A COMPLEMENTARITY VIEW ON UNCERTAINTY

YIQIS 2020

MARCOS BASSO[a] & JONAS MAZIERO[b]

[a]marcoslwbasso@mail.ufsm.br
[b]Universidade Federal de Santa Maria (UFSM).

INTRODUCTION

- Uncertainty principle $\rightarrow$ Uncertainty relations.
- For mixed states, the variance is a hybrid of quantum and classical uncertainties.
- Complementarity Principle $\rightarrow$ Complementarity relations.
- For (maximally) mixed states, no information about the wave-particle aspects of the system can be obtained. Therefore, one also has to consider its correlations with another systems.
- Complementarity $\leftrightarrow$ Uncertainty.

LUO'S CRITERIA

Luo[a] proposed to split the variance $\mathcal{V}(\rho, A)$ in its quantum and classical parts

$$\mathcal{V}(\rho, A) = \mathcal{Q}(\rho, A) + \mathcal{C}(\rho, A), \quad (8)$$

$$\mathcal{Q}(\rho, A) := \mathcal{I}_{wy}(\rho, A) = -\frac{1}{2}\operatorname{Tr}([\sqrt{\rho}, A_0]^2), \quad (9)$$

$$\mathcal{C}(\rho, A) := \mathcal{V}(\rho, A) - \mathcal{Q}(\rho, A) = \operatorname{Tr}\sqrt{\rho}A_0\sqrt{\rho}A_0,$$

where $\mathcal{Q}(\rho, A)$ and $\mathcal{C}(\rho, A)$ correspond to the quantum and classical uncertainties, respectively, and $\mathcal{I}_{wy}$ is the Wigner-Yanase skew information. Luo's criteria are given by:

L.1 If ρ is pure, then $\mathcal{V}(\rho, A) = \mathcal{Q}(\rho, A)$ and $\mathcal{C}(\rho, A) = 0$, because there is no classical mixing and all uncertainties are intrinsically quantum.

L.2 If $[\rho, A] = 0$, both are diagonal in the same basis and ρ and A behave like classical variables. Hence, all uncertainties are classical, i.e., $\mathcal{Q}(\rho, A) = 0$ and $\mathcal{V}(\rho, A) = \mathcal{C}(\rho, A)$.

L.3 $\mathcal{C}(\rho, A)$ must be concave in ρ, while $\mathcal{Q}(\rho, A)$ must be convex, i.e., $\mathcal{C}(\sum_i \lambda_i \rho_i, A) \geq \sum_i \lambda_i \mathcal{C}(\rho_i, A)$ and $\mathcal{Q}(\sum_i \lambda_i \rho_i, A) \leq \sum_i \lambda_i \mathcal{Q}(\rho_i, A)$, with $\sum_i \lambda_i = 1$, $\lambda_i \in [0, 1]$ and ρ_i are valid quantum states.

[a]S. L. Luo, Theor. Math. Phys. 143, 681 (2005).

DÜRR'S CRITERIA

Also, Dürr[a] established criteria for predictability measures $P(\rho)$ and for interference pattern visibility quantifiers $V(\rho)$:

D.1 P, V must be a continuous function of the diagonal elements of the density matrix.

D.2 P, V must be invariant under permutations of the states indexes.

D.3 If $\rho_{jj} = 1$ for some j, then P (V) must reach its maximum (minimum) value.

D.4 If $\{\rho_{jj} = 1/d\}_{j=1}^{d}$, then P must reach its minimum value. In addition, if ρ is pure, then V must reach its maximum value.

D.5 P, V must be a convex function, since classical mixing does not increase the predictability and the visibility.

For instance, the following measures satisfies Dürr's criteria[b]

$$P_l(\rho) := S_l^{max} - S_l(\rho_{diag}) = \sum_j \rho_{jj}^2 - 1/d, \quad (10)$$

$$C_{wy}(\rho) := \sum_j \mathcal{I}_{wy}(\rho, |j\rangle\langle j|) = \sum_{j\neq k} |(\sqrt{\rho})_{jk}|^2,$$

where S_l is the linear entropy.

[a]S. Dürr, Phys. Rev. A, 64, 042113 (2001).
[b]M. L. W. Basso, D. S. S. Chrysosthemos, J. Maziero, Quant. Inf. Process. 19, 254 (2020).

A COMPLEMENTARITY VIEW ON UNCERTAINTY

Considering the observable A as the projection onto one of the paths (or slit) of the interferometer, i.e., $A = |j\rangle\langle j|$, for some path label j. The quantum and classical uncertainties of the path j are given by

$$\mathcal{Q}(\rho, |j\rangle\langle j|) = \langle j|\rho|j\rangle - \langle j|\sqrt{\rho}|j\rangle^2, \quad (1)$$

$$\mathcal{C}(\rho, |j\rangle\langle j|) = \langle j|\sqrt{\rho}|j\rangle^2 - \langle j|\rho|j\rangle^2, \quad (2)$$

meanwhile the quantum and classical uncertainties of all paths are expressed by

$$\mathcal{U}_q := \sum_j \mathcal{Q}(\rho, |j\rangle\langle j|) = C_{wy}(\rho), \quad (3)$$

