Anyonic molecules in atomic fractional quantum Hall liquids:

a quantitative probe of
fractional charge and anyonic statistics

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Quantum Science and Technology in Trento

PROVINCIA AUTONOMA DI TRENTO

BEC

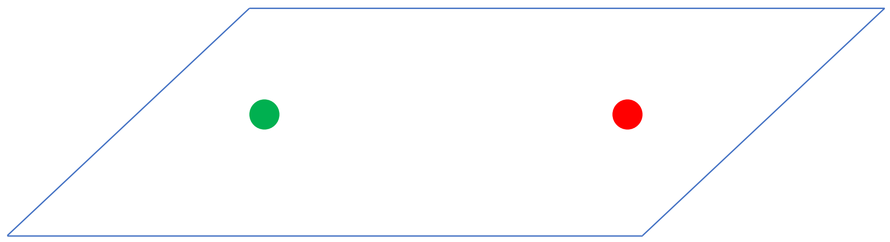
UNIVERSITÀ DI TRENTO

Fractional statistics

Anyons: quasiparticles which are neither bosons nor fermions can exist in 2D

3D

$\psi(\mathbf{r}_1, \mathbf{r}_2)$



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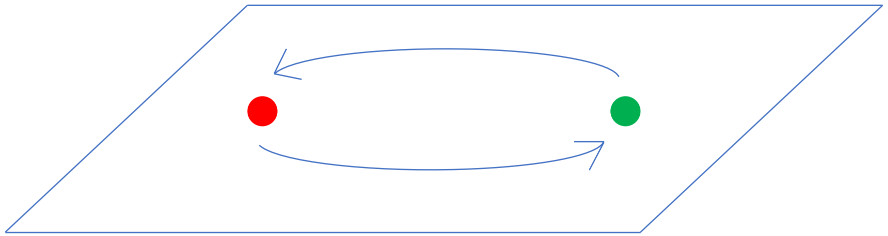
$$\psi(\mathbf{r}_2, \mathbf{r}_1) = e^{i\pi\alpha}\,\psi(\mathbf{r}_1, \mathbf{r}_2)$$



$$|\psi(\mathbf{r}_1, \mathbf{r}_2)|^2 = |\psi(\mathbf{r}_2, \mathbf{r}_1)|^2$$

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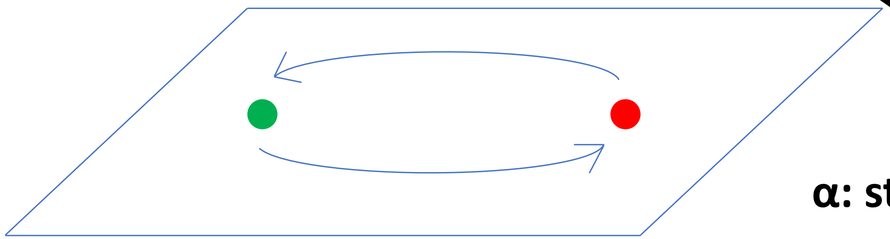
$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{2i\pi\alpha}\psi(\mathbf{r}_1, \mathbf{r}_2) \longrightarrow e^{2\pi i\alpha} = 1$$

$\alpha = 0$ (bosons)

$\alpha = 1$ (fermions)



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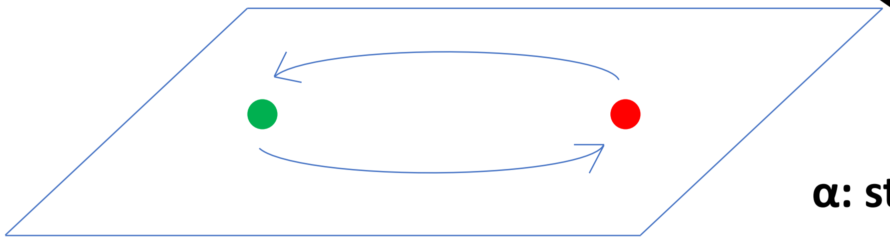

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This is topologically equivalent to not moving the particles at all

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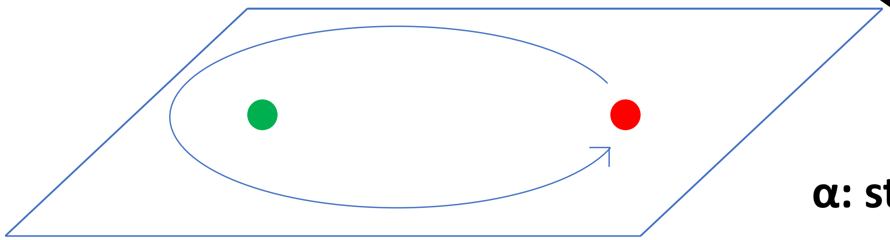

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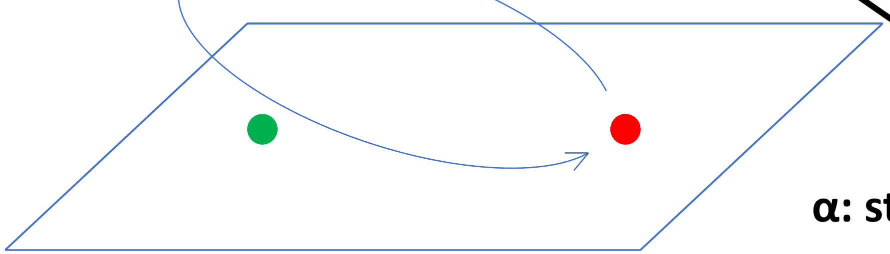

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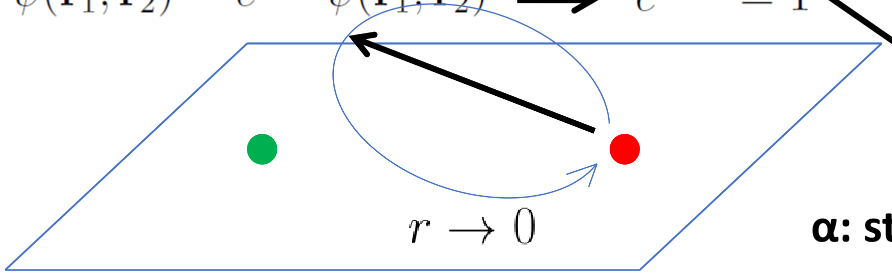

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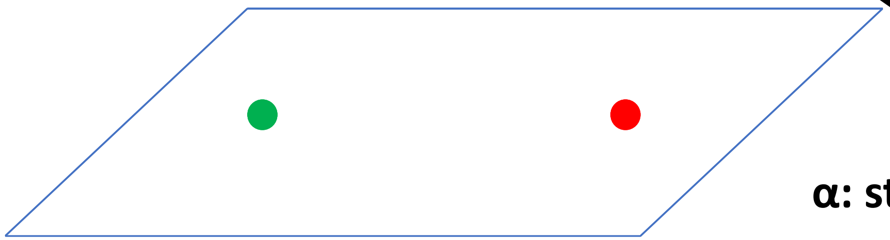

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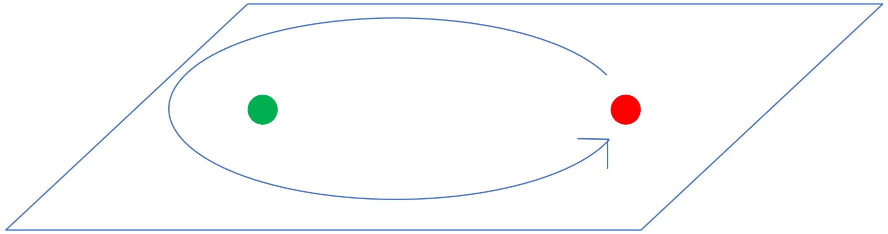

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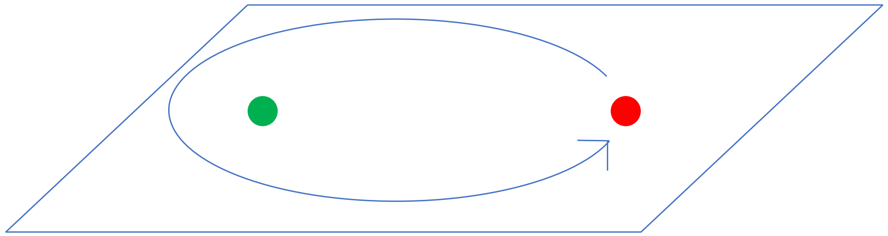


$\alpha \in [0, 2)$ $\quad e^{i\pi\alpha} \in U(1)$

- ***Braiding*** 2 particles produces a **unitary transformation** on the system
- But it's **trivial**: global phase for wavefunction

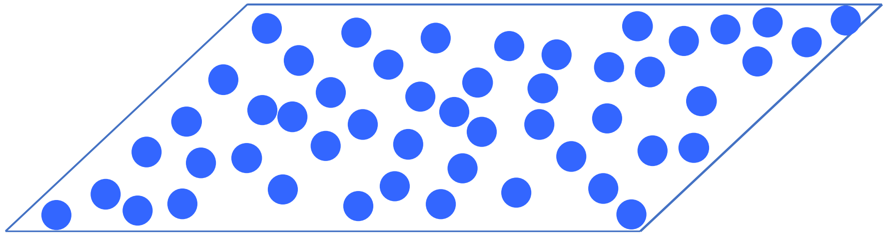
→ **ABELIAN ANYONS**

[*Leinaas and Myrheim, Il Nuovo Cimento B 1977*]

[*Wilczek, PRL 1982*]

