Quantifying the efficiency of state preparation via quantum variational eigensolvers

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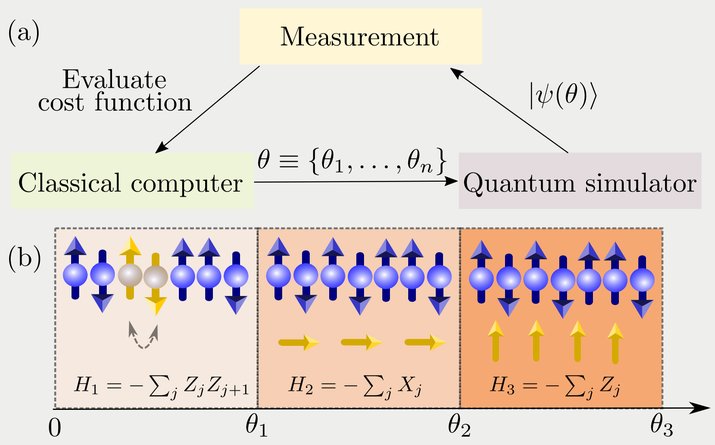
Quantum Approximate Optimization Algorithm (QAOA)

QAOA is a variational algorithm for preparing quantum states using low-depth quantum circuits [1]. It optimizes over a set of angles $\boldsymbol{\theta}$ in order to minimize some cost function, e.g. the energy $\langle\psi(\boldsymbol{\theta})|H|\psi(\boldsymbol{\theta})\rangle$, where

$$|\psi(\boldsymbol{\theta})\rangle = e^{-i\theta_{p,1}H_1}\ldots e^{-i\theta_{p,M}H_M}\ldots e^{-i\theta_{1,1}H_1}\ldots e^{-i\theta_{1,M}H_M}\,|\psi_0\rangle\,, \qquad (1)$$

$|\psi_0\rangle$ is some easily preparable initial state, $\{H_j\}_{j\in\{1,\ldots,M\}}$ is some set of Hamiltonians, and p is the number of steps.



QAOA is able to prepare non-trivial quantum states [2] e.g. the GHZ state.

Interaction distance

Given some density matrix $\boldsymbol{\rho}$, we define the *interaction distance of* $\boldsymbol{\rho}$ [4] as

$$\mathcal{D}_{\mathcal{F}}(\boldsymbol{\rho}) := \min_{\sigma\in\mathcal{F}} \frac{1}{2}\mathrm{Tr}\left(\sqrt{(\boldsymbol{\rho}-\boldsymbol{\sigma})^2}\right), \qquad (2)$$

where $\mathcal{F}$ is the manifold of Gaussian density matrices, which are of the form $\boldsymbol{\sigma} = e^H$, with H quadratic. Interaction distance measures how far a density matrix is from a Gaussian density matrix.

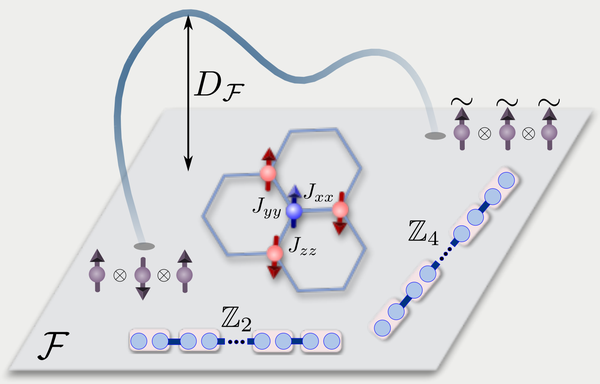


Figure 1: Interaction distance can be interpreted as a distance to the closest free fermion model [3]. The ground states of free fermion models are always Gaussian.

