The effects of the external magnetic field in a qutrit–qubit mixed spin chain on thermal entanglement, quantum discord based on linear entropy, and measurement-induced disturbance

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Abstract

Quantum entanglement plays important roles in many areas of quantum information processing (QIP). Nevertheless, quantum entanglement is not the only form of quantum correlation that is useful for QIP. In fact, some separable states may also speed up certain quantum tasks, relative to their classical counterparts. Example of such quantum correlations, is a quantity, called quantum discord (QD), which can effectively capture all quantum correlations present in various kinds of quantum systems. The quantum discord involves a minimization procedure that is difficult to solve in general. To overcome the difficulty encountered with the computability of quantum discord based on von Neumann entropy, we propose a reliable analytical method to evaluate the quantum discord based on linear entropy for an arbitrary qudit-qubit quantum [1]. The quantum discord based on linear entropy is employed to derive the amount of quantum correlations in a qutrit-qubit mixed spin system in the thermal equilibrium at temperature T [1]. We investigated also the situation when the system is embedded in an external magnetic field B. The obtained amount of quantum discord is then compared with the measurement-induced disturbance (MID) and logarithmic negativity (LN).

Model and solution

We consider qutrit–qubit mixed spin (1, 1/2) Heisenberg chain with only nearest-neighbor interactions under the influence of an external magnetic field. The Hamiltonian describing this system can be written as

$$H = J(S_1^x s_2^x + S_1^y s_2^y + S_1^z s_2^z) + B s_2^z$$

where J is the coupling coefficient between spin-1 and spin-1/2 particles. For $J > 0$ (resp. $J < 0$) corresponds to the antiferromagnetic (resp. ferromagnetic) phase. B is the external magnetic field; S_1^α ($\alpha = x, y, z$) denotes the spin-1 operator and s_2^α are the familiar Pauli matrices.

For a system in thermal equilibrium, the state of the system is given by the density operator

$$\rho = \exp(-\beta H)/Z$$

where $Z = Tr(\exp(-\beta H))$ is the partition function and $\beta = 1/k_B T$. Hereafter, the Boltzmann's constant is set to unit for simplicity, i.e., $k_B = 1$.

Quantum correlations

1. Logarithmic negativity

The logarithmic negativity $\mathcal{E}_{\mathcal{N}}(\rho)$ is based on the definition of negativity $\mathcal{N}(\rho)$ defined as the absolute sum of the negative eigenvalues of the partial transposed density matrix with respect to the first sub-system, i.e.,

$$\mathcal{N}(\rho) = \sum_i |\vartheta_i|,$$

where ϑ_i are the negative eigenvalues of ρ^{T_A}. The logarithmic negativity writes as

$$\mathcal{E}_{\mathcal{N}}(\rho) = \log_2[2\mathcal{N}(\rho) + 1].$$

2. Measurement-induced disturbance

Let $\{\Pi_i^A\}$ (resp. $\{\Pi_j^B\}$) be complete projective measurements (with $\Pi_i \Pi_j = \delta_{ij}$ and $\sum_i \Pi_i = 1$) consisting of one-dimension. Then, the state ρ after the measurement al orthogonal projections on the subsystem A (resp. B). becomes

$$\Pi(\rho) = \sum_{ij} (\Pi_i^A \otimes \Pi_j^B) \rho (\Pi_i^A \otimes \Pi_j^B).$$

The quantum correlations quantified by the measurement-induced disturbance is given as

$$\mathrm{MID}(\rho) = \mathfrak{T}(\rho) - \mathfrak{T}(\Pi(\rho)).$$

where $\mathfrak{T}(.)$ is the quantum mutual information.

3. Quantum discord based on linear entropy

The quantum discord $Q(\rho_{AB})$ for a bipartite system composed of partitions A and B is written

$$Q(\rho_{AB}) = \mathfrak{T}(\rho_{AB}) - \mathcal{J}(\rho_{AB}), \qquad (1)$$

with $\mathfrak{T}(\rho_{AB})$ is the quantum mutual information and $\mathcal{J}(\rho_{AB})$ denotes the classical correlation, which writes as

$$\mathcal{J}(\rho_{AB}) = \max_{B_j} [S(\rho_A) - \textstyle\sum_j p_j S(\rho_{A|j})] \qquad (2)$$

Where $\{B_j\}$ is a set of positive operator-valued measurements (POVM) performed on the subsystem Bonly, with $p_j = Tr[(I_A \otimes B_j)\rho_{AB}(B_j \otimes I_A)]$ and $\rho_{A|j} = Tr_B[(I_A \otimes B_j)\rho_{AB}(B_j \otimes I_A)/p_j$ are respectively the probability and the conditional state of the system A when the outcome j occurs. I_A is the identity operator acting on the subsystem A.

