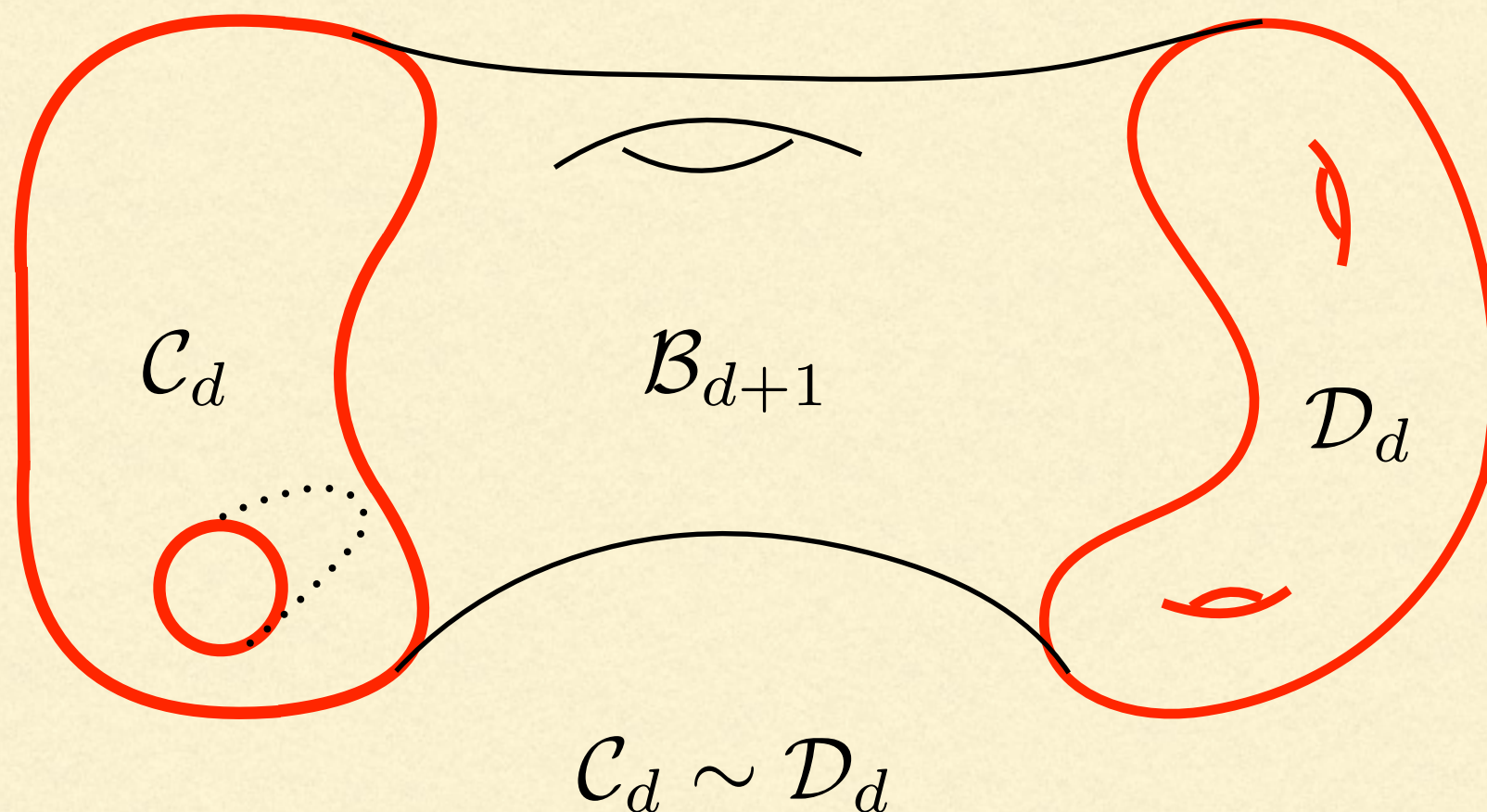
**Bordism groups**

- 
- Two manifolds  $C, D$  are in the same **bordism class** if their union is a boundary

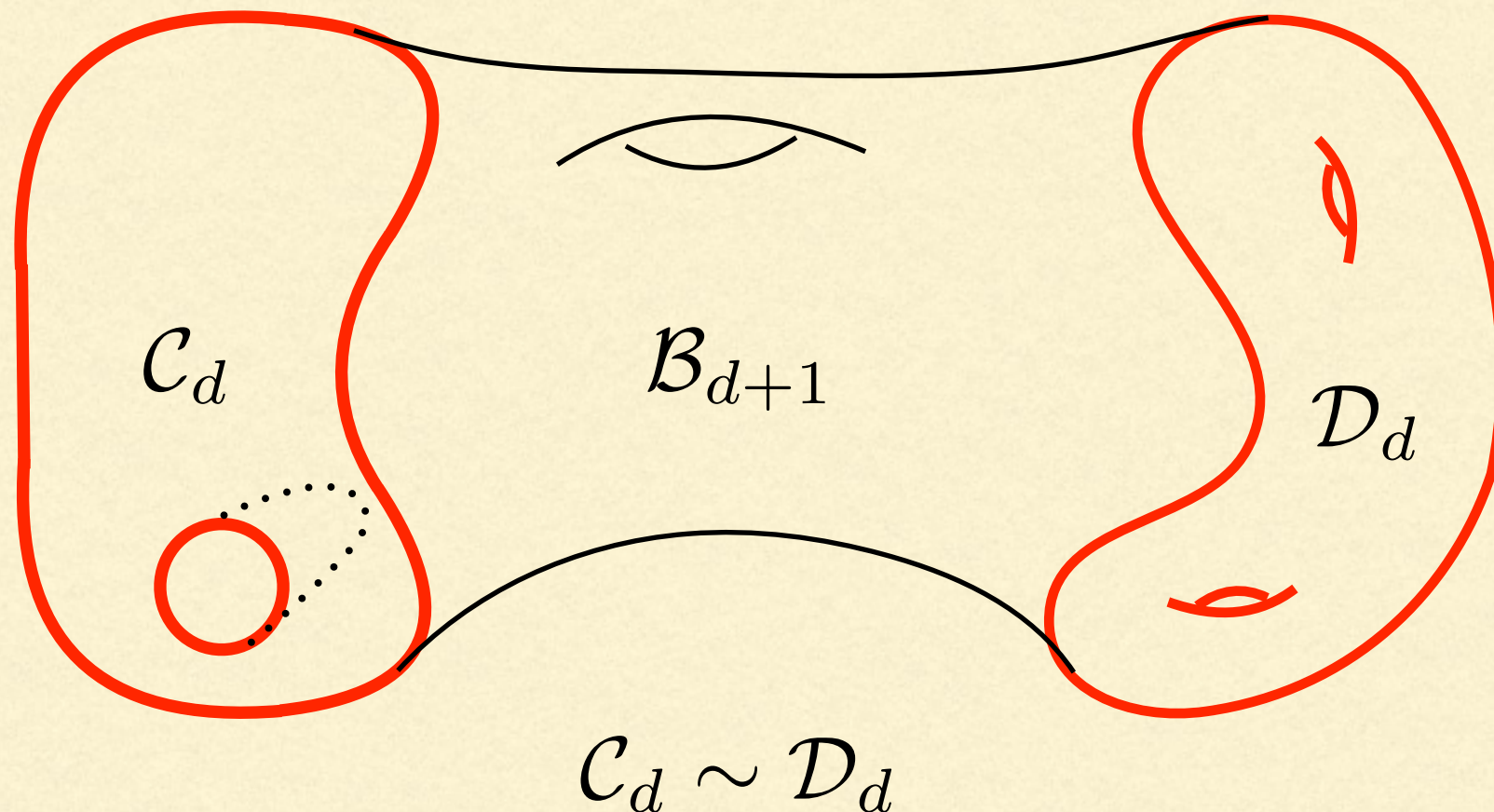
- 
- Two manifolds  $C, D$  are in the same **bordism class** if their union is a boundary



- Two manifolds  $C, D$  are in the same **bordism class** if their union is a boundary

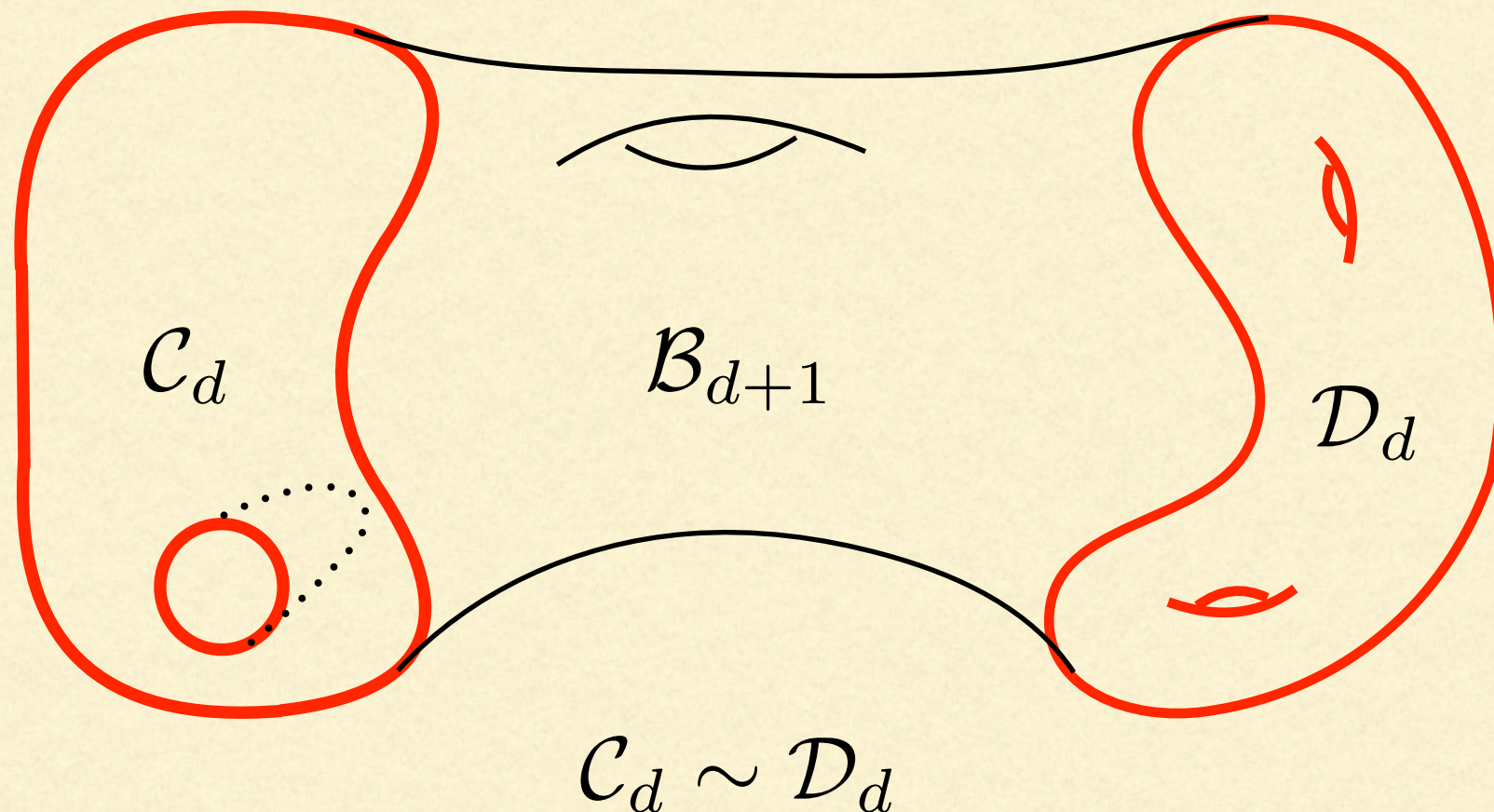


- Two manifolds  $C, D$  are in the same **bordism class** if their union is a boundary



