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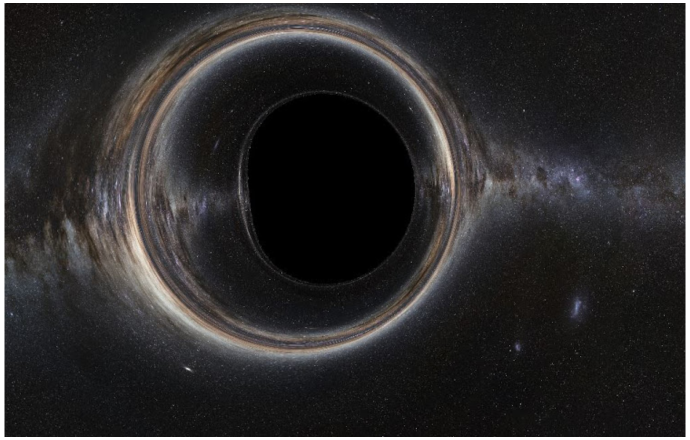
Updates on the search for Multicenter AdS$_4$ solutions

Roma Tor Vergata, Zoom Seminar, April 12, 2021

Based on work to appear with R. Monten

Black holes and quantum gravity

Black holes are often seen as a theoretical laboratory for quantum gravity



Gravity knows about thermodynamics, and it is holographic

$$S_{BH} = \frac{c^3}{G_N \hbar} \frac{A}{4}$$

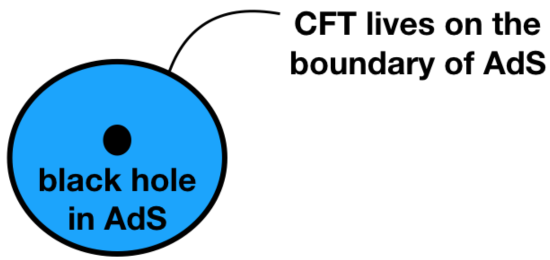
Black holes preserving susy provide a very valuable framework

- one can construct explicit solutions (most often analytical)
- String theory allows to identify the microscopic d.o.f. responsible for their entropy

Supersymmetric (BPS) black holes

Many studies in the context of asymptotically flat black holes have shown a remarkable agreement between macroscopic and microscopic picture [Strominger, Vafa '96]

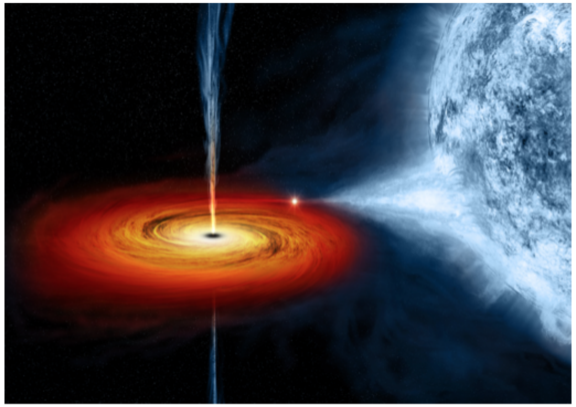
Quite recently, this was extended to asymptotically AdS black holes: entropy related to the **counting of states** in the dual CFT, living on the boundary.



When bulk and boundary are supersymmetric perform detailed counting of states! Exact quantities (i.e. partition function, indices) computed via *supersymmetric localization* in the dual theory

Rotating BPS AdS black holes

AdS_4 extremal rotating black holes can preserve susy! Impossible in 4D Minkowski



BPS bound:

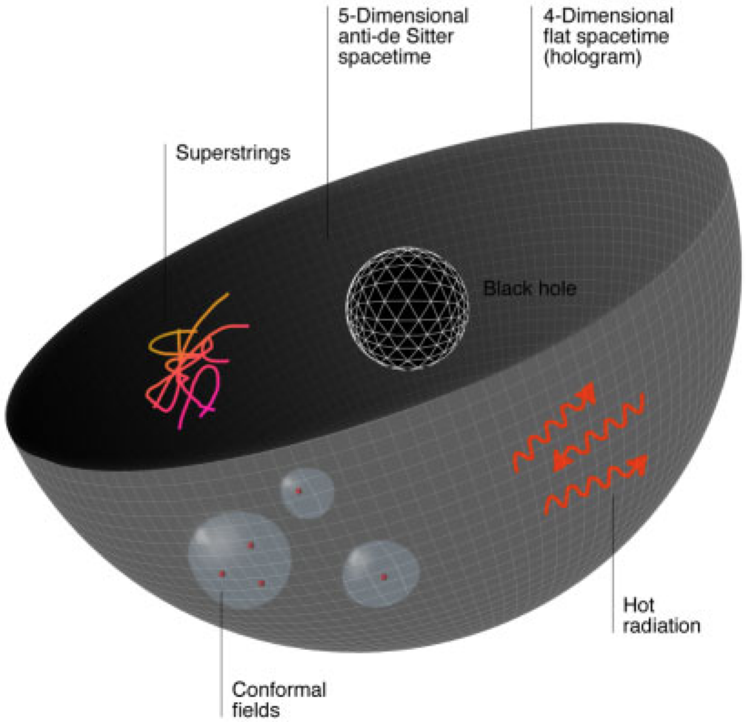
$$M = Q + \frac{J}{l_{AdS}}$$

compatible with extremality bound

Extremal rotating AdS_4 black holes have Near Horizon geometry in the same class as the Near-Horizon Extremal Kerr (NHEK), present in our universe.

