"COMPUTING SUPERGRAVITY SCATTERING AMPLITUDES FROM CFT"

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BASED ON A SERIES OF PAPERS w/ Aharony, Alday, Fardelli, Georgoudis, Perlmutter

IDEA

AMPLITUDES IN AdS

AdS CFT correspondence

CFT CORRELATION FUNCTION AT LARGE N

- UNDERSTAND THE STRUCTURE OF AMPLITUDES IN CURVED SPACE
- DIFFICULT TO ACCESS WITH USUAL FEYNMAN DIAGRAMS TECHNIQUES
- UNDERSTAND BETTER THE DYNAMICS OF HOLOGRAPHIC CFTS, BEYOND LEADING ORDER.

METHOD

- STUDY CFT CONSISTENCY CONDITIONS AND USE CONFORMAL BOOTSTRAP TECHNIQUES (IN ADDITION TO SUSY) TO FIND SCATTERING AMPLITUDES IN AdS.

PLAN

- REVIEW THE METHOD & SEE HOW TO USE THE METHOD TO STUDY HOLOGRAPHIC CFTs
- INTRODUCE SUSY AND SEE WHAT CHANGES TO MAKE
- CONNECT THE CFT WITH SUGRA

METHOD

1. CONSIDER A SCALAR OPERATOR θ OF CONFORMAL DIMENSION Δ_θ IN A D-DIMENSIONAL CFT.

2. THE FOUR POINT FUNCTION OF 4 IDENTICAL θ IS FIXED BY CONFORMAL SYMMETRY TO BE

$$\langle \theta(x_1)\,\theta(x_2)\,\theta(x_3)\,\theta(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta\theta}\;x_{34}^{2\Delta\theta}}$$

WITH $u = \dfrac{x_{12}^2\,x_{34}^2}{x_{13}^2\,x_{24}^2}$ $\quad v = \dfrac{x_{14}^2\,x_{23}^2}{x_{13}^2\,x_{24}^2}$ $\rightarrow$ CROSS RATIOS

3. USE ASSOCIATIVITY OF THE OPE:

$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}(x_4)\rangle = \langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}(x_4)\rangle$$

$x_1 \leftrightarrow x_3$

$$\frac{G(u,v)}{x_{12}^{2\Delta_{\mathcal{O}}}\,x_{34}^{2\Delta_{\mathcal{O}}}} = \frac{G(v,u)}{x_{23}^{2\Delta_{\mathcal{O}}}\,x_{14}^{2\Delta_{\mathcal{O}}}}$$

$$\boxed{G(u,v) = \left(\frac{u}{v}\right)^{\Delta_{\mathcal{O}}} G(v,u)}$$

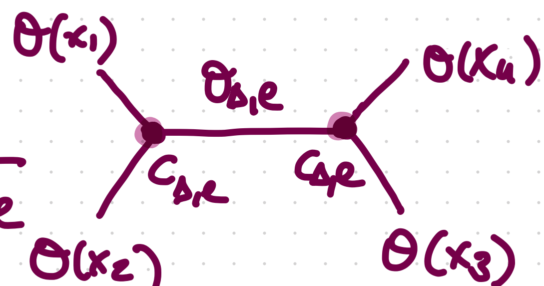
4. USE CONFORMAL BLOCK DECOMPOSITION TO EXPRESS:

$$G(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell} \, \underbrace{u^{\frac{\Delta-\ell}{2}} \, g_{\Delta\ell}(u,v)}_{\text{CONFORMAL BLOCKS:}}$$