This minimization can be restricted to the Gaussian states simultaneously diagonalizable with $\boldsymbol{\rho}$, parameterized by the free energies $\boldsymbol{\epsilon}$:

$$\mathcal{D}_{\mathcal{F}}(\boldsymbol{\rho}) = \min_{\epsilon} \frac{1}{2}\sum_k \left| e^{-\beta\rho_k} - \frac{e^{-\beta\sigma_k(\epsilon)}}{\sum_j e^{-\beta\sigma_j(\epsilon)}} \right|,$$

$$\boldsymbol{\sigma}_k(\boldsymbol{\epsilon}) = \sum_{j=0}^{N-1} \epsilon_j n_j^{(k)}, \qquad n_j^{(k)} \in \{0,1\} \qquad (3)$$

$\boldsymbol{\rho}_k$ is the entanglement spectrum of $\boldsymbol{\rho}$ in descending order; k also labels the Fock basis such that the $\boldsymbol{\sigma}_k$ are in descending order, and $n_j^{(k)}$ is the occupancy number of the jth site.

Ising model

We consider the non-integrable Ising model with PBC

$$H = -\sum_{i=0}^{N-1} \pm\sigma_i^z\sigma_{i+1 \bmod N}^z - h_z\sum_{i=0}^{N-1}\sigma_i^z - h_x\sum_{i=0}^{N-1}\sigma_i^x, \qquad (4)$$

In [2], it was argued that the ground states of the transverse field Ising model ($h_x = 0$) can be exactly prepared with $p = \frac{N}{2}$, using $H_1 = -\sum_{i=0}^{N-1}\sigma_i^x$, $H_2 = -\sum_{i=0}^{N-1}\sigma_i^z\sigma_{i+1 \bmod N}^z$.

References

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Results

We want to quantify how efficient QAOA is in preparing the ground state of (4) for $h_x, h_z \in [0,3]$.

- We use $H_1' = -\sum_{i=0}^{N-1}\sigma_i^z$, $H_2' = H_1$, $H_3' = H_2$.
- We simulate the quantum evolution step classically using $1-f$ as the cost function, where $f = \langle\psi_{GS}|\psi(\boldsymbol{\theta})\rangle^2$ and ψ_{GS} is the ground state of H.
- `scipy.minimize.basinhopping` was used to avoid local minima.
- We computed $\mathcal{D}_{\mathcal{F}}$ across the phase diagram of (4) and found good correlation with $1-f$ (Figures 2 and 3).