Anyons in real life

2D electron gas



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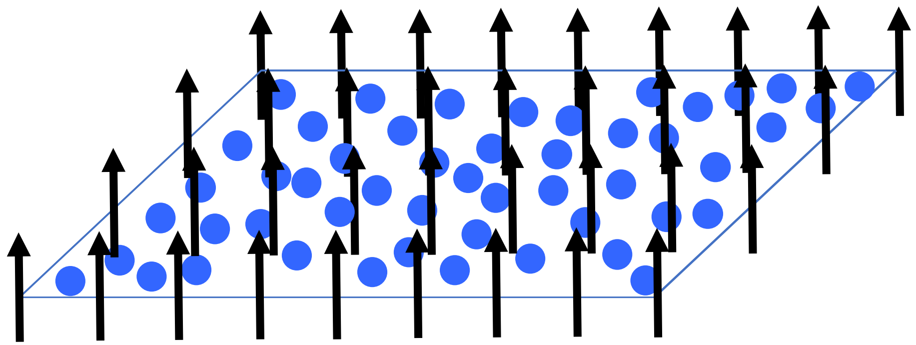

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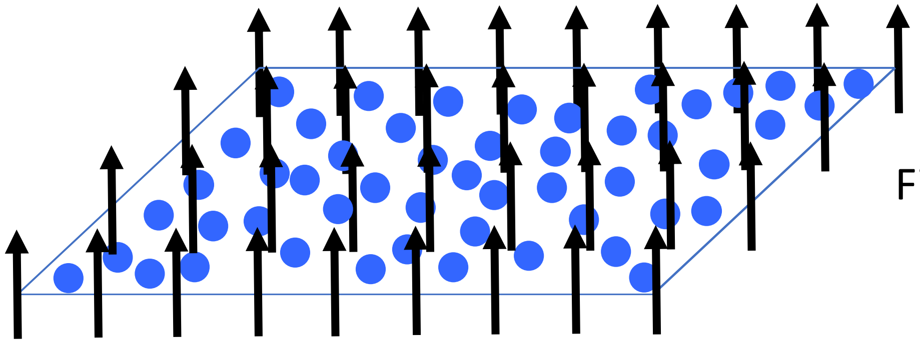
Fractional quantum Hall liquid [*Willett et al., PRL 1987*]



Filling fraction

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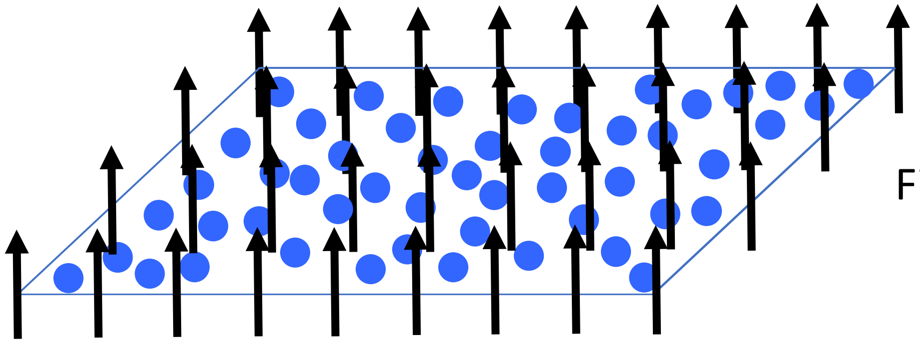

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- Ansatz for ground state [*Laughlin, PRL 1983*]
- **Excitations:** quasi-holes (QHs) and quasi-particles (QPs) with
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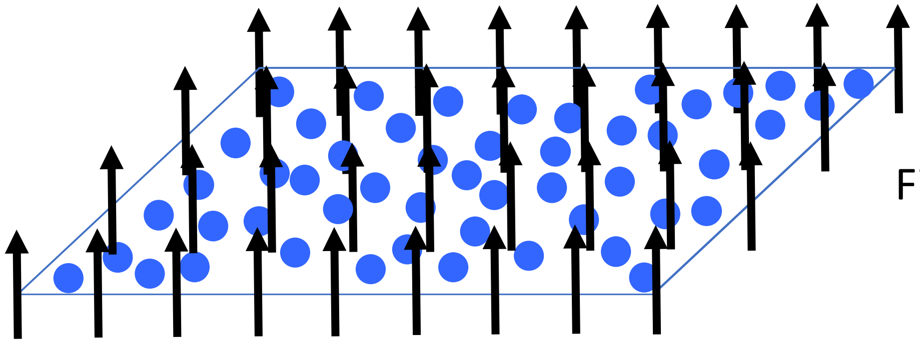

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Shot-noise / interferometric experiments in this setup are difficult to modellize and perform

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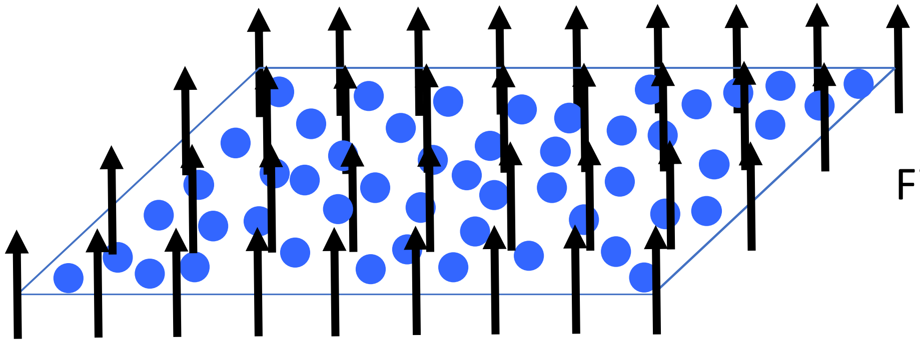

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2D synthetic material

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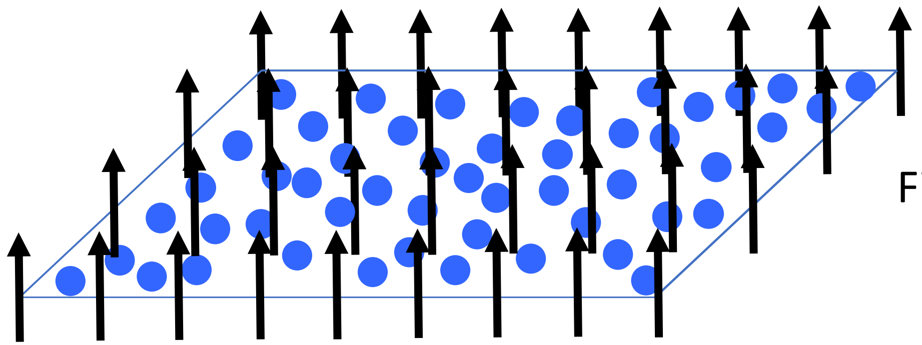

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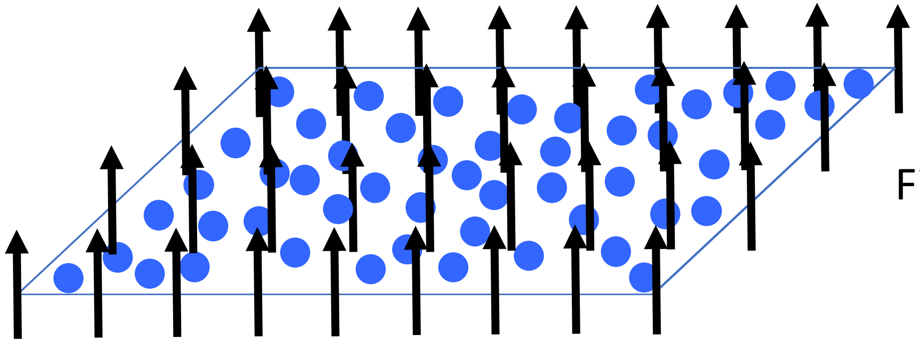

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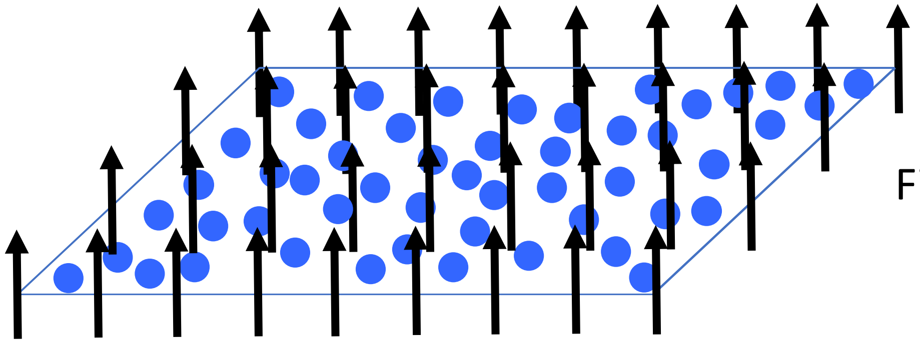

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+ Strong interactions
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Fractional quantum Hall liquid

- **Ultracold atoms**
  - Tunable interactions (Feshbach resonance)
  - Rotation: Coriolis force = Lorentz force
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[*Dalibard, RMP 2011*]

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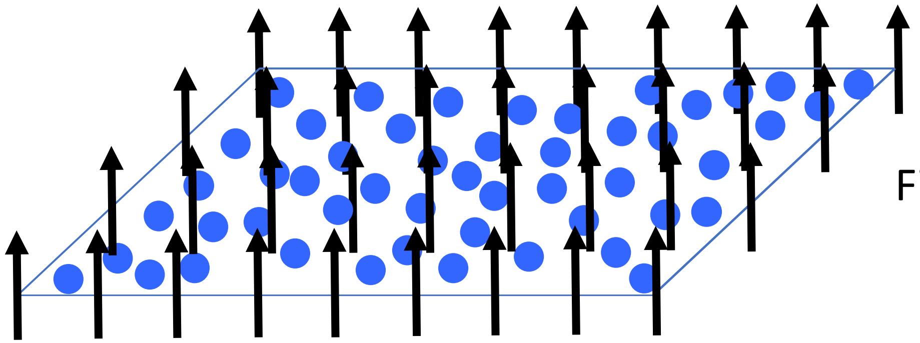