1. REPACK THE CONTRIBUTION OF A PRIMARY OF DIMENSION Δ AND SPIN ℓ AND ALL ITS DESCENDANTS

2. COMPLETELY FIXED BY CONFORMAL SYMMETRY

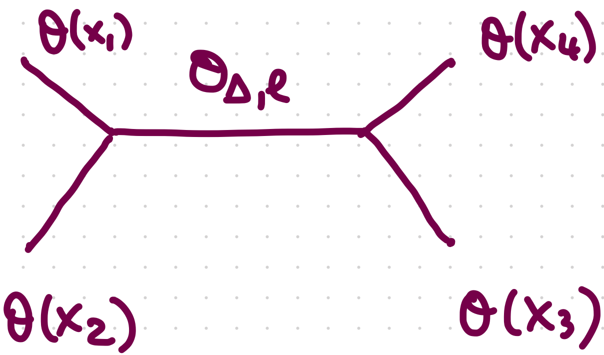
$$G(u,v) = \sum_{\Delta\ell}$$



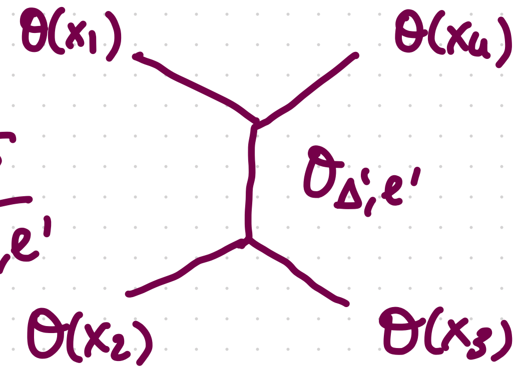
$$C^2_{\Delta,\ell} = a_{\Delta,\ell}$$

4.1 THUS WE CAN REWRITE ASSOCIATIVITY OF THE OPERATOR PRODUCT EXPANSION AS

$\sum_{\Delta,\ell}$



$= \sum_{\Delta',\ell'}$



5. PROPERTIES OF CONFORMAL BLOCKS

$$\lim_{u\to 0} u^{\frac{\Delta-\ell}{2}} g_{\Delta\ell}(u,v) \quad \text{IS EXPLICIT SINCE} \quad g_{\Delta,\ell}(u,v) \sim u^{\#} v^{\#}$$

$$\lim_{v\to 0} u^{\frac{\Delta-\ell}{2}} g_{\Delta,\ell}(u,v) = A_{\Delta,\ell}(u,v) + B_{\Delta\ell}(u,v) \log v$$

6. SEE THE CONSEQUENCES OF THESE PROPERTIES TO THE CONFORMAL BOOTSTRAP EQUATION:

$$\mathcal{G}(u,v) = \left(\frac{u}{v}\right)^{\Delta_\phi} \mathcal{G}(v,u)$$

$$\sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, g_{\Delta\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_\phi} \sum_{\Delta\ell} a_{\Delta\ell}\, v^{\frac{\Delta-\ell}{2}}\, g_{\Delta\ell}(v,u)$$

$$= \left(\frac{u}{v}\right)^{\Delta_\phi}\left(a_{0,0}\, g_{0,0}(v,u) + a_{2,0}\, v\, g_{2,0}(v,u) + \dots\right)$$

TAKE THE LIMIT $v \to 0$

EACH $g_{\Delta,\ell}(u,v) \to \log v$ $\quad u^{\Delta_\phi} v^{-\Delta_\phi} \dots + u^{\Delta_\phi} v^{-\Delta_\phi+1} + \dots$

NEED INFINITELY MANY OPERATORS!

7. WE NEED TO REPRODUCE POWER LAW DIVERGENCES ON THE LHS WITH LOGARITMS ON RHS

$$\sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, g_{\Delta,\ell}(u,v)$$

$$\downarrow$$

- NEED TO HAVE INFINITELY MANY OPERATORS WITH $\Delta = 2\Delta_\phi + \ell + 2n$ AND WITH LARGE SPIN

$$\downarrow$$

- FIXES THE $a_{\Delta,\ell}$ FOR LARGE ℓ AND THE CORRECTIONS TO THE DIMENSIONS AS POWERS OF $\ell^{-\pi}$.

THIS METHOD CAN BE CARRIED OUT FOR EACH INVERSE POWER OF ℓ, AND THE SERIES CAN BE RESUMMED AND IT IS VALID ALSO FOR FINITE SPINS. (ANALYTICITY IN THE SPIN).

LARGE N EXPANSION - SETUP

- CONSIDER A CFT WITH A LARGE GAP WHICH ADMITS A LARGE N EXPANSION. ASSUME THAT THE OPE:

$$\mathcal{O} \times \mathcal{O} = 1 + \cancel{\mathcal{O}} + \underbrace{\cancel{T_{\mu\nu}}}_{\substack{\text{STRESS} \\ \text{TENSOR}, \, \Delta = d \;\; \ell = 2}} + [\mathcal{O}\mathcal{O}]_{n,\ell} + \cancel{[T\mathcal{O}]}_{n,\ell} + \cancel{[TT]}_{n,\ell} + \dots$$