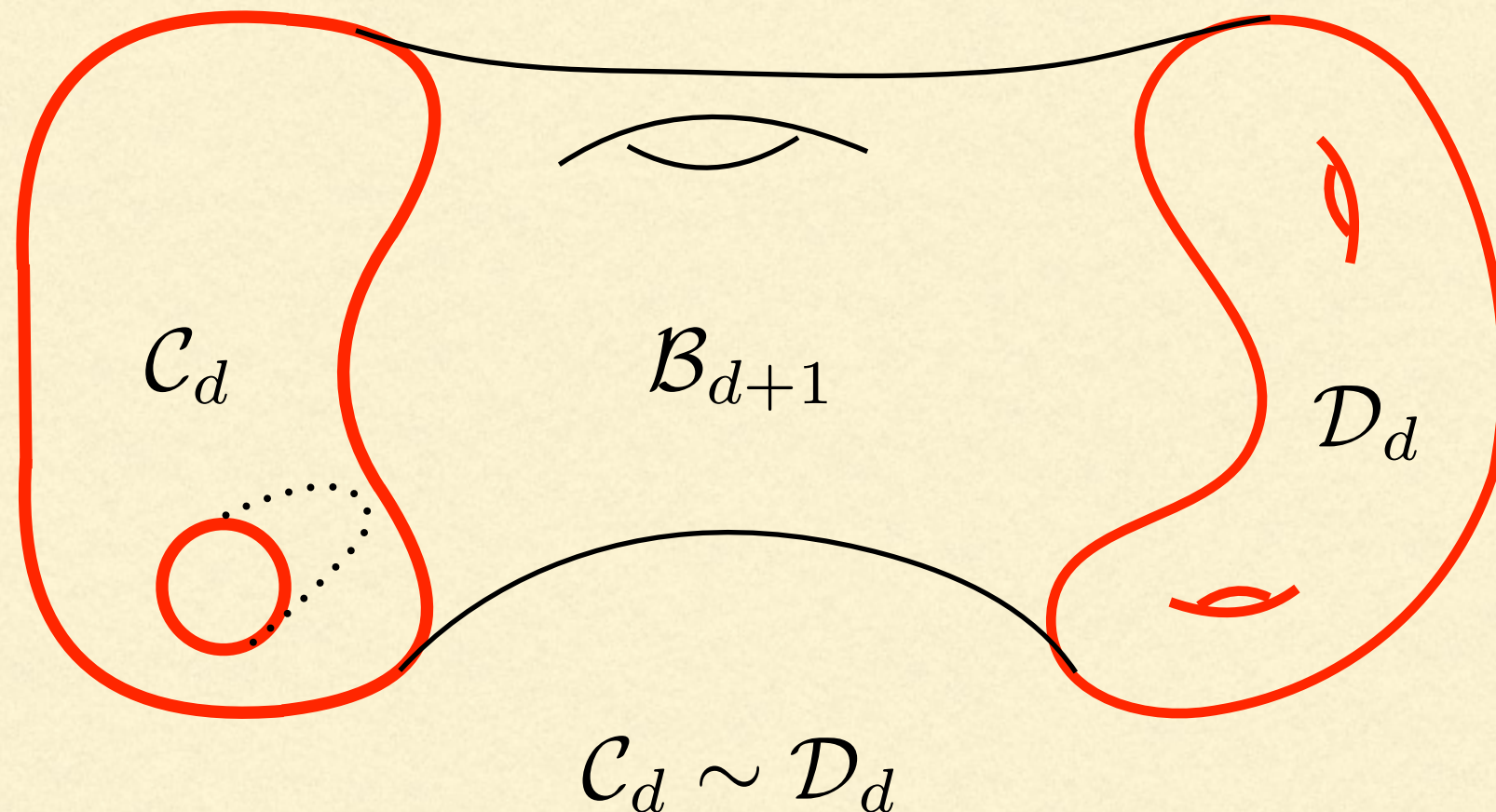
- The set of equivalence classes has an **abelian group** structure

- Two manifolds  $C, D$  are in the same **bordism class** if their union is a boundary



- The set of equivalence classes has an **abelian group** structure
- Can be **decorated** with e.g. a spin structure

- Two manifolds  $C, D$  are in the same **bordism class** if their union is a boundary



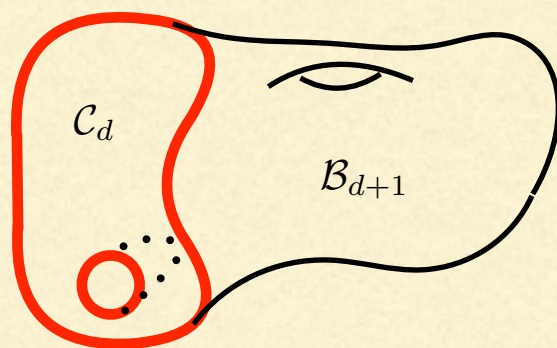
- The set of equivalence classes has an **abelian group** structure
- Can be **decorated** with e.g. a spin structure

$$\Omega_d^{\text{Spin}}$$

---

A manifold can only admit a bubble of nothing if  
it belongs to the

**trivial class**



$$\mathcal{C}_d = \partial \mathcal{B}_{d+1}$$

in  $\Omega_d^{\text{Spin}}$

i.e. if it is a **boundary**

(so that it can "shrink to a point")

[Bais, Gomez, Rubakov '87]  
[Mazur '87]

---

- 
- The **bordism** classification matches nicely with Witten's BoN!

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Circle with periodic b.c.

Trivial class: Circle with antiperiodic b.c.

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Circle with periodic b.c.

**No bubble**

Trivial class: Circle with antiperiodic b.c.

**Bubble**

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Circle with periodic b.c.

**No bubble**

Trivial class: Circle with antiperiodic b.c.

**Bubble**

- We can now study BoN's in **higher dimensions!**

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Circle with periodic b.c.

**No bubble**

Trivial class: Circle with antiperiodic b.c.

**Bubble**

- We can now study BoN's in **higher dimensions!**

$$\Omega_2^{\text{Spin}} = \mathbb{Z}_2$$

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Circle with periodic b.c.

**No bubble**

Trivial class: Circle with antiperiodic b.c.

**Bubble**

- We can now study BoN's in **higher dimensions!**

$$\Omega_2^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Torus with periodic b.c

Nontrivial class: Torus with antiperiodic b.c.

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Circle with periodic b.c.

**No bubble**

Trivial class: Circle with antiperiodic b.c.

**Bubble**

- We can now study BoN's in **higher dimensions!**

$$\Omega_2^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Torus with periodic b.c

Nontrivial class: Torus with antiperiodic b.c.

- We could apply “**physicist's induction**”
-

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Circle with periodic b.c.

**No bubble**

Trivial class: Circle with antiperiodic b.c.

**Bubble**

- We can now study BoN's in **higher dimensions!**

$$\Omega_2^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Torus with periodic b.c

Nontrivial class: Torus with antiperiodic b.c.

- We could apply “**physicist's induction**”  $\Omega_d^{\text{Spin}} \supset \mathbb{Z}_2$

- 
- The **bordism** classification matches nicely with Witten's BoN!

$$\Omega_1^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Circle with periodic b.c.

**No bubble**

Trivial class: Circle with antiperiodic b.c.

**Bubble**

- We can now study BoN's in **higher dimensions!**

$$\Omega_2^{\text{Spin}} = \mathbb{Z}_2$$

Nontrivial class: Torus with periodic b.c

Nontrivial class: Torus with antiperiodic b.c.

- We could apply “**physicist's induction**”

$$\Omega_d^{\text{Spin}} \supset \mathbb{Z}_2$$

But in  
fact...

$$\Omega_3^{\text{Spin}} = 0$$



- 
- The fact that  $\Omega_3^{\text{Spin}} = 0$  has **profound** implications for our understanding of BoN's.

- 
- The fact that  $\Omega_3^{\text{Spin}} = 0$  has **profound** implications for our understanding of BoN's.
  - It means that  $\mathbf{T}^3$  with SUSY boundary conditions admits a bubble.
-

- 
- The fact that  $\Omega_3^{\text{Spin}} = 0$  has **profound** implications for our understanding of BoN's.
  - It means that  $\mathbf{T}^3$  with SUSY boundary conditions admits a bubble.
  - But SUSY vacua are exactly stable, so what **prevents the decay?**
-

- 
- The fact that  $\Omega_3^{\text{Spin}} = 0$  has **profound** implications for our understanding of BoN's.
  - It means that  $\mathbf{T}^3$  with SUSY boundary conditions admits a bubble.
  - But SUSY vacua are exactly stable, so what **prevents the decay?**
  - To construct a bubble, a manifold is not enough, we need **a metric**.
-

- 
- The fact that  $\Omega_3^{\text{Spin}} = 0$  has **profound** implications for our understanding of BoN's.
  - It means that  $\mathbf{T}^3$  with SUSY boundary conditions admits a bubble.
  - But SUSY vacua are exactly stable, so what **prevents the decay?**
  - To construct a bubble, a manifold is not enough, we need **a metric.**

Not possible for SUSY theories

---

- 
- The fact that  $\Omega_3^{\text{Spin}} = 0$  has **profound** implications for our understanding of BoN's.
  - It means that  $\mathbf{T}^3$  with SUSY boundary conditions admits a bubble.
  - But SUSY vacua are exactly stable, so what **prevents the decay?**
  - To construct a bubble, a manifold is not enough, we need **a metric**.

Not possible for SUSY theories
  - More generally, there is a **dynamical obstruction** coming from the **Positive Energy Theorem**

[Schoen & Yau '78, Witten '82]

---

---

**PET:** As long as

---

**PET:** As long as

- We have **asymptotically covariantly constant spinors**

---

**PET:** As long as

- We have **asymptotically covariantly constant spinors**
- We satisfy the **Dominant Energy Condition**

$-T^M_N k^N$  is causal and future-pointing

---

**PET:** As long as

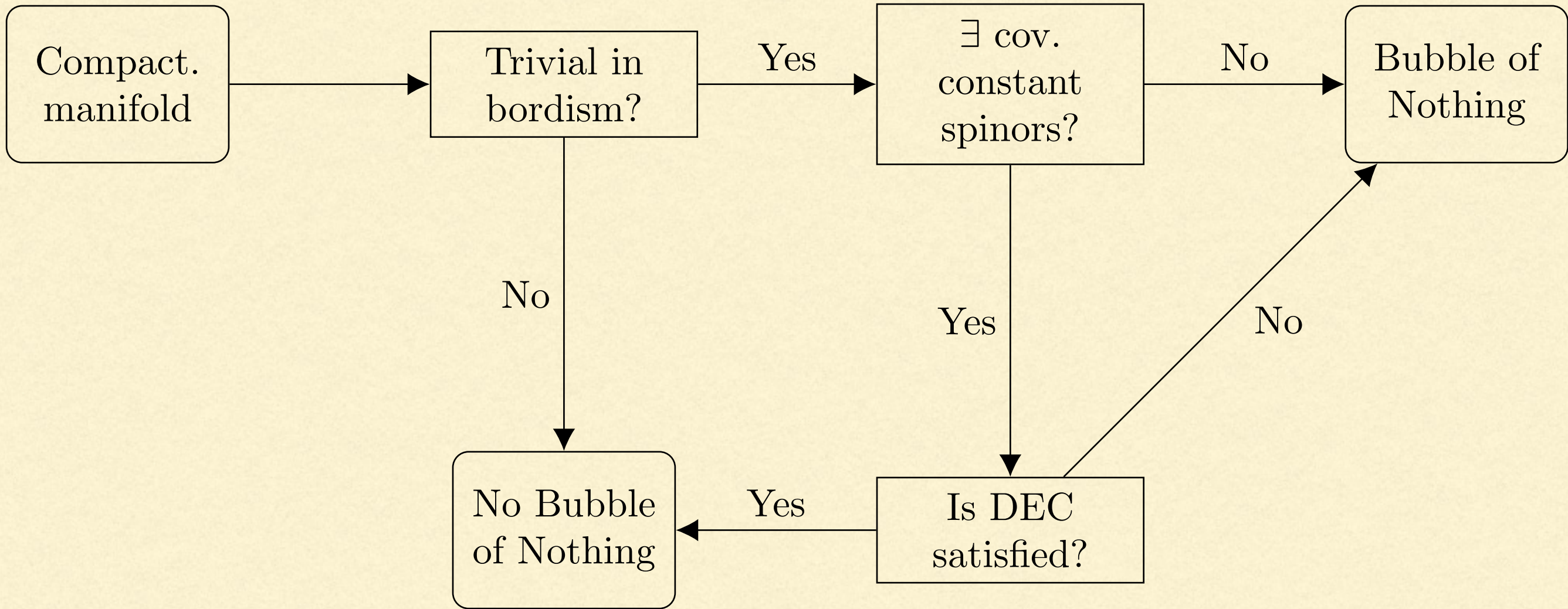
- We have **asymptotically covariantly constant spinors**
- We satisfy the **Dominant Energy Condition**