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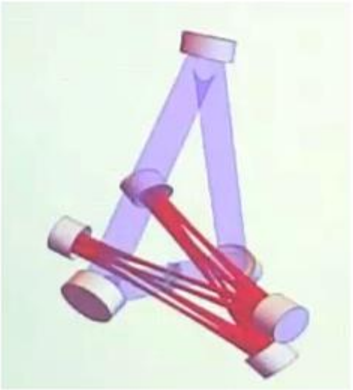
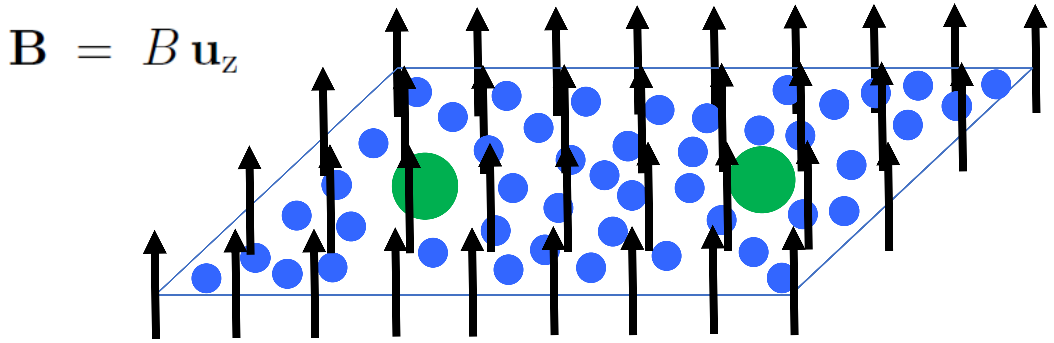
- **Photons in twisted cavity** [*Clark et al., arXiv 2019*]



- Control laser
  - Excitation to Rydberg state
  - Cavity Rydberg polaritons: **strong interactions**
- Probe twisted laser
  - **Artificial gauge field**

System and model

Impurities inside **FQH atomic liquid**



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	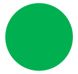	
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Charge	Q	q
Mass	M	m

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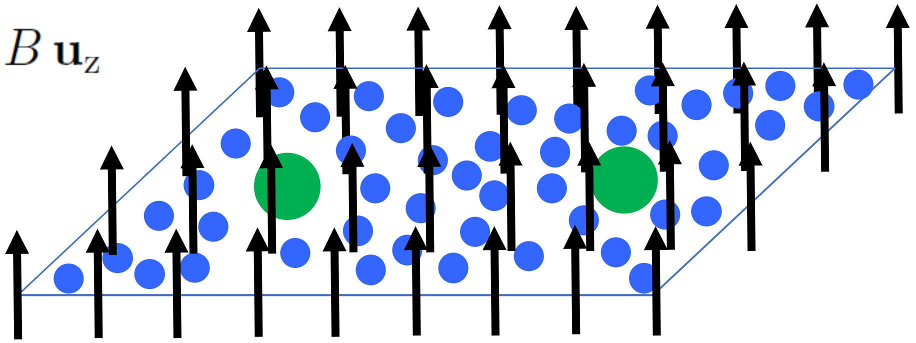
$\mathbf{B} = B\,\mathbf{u}_z$



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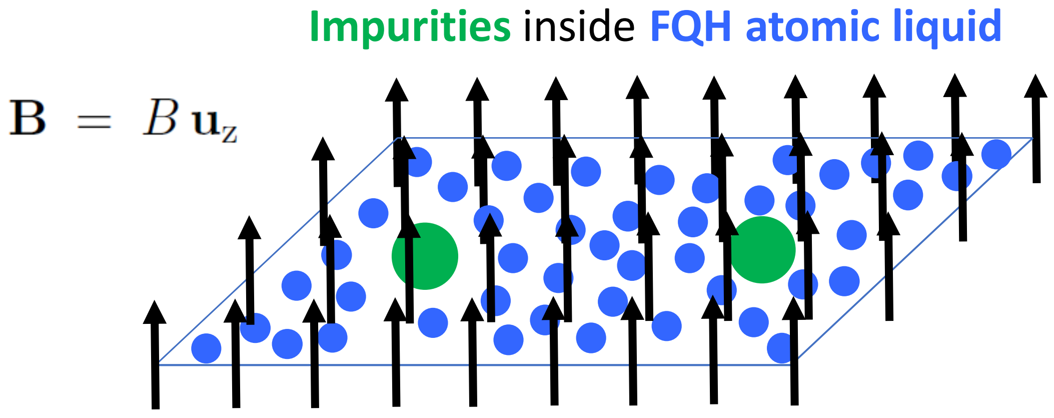

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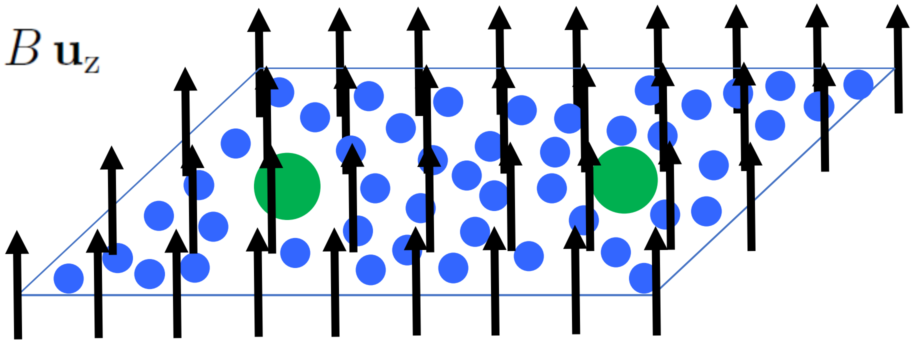

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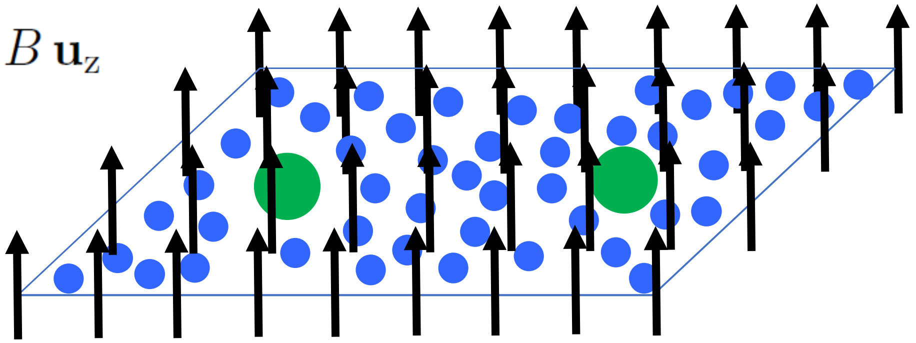

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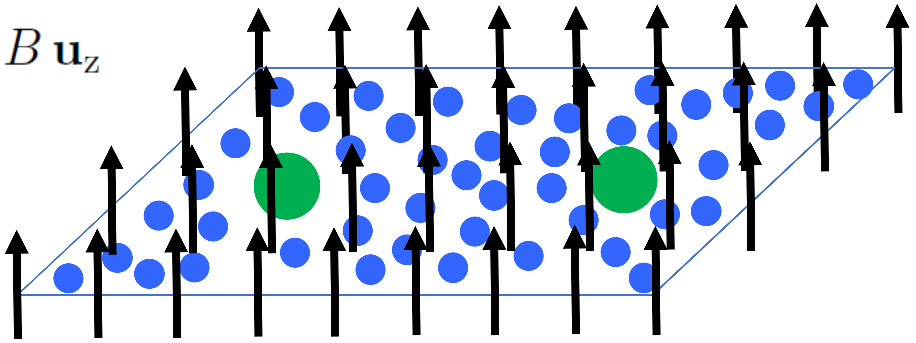

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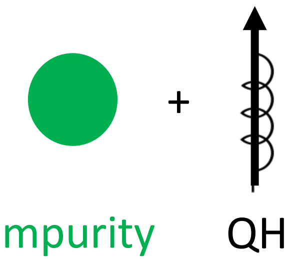

= **“Anyonic molecule”**

Born-Oppenheimer approximation

- **Total wavefunction**: (under BO approx.)

$$\psi(\{\mathbf{r}_\mathrm{i}\},\{\mathbf{R}_\mathrm{i}\},t) = \underbrace{\varphi^{(0)}_{\{\mathbf{R}_\mathrm{i}\}}(\{\mathbf{r}_\mathrm{i}\},t)}_{\text{Atoms}}\,\underbrace{\chi(\{\mathbf{R}_\mathrm{i}\},t)}_{\text{Impurities}}$$

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- QHs at impurities positions

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1st correction to BO approx.