Impressive progress in microstate counting via SCIs, also in higher dimensions (e.g. AdS_5) in the past 3-4 years

AdS black holes thermodynamics



Black hole with temperature in the bulk corresponds to a thermal QFT in the boundary

Model strongly coupled field theory processes by gravity dual via AdS/CFT

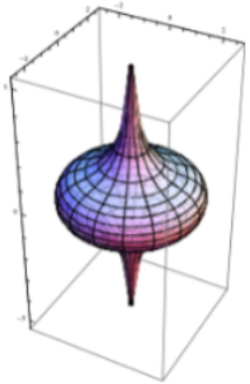
i.e. superconductivity modeled by AdS black holes with scalar hair [Gubser '08], [Hartnoll, Herzog, Horowitz '08]

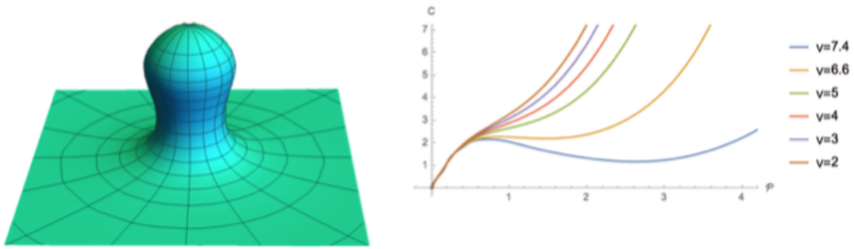
Black hole phase transition manifests itself as reorganization of the geometry and the matter fields outside the horizon

Deformed horizons

The horizon of an AdS back hole can be deformed by tuning conditions at boundary

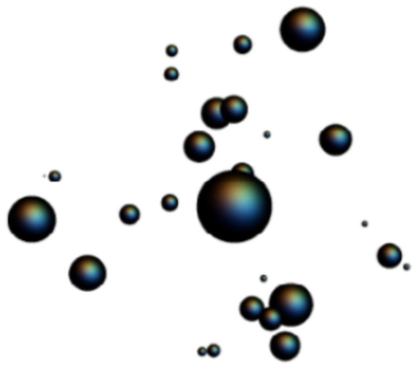
Ultraspinning black hole [Hristov, Gnecchi, Klemm, **Toldo**, Vaughan '18]





Black globules [Horowitz, Santos, **Toldo** '18]

Multi-center black hole

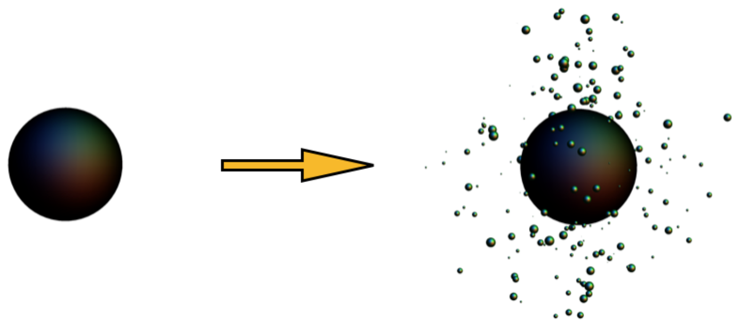


Fragmented geometries: disordered yet rigid/ exponentially many local free energy minima. Disorderly frozen matter distribution arising upon cooling

Multicenter black holes and holographic glass

Glass: disordered yet rigid/ exponentially many local free energy minima. Poorly understood.

Black hole bound states model glassy phase of matter in the dual theory [Denef et al.'13]



High T: unique, **single** center black hole $\rightarrow$ liquid phase

Low T: zoo of **multi** center black holes $\rightarrow$ glass phase

Multiple black holes induce electric and magnetic dipole charges, corresponding to inhomogeneities in charge densities and magnetic fields in the dual field theory $\rightarrow$ High viscosity

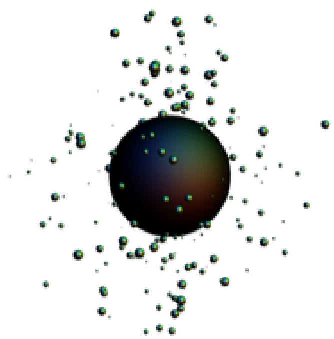
Multicenter black holes

Asymptotically flat multicenter black holes exist and well studied in string theory (e.g. [Denef, '99] + connection to interesting mathematics [Kontsevich, Soibelman '08])

Multicenter in AdS spacetime: long standing challenge. Presence of potential might spoil equilibrium conditions between the centers.

Nevertheless, composite AdS configurations exist: hovering black holes [Iqbal,Horowitz, Santos '14; Horowitz, Santos, **Toldo** '18] + dynamical multi black holes in spaces with negative Λ [Chimento, Klemm '13]

- Work in probe approximation: small probe black hole around a big, massive one [Denef et al, '11–'13]



Outline

Multicenter black holes in AdS_4 in the probe approximation

- Background black hole
  - Construction of AdS_4 thermal black holes from M-theory on Q^{111}
  - Background thermodynamics
- Probe black holes
  - Stability of spacetime filling and internally wrapped branes
  - Supersymmetric limit
- Microstate counting: a status report

Multicenter BHs in probe approximation

Exact solutions of Multi-center black holes found [Denef '99] by completing the squares in the sugra Lagrangian and solving the resulting first order equations. Presence of scalar potential obstructs this procedure.

Probe analysis: Stable and metastable probes exist in the background of a $T > 0$ 4d dyonic black hole with scalar profile (neutral scalars) [Anninos, Anous, Denef, Peeters '13]. **True both in Minkowski and AdS.**

In the AdS_4 compactification dual to ABJM theory, one linear combination of the gauge fields is Higgsed, thus massive [Aharony, Bergman, Jafferis, Maldacena '08].

Aim: study probe stability in a more general black hole background - charged scalars and massive vector field.