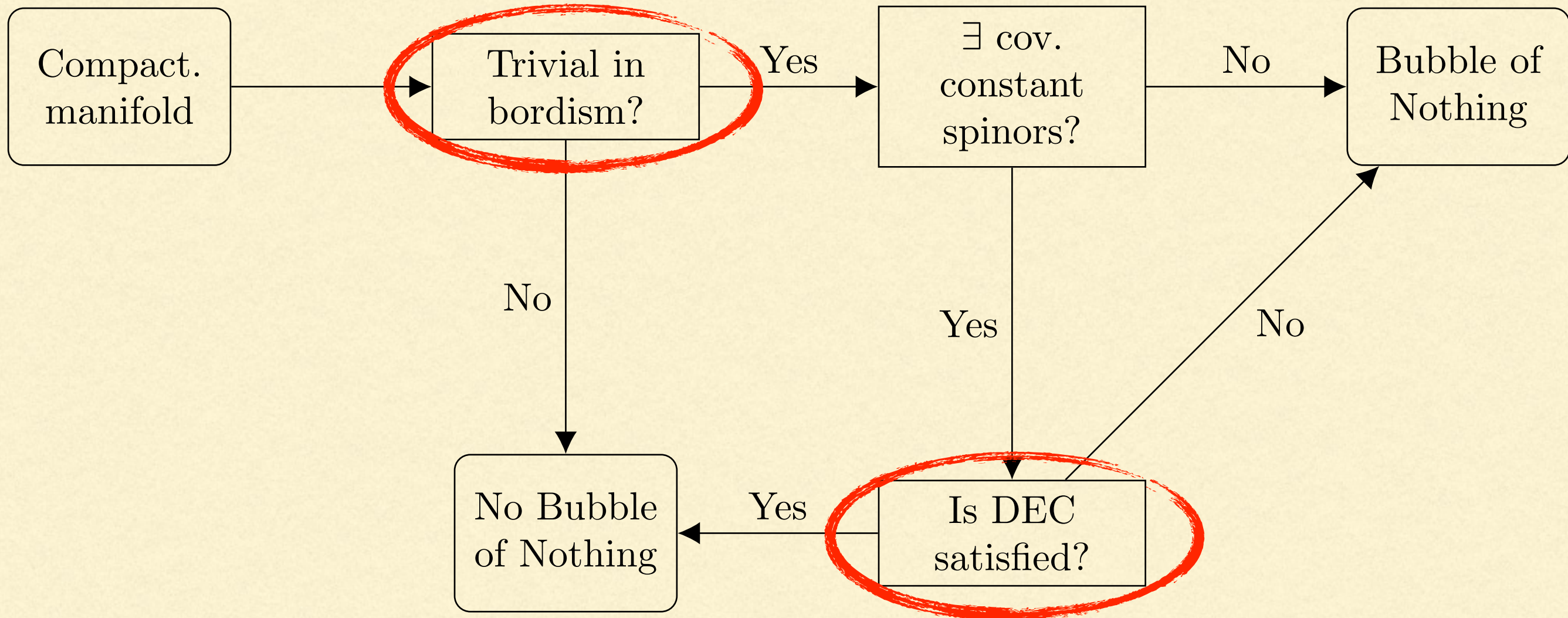
$-T^M_N k^N$  is causal and future-pointing

all nontrivial solutions of Einsteins eqs. have **positive mass**.

(A bubble has zero mass, so it can't exist if PET holds)

---





Generically **no** when SUSY is broken!  
Casimir energies/quantum effects, higher  
derivative terms...

---

# A BUBBLE FOR $T^3$

---

For **concreteness**, we construct a bubble for  $T^3 \times \mathbb{R}^4$

---

---

# A BUBBLE FOR $T^3$

---

For **concreteness**, we construct a bubble for  $T^3 \times \mathbb{R}^4$

- **Topology** (find a manifold  $B_4$  such that  $\partial B_4 = T^3$ )

---

# A BUBBLE FOR $T^3$

---

For **concreteness**, we construct a bubble for  $T^3 \times \mathbb{R}^4$

- **Topology** (find a manifold  $B_4$  such that  $\partial B_4 = T^3$ )

Answer: Half a  $K3$

---

---

# A BUBBLE FOR $T^3$

---

For **concreteness**, we construct a bubble for  $T^3 \times \mathbb{R}^4$

- **Topology** (find a manifold  $B_4$  such that  $\partial B_4 = T^3$ )

Answer: Half a  $K3$

- **Dynamics** (find a suitable metric on  $B_4$ )
-

---

# A BUBBLE FOR $T^3$

---

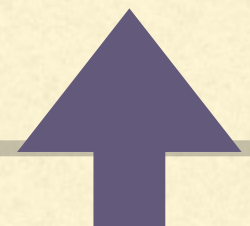
For **concreteness**, we construct a bubble for  $T^3 \times \mathbb{R}^4$

- **Topology** (find a manifold  $B_4$  such that  $\partial B_4 = T^3$ )

Answer: Half a K3

- **Dynamics** (find a suitable metric on  $B_4$ )

$$S = -\frac{g_s^2}{16\pi G_D} \int d^D x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla\phi)^2 - \frac{1}{12} H^2 + \frac{\alpha}{8} R_{GB}^2 \right]$$



---

**Problem statement:** Find a solution of Einstein's eqs

$$R_{MN} = -2\nabla_M \nabla_N \phi - \frac{1}{4}\alpha \left[ R_{MNST} R_N{}^{RST} \right. \\ \left. - 2R_{MSNT} R^{ST} - 2R_M{}^S R_{NS} + R R_{MN} \right]$$

$$\nabla_{(D)}^2 \phi - 2(\nabla \phi)^2 = \frac{\alpha}{16} R_{GB}^2$$

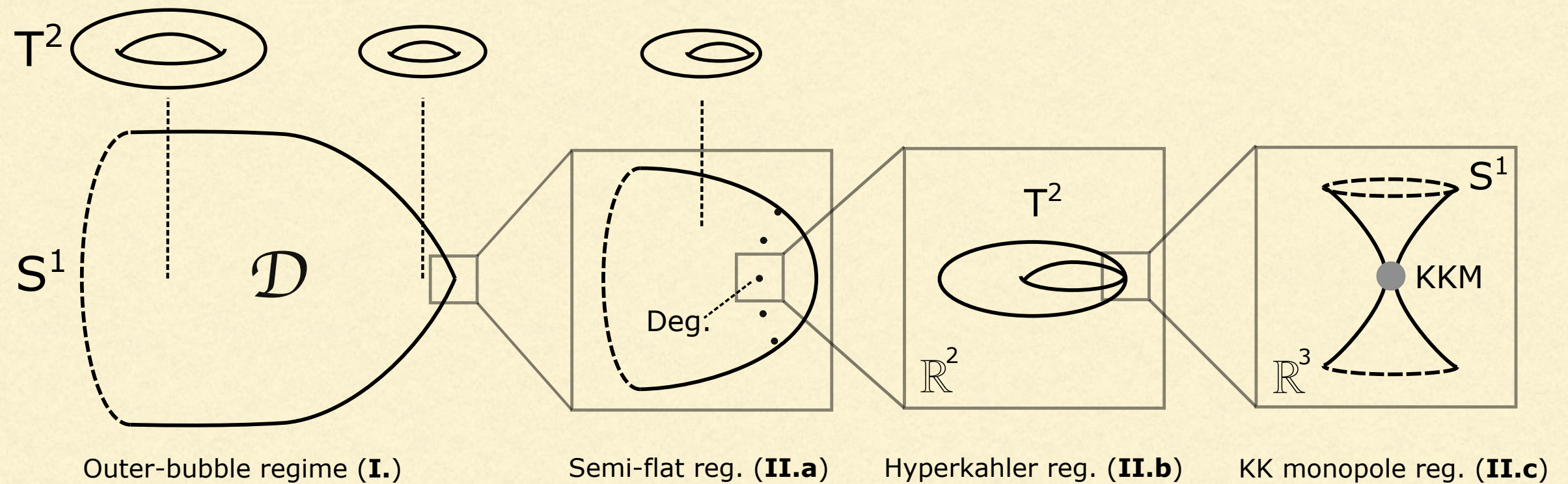
on

$$S^3 \times \mathcal{B}_4$$

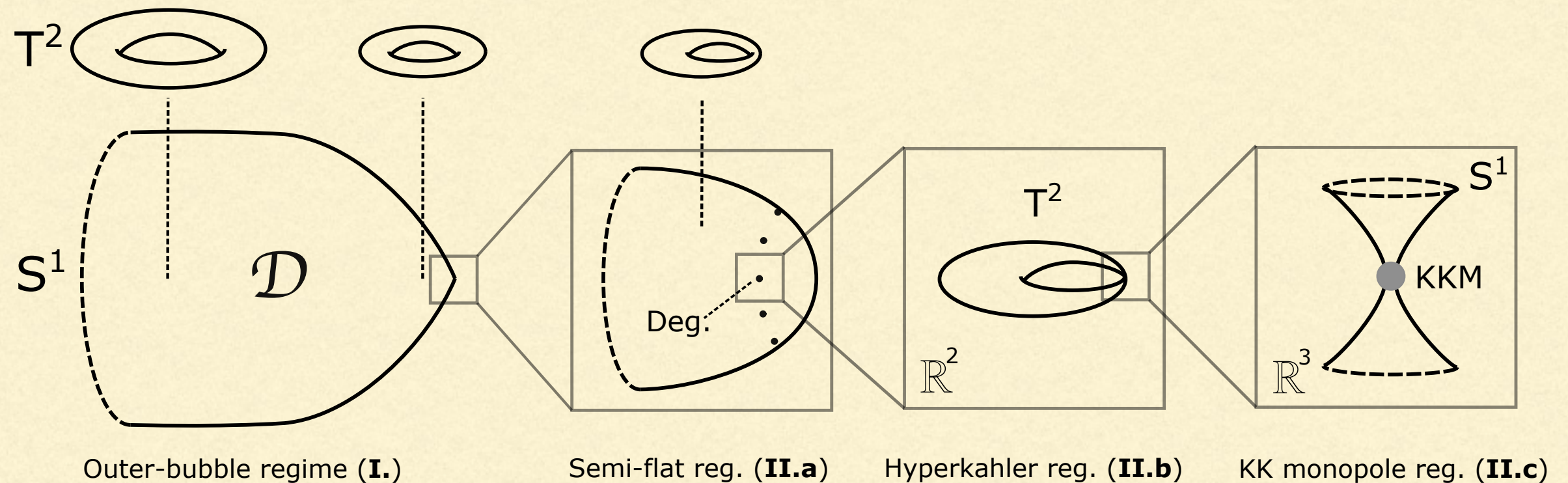
with boundary conditions

$$\phi \rightarrow \phi_0, \quad ds^2 \rightarrow dr^2 + ds_{T^3}^2 + r^2 d\Omega_3^2$$

---



We solve **eoms** independently in each layer



We solve **eoms** independently in each layer  
 then **glue** the different layers using **junction conditions**.