[*Scherrer et al., PRX 2017*]

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$$\boxed{\frac{\Delta M}{M} \simeq \frac{m}{M}\frac{\omega_{\mathrm{cycl}}}{\Delta\omega_{-1}}}$$

$\Delta\omega_{-1}$ (1st excited state energy)

$\omega_{\mathrm{cycl}} = qB/m$

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- 1 impurity **bound to 1 quasihole**

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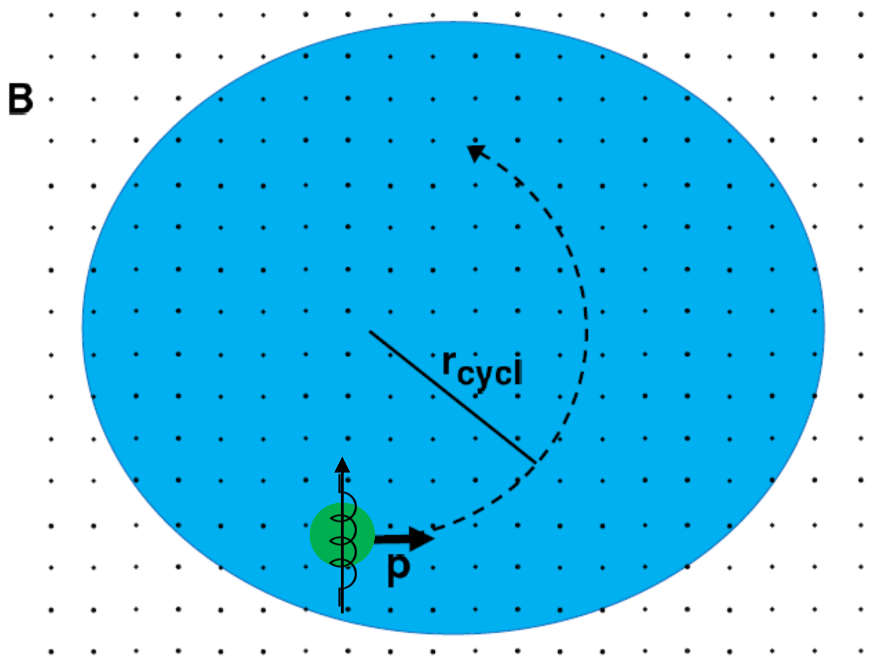

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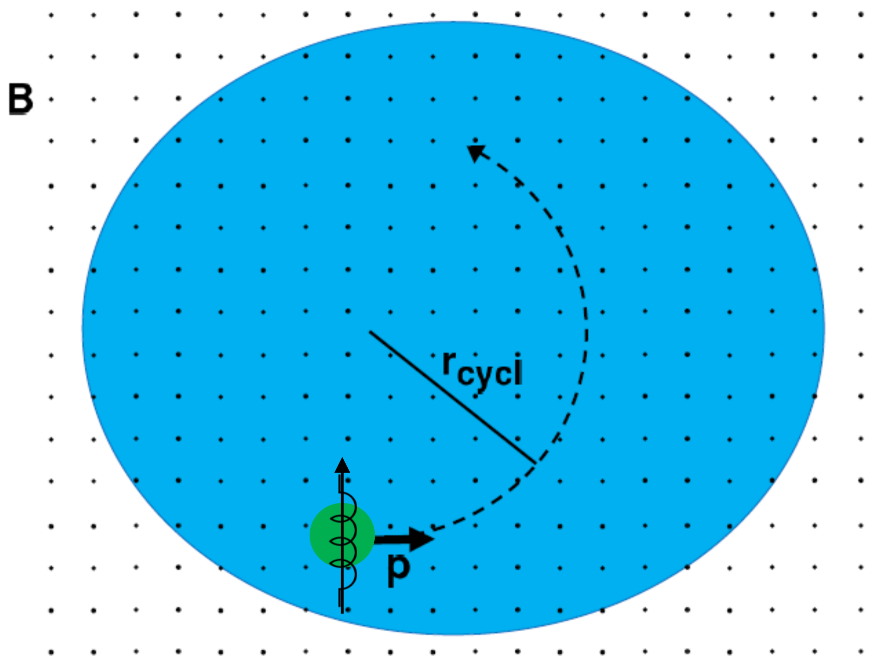

- Give molecule momentum kick $\mathbf{p} = \mathcal{M}\mathbf{v}$
- Cyclotron orbit with $r_{\text{cycl}} = \dfrac{\mathcal{M}v}{\mathcal{Q}B}$
- Image impurity's position at different times after deterministic preparation

→ Reconstruct trajectory → Measure $\mathcal{M}$, $\mathcal{Q}$

Two impurities

- 1 QH bound to each impurity

  → $\alpha = \nu$ **(bosons)**, $\alpha = 1 + \nu$ **(fermions)**

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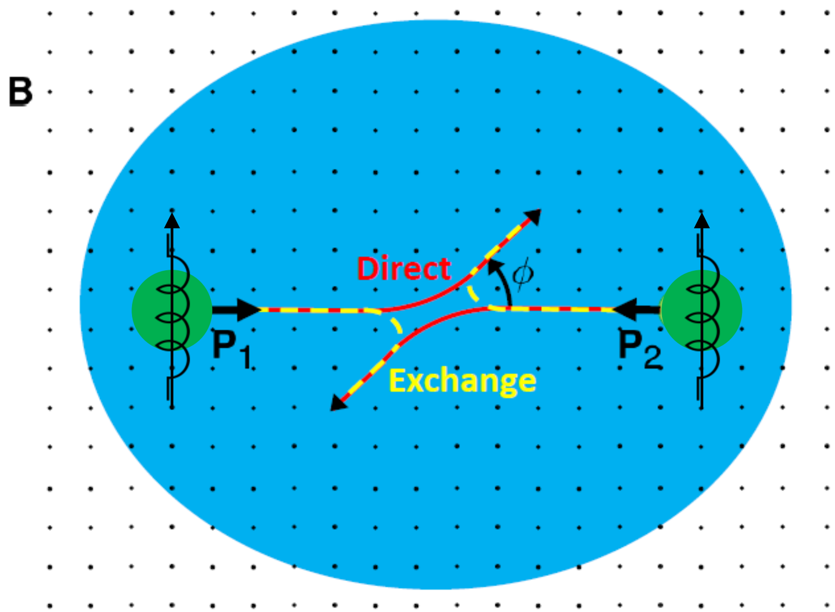
- **2D scattering between 2 anyonic molecules**