The Model: M theory on Q^{111}

M–theory truncation on homogeneous SE_7 manifold Q^{111}: 4d gauged supergravity with hyper-multiplets with susy AdS_4 vacuum [Cassani, Koerber, Varela '12]

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - g_{i\bar{j}}\partial_\mu z^i \partial^\mu z^{\bar{j}} - h_{uv}D_\mu q^u D^\mu q^v + I_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\mu\nu,\Sigma} + \epsilon_{\mu\nu\rho\sigma}R_{\Lambda\Sigma}F^{\mu\nu,\Lambda}F^{\rho\sigma,\Sigma} - V\right)$$

Content: gravity, 3 VM and universal hypermultiplet, prepotential $F = -2i\sqrt{X^0X^1X^2X^3}$

$$k^a_\Lambda = -\{e_0, 2, 2, 2\} \qquad P^3_\Lambda = \{4 - \frac{1}{2}e^{2\phi}e_0, -e^{2\phi}, -e^{2\phi}, -e^{2\phi}\}$$

$$Dq^u = dq^u + k^u_\Lambda A^\Lambda$$

Dual is a $\mathcal{N} = 2$ Chern–Simons matter theory [Benini, Closset, Cremonesi '09], [Jafferis '09]

The Model: M theory on Q^{111}

The linear combination $\zeta = 6A_0 + 2A_1 + 2A_2 + 2A_3$ becomes massive via Higgs mechanism.

Black hole (branes) solutions with $T \geqslant 0$: background for probe analysis. BPS solutions found in [Gauntlett,Donos, '12] [Halmagyi, Petrini, Zaffaroni '14]. Black holes are M2 and M5 branes wrapping noncontractible cycles of the internal manifold

Ansatz: All hypermultiplets scalars except one set to zero. Scalar modes have masses $m^2 = (16, 10, 4, -2, -2, -2, -2)$ corresponding to $\Delta = (6, 5, 4, (2, 1) \times 4)$

Fermions are electrically charged: Dirac-like quantization condition on the black hole magnetic charges

$$P^\Lambda P^3_\Lambda(\overline{u}) \in \mathbb{Z} \qquad P^\Lambda k^u_\Lambda(\overline{u}) \in \mathbb{Z}$$

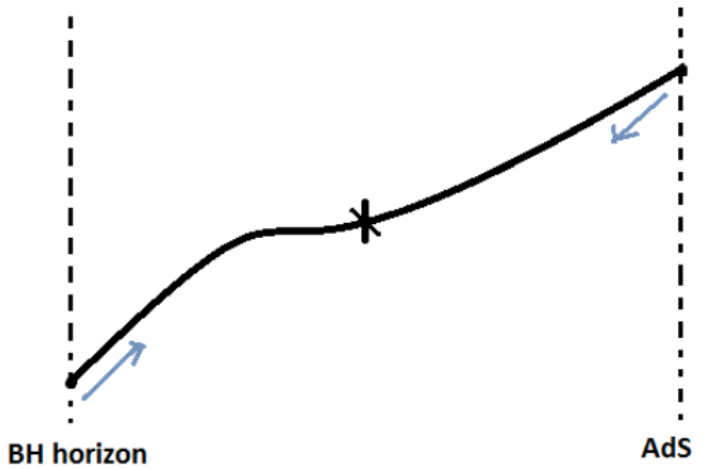
Black hole solutions

Static and spherically symmetric ansatz:

$$ds^2 = -e^{-\beta(r)}h(r)dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\phi_I = \phi_I(r) \qquad A^\Lambda = \tilde{q}^\Lambda dt - P^\Lambda \cos\theta d\phi$$

Maxwell's equations yield $P^\Lambda k^a_\Lambda = 0$. Massive vector is purely electric.

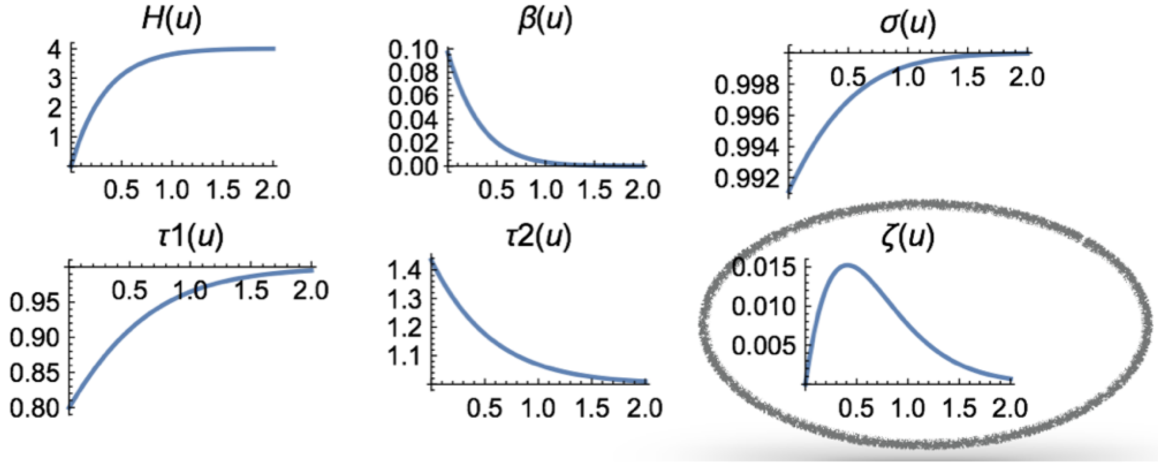


Expansion in series at the black hole horizon and at infinity. Demand to match in between.