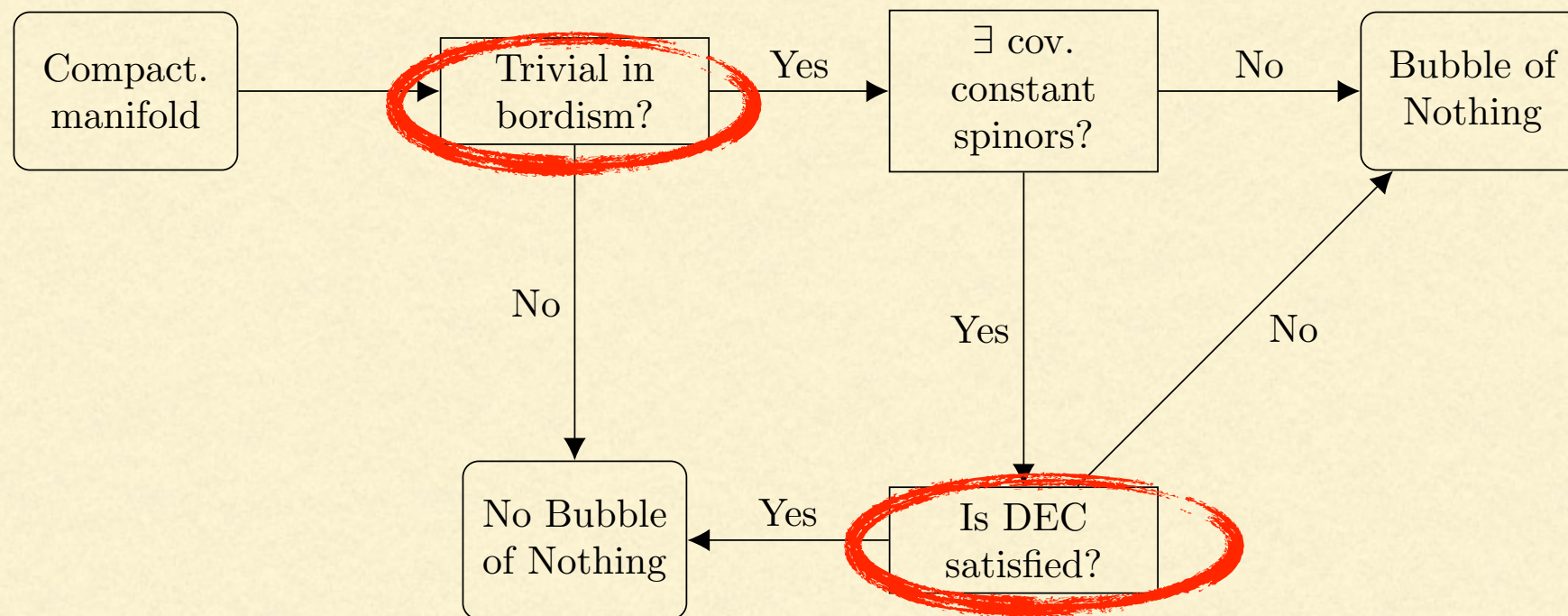
(to any given order in a consistent perturbative expansion)

---

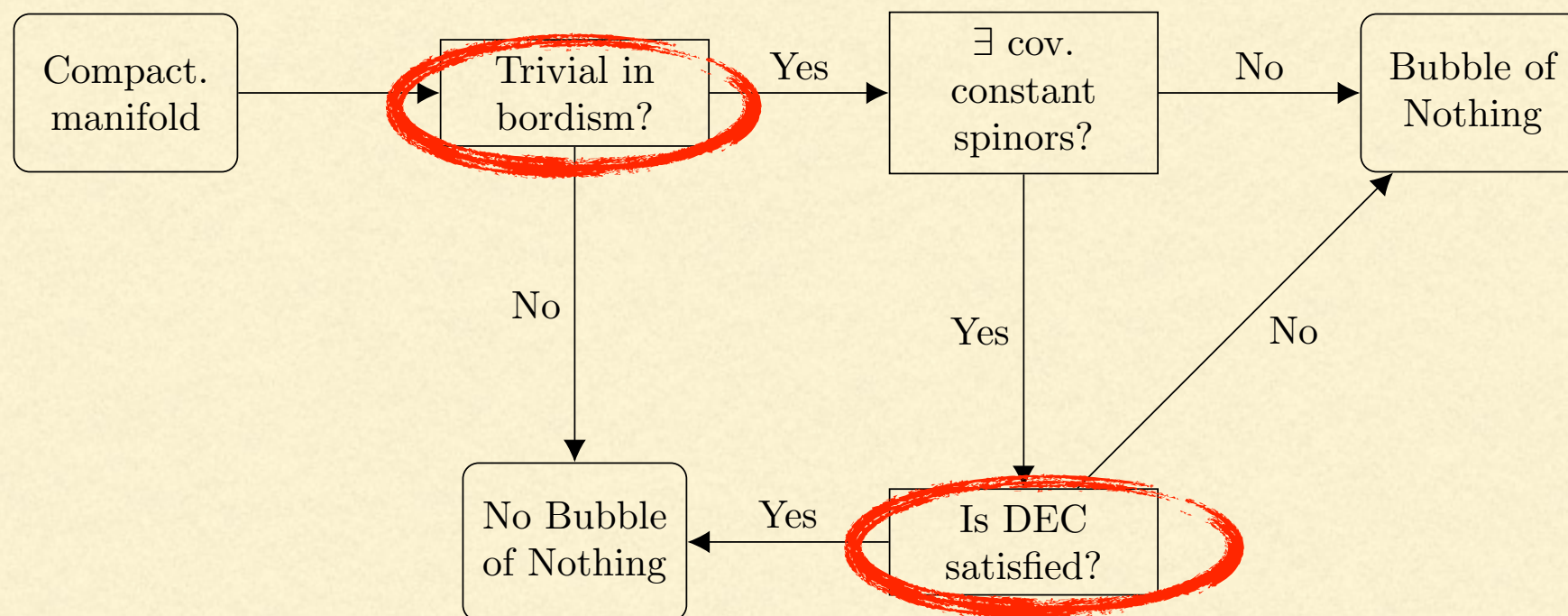
Long story short ...

$$\mathcal{R} \propto \left( \frac{24\pi^2}{\mathcal{V}_{T^3}} \alpha \right)^{-1}, \quad S_{\text{BON}} \propto \left( \frac{24\pi^2}{\mathcal{V}_{T^3}} \alpha \right)^{-2}$$

- Both radius and action **blow up** as  $\alpha \rightarrow 0$
  - $O(1)$  prefactor to be determined numerically, case-by-case
  - Can be embedded e.g. in type I on T3, heterotic...
-

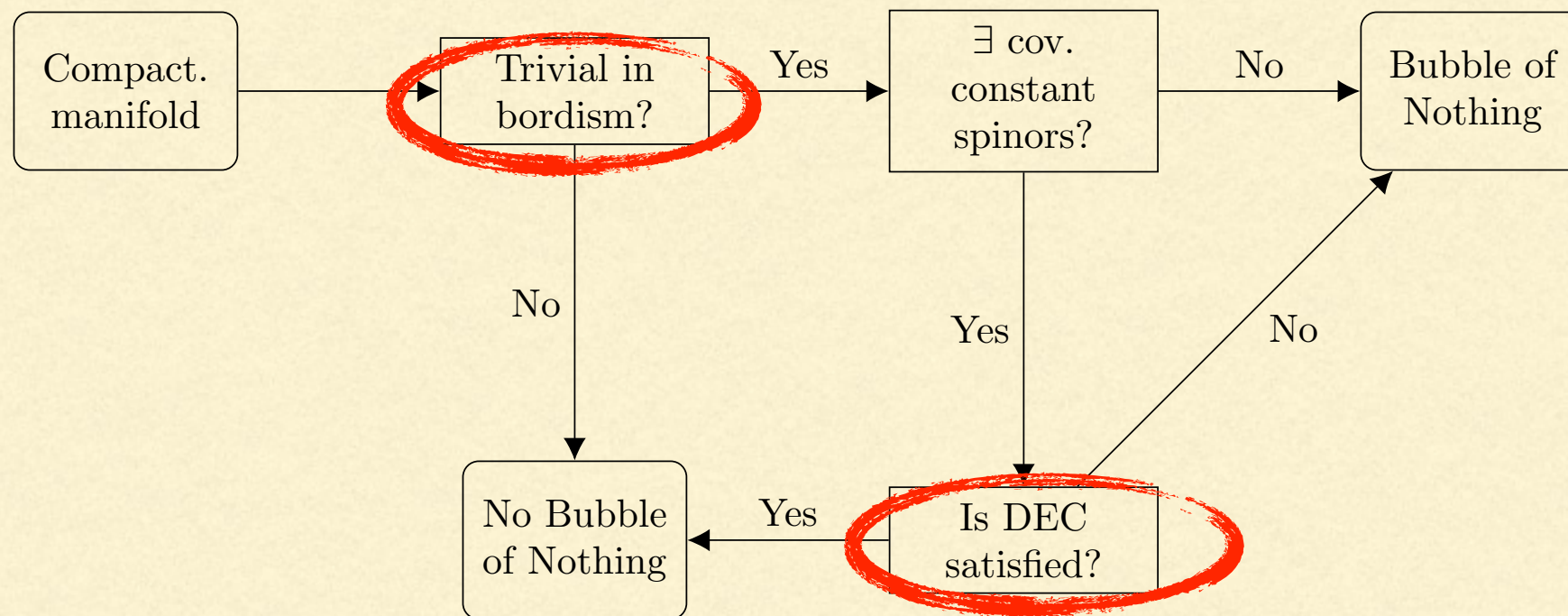


- We have a **proof of concept**. But how general?



- We have a **proof of concept**. But how general?

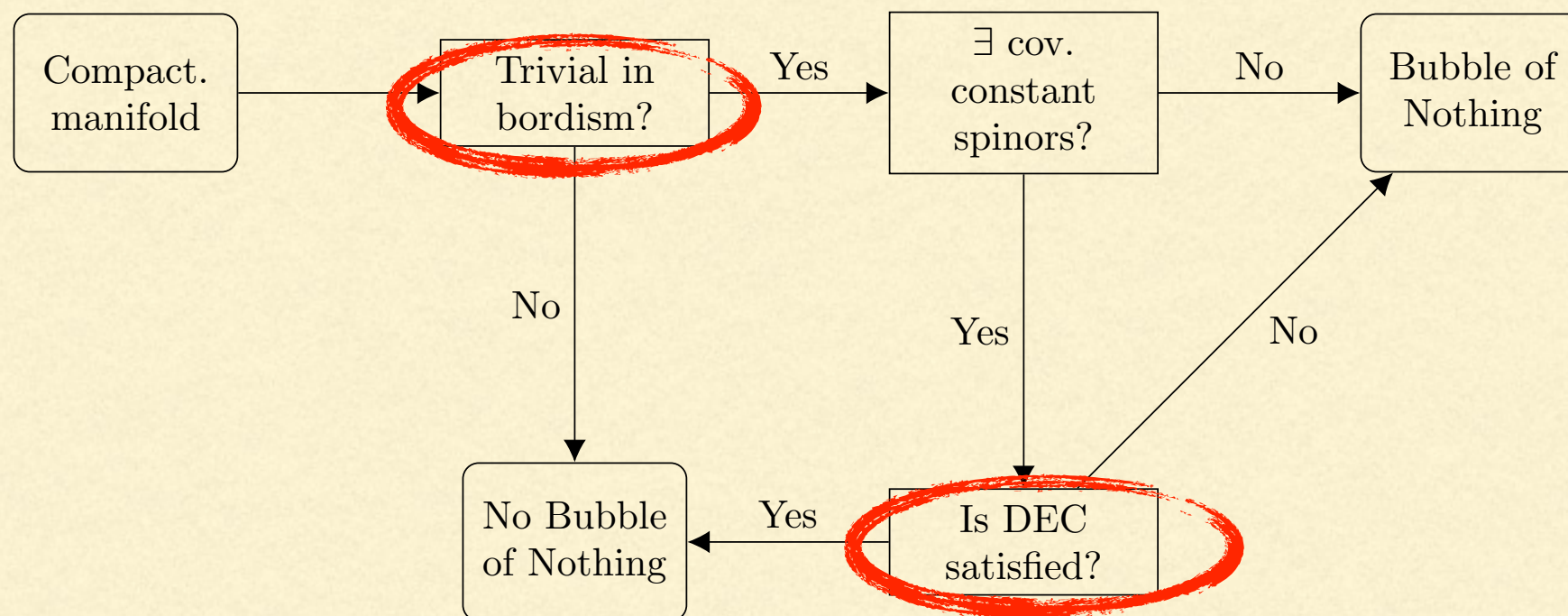
| $d$                      | 0            | 1              | 2              | 3 | 4            | 5 | 6 | 7 | 8             | 9               |
|--------------------------|--------------|----------------|----------------|---|--------------|---|---|---|---------------|-----------------|
| $\Omega_d^{\text{spin}}$ | $\mathbb{Z}$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | 0 | $\mathbb{Z}$ | 0 | 0 | 0 | $2\mathbb{Z}$ | $2\mathbb{Z}_2$ |