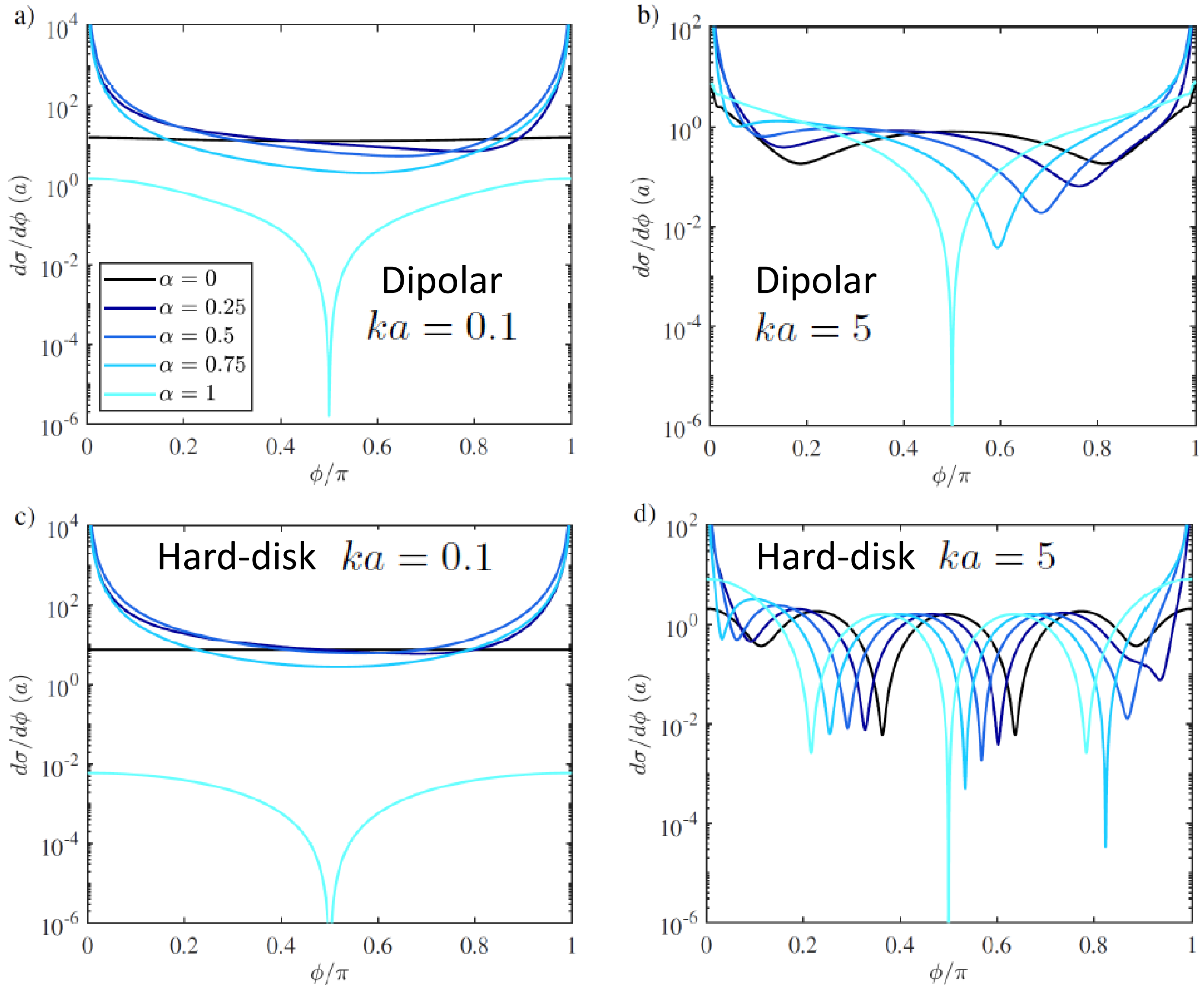


- Solve Schrödinger equation
- **Differential scattering cross section**

$$\frac{d\sigma}{d\phi}$$

2D scattering: results

Indistinguishable bosonic impurities



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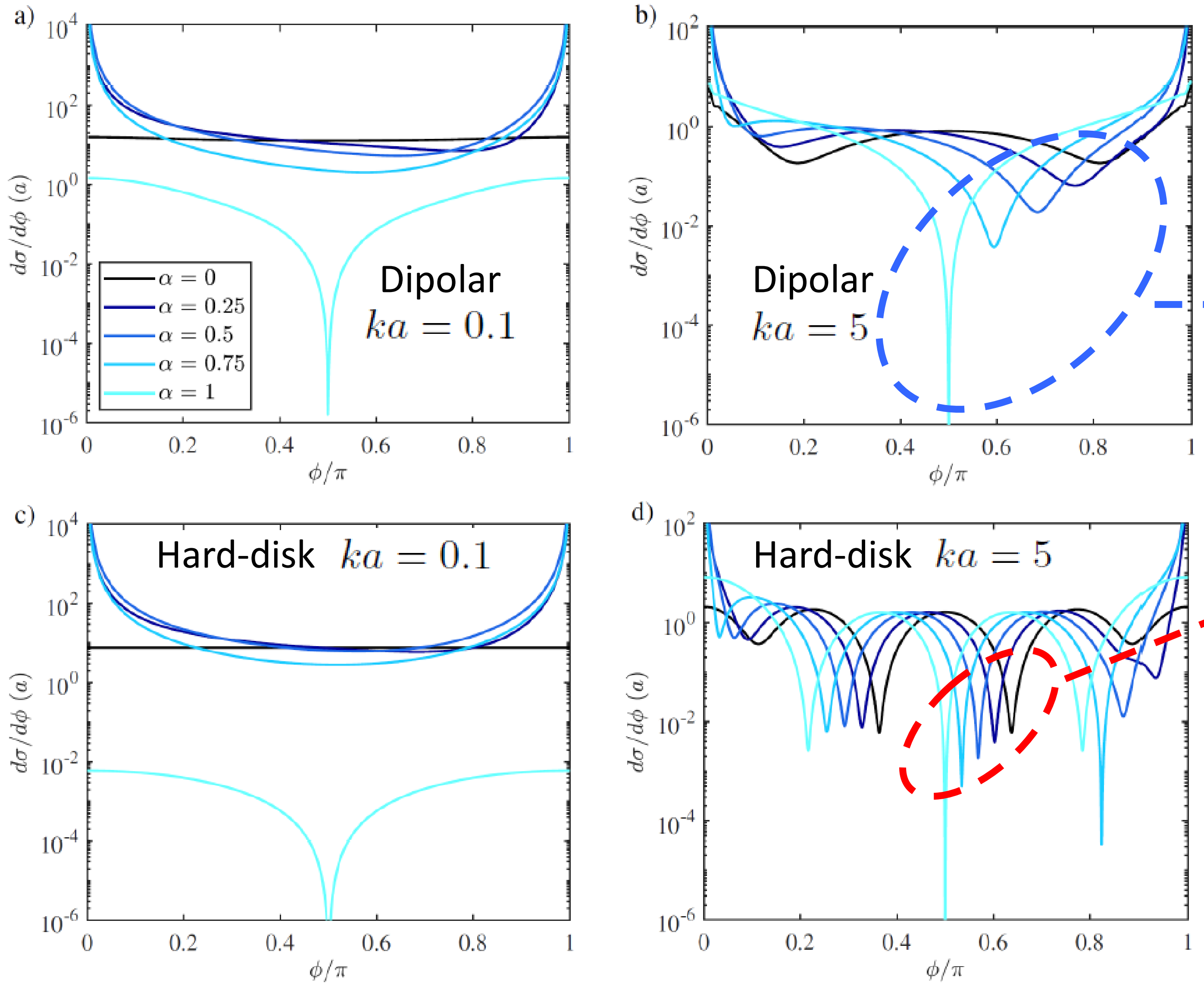
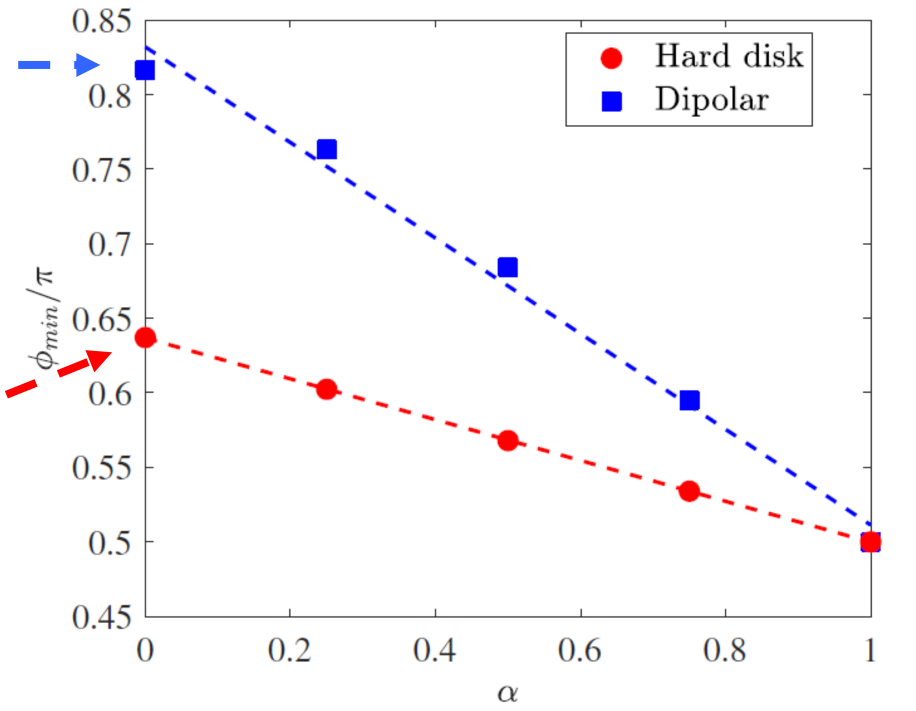


Proposed experiment for bosonic impurities



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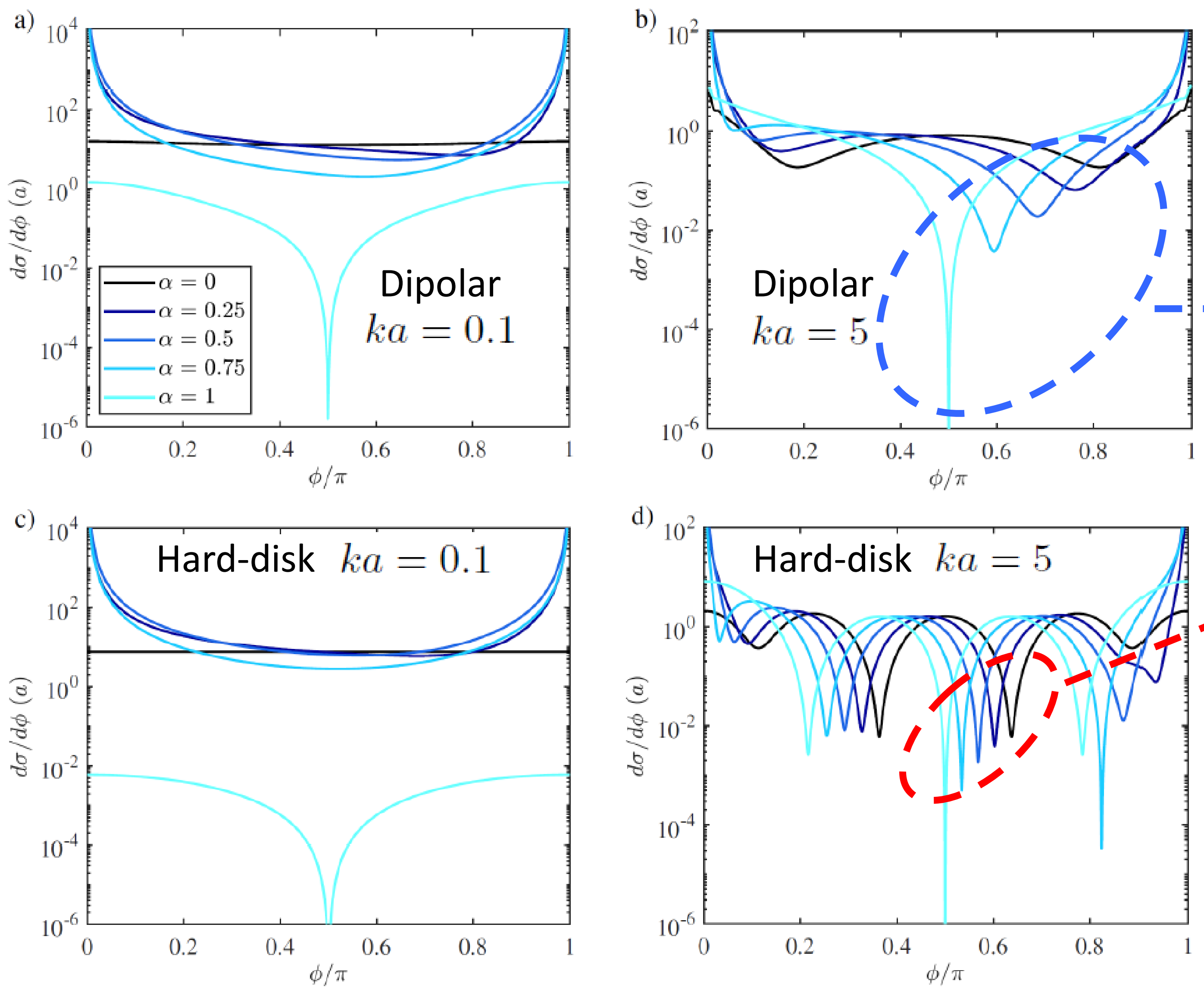