In total there are 14 equations. At the end of the day, 7 free parameters = BH charges

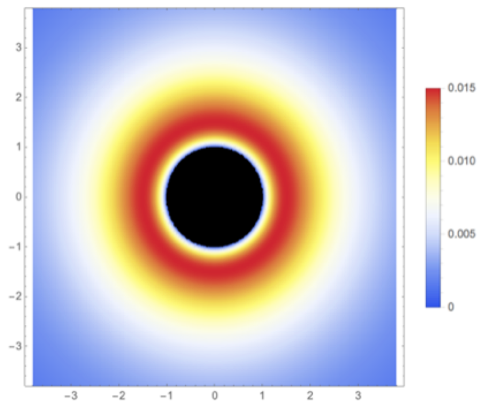
Black hole solutions [Monten, CT, ’17]

Example of electric BH with massive vector and nontrivial scalar profile:



massive vector field

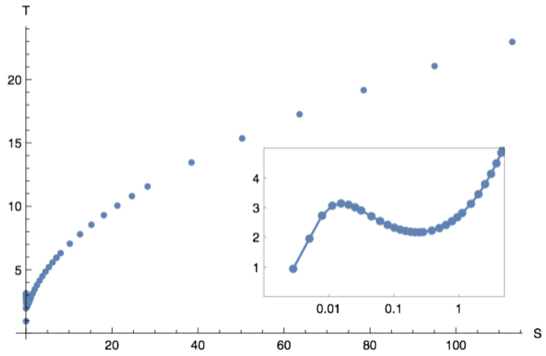
Black hole is surrounded by a massive vector "atmosphere", which hovers outside the black hole horizon

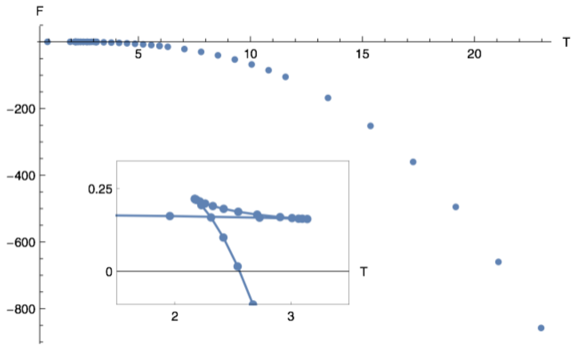


Accuracy of the numerics verified by checking the first law:

$$\delta M = T\delta S + \phi^{\Lambda}\delta Q_{\Lambda} - \chi_{\Lambda}\delta P^{\Lambda}$$

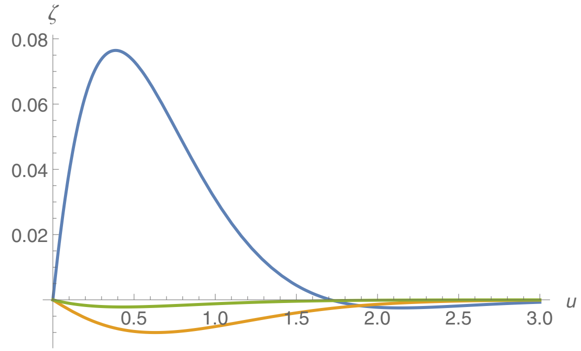
Background thermodynamics





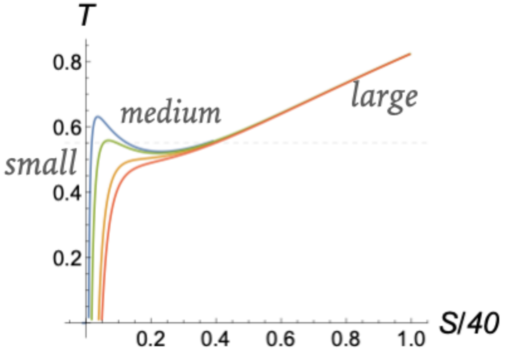
First order small/large black hole phase transition, similar to [Chamblin, Emparan, Johnson, Myers, '99]

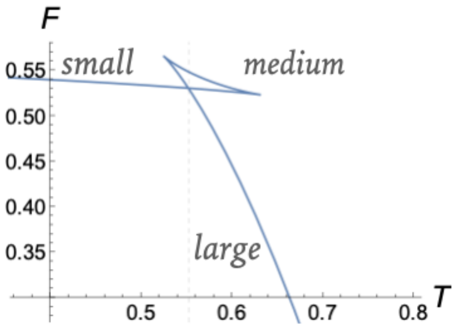
Radial profile for the massive vector field for small black holes, medium and large ones



The condensate is never vanishing, massive vector field always switched on. No restoring of the broken symmetry for a finite temperature $\neq$ holographic superconductor

Background thermodynamics





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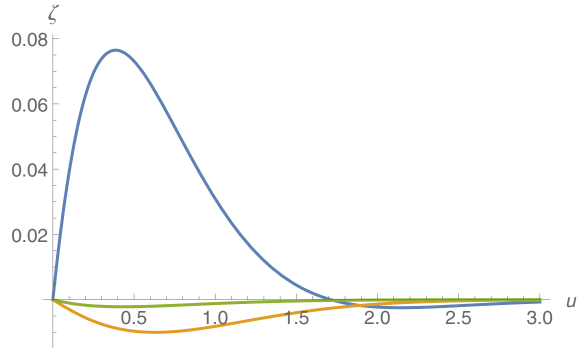

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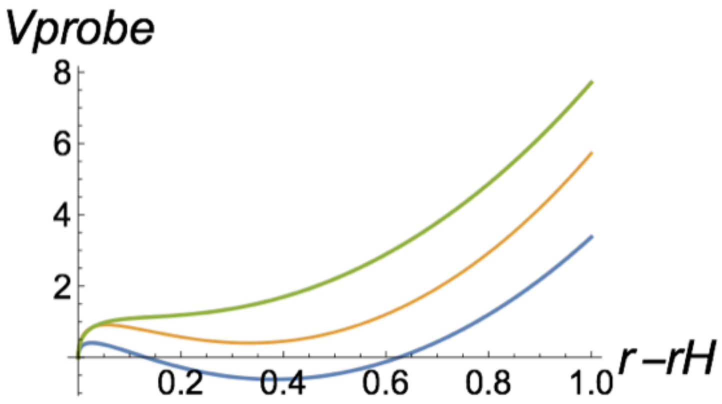
Probes

Expectation: at high temperatures the single-center horizon will be thermodynamically favored (liquid phase), probe will enter the black hole



Vprobe is zero at the black hole horizon.