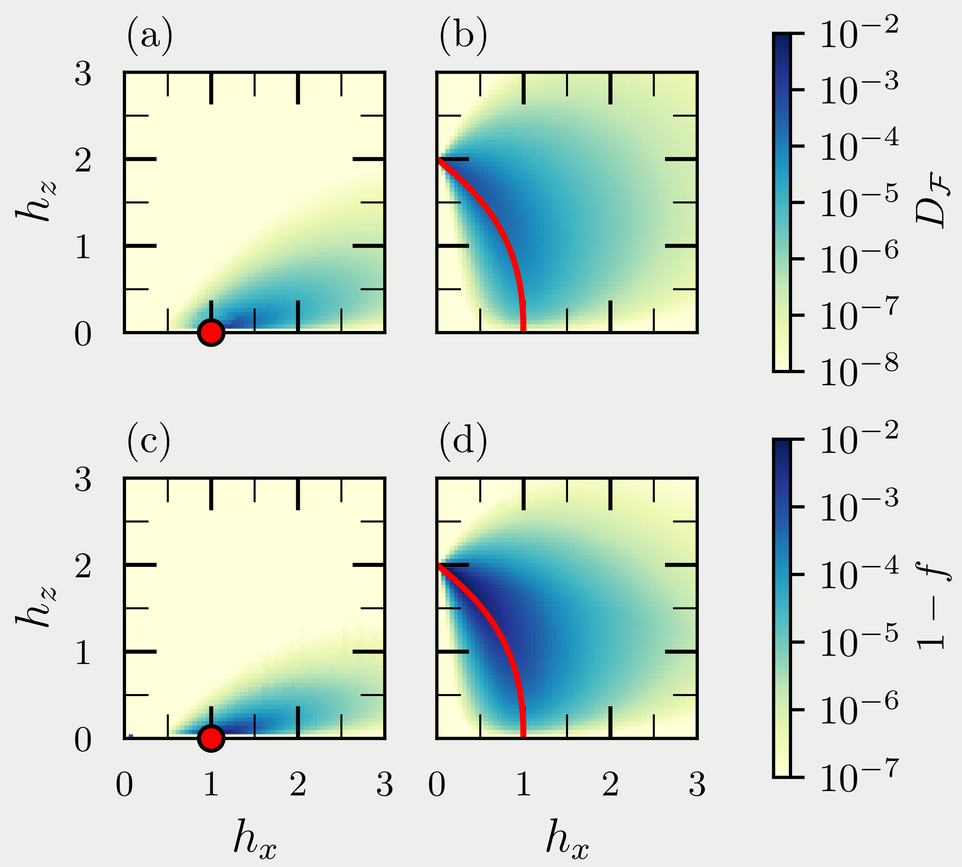

Figure 2: Ferromagnetic (left) and antiferromagnetic (right) Ising model.

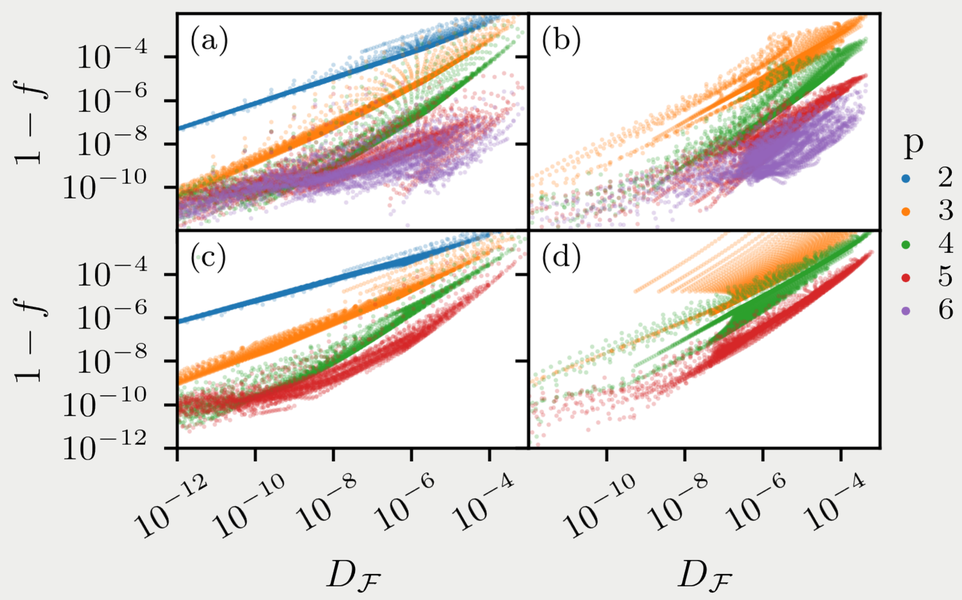

Figure 3: Correlation between $\mathcal{D}_{\mathcal{F}}$ and $1-f$ in ferromagnetic (left) and antiferromagnetic (right) Ising model, for $N = 6$ (top) and $N = 8$ (bottom) for the data in Figure 2.

Analysis

- We have found a correlation between $\mathcal{D}_{\mathcal{F}}$ and QAOA around $p = \frac{N}{2}$ in the Ising model (4).
- This can be explained by the perturbatively small angles associated with H_1' (ask me about this!).
- We relate how hard it is to prepare a ground state with how free it is, as measured by $\mathcal{D}_{\mathcal{F}}$
- This correlation is also present in the three-spin Ising model.
- This is a hard minimization problem. The initial guess and the classical simulation of the quantum evolution can be improved.

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