- We have a **proof of concept**. But how general?

| $d$                      | 0            | 1              | 2              | 3 | 4            | 5 | 6 | 7 | 8             | 9               |
|--------------------------|--------------|----------------|----------------|---|--------------|---|---|---|---------------|-----------------|
| $\Omega_d^{\text{spin}}$ | $\mathbb{Z}$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | 0 | $\mathbb{Z}$ | 0 | 0 | 0 | $2\mathbb{Z}$ | $2\mathbb{Z}_2$ |

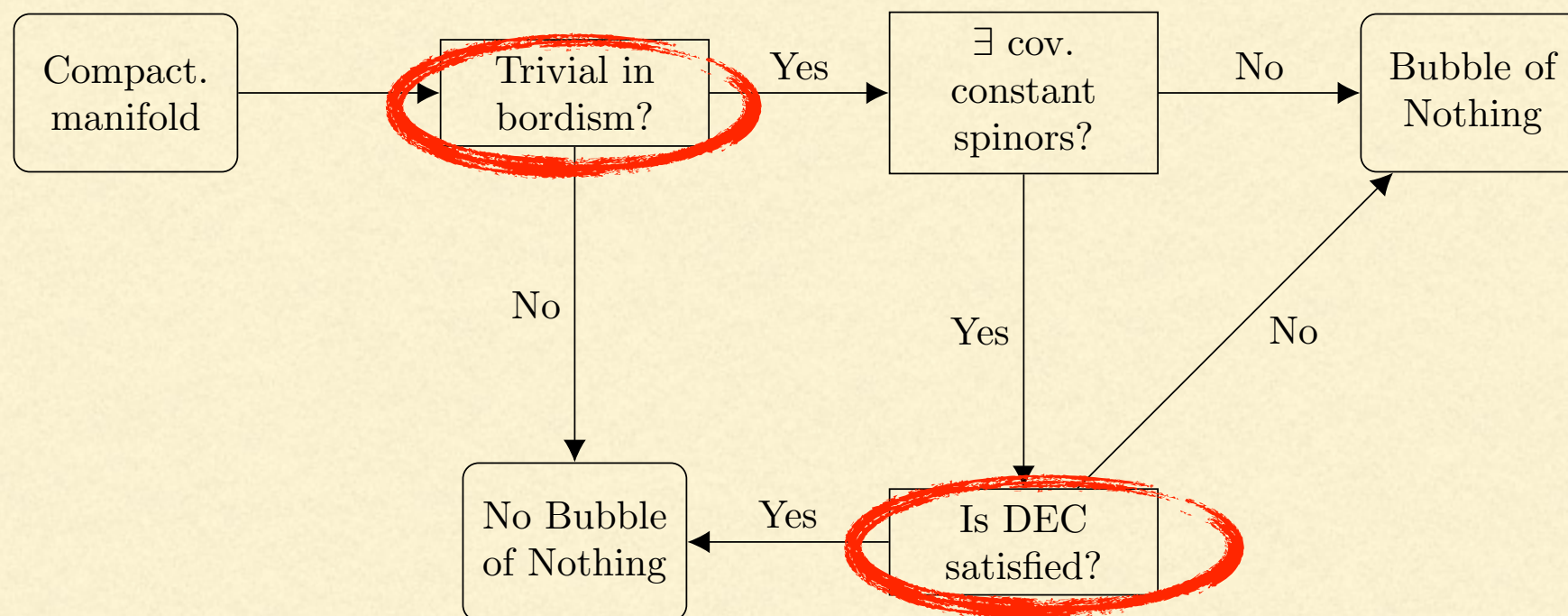

 M th on G2



- We have a **proof of concept**. But how general?

| $d$                      | 0            | 1              | 2              | 3 | 4            | 5 | 6 | 7 | 8             | 9               |
|--------------------------|--------------|----------------|----------------|---|--------------|---|---|---|---------------|-----------------|
| $\Omega_d^{\text{spin}}$ | $\mathbb{Z}$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | 0 | $\mathbb{Z}$ | 0 | 0 | 0 | $2\mathbb{Z}$ | $2\mathbb{Z}_2$ |

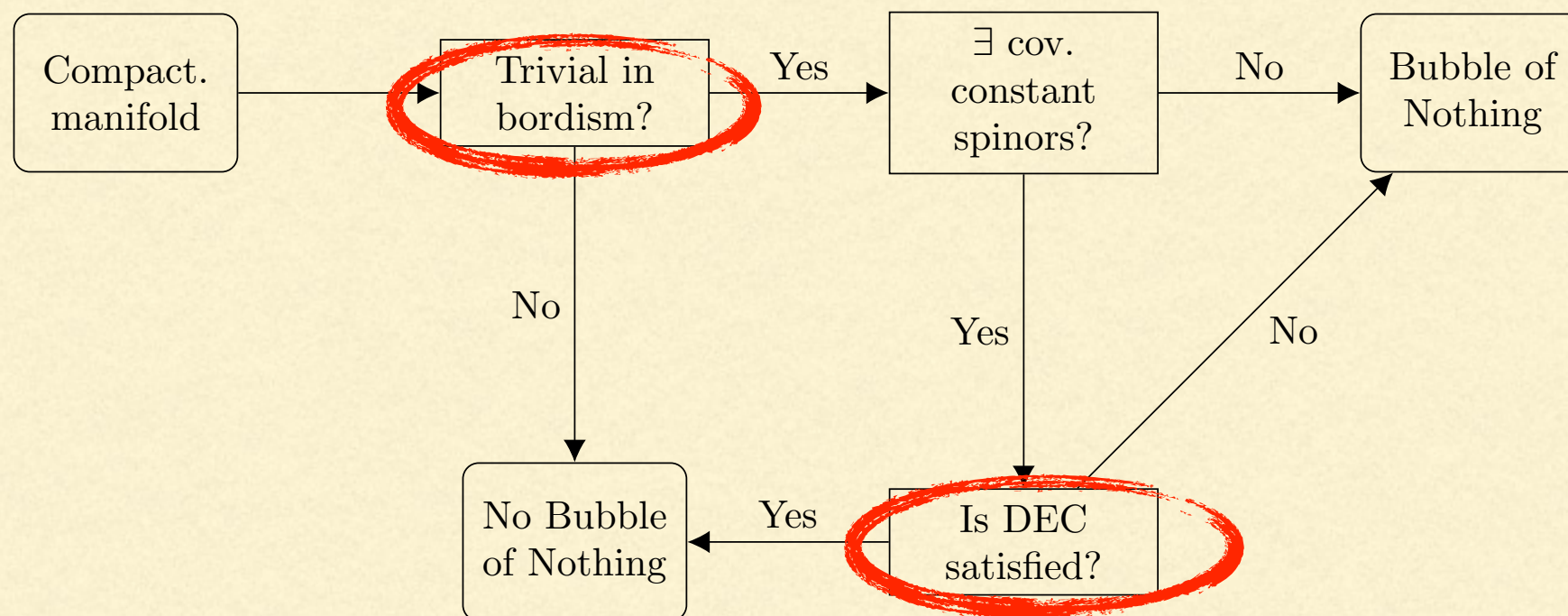
$\uparrow$  CY3  
 $\uparrow$  M th on G2



- We have a **proof of concept**. But how general?

| $d$                      | 0            | 1              | 2              | 3 | 4            | 5 | 6 | 7 | 8             | 9               |
|--------------------------|--------------|----------------|----------------|---|--------------|---|---|---|---------------|-----------------|
| $\Omega_d^{\text{spin}}$ | $\mathbb{Z}$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | 0 | $\mathbb{Z}$ | 0 | 0 | 0 | $2\mathbb{Z}$ | $2\mathbb{Z}_2$ |

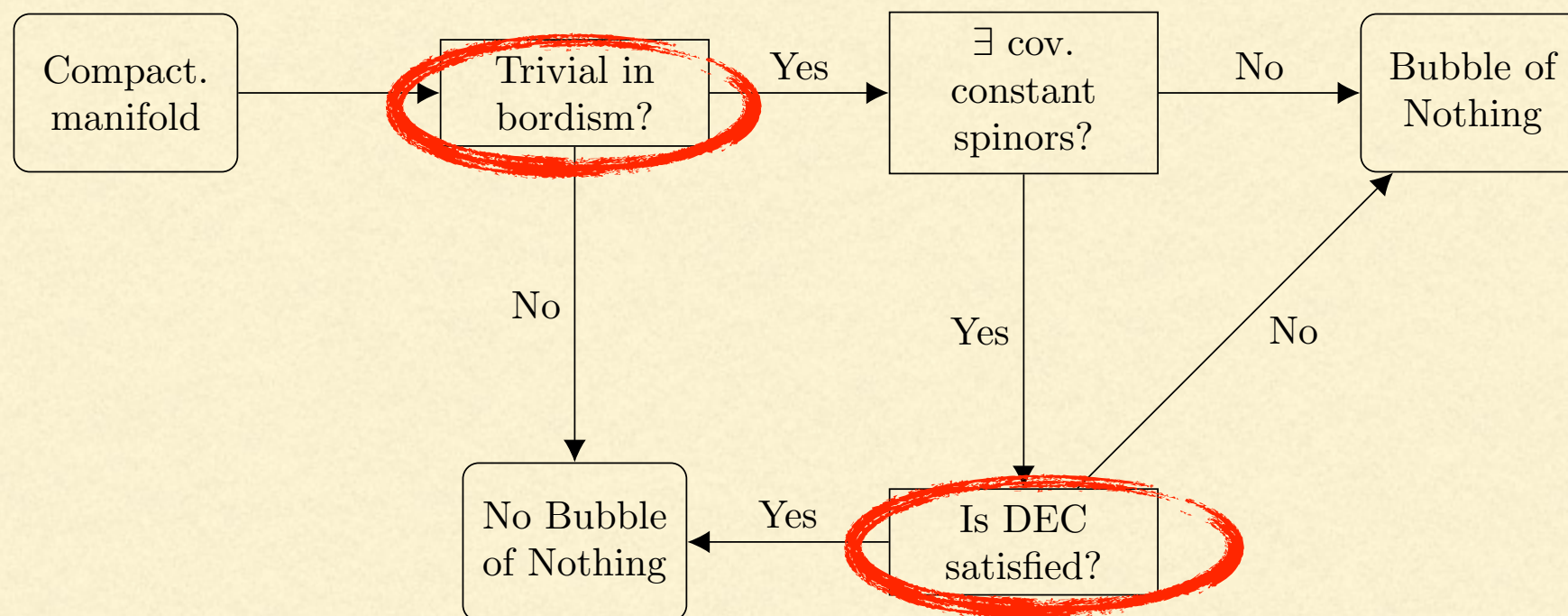
Sasaki-Einstein  $\nearrow$  (pointing to  $d=5$ )  
 CY3  $\nearrow$  (pointing to  $d=6$ )  
 M th on G2  $\nearrow$  (pointing to  $d=7$ )



- We have a **proof of concept**. But how general?

| $d$                      | 0            | 1              | 2              | 3 | 4            | 5 | 6 | 7 | 8             | 9               |
|--------------------------|--------------|----------------|----------------|---|--------------|---|---|---|---------------|-----------------|
| $\Omega_d^{\text{spin}}$ | $\mathbb{Z}$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | 0 | $\mathbb{Z}$ | 0 | 0 | 0 | $2\mathbb{Z}$ | $2\mathbb{Z}_2$ |

K3 → (points to  $\mathbb{Z}$  at  $d=4$ )  
 Sasaki-Einstein → (points to 0 at  $d=5$ )  
 CY3 → (points to 0 at  $d=6$ )  
 M th on G2 → (points to 0 at  $d=7$ )