Probes can be unstable, stable and metastable depending on the minimum of the potential

- Chart parameter space to see where stable probes arise
- Compare with the previous case of uncharged scalar. Effect of the interaction probes - condensate

Probe analysis in various cases [Monten, CT, to appear]

We considered various horizon topologies (planar, spherical, higher genus horizon)

Consider instability towards nucleation of spacetime filling M2 probes or internally wrapped branes (M2 branes of fluxed D6 obtained upon reduction to IIA)

Uplift to 11d SE_7 manifold $Q^{111} = SU(2)^3/U(1)^2$ for solutions **without axions**:

$$ds^2_{11} = e^{2V}\mathcal{K}^{-1}ds^2_4 + e^{-V}ds^2(B_6) + e^{2V}(\theta + A_0)^2 \qquad \theta = d\psi + \frac{1}{4}(\cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 + \cos\theta_3 d\phi_3)$$

with

$$ds^2(B_6) = e^{2U_1}ds^2_{V_1} + e^{2U_2}ds^2_{V_2} + e^{2U_3}ds^2_{V_3} \qquad ds^2_{V_i} = d\theta_i^2 + \sin^2\theta_i d\phi_1^2$$

and

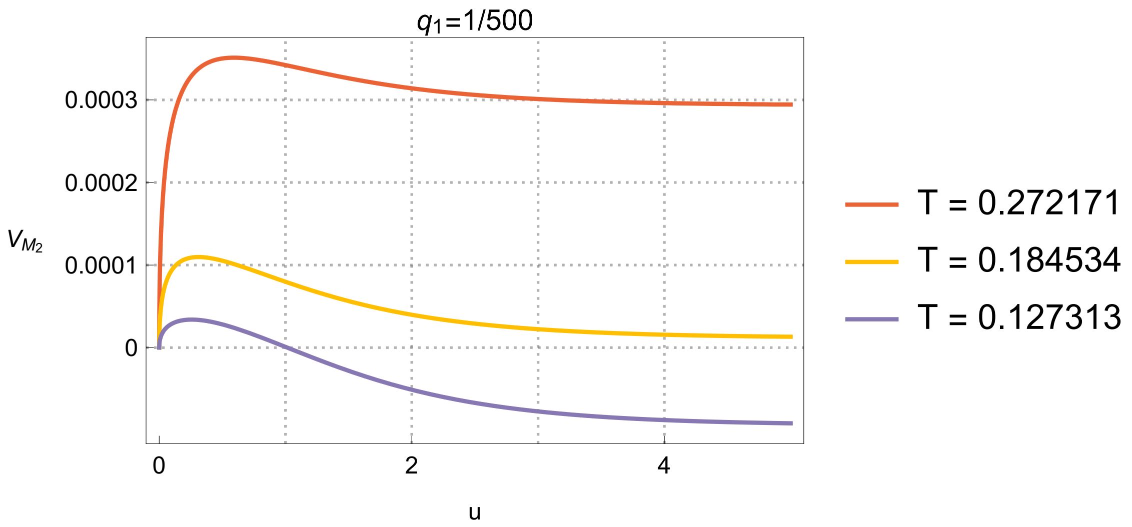
$$F_4 = 6\mathcal{K}^{-1}e^{4\phi} \star 1 + d\tilde{B} \wedge (\theta + A_0) + dA^i \wedge \omega_i \qquad \omega_i = \frac{1}{8}\sin\theta_i d\theta_i d\phi_i$$

Black branes and spacetime filling brane nucleation

Consider M2 branes that fill the spacetime directions t, x, y

$$S_{M2} = -\tau_{M_2}\int d^3x\sqrt{G} \pm \tau_{M_2}\int A_3 = -\tau_{M_2}\int dt dx^1 dx^2 (V_g + V_e)$$

Electric black branes manifest instability towards nucleation of **spacetime filling** M2 branes.