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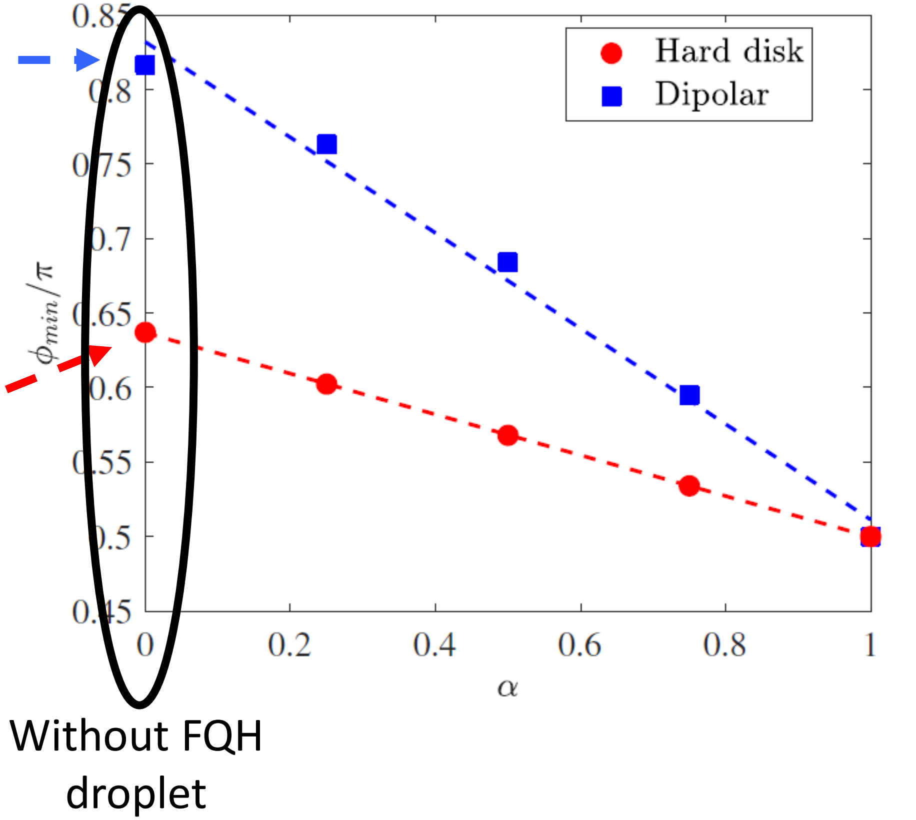

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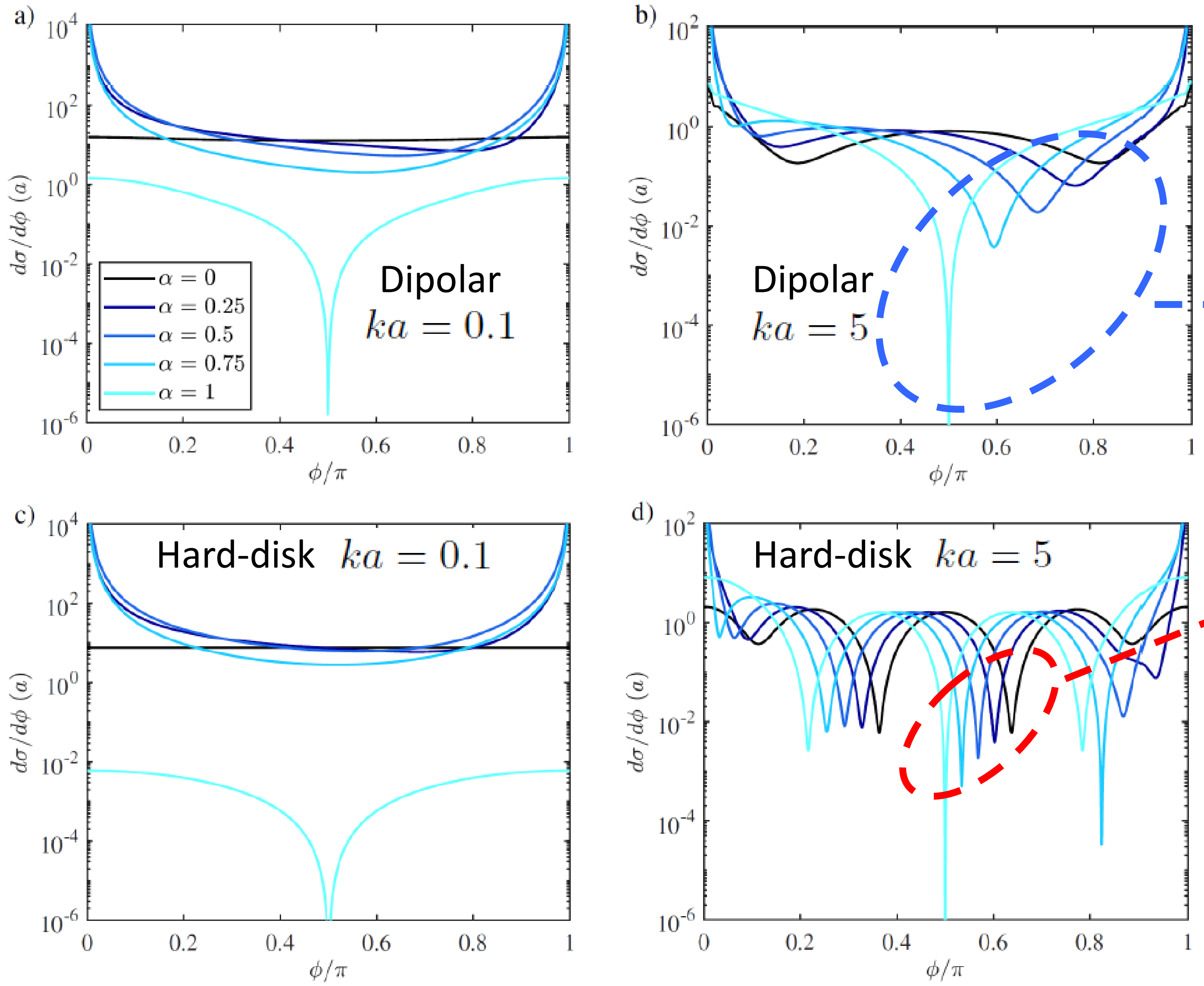

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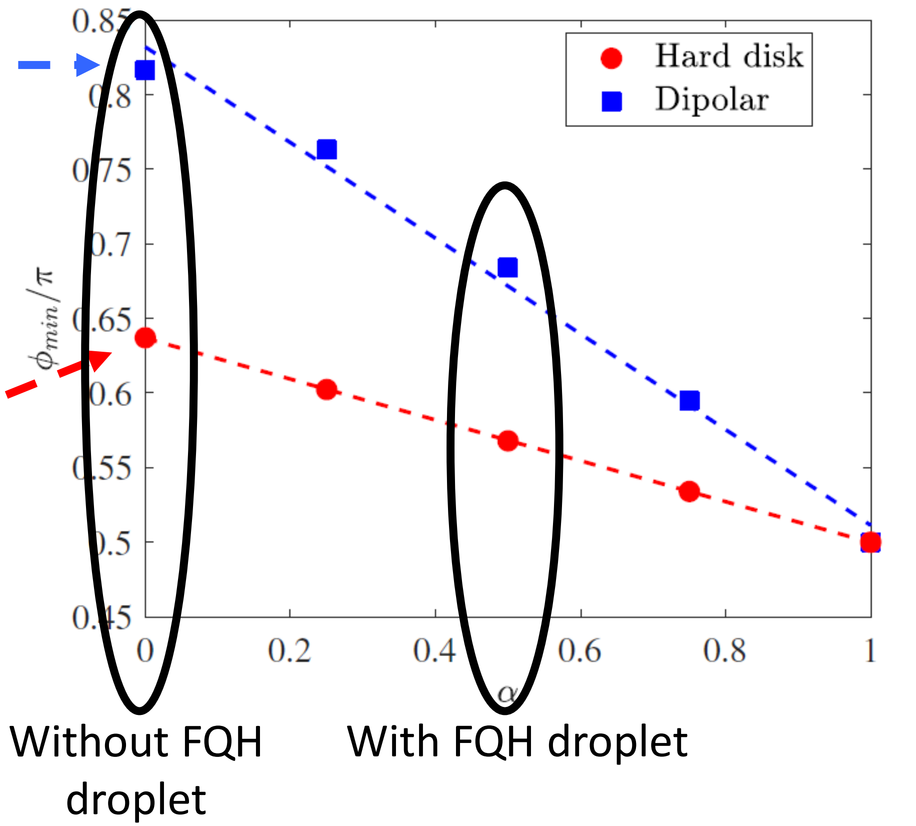