$[\mathcal{O}\mathcal{O}]_{n,\ell} \to$ DOUBLE TRACE OPS

$$\mathcal{O} \, \Box^n \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O} + \dots$$

$$2\Delta_{\mathcal{O}} + 2n + \ell$$

- ASSUME THAT THERE IS A $\mathbb{Z}_2$ SYMMETRY
- ASSUME THE SIMPLEST SETUP IN WHICH $T_{\mu\nu}$ DOES NOT APPEAR
- GENERICALLY THERE ARE ALSO HIGHER TRACES

EXPANSION

- EXPAND THE CFT DATA AND THE FOUR POINT FUNCTION FOR LARGE N:

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \ldots$$

$$\Delta_{n,\ell} = \Delta^{(0)}_{n,\ell} + \frac{1}{N^2}\gamma^{(1)}_{n,\ell} + \frac{1}{N^4}\gamma^{(2)}_{n,\ell} + \ldots$$

$$a_{n,\ell} = a^{(0)}_{n,\ell} + \frac{1}{N^2}a^{(1)}_{n,\ell} + \frac{1}{N^4}a^{(2)}_{n,\ell} + \ldots$$

- REMEMBER THAT CROSSING NEEDS TO BE SATISFIED ORDER BY ORDER IN N.

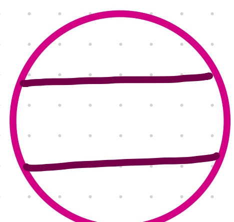
ORDER N^0

- THE FOUR POINT FUNCTION AT THIS ORDER IS ONLY MADE BY THE DISCONNECTED PART:

$$\mathcal{G}^{(0)}(u,v) = 1 + u^{\Delta_\phi} + \left(\frac{u}{v}\right)^{\Delta_\phi}$$

- WE CAN DECOMPOSE THIS IN CONFORMAL BLOCKS TO FIND $a^{(0)}_{n,\ell}$ AND $\Delta^{(0)}_{n,\ell}$

$$\Delta^{(0)}_{n,\ell} = 2\Delta_\phi + 2n + \ell \qquad a^{(0)}_{n,\ell}$$

- WITTEN DIAGRAM  + CROSSED

ORDER $1/N^2$

- WE HAVE TO EXPAND THE PIECE $\sum_{n,\ell} a_{n,\ell}\, u^{\frac{\Delta-\ell}{2}} g_{n,\ell}(u,v)$ AT ORDER N^{-2}

$$G^{(1)}(u,v) = \sum_{n,\ell} u^{\Delta_0+n} \left(a^{(1)}_{n,\ell} + \frac{1}{2} a^{(0)}_{n,\ell} \gamma^{(1)}_{n,\ell} \left(\log u + \frac{\partial}{\partial n} \right) \right) g_{2\Delta_0+n+\ell,\ell}(u,v)$$

- WE WOULD LIKE TO UNDERSTAND HOW TO COMPUTE $\gamma^{(1)}_{n,\ell}$ AND $a^{(1)}_{n,\ell}$

$\downarrow$

- USE CROSSING SYMMETRY
- THERE ARE 4 TERMS : $\log u$
  $\log v$
  $\log u \log v$
  no logs
- ALL OF THEM CAN BE REPRODUCED BY FINITELY MANY BLOCKS;

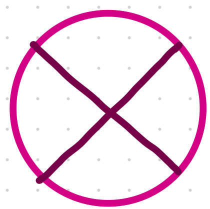
ORDER $1/N^2$. 1

- THIS MEANS THAT $\gamma^{(1)}_{n,\ell} \neq 0$ FOR $\ell = 0,2,4 \ldots L$

  $a^{(1)}_{n,\ell} \neq 0$ FOR $\ell = 0,2,4 \ldots L$

- IT IS POSSIBLE TO SEE THAT

$$a^{(1)}_{n,\ell} = \frac{1}{2} \frac{\partial}{\partial n} \left(a^{(0)}_{n,\ell} \, \gamma^{(1)}_{n,\ell} \right)$$