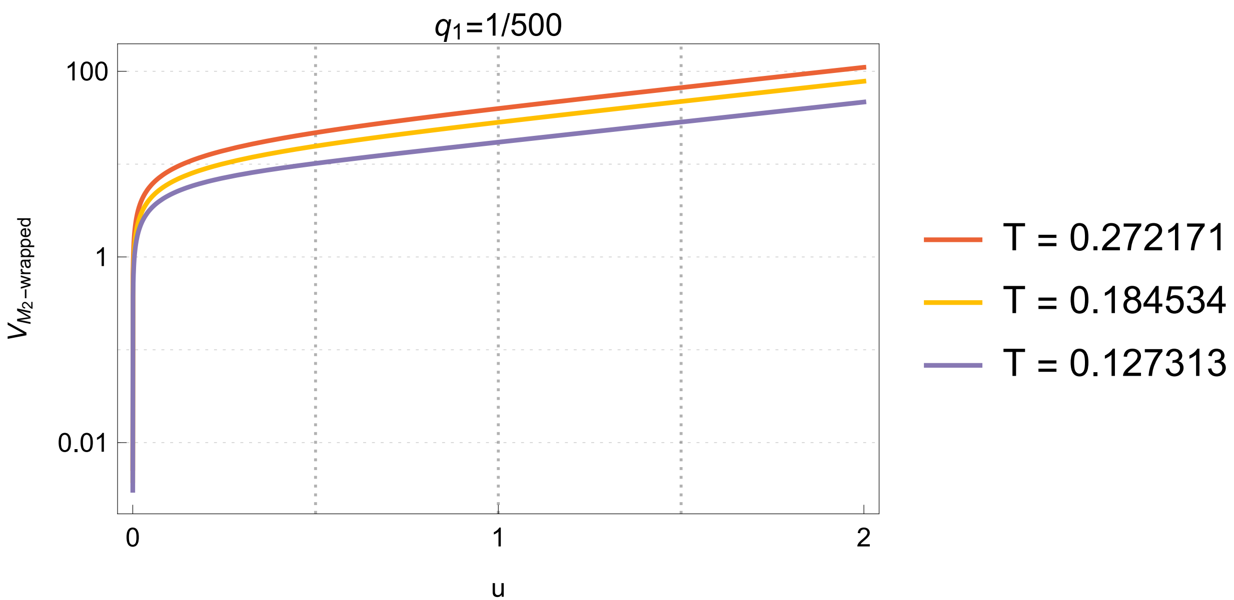


Lower energy by moving away towards the boundary of spacetime. Instability is due to the fact that we switched on the R−symmetry field

Probes: first results for wrapped M2

First easy case: Purely electric spherical black holes with no R-charge.

M2 branes wrapped on minimal volume cycles [Klebanov, Pufu, Tesileanu '12] are **not stable**: no minima of the potential outside horizon



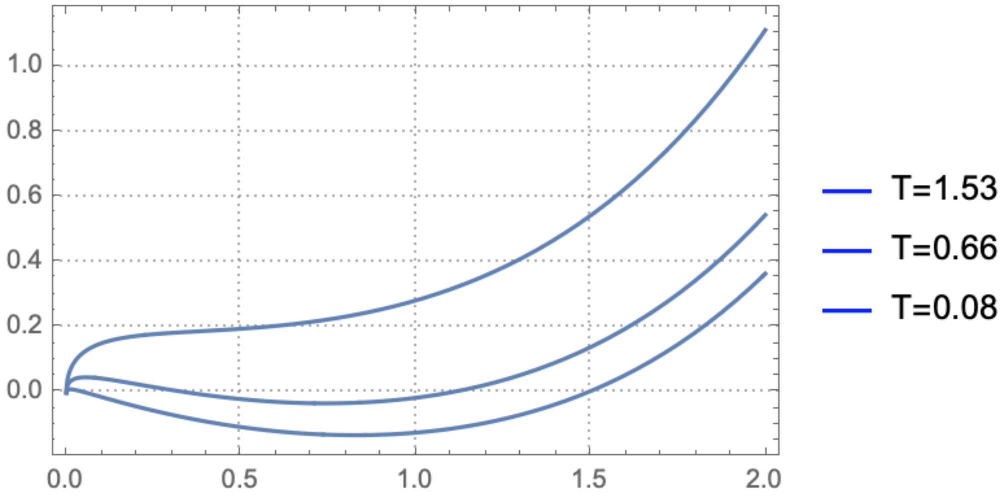
Need to have mutually nonlocal charges. Need to consider other probes

More general probes: preliminary results [Monten, CT, to appear]

DBI action for Fluxed D6 brane (combination of M2, M5 KK monopole)

$$V_{D6} = -\int m_\gamma(z) - \int (q_I A^I - p^I B_I) \qquad m_\gamma = |Z(\gamma, z, \bar{z})|$$

as in [Asplund, Denef, Dzienkowski '15]



Potential for fluxed D6 with $\kappa = 1$ ($q_0 = -.01$, $q_1 = q_3 = .011$, $q_2 = .012$)

Intuition from the FI case on the parameter space where we can find stable probes [Denef et al. '13]

Notice that we set to zero the magnetic component of the Higgsed U(1)

Supersymmetric solutions for Q^{111}

1/4 BPS black hole solutions found by [Halmagyi, Petrini, Zaffaroni '13]. NH geometry of the form $AdS_2 \times \Sigma_g$:

$$ds^2 = \frac{r^2}{R_1^2}dt^2 - \frac{R_1^2 dr^2}{r^2} - R_2^2(d\theta + f'(\theta)d\phi^2)$$

$$A^\Lambda = \tilde{q}_0^\Lambda \, r \, dt + p^\Lambda f(\theta) d\phi \qquad z^i = \tau^i - i b^i$$

NH analytic, full solution found numerically. Conditions for susy (topological twist):

$$p^\Lambda P_\Lambda^3 = -\kappa \qquad p^\Lambda k_\Lambda^u = 0$$

and

$$\mathcal{L}_r^\Lambda P_\Lambda^3 = 0 \qquad \mathcal{L}_i^\Lambda k_\Lambda^u = 0$$

with

$$\mathcal{L}^\Lambda = \mathcal{L}_r^\Lambda + i\mathcal{L}_i^\Lambda = -e^{\mathcal{K}/2} X^\Lambda \, .$$

Supersymmetric solutions for Q^{111}

Solutions without axions and with purely magnetic charges exist for $\kappa = -1$.