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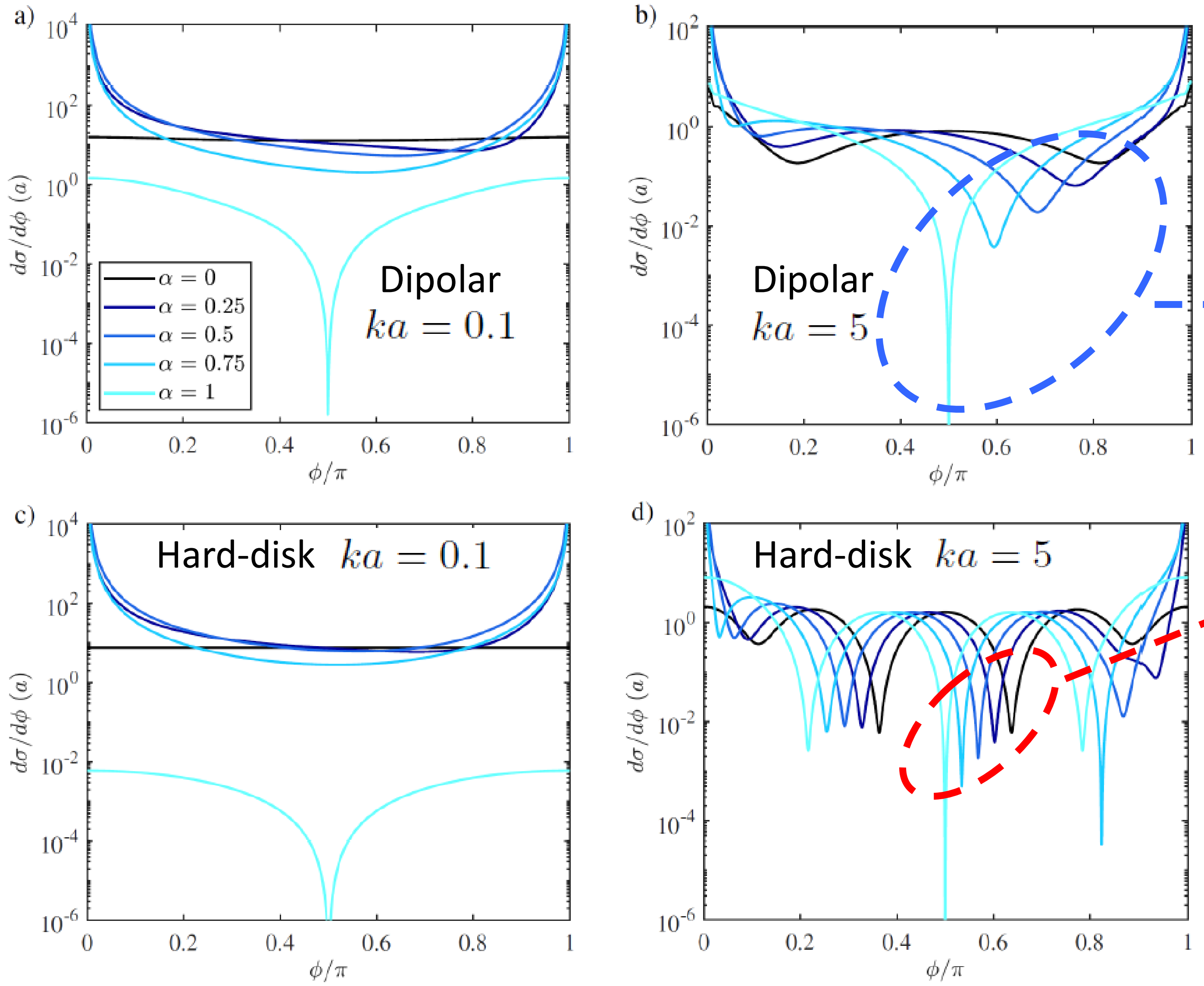

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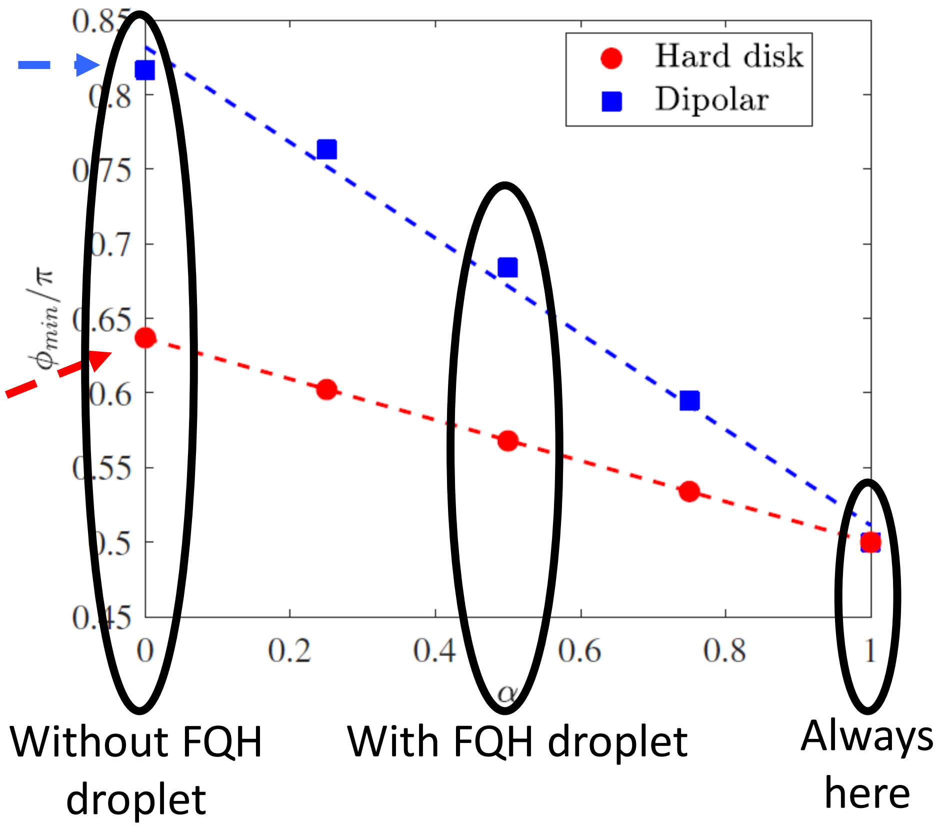

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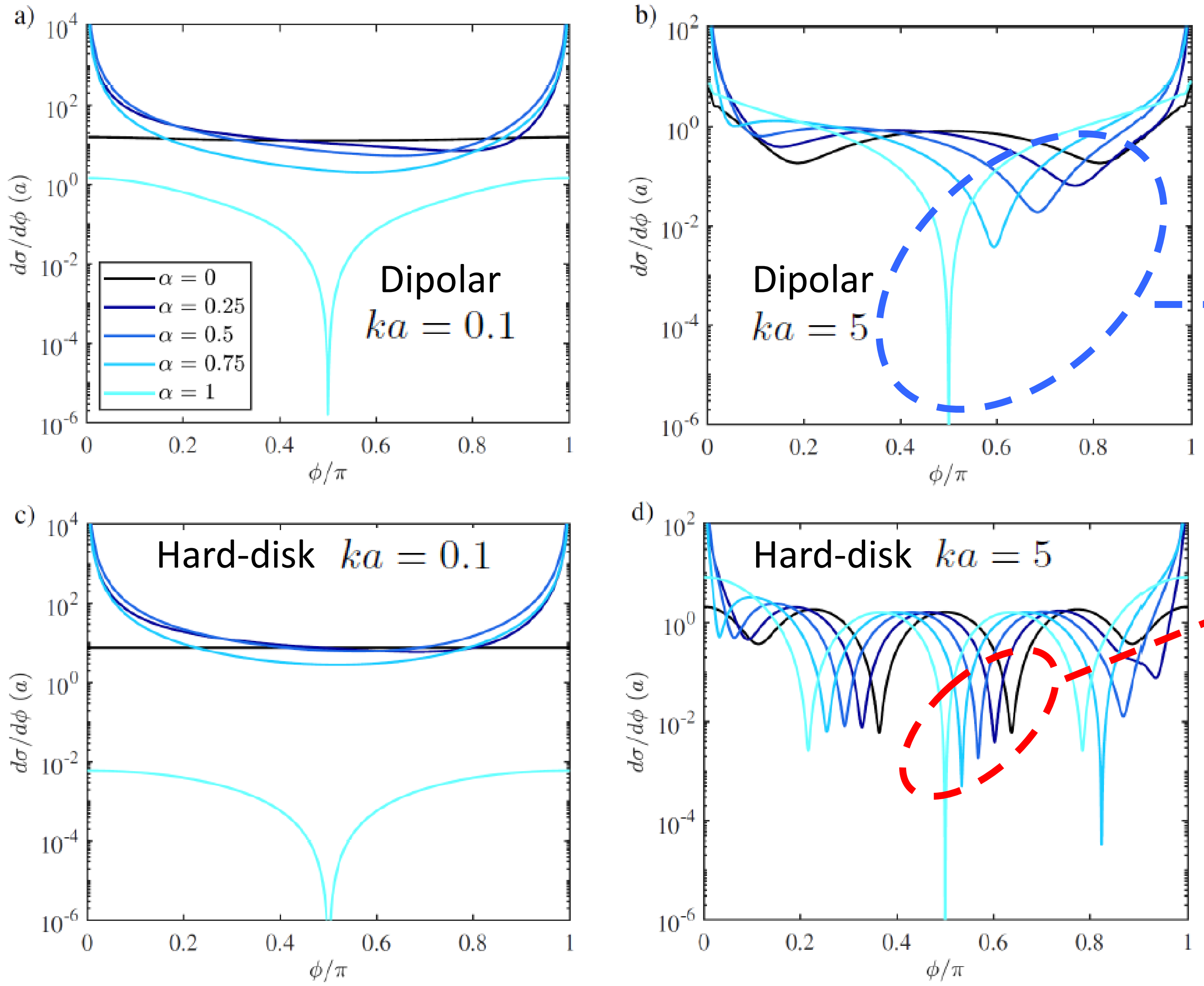