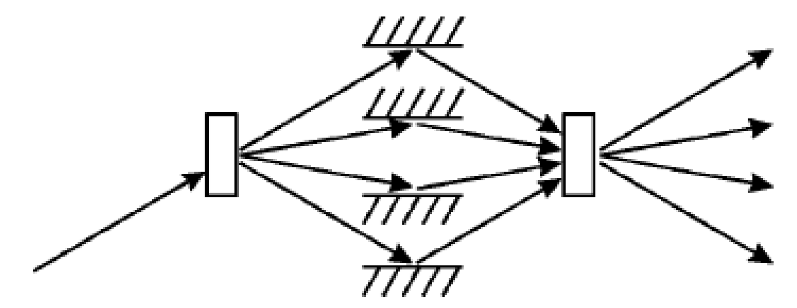
$$\mathcal{U}_c := \sum_j \mathcal{C}(\rho, |j\rangle\langle j|) = \sum_j (\langle j|\sqrt{\rho}|j\rangle^2 - \langle j|\rho|j\rangle^2),$$

Now, summing both uncertainties, we have a complementarity relation between classical and quantum uncertainties

$$\mathcal{U}_q + \mathcal{U}_c = S_l(\rho_{diag}) \leq S_l^{max}. \quad (4)$$

Exploring Eq. (4) even further, it's possible to obtain a complete complementarity relation (CCR) between uncertainty and predictability:

$$\mathcal{U}_q + \mathcal{U}_c + P_l = S_l^{max}. \quad (5)$$



AN UNCERTAINTY VIEW OF COMPLEMENTARITY

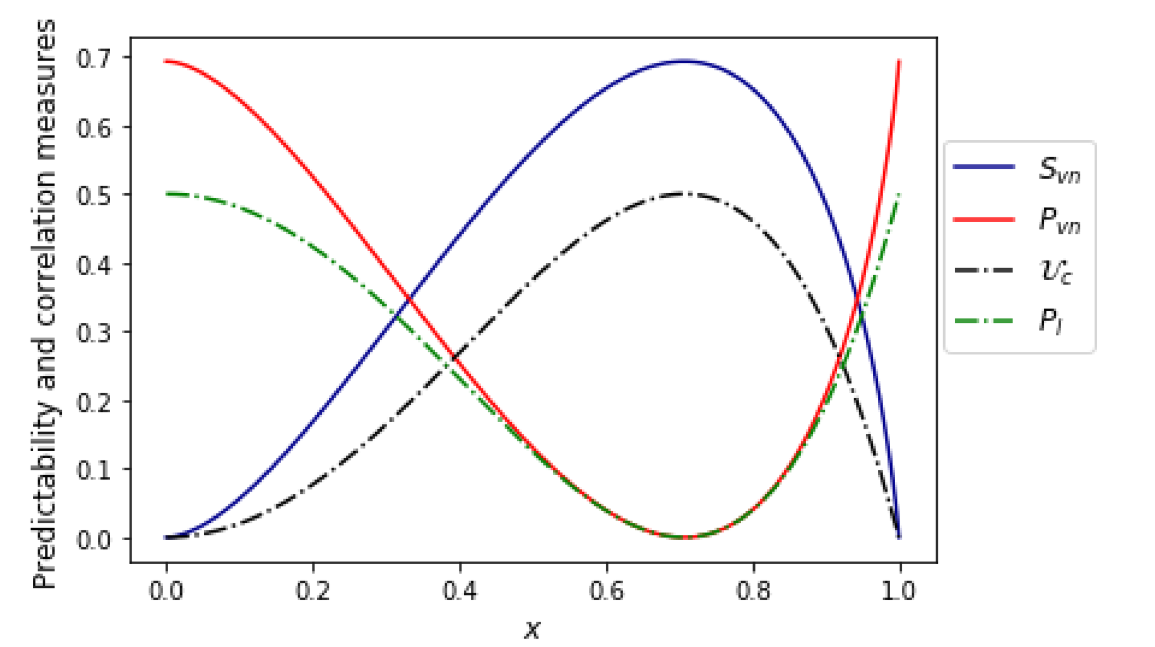
Within this framework, any CCR can be interpreted in terms of uncertainty [1]. For instance,

$$C_{re}(\rho) + S_{vn}(\rho) + P_{vn}(\rho) = S_{vn}^{max} \quad (6)$$

can be taken as a complete complementarity relation, if we consider the quanton as part of a bipartite pure quantum system, where $C_{re}(\rho)$ is the relative entropy of coherence, $S_{vn}(\rho)$ is the von-Neumann entropy, and $P_{vn}(\rho)$ is a measure of predictability. Besides, we can take $C_{re}(\rho)$ as a measure of quantum uncertainty and $S_{vn}(\rho)$ as a measure of classical uncertainty[a] Since $P_{vn}(\rho) \geq 0$, we have the following complementarity relation between quantum and classical entropic uncertainty

$$C_{re}(\rho) + S_{vn}(\rho) \leq S_{vn}^{max}. \quad (7)$$

In addition, we showed that *the coherences of ρ give rise to quantum uncertainties and the classical uncertainty is due to the possible correlations with others systems, if we consider ρ as part of a pure multipartite quantum system. Conversely, classical uncertainties are signatures of quantum correlations and quantum uncertainties are signatures of quantum coherence.*



: Predictability and correlation measures of the quanton A as part of a bipartite quantum system in the state $|\Psi\rangle_{A,B} = x|0,1\rangle_{A,B} + \sqrt{1-x^2}|1,0\rangle_{A,B}$, with $x \in [0, 1]$.

[a]K. Korzekwa, M. Lostaglio, D. Jennings, T. Rudolph, Phys. Rev. A 89, 042122 (2014).

REFERENCES

[1] M. L. W. Basso and J. Maziero. An uncertainty view on complementarity and a complementarity view on uncertainty. *arXiv:2007.05053*, 2020.

CONCLUSIONS

- $\mathcal{U}_c$ as a measure of the quantum correlations.
- Duality between Uncertainty and Complementarity measures.

ACKNOWLEDGMENTS

QIEP Quantum Information and emergent Phenomena Group
CAPES