Condition for $b^i = q^\Lambda = 0$ is

$$\tau_2 = \frac{3-\tau_1^2}{2\tau_1} \qquad \tau_3 = \tau_1$$

charges are

$$p^0 = -\frac{1}{4\sqrt{2}} \qquad p^2 = \frac{3}{4\sqrt{2}} - 2p^1 \qquad p^1 = -\frac{\tau_1^2\left(\tau_1^2-3\right)}{4\sqrt{2}\left(\tau_1^2+1\right)}$$

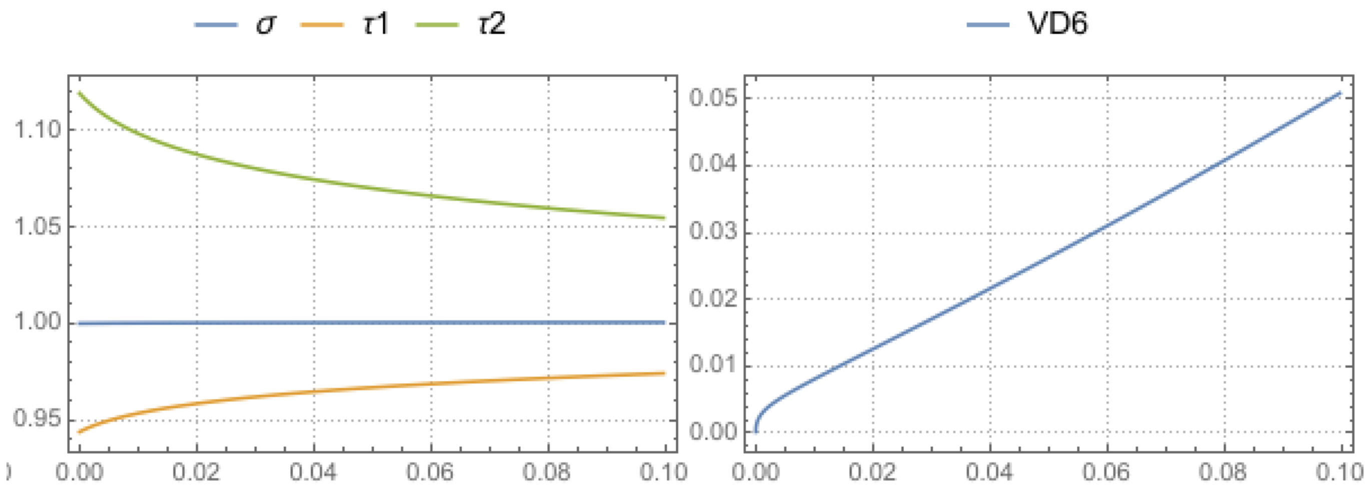
We can read off the NH geometry $AdS_2 \times \Sigma_g$

$$R_1^2 = \frac{\tau_1}{32}\left(3-\tau_1^2\right) \qquad R_2^2 = \frac{\kappa\tau_1^2\left(\tau_1^2-3\right)\left(\tau_1^4-2\tau_1^2+9\right)}{1024\left(\tau_1^2+1\right)}$$

Supersymmetric solutions: probe action

Starting from this NH geometry (and charges & value of the scalars at horizon) we solved the EOMs for solutions to $T = 0$. We managed to get "close" to extremality

$$R_2 \sim 1.0012 \times R_{2,\mathrm{BPS}} \qquad\qquad T \sim 0.005$$

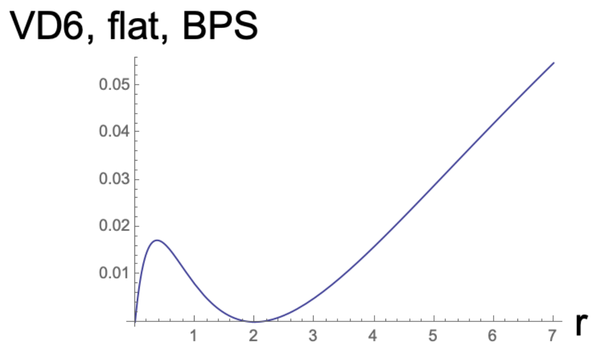


Preliminary results: for these magnetic solutions we do not find stable probes.

Multicenter black holes in the probe approximation

Small black hole / asymptotically flat limit should reproduce the BPS equilibrium separation formula. "Caged wall crossing" due to AdS asymptotics [Denef et al '13]: radius cannot diverge.

$$r_{\rm BPS} = \frac{q_0 - q_1\kappa^2 - q_2\kappa^2 - q_3\kappa^2}{3\kappa^2 - 1}$$



Recap:

- Instability for black branes towards nucleation of spacetime filling probes
- Stable and metastable wrapped fluxed D6 probes for purely electric configurations (D0-D4 in flat space) at small T in a small range of parameters
- For the moment, no stable probes on supersymmetric backgrounds

Microstate counting

Possible evidence for "new physics" (or new saddles) from 4d superconformal index [Benini, Milan '18], [Cabo-Bizet, Murthy '19] for AdS_5 black holes

[Ardehali, Hong, Liu '20] found micro-canonical entropies $S_C(J_{1,2}, Q_a) = S_{BH}(J_{1,2}, Q_a)/C$ for $C = 2, 3, 4, 5$ presumably corresponding to entropies of new black objects in the bulk.

Believed to represent supersymmetric solutions with Lens space horizons (in analogy to the Minkowski supersymmetric Lens [Kunduri, Lucietti '08])

Status for AdS_4?

Entropy matching for static AdS$_4$ black holes

Static AdS$_4$ black holes with uplift in M-theory on S^7 known from [Cacciatori, Klemm '09]

- black holes are 1/4 BPS flows from AdS$_4$ to AdS$_2 \times \Sigma_g$ near horizon geometry
- magnetic gauge field cancels spin connection in the susy equations (topological twist)
- their entropy is function of the charges $S_{BH} = S(Q^I, J)$

Boundary is $S^1 \times \Sigma_g$: ABJM partition function on $S^1 \times \Sigma_g$ with magnetic fluxes s_i on Σ_g computed via susy localization, in the large N limit [Benini, Hristov, Zaffaroni '15], [Benini, Zaffaroni '16]

$$\log Z_{S^1 \times S^2} \approx -\frac{2\pi N^{3/2}}{3} \sqrt{2\Delta_1 \Delta_2 \Delta_3 \Delta_4} \sum_{i=1}^{4} \frac{s_i}{\Delta_i} \qquad\qquad \sum \Delta_i = 2\pi$$

reproduces Bekenstein-Hawking entropy upon extremization on Δ_i.