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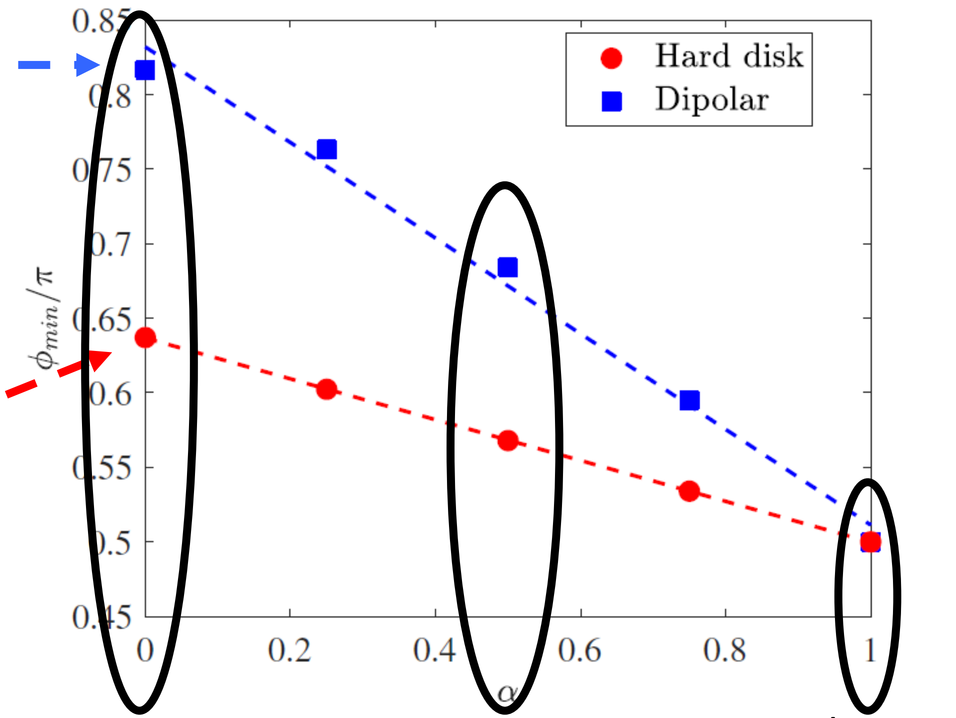


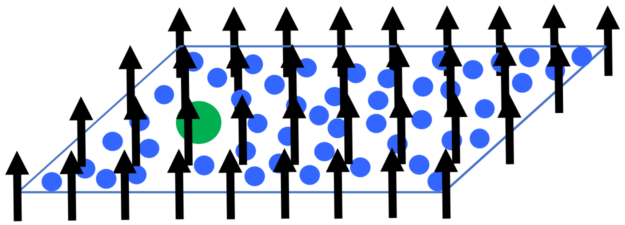
Without FQH droplet

With FQH droplet
→ **interpolate**

Always here

Conclusions

- **Fractional charge and statistics from quantum dynamics of impurities in FQH droplet**



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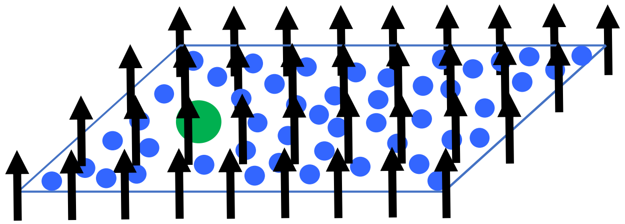

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  - Renormalization of **mass and charge**
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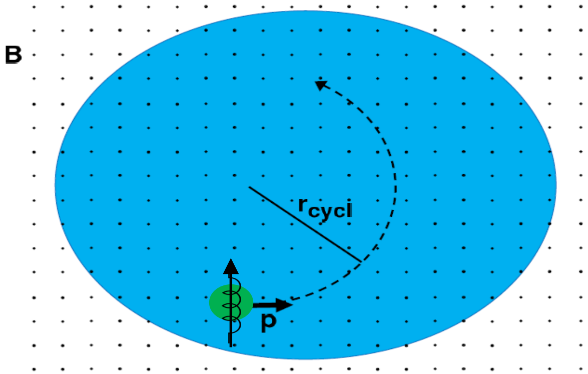

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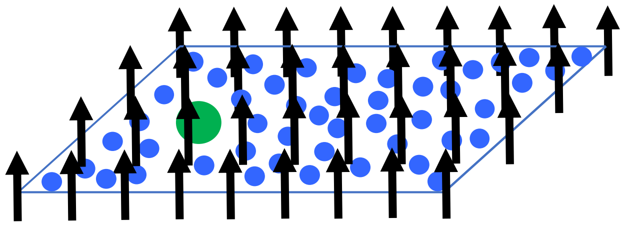

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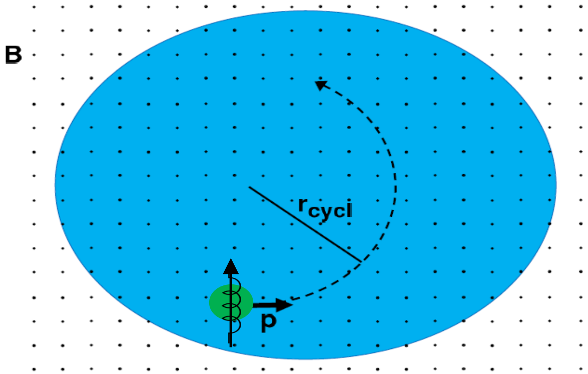

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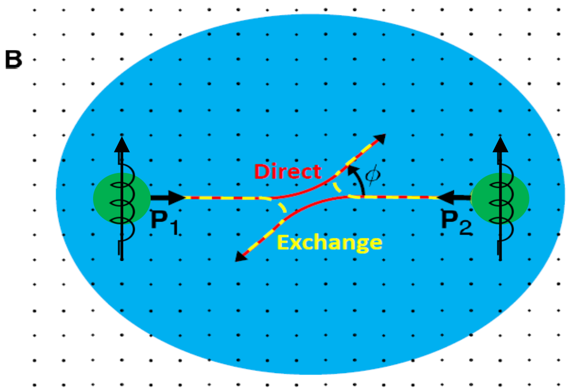
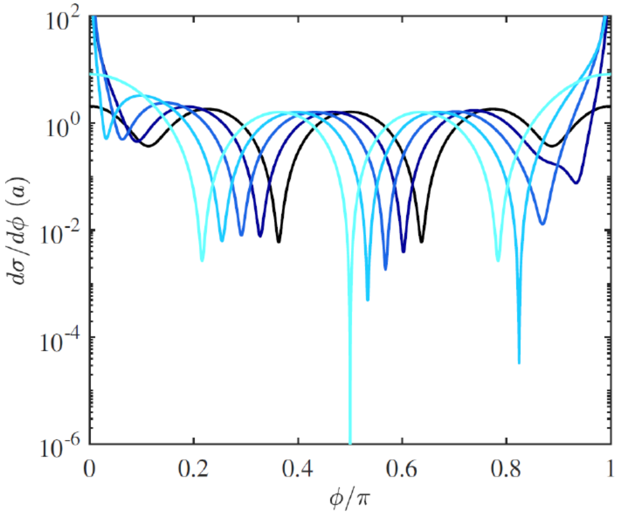
  - Differential scattering cross section displays **oscillatory pattern**
  - **Fractional statistics is observable** from rigid shift in its angular dependence





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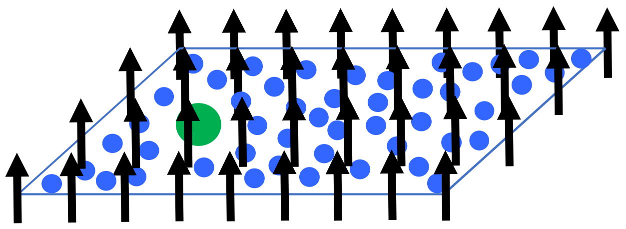

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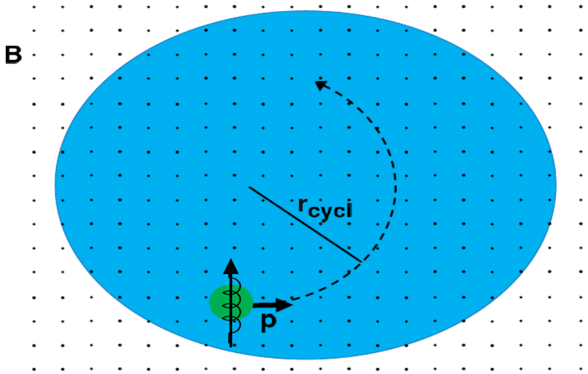

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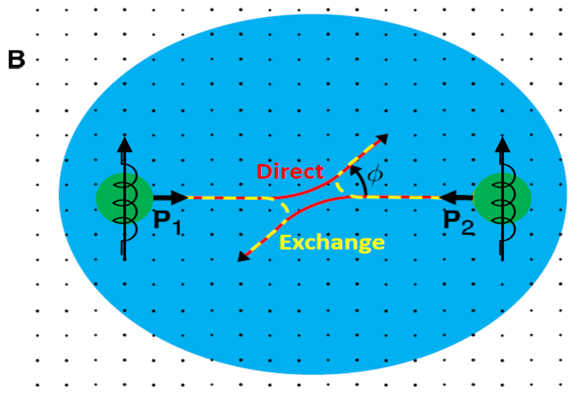

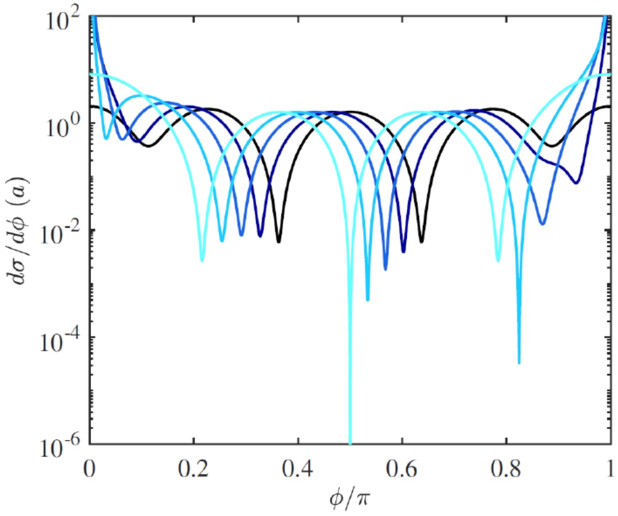

- **Future perspectives**
  - FQH fluids with **non-Abelian** excitations

[*Nayak, RMP 2008*]