CONTACT DIAGRAMS

ORDER $1/N^4$

- NOW LET'S GO TO NEXT ORDER (SAME IDEA AS BEFORE)

$$g^{(2)}(u,v) = \sum_{n,\ell} u^{\Delta_\phi + n} \Big(a^{(2)}_{n,\ell} + \frac{1}{2} a^{(0)}_{n,\ell} \gamma^{(2)}_{n,\ell} \left(\log u + \frac{\partial}{\partial n} \right)$$

$$+ \frac{1}{2} a^{(1)}_{n,\ell} \gamma^{(1)}_{n,\ell} \left(\log u + \frac{\partial}{\partial n} \right)$$

$$+ \frac{1}{8} a^{(0)}_{n,\ell} \left(\gamma^{(1)}_{n,\ell}\right)^2 \left(\log^2 u + 2 \log u \frac{\partial}{\partial n} + \frac{\partial^2}{\partial n^2} \right) \Big)$$

$$g_{2\Delta_\phi + 2n + \ell, \ell}(u,v)$$

- SAME AS PREVIOUS ORDER BUT $\left(a^{(1)}_{n,\ell}, \gamma^{(1)}_{n,\ell}\right) \leftrightarrow \left(a^{(2)}_{n,\ell}, \gamma^{(2)}_{n,\ell}\right)$
- UNDER CROSSING THIS TERM MAPS INTO a $\log^2 v$

ORDER $1/N^4$. 1

- LET'S TRY TO UNDERSTAND HOW TO COMPUTE $\gamma^{(2)}_{n,\ell}$

1) REMEMBER THAT EACH CONFORMAL BLOCK AS $v \to 0$ DIVERGES AS $\log v$

2) WE ARE BACK TO THE PROBLEM THAT WE HAD BEFORE. IN ORDER TO REPRODUCE THE $\log^2 v$ WE NEED TO HAVE INFINITELY MANY CONFORMAL BLOCKS.

THE INTERESTING FACT IS THAT

$$\mathcal{G}^{(2)}(u,v)\Big|_{\log^2 u} = \sum_n \sum_{\ell=0}^{L} u^{\Delta_0+n} \frac{1}{8} \underbrace{a^{(0)}_{n,\ell} \left(\gamma^{(1)}_{n,\ell}\right)^2}_{} g_{2\Delta_0+2n+\ell,\ell}(u,v)$$

$$= u^{\Delta_0}\left(f(u,v) \log v + g(u,v) \right)$$

$\hookrightarrow$ DEPENDS ON DATA AT ORDER N^0 AND N^{-2}

3) CROSSING MAPS $\quad G^{(2)}(u,v) = \left(\frac{u}{v}\right)^{\Delta_0} G^{(2)}(v,u)$

THUS $G^{(2)}(u,v)$ SHOULD CONTAIN A TERM LIKE:

$$G^{(2)}(u,v) = u^{\Delta_0} \log^2 v \left(f(v,u) \log u + g(v,u) \right) + \dots$$

Q: FROM WHICH TERM OF THE CONFORMAL BLOCK DECOMPOSITION THIS CAN COME FROM?

$$G^{(2)}(u,v) = \sum_{n,\ell} u^{\Delta_0+n} \Big(a^{(2)}_{n,\ell} + \boxed{\frac{1}{2} a^{(0)}_{n,\ell} \gamma^{(2)}_{n,\ell} \left(\log u + \frac{\partial}{\partial n}\right)} \;!$$

$$+ \frac{1}{2} a^{(1)}_{n,\ell} \gamma^{(1)}_{n,\ell} \left(\log u + \frac{\partial}{\partial n}\right)$$

$$+ \frac{1}{8} a^{(0)}_{n,\ell} \left(\gamma^{(1)}_{n,\ell}\right)^2 \left(\log^2 u + 2 \log u \frac{\partial}{\partial n} + \frac{\partial^2}{\partial n^2}\right)\Big)$$

$$g_{2\Delta_0+2n+\ell,\ell}(u,v)$$

TRUNCATED IN THE SPIN $\downarrow$ no $\log^2 v$!