Refinement by angular momentum

Constructed two analytic classes of supersymmetric stationary AdS black holes [Hristov, Katmadas, **Toldo** '18 '19].

Solution fully determined in terms of sections $e^{\psi}\mathcal{I} = H(r) + j\cos\theta$ and gauge fields, with

$$ds_4^2 = -e^{2U}(dt + \omega\, d\phi)^2 + e^{-2U}(d\rho^2 + e^{2\varphi}(dx^2 + dy^2))$$

Entropy derived from Entropy function $\mathcal{S}$

$$\mathcal{S}(\omega, \Delta_i) = i\frac{4\sqrt{2}}{3}N^{3/2}\frac{\sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}}{\omega} + \sum_I \Delta_I q_I + \omega J \qquad \sum_I 2g_I\Delta^I - \omega = 2\pi i$$

coincides with large N limit of the ABJM superconformal index [Choi, Hwang, Kim, '19] (index found to be $\sim O(1)$ [S. Kim, '09] when fugacities are real)

Exciting! But still some things to understand (i.e. complex fugacities...)

Status of microstate counting: evidence for multi-center?

- Presence of baryonic symmetries in quivers dual to Q^{111} and M^{111} makes the computation of the index problematic

- Still too early to say – also for ABJM – so far no studies revealed new saddles

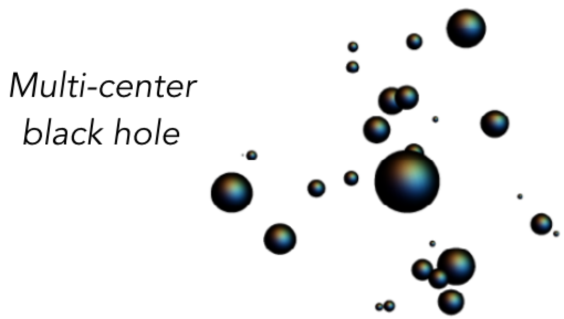
- Need for a better understanding of the origin of the complex fugacities (progress in [Cabo-Bizet et al '18] [Cassani, Papini '19])

Conclusions and perspectives/1

Chart the parameter space of probe charges looking for stable and metastable probes. We have found stable (non-supersymmetric) probes which upon backreaction could turn into black hole bound states

- Interpretation of the instability for spacetime filling branes nucleation (similar to [Henriksson, Hoyos, Jokela '19] for branes in T^{11}).

- Multiple black holes induce electric and magnetic dipole charges, corresponding to inhomogeneities in charge densities and magnetic fields in the dual field theory
  $\rightarrow$ High viscosity? to be verified



Conclusions and perspectives/2

More...

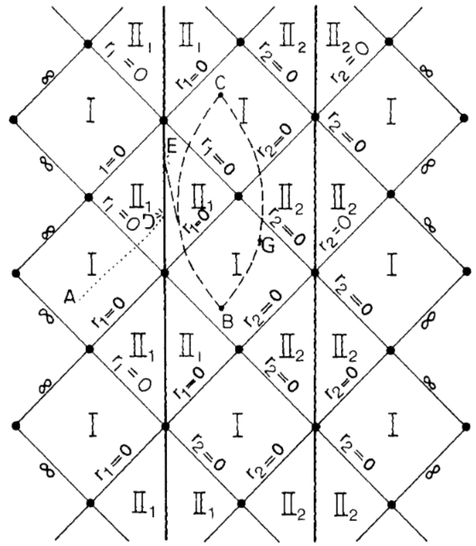
- Subtleties regarding boundary conditions and baryon operators in ABJM-like theories [Bergman, Tachikawa, Zafrir '20]
- Understand microstates for quivers such as Q^{111} in presence of baryonic and mesonic charges

More generally:

- more general 5d supersymmetric solutions? e.g. wrapped branes on solutions from $T^{1,1}$
- exotic horizons i.e. susy black saturns in 5d [Armas, Gnecchi, **Toldo**, to appear]

the end. Thank you!

Penrose diagram for Majumdar-Papapetrou solutions
[Hartle-Hawking, Comm.Math.Phys 1972]



BPS squaring for stationary configurations, gauged sugra:

$$\begin{aligned} S_{4D} = -\frac{1}{16\pi}\int dt \int_{\mathbb{R}^3} \Big[& (\mathcal{E},\mathcal{E}) - 4\,(Q + d\alpha + 2e^{-U}\,\mathrm{Re}\,Z(\star\mathcal{G}) + \tfrac{1}{2}e^{2U}\star d\omega) \wedge \mathrm{Im}\langle \mathcal{E}, e^{U}e^{-i\alpha}\mathcal{V}\rangle \\ & - 2\left[\langle \mathcal{F} + 2\,\mathrm{Re}\,d(e^{U}e^{-i\alpha}\mathcal{V}\,\omega), \star\mathcal{G}\rangle - \star 1\right] \\ & - 2\left(\star d\psi - 2\,e^{-U}\,\mathrm{Im}(e^{-i\alpha}Z(\mathcal{G}))\right)\wedge\left(d\psi - 2\,e^{-U}\,\mathrm{Im}(e^{-i\alpha}Z(\star\mathcal{G}))\right) \\ & + 4e^{-U}e^{2\psi}\,\mathrm{Im}(e^{-i\alpha}Z(G))\,d\left(e^{-2\psi}\star\eta\right)\Big]. \end{aligned}$$