To calculate the quantum discord, instead of the von Neumann entropy one uses the linear entropy. The linear entropy of a state ρ is given by

$$S_2(\rho) = 2[1 - Tr(\rho^2)],$$

Based on the results reported in [1], The classical correlation under linear entropy for any arbitrary d $\otimes$ 2 quantum state has the following form

$$\mathcal{J}_2(\rho_{AB}) = \frac{4}{d^2} \lambda_{max}(L^T L)\ S_2(\rho_B), \qquad (3)$$

where $\lambda_{max}(L^T L)$ denotes the largest eigenvalue of the matrix $L^T L$. Hence, having the matrix L, the classical correlation (3) as well as the quantum discord can readily be computed.

Conclusion

In summary, a comparison between these three quantum correlations quantifiers is presented. The quantum discord based on the linear entropy behaves like measurement-induced disturbance. This indicates that this variant of quantum discord is a useful tool to deal with quantum correlations between multi-component systems of higher-dimensional Hilbert spaces. Besides, both QD and MID present a strong behavior against thermal effects. Further, QD and MID are able to detect the critical points of transition QPT, while the logarithmic negativity cannot reveal such transition between ferromagnetic and antiferromagnetic phases.

References

[1] Benabdallah, F., Slaoui, A. and Daoud, M.: Quantum discord based on linear entropy and thermal negativity of qutrit-qubit mixed spin chain under the influence of external magnetic field . Quantum Inf. Process. 19, 252 (2020).

Results and Discussion

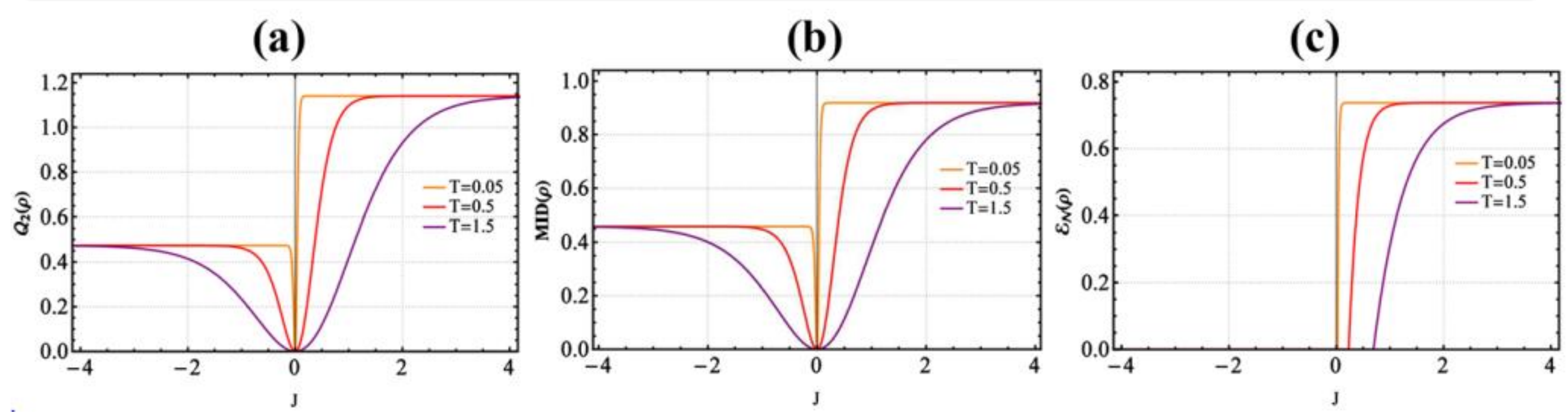

Fig. 1. Quantum discord $Q_2(\rho)$, measurement-induced disturbance MID(ρ) and logarithmic negativity $\mathcal{E}_{\mathcal{N}}(\rho)$ as function of the temperature T and the exchange coupling J when $B = 0$.

The results for $B = 0$ are shown (**Fig. 1**). The three quantifiers decrease with increasing values of the temperature. We notice that LN disappears at some critical temperature in contrast with QD and MID which continue to be non zero with increasing values of temperature. They tend to be zero for higher values of temperature (Fig.1a and Fig.1b). Therefore, the destructive effects of the temperature are much stronger for LN than those for thermal quantum correlations measured by both QD and MID. Moreover, entanglement is zero for the ferromagnetic case (Fig.1c), but the QD and