4) PROVIDES AN EQUATION FOR $\gamma^{(2)}_{n,\ell}$

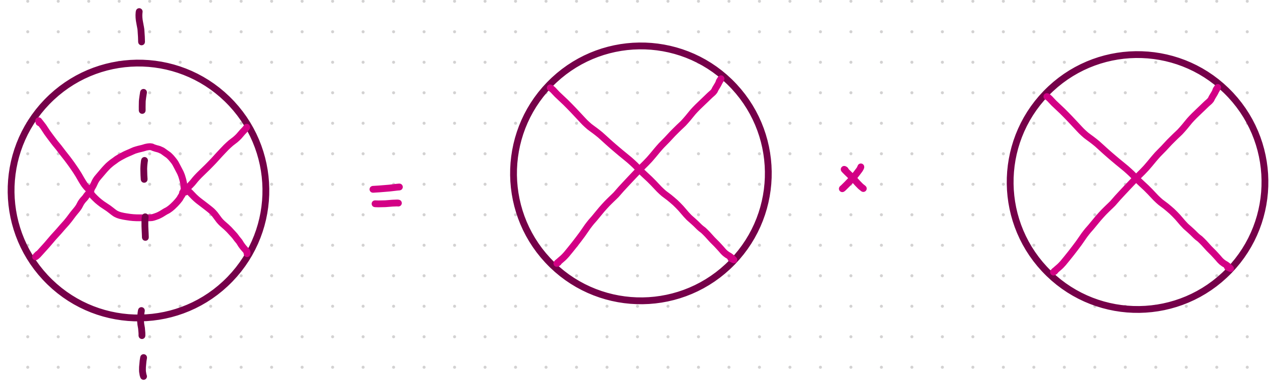
$$\boxed{\sum_{n,\ell} u^n\, a^{(0)}_{n,\ell}\, \gamma^{(2)}_{n,\ell}\, g_{2\Delta_\phi+2n+\ell,\ell}(u,v)\Big|_{\log^2 v} = f(v,u)}$$

5) THERE IS A SYSTEMATIC WAY TO EXRACT $\gamma^{(2)}_{n,\ell}$ (and $a^{(2)}_{n,\ell}$) FROM THIS EQUATION.

MAIN MESSAGE: FROM $a^{(0)}_{n,\ell}$ AND $\gamma^{(1)}_{n,\ell}$ IT IS POSSIBLE TO COMPUTE $a^{(2)}_{n,\ell}$ AND $\gamma^{(2)}_{n,\ell}$.

AMPLITUDES

THIS OBSERVATION IS VERY REMINISCENT OF THE UNITARITY METHODS IN AMPLITUDES.



AdS/CFT CORRESPONDENCE

TYPE IIB SUPERSTRING THEORY ON $AdS_5 \times S^5$

$\mathcal{N}=4$ SYM in 4 DIMENSIONS WITH GAUGE GROUP $SU(N)$

- SCATTERING AMPLITUDES OF STRING STATES

$\downarrow$

GRAVITON

- CORRELATION FUNCTIONS OF LOCAL OPERATORS

$\downarrow$

STRESS TENSOR

(g_s, α') $\quad \alpha' = \frac{R^2}{\sqrt{\lambda}}$ $\quad \lambda = g_{YM}^2 N$ $\quad (g_{YM}, N)$

$\boxed{\text{SUPERGRAVITY APPROXIMATION}}$: LARGE N AND LARGE λ

ADD SUSY

- IT IS POSSIBLE TO APPLY THIS METHOD TO CFTs WITH SUSY.
- LET'S CONSIDER $\mathcal{N}=4$ SYM IN 4 DIMENSIONS.
- SINCE WE WOULD LIKE TO UNDERSTAND HOW TO DEAL WITH AMPLITUDES OF GRAVITONS, WE HAVE TO CONSIDER THE 4 POINT FUNCTION OF $T_{\mu\nu}$ (STRESS TENSOR).
- LUCKILY, SUPERSYMMETRY HELPS US AND THE SUPERCONFORMAL PRIMARY OF THE SUPERMULTIPLET OF $T_{\mu\nu}$ IS A $\frac{1}{2}$ BPS SCALAR OPERATOR $\mathcal{O}^{(2)}$

$\mathcal{N}=4$ SYM

- CONSIDER THE 4 POINT FUNCTION OF $\mathcal{O}^{(2)}$ → PROTECTED OPERATOR TRANSFORMING UNDER THE $\underline{20'}$ OF $SO(6)$ R-SYMMETRY $([0,2,0])$

$$\langle \mathcal{O}^{(2)}(x_1,y_1)\,\mathcal{O}^{(2)}(x_2,y_2)\,\mathcal{O}^{(2)}(x_3,y_3)\,\mathcal{O}^{(2)}(x_4,y_4)\rangle =$$

y_i ↓ R SYMMETRY COORDINATES (NULL VECTORS)

$$= \frac{(y_1\cdot y_2)(y_3\cdot y_4)}{x_{12}^4\;x_{34}^4} \sum_{R} \mathcal{G}^{(R)}(u,v,y_i)$$