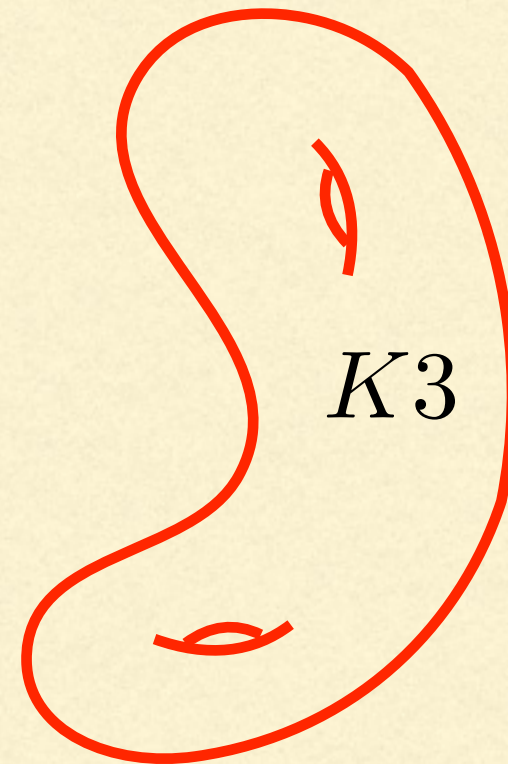
- We have a **proof of concept**. But how general?

| $d$                      | 0            | 1              | 2              | 3 | 4            | 5 | 6 | 7 | 8             | 9               |
|--------------------------|--------------|----------------|----------------|---|--------------|---|---|---|---------------|-----------------|
| $\Omega_d^{\text{spin}}$ | $\mathbb{Z}$ | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | 0 | $\mathbb{Z}$ | 0 | 0 | 0 | $2\mathbb{Z}$ | $2\mathbb{Z}_2$ |

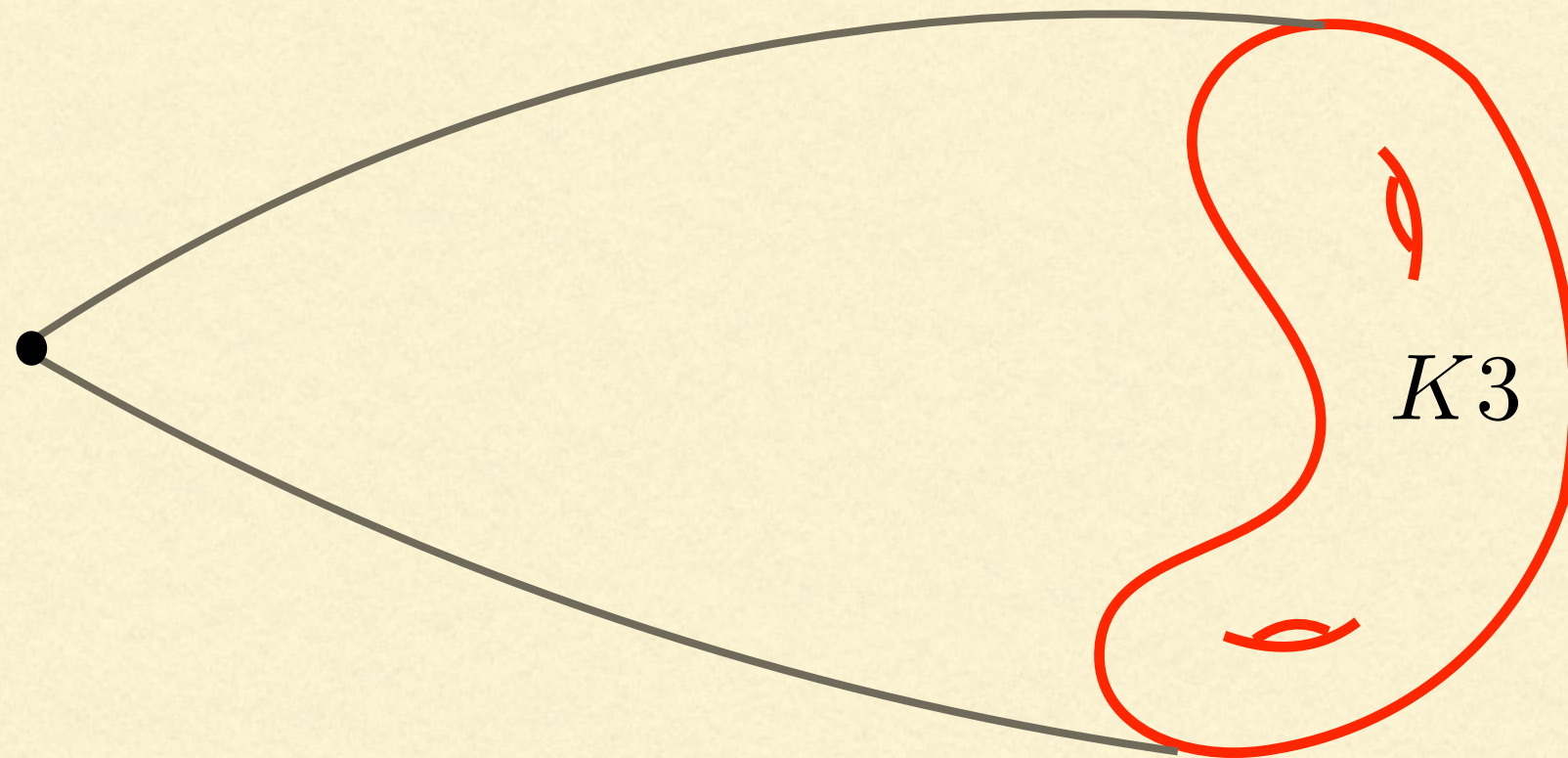
K3 → (points to  $\mathbb{Z}$  at  $d=4$ )  
 Sasaki-Einstein → (points to 0 at  $d=5$ )  
 CY3 → (points to 0 at  $d=6$ )  
 M th on G2 → (points to 0 at  $d=7$ )

- **Flux** introduces additional complications.

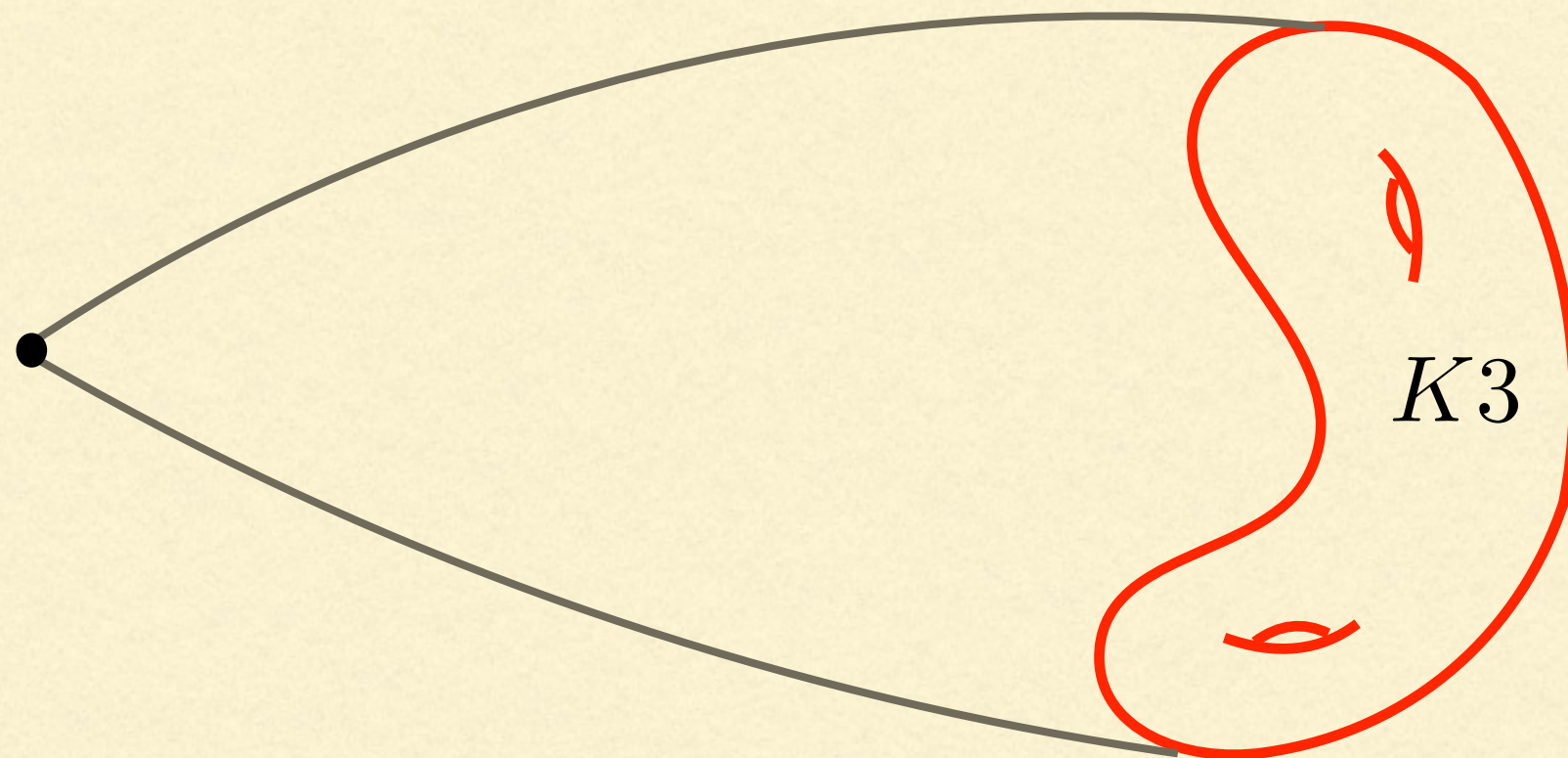
- 
- In order to **kill** this charge, we need to be able to “shrink  $K3$  to a point” in the full theory:



- 
- In order to **kill** this charge, we need to be able to “shrink  $K3$  to a point” in the full theory:



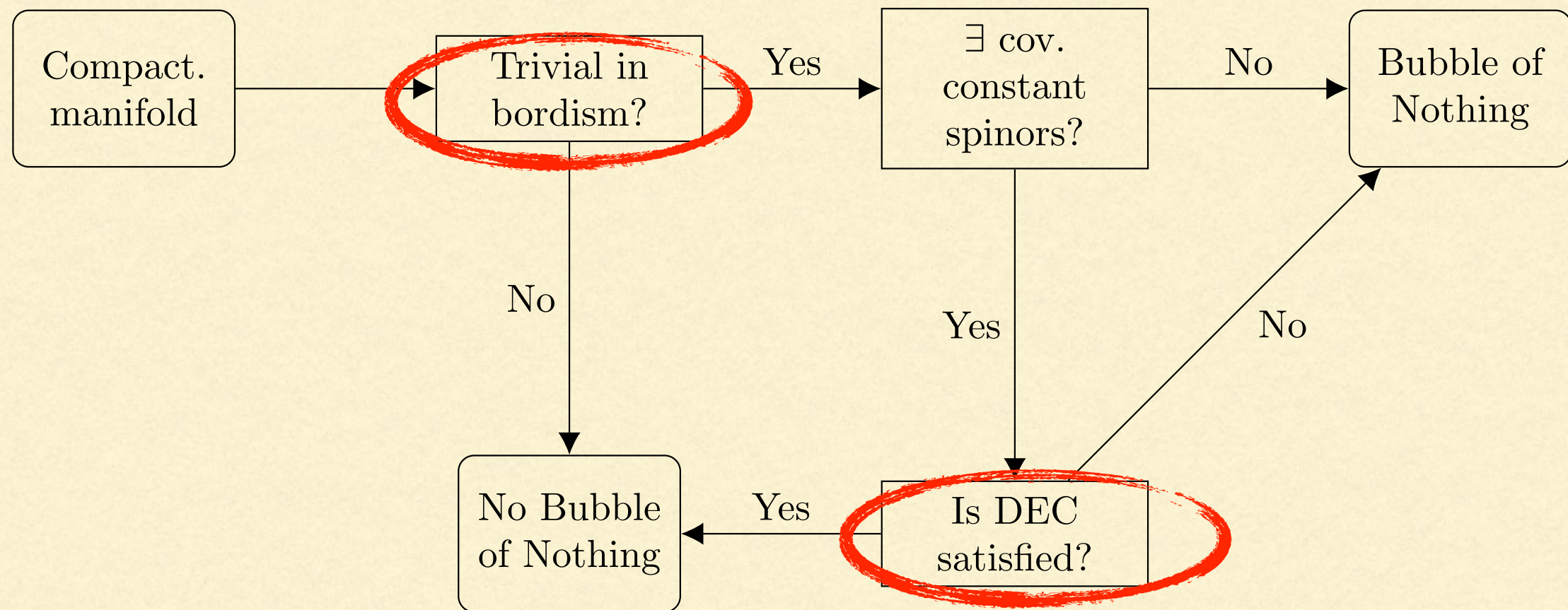
- In order to **kill** this charge, we need to be able to “shrink K3 to a point” in the full theory:



- **Cobordism conjecture** [Mcnamara-Vafa '19]: Can **always** do this

$$\Omega_{QG} = 0$$

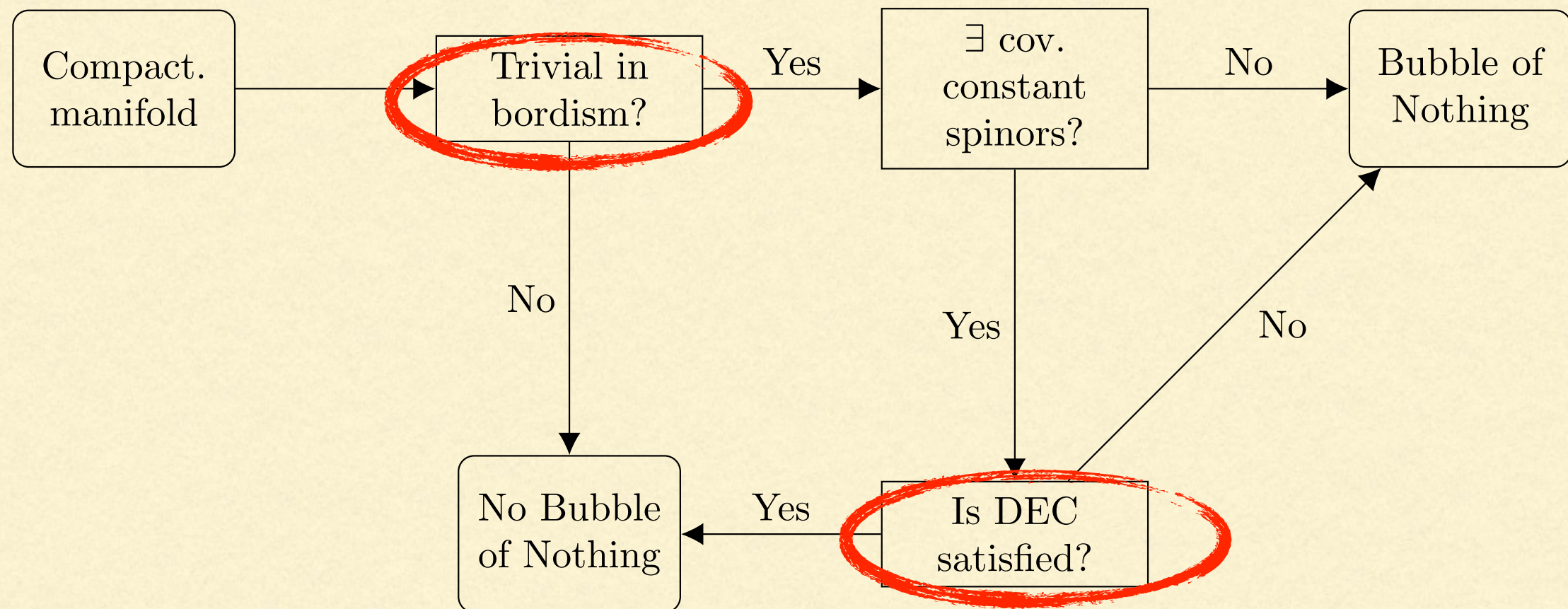
- Geometrical version of “no global symmetries”



Generically violated  
by quantum effects

Always true, due to  
cob. conjecture

(in the full,  
UV theory)



Generically violated  
by quantum effects

- In order to **kill** this charge, we need to be able to “shrink K3 to a point” in the full theory:



Killing the global symmetry forces the introduction of  
**defects**

- In order to **kill** this charge, we need to be able to “shrink K3 to a point” in the full theory:



Killing the global symmetry forces the introduction of  
**defects**

These defects can be potentially **anomalous**



---

If cobordisms predict an

**anomalous object**

in some low-energy EFT, and the anomaly  
cannot be cancelled somehow

then the theory is in the **Swamp**

We will use this to get **new constraints** on theories with 16  
supercharges, in 9,8,7d.

---

---

# Flash review: SUGRA w. 16 supercharges

---

# Flash review: **SUGRA** w. 16 supercharges

- Minimal SUSY in  $d > 6$ .

---

# Flash review: SUGRA w. 16 supercharges

- Minimal SUSY in  $d > 6$ .
- Life is simple: There are just **two multiplets**

---

# Flash review: SUGRA w. 16 supercharges

- Minimal SUSY in  $d > 6$ .
- Life is simple: There are just **two multiplets**
  - Gravity:  $g, A_i, B, \phi$

---

# Flash review: SUGRA w. 16 supercharges

- Minimal SUSY in  $d > 6$ .
- Life is simple: There are just **two multiplets**
  - Gravity:  $g, A_i, B, \phi$
  - Gauge:  $A, \phi_i$

---

# Flash review: SUGRA w. 16 supercharges

- Minimal SUSY in  $d > 6$ .
- Life is simple: There are just **two multiplets**
  - Gravity:  $g, A_i, B, \phi$
  - Gauge:  $A, \phi_i$
- Low-energy interactions are **completely fixed** by susy.

---

# Flash review: SUGRA w. 16 supercharges

- Minimal SUSY in  $d > 6$ .
- Life is simple: There are just **two multiplets**
  - Gravity:  $g, A_i, B, \phi$
  - Gauge:  $A, \phi_i$
- Low-energy interactions are **completely fixed** by susy.
- There is a **Narain** moduli space parametrized by the scalars

$$\frac{SO(10 - d, r)}{O(\Gamma) \text{ “}\times\text{” } SO(10 - d) \times SO(r)}$$

---

- 
- On a **generic point** in moduli space, the group is  $U(1)^{r+d}$

- 
- On a **generic point** in moduli space, the group is  $U(1)^{r+d}$
  - At some points in moduli space, **symmetry enhancement** can occur.

- 
- On a **generic point** in moduli space, the group is  $U(1)^{r+d}$
  - At some points in moduli space, **symmetry enhancement** can occur.
  - So a sugra is **fully specified** by the **rank** of the gauge group...

- 
- On a **generic point** in moduli space, the group is  $U(1)^{r+d}$
  - At some points in moduli space, **symmetry enhancement** can occur.
  - So a sugra is **fully specified** by the **rank** of the gauge group...  
...and a choice of **Lie algebra** (at special points in moduli space)
-

- 
- On a **generic point** in moduli space, the group is  $U(1)^{r+d}$
  - At some points in moduli space, **symmetry enhancement** can occur.
  - So a sugra is **fully specified** by the **rank** of the gauge group...  
...and a choice of **Lie algebra** (at special points in moduli space)
  - The **Swampland** is just figuring out which combinations have a consistent UV embedding!
-

- 
- On a **generic point** in moduli space, the group is  $U(1)^{r+d}$
  - At some points in moduli space, **symmetry enhancement** can occur.
  - So a sugra is **fully specified** by the **rank** of the gauge group...

...and a choice of **Lie algebra** (at special points in moduli space)

- The **Swampland** is just figuring out which combinations have a consistent UV embedding!
- In 10d, only 4 possibilities:

$$E_8 \times E_8, \text{Spin}(32)/\mathbb{Z}_2, E_8 \times U(1)^{248}, U(1)^{496}$$

---

- On a **generic point** in moduli space, the group is  $U(1)^{r+d}$
- At some points in moduli space, **symmetry enhancement** can occur.
- So a sugra is **fully specified** by the **rank** of the gauge group...

...and a choice of **Lie algebra** (at special points in moduli space)

- The **Swampland** is just figuring out which combinations have a consistent UV embedding!
- In 10d, only 4 possibilities:

$$E_8 \times E_8, \text{Spin}(32)/\mathbb{Z}_2, E_8 \times U(1)^{248}, U(1)^{496}$$

[Adams-De Wolfe- Taylor '10, Kim-Shiu-Vafa '19]

We get **string lamppost principle/string universality**

- 
- Let us look at a **generic point** in the Coulomb branch. Only data is the **rank**

- 
- Let us look at a **generic point** in the Coulomb branch. Only data is the **rank**

$$\text{rank} \leq 26 - d \quad [\text{Kim-Tarazi-Vafa '19}]$$

(from a strong version of SDC)

- 
- Let us look at a **generic point** in the Coulomb branch. Only data is the **rank**

$$\text{rank} \leq 26 - d \quad [\text{Kim-Tarazi-Vafa '19}]$$

(from a strong version of SDC)

| $d$ | $r$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|-----|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

---

- Let us look at a **generic point** in the Coulomb branch. Only data is the **rank**

$$\text{rank} \leq 26 - d \quad [\text{Kim-Tarazi-Vafa '19}]$$

(from a strong version of SDC)

| $d$ | $r$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|-----|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

- Let us look at a **generic point** in the Coulomb branch. Only data is the **rank**

$$\text{rank} \leq 26 - d \quad [\text{Kim-Tarazi-Vafa '19}]$$

(from a strong version of SDC)

| $d$ | $r$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|-----|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

- Let us look at a **generic point** in the Coulomb branch. Only data is the **rank**

$$\text{rank} \leq 26 - d \quad [\text{Kim-Tarazi-Vafa '19}]$$

(from a strong version of SDC)

| $d$ | $r$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|-----|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

- **Red:** Theory is **inconsistent** due to global grav. anomaly  
[Alvarez Gaumé - Witten '82]

- Let us look at a **generic point** in the Coulomb branch. Only data is the **rank**

$$\text{rank} \leq 26 - d \quad [\text{Kim-Tarazi-Vafa '19}]$$

(from a strong version of SDC)

| $d$ | $r$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|-----|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

- Red:** Theory is **inconsistent** due to global grav. anomaly  
[Alvarez Gaumé - Witten '82]
- Yellow:** No **known** inconsistency, but no construction either

- Let us look at a **generic point** in the Coulomb branch. Only data is the **rank**

$$\text{rank} \leq 26 - d \quad [\text{Kim-Tarazi-Vafa '19}]$$

(from a strong version of SDC)

| $d$ | $r$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|-----|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

- **Red:** Theory is **inconsistent** due to global grav. anomaly  
[Alvarez Gaumé - Witten '82]
- **Yellow:** No **known** inconsistency, but no construction either
- **Green:** Theory is in the landscape

| $d$ | $r$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|-----|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

| $d$ | $r$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|-----|-----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7   | 0   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

|     |      |   |   |   |   |   |   |   |   |   |      |    |    |    |    |    |    |    |    |    |  |
|-----|------|---|---|---|---|---|---|---|---|---|------|----|----|----|----|----|----|----|----|----|--|
|     | $+8$ |   |   |   |   |   |   |   |   |   | $+8$ |    |    |    |    |    |    |    |    |    |  |
| $d$ |      |   |   |   |   |   |   |   |   |   | $r$  |    |    |    |    |    |    |    |    |    |  |
| 9   | 0    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10   | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |  |
| 8   | 0    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10   | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |  |
| 7   | 0    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10   | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |  |

|     |   |    |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |  |
|-----|---|----|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|--|
|     |   | +8 |   |   |   |   |   |   |   |   | +8 |    |    |    |    |    |    |    |    |    |  |
| $d$ |   |    |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |  |
| 9   | 0 | 1  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |  |
| 8   | 0 | 1  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |  |
| 7   | 0 | 1  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |  |

- We explain the **mod 8 periodicity** in  $8d$  and  $9d$  using the

## Cobordism Conjecture

- We partly explain the pattern in  $7d$ .

- 
- Fact: The 9d, 8d supergravity lagrangians have a **parity symmetry** anywhere on the Coulomb branch

$$A \rightarrow -A, \quad B \rightarrow B, \quad \phi \rightarrow \phi$$

- 
- Fact: The 9d, 8d supergravity lagrangians have a **parity symmetry** anywhere on the Coulomb branch

$$A \rightarrow -A, \quad B \rightarrow B, \quad \phi \rightarrow \phi$$

- Fact: This symmetry squares to  $(-1)$  on fermions. It provides a **Pin- structure**.

- 
- Fact: The 9d, 8d supergravity lagrangians have a **parity symmetry** anywhere on the Coulomb branch

$$A \rightarrow -A, \quad B \rightarrow B, \quad \phi \rightarrow \phi$$

- Fact: This symmetry squares to  $(-1)$  on fermions. It provides a **Pin- structure**.
  - Fact: In all known stringy compactifications to 8d and 9d this is an **exact symmetry**
-

- 
- Fact: The 9d, 8d supergravity lagrangians have a **parity symmetry** anywhere on the Coulomb branch

$$A \rightarrow -A, \quad B \rightarrow B, \quad \phi \rightarrow \phi$$

- Fact: This symmetry squares to  $(-1)$  on fermions. It provides a **Pin- structure**.
  - Fact: In all known stringy compactifications to 8d and 9d this is an **exact symmetry**
  - Fact: This means the theory can be consistently put on certain **non-orientable manifolds**
-

- 
- Fact: The 9d, 8d supergravity lagrangians have a **parity symmetry** anywhere on the Coulomb branch

$$A \rightarrow -A, \quad B \rightarrow B, \quad \phi \rightarrow \phi$$

- Fact: This symmetry squares to  $(-1)$  on fermions. It provides a **Pin- structure**.
- Fact: In all known stringy compactifications to 8d and 9d this is an **exact symmetry**
- Fact: This means the theory can be consistently put on certain **non-orientable manifolds**

**Assumption: Every N=1 theory in 7d, 8d, 9d makes sense on non-orientable manifolds**

---

- 
- If the theory makes sense on  $\text{Pin-}$  manifolds, there is a **nonvanishing bordism group**

- 
- If the theory makes sense on Pin- manifolds, there is a **nonvanishing bordism group**

$$\Omega_2^{\text{Pin}^-} = \mathbb{Z}_8$$

- 
- If the theory makes sense on Pin- manifolds, there is a **nonvanishing bordism group**

$$\Omega_2^{\text{Pin}^-} = \mathbb{Z}_8$$

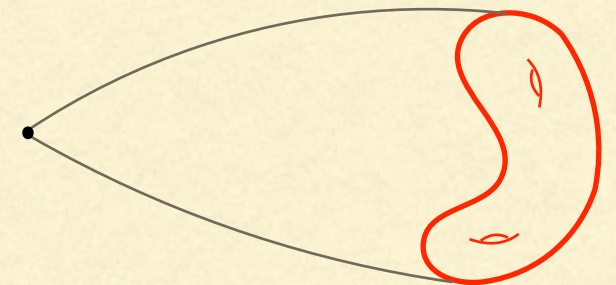
- The manifold carrying this charge is  $\mathbb{RP}^2$
-

- 
- If the theory makes sense on  $\text{Pin-}$  manifolds, there is a **nonvanishing bordism group**

$$\Omega_2^{\text{Pin}^-} = \mathbb{Z}_8$$

- The manifold carrying this charge is  $\mathbb{RP}^2$
- Cancelling this charge requires the existence of **defects** that look like

$$\mathbb{R}^3 / \mathbb{Z}_2$$



These are **orbifolds** or **orientifolds** in examples

We call them **I-folds**

---

- 
- Existence of Ifolds means that **it is consistent to compactify the 9d theory on**

$$T^3/\mathbb{Z}_2$$

- 
- Existence of Ifolds means that **it is consistent to compactify the 9d theory on**

$$T^3/\mathbb{Z}_2$$

- Each fixed point contributes the same (Subtlety involving tadpole cancellation)
-

- 
- Existence of Ifolds means that **it is consistent to compactify the 9d theory on**

$$T^3/\mathbb{Z}_2$$

- Each fixed point contributes the same (Subtlety involving tadpole cancellation)
- This is a 6d model. Bulk supersymmetric, but SUSY can be broken by the I-folds. Gravitational anomaly cancellation demands

$$273 = n_T + n_H - n_V$$

$$n_T = 8n_I^T + 1, \quad n_H = 3 + r + 8n_H^I, \quad n_V = 8n_V^I$$

- [Bianchi-Sagnotti '90 ]

$$r \equiv 1 \pmod{8}$$

---

- 
- Type I I-fold compactification: **O5<sup>+</sup> planes + 8 anti D5 branes. Ifold compactification is nonsusy!**

$$n_T = 1, \quad n_H = 3 + 17, \quad (n_H - n_V)^{\overline{D5}} = -240$$

- [Angelantonj, Blumenhagen, Lust, Font, Dudas, Mourad '98,'99'00 ]

- 
- Type I I-fold compactification: **O5<sup>+</sup> planes + 8 anti D5 branes. Ifold compactification is nonsusy!**

$$n_T = 1, \quad n_H = 3 + 17, \quad (n_H - n_V)^{\overline{D5}} = -240$$

- [Angelantonj, Blumenhagen, Lust, Font, Dudas, Mourad '98,'99'00 ]

- Also **heterotic, CHL I-folds are nonsusy**
-

- 
- Type I I-fold compactification: **O5<sup>+</sup> planes + 8 anti D5 branes. Ifold compactification is nonsusy!**

$$n_T = 1, \quad n_H = 3 + 17, \quad (n_H - n_V)^{\overline{D5}} = -240$$

- [Angelantonj, Blumenhagen, Lust, Font, Dudas, Mourad '98,'99'00 ]

- Also **heterotic, CHL I-folds are nonsusy**
- Rank I I-fold is supersymmetric: Its just M-theory on

$$(K3 \times S^1)/\mathbb{Z}_2 \quad \blacksquare \text{ [Sen '96]}$$

or the DP orientifold of an orbifold K3 [Dabholkar-Park '96]

---

- 
- Similar story in 8d: Compactify on

$$T^3/\mathbb{Z}_2$$

to get a 5d theory with a **modulo 16** parity anomaly.

- 
- Similar story in 8d: Compactify on

$$T^3/\mathbb{Z}_2$$

to get a 5d theory with a **modulo 16** parity anomaly.

- I-folds contribute multiple of 8 to the anomaly
-

- 
- Similar story in 8d: Compactify on

$$T^3/\mathbb{Z}_2$$

to get a 5d theory with a **modulo 16** parity anomaly.

- I-folds contribute multiple of 8 to the anomaly
- SUGRA fields contribute  $6+r$ , so

$$r \equiv 2 \pmod{8}$$

---

- 
- Similar story in 8d: Compactify on

$$T^3/\mathbb{Z}_2$$

to get a 5d theory with a **modulo 16** parity anomaly.

- I-folds contribute multiple of 8 to the anomaly
- SUGRA fields contribute  $6+r$ , so

$$r \equiv 2 \pmod{8}$$

- Possible subtlety involving a **topological Green-Schwartz mechanism** [Garcia-Etxebarria-Hayashi-Ohmori-Tachikawa-Yonekura '17]
-

- 
- In 7d, compactification on  $S^1/Z_2$  yields

$$r \equiv 1 \pmod{2}$$

so we don't explain the pattern in this case.

(observed pattern is  $r=3+k$ ,  $k=0,2,4,8,16$ )

---

---

|   |   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

---

---

In 8d, 9d, we achieve Coulomb branch  
**string universality**

|   |   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |
|---|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |    |    |
| 8 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |    |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

**String theory compactifications**  
exactly match  
**Swampland constraints**

---

- Twisting by magnetic flux, we can constrain **global form of the gauge group**

| Algebra | $\dim(\mathfrak{g}) + \text{rank}(\mathfrak{g})$ | $\neq 0 \bmod 8$ ?          | Group                   | Real reps?   |
|---------|--------------------------------------------------|-----------------------------|-------------------------|--------------|
| $A_k$   | $k^2 + 3k$                                       | $k \not\equiv 0, 5 \bmod 8$ | $SU(k+1)$               | $\times$     |
|         |                                                  |                             | $PSU(k+1)$              | $\checkmark$ |
| $B_k$   | $2k(2k+1)$                                       | $k \equiv 1, 2 \bmod 4$     | $Spin(2k+1)$            | $\times$     |
|         |                                                  |                             | $SO(2k+1)$              | $\checkmark$ |
| $C_k$   | $2k(2k+1)$                                       | $k \equiv 1, 2 \bmod 4$     | $Sp(k)$                 | $\times$     |
|         |                                                  |                             | $Sp(k)/\mathbb{Z}_2$    | $\checkmark$ |
| $D_k$   | $2k^2$                                           | $k \text{ odd}$             | $Spin(2k)$              | $\times$     |
|         |                                                  |                             | $SO(2k)$                | $\checkmark$ |
|         |                                                  |                             | $Spin(2k)/\mathbb{Z}_4$ | $\checkmark$ |
| $E_6$   | 84                                               | $\checkmark$                | $E_6$                   | $\times$     |
|         |                                                  |                             | $E_6/\mathbb{Z}_3$      | $\checkmark$ |
| $E_7$   | 140                                              | $\checkmark$                | $E_7$                   | $\times$     |
|         |                                                  |                             | $E_7/\mathbb{Z}_2$      | $\checkmark$ |
| $E_8$   | 256                                              | $\times$                    | $E_8$                   | $\checkmark$ |
| $F_4$   | 56                                               | $\times$                    | $F_4$                   | $\checkmark$ |
| $G_2$   | 16                                               | $\times$                    | $G_2$                   | $\checkmark$ |

- Twisting by magnetic flux, we can constrain **global form of the gauge group**

| Algebra | $\dim(\mathfrak{g}) + \text{rank}(\mathfrak{g})$ | $\neq 0 \bmod 8?$           | Group                   | Real reps?   |
|---------|--------------------------------------------------|-----------------------------|-------------------------|--------------|
| $A_k$   | $k^2 + 3k$                                       | $k \not\equiv 0, 5 \bmod 8$ | $SU(k+1)$               | $\times$     |
|         |                                                  |                             | $PSU(k+1)$              | $\checkmark$ |
| $B_k$   | $2k(2k+1)$                                       | $k \equiv 1, 2 \bmod 4$     | $Spin(2k+1)$            | $\times$     |
|         |                                                  |                             | $SO(2k+1)$              | $\checkmark$ |
| $C_k$   | $2k(2k+1)$                                       | $k \equiv 1, 2 \bmod 4$     | $Sp(k)$                 | $\times$     |
|         |                                                  |                             | $Sp(k)/\mathbb{Z}_2$    | $\checkmark$ |
| $D_k$   | $2k^2$                                           | $k \text{ odd}$             | $Spin(2k)$              | $\times$     |
|         |                                                  |                             | $SO(2k)$                | $\checkmark$ |
|         |                                                  |                             | $Spin(2k)/\mathbb{Z}_4$ | $\checkmark$ |
| $E_6$   | 84                                               | $\checkmark$                | $E_6$                   | $\times$     |
|         |                                                  |                             | $E_6/\mathbb{Z}_3$      | $\checkmark$ |
| $E_7$   | 140                                              | $\checkmark$                | $E_7$                   | $\times$     |
|         |                                                  |                             | $E_7/\mathbb{Z}_2$      | $\checkmark$ |
| $E_8$   | 256                                              | $\times$                    | $E_8$                   | $\checkmark$ |
| $F_4$   | 56                                               | $\times$                    | $F_4$                   | $\checkmark$ |
| $G_2$   | 16                                               | $\times$                    | $G_2$                   | $\checkmark$ |

See [Cvetic-Dierigl-Lin-Zhang '20] for similar constraints from a different viewpoint and [Font-Fraiman-Graña-Núñez-Parra'20] for a list of all maximal groups in heterotic. **We all agree**

---

# TO WRAP UP

---

- **Cobordisms are the right tool to understand topology-changing processes in quantum gravity**
  - Bubbles of nothing are much more general than what we thought
    - Explicit T3 example, many more constructible
    - Universal instability of nonsusy vacua?
  - **Absence of Cobordism + Anomalies = Swampland**
    - Enough to “bootstrap” features of ST: CB universality in 8d, 9d
    - Apply to 6d N=1, or even the Standard Model!
-

---

**Thank you!**

**Grazie mille!**

---