R ↳ $[0,2,0]\times[0,2,0]$

SUPERSYMMETRY & CAVEATS

- SUPERCONFORMAL WARD IDENTITIES PROVIDE A SET OF EQUATIONS WHICH ALLOW US TO WRITE THE SIX COMPONENTS $\mathcal{G}^{(R)}(u,v,y_i)$ IN TERMS OF ONLY ONE FUNCTION

$$\mathcal{G}(u,v) \equiv \mathcal{G}^{[0,4,0]}(u,v) = \mathcal{G}^{105}(u,v)$$

- PRESENCE OF PROTECTED OPS IN THE OPE $\left(\frac{1}{2} \text{ and } \frac{1}{4} \text{ BPS}\right)$

$$\theta^{(2)} \times \theta^{(2)} = \text{LONG} + \text{SHORT}$$

$\downarrow$

- DIMENSION PROTECTED
- $\langle \theta^{(2)}\, \theta^{(2)}\ \tfrac{1}{2}\text{ BPS} \rangle$
  $\langle \theta^{(2)}\, \theta^{(2)}\ \tfrac{1}{4}\text{ BPS} \rangle$

ONLY FUNCTIONS OF N (NOT λ)

$$\mathcal{G}(u,v) = \mathcal{G}^{\text{SHORT}}(u,v) + \mathcal{G}^{\text{LONG}}(u,v)$$

CAVEATS . 2

- $$\mathcal{G}^{\text{LONG}}(u,v) = \sum_{\Delta,\ell} a^{\text{LONG}}_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, g_{\Delta+4,\ell}(u,v)$$

$\sum_{\Delta,\ell}$ → DOUBLE TRACE OPERATORS
(ONLY LONG OPERATORS IN SUGRA)

$g_{\Delta+4,\ell}$ ← SUPERCONFORMAL BLOCKS

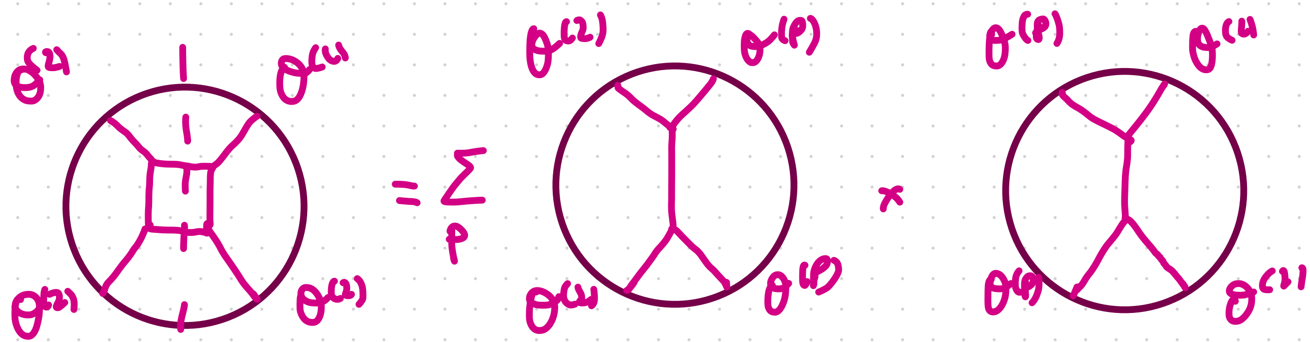
$$[\mathcal{O}^{(2)}\mathcal{O}^{(2)}]_{n,\ell} \rightarrow 2\cdot 2 + 2n + \ell \qquad [\mathcal{O}^{(3)}\mathcal{O}^{(3)}]_{n-1,\ell}$$
$$3+3+2(n-1)+\ell$$

MIXING PROBLEM → THERE IS AN INFINITE NUMBER OF OPS WITH THE SAME QUANTUM NUMBERS → MIXING

$$a^{(0)}_{n,\ell,I} \qquad \gamma^{(1)}_{n,\ell,I} \qquad \langle \mathcal{O}^{(2)}\mathcal{O}^{(2)}\mathcal{O}^{(p)}\mathcal{O}^{(p)}\rangle$$

STRATEGY

- USE EXACTLY THE SAME METHOD AS BEFORE TO COMPUTE $\gamma^{(2)}_{n,\ell}$ AND EVENTUALLY EVEN SOME ALL LOOPS $\gamma^{(k)}_{n,\ell}$
- COMPUTE THE 4 POINT FUNCTION ⇒ AMPLITUDE
- CHECKS IN FLAT SPACE
- AGAIN SIMILAR UNITARITY RULES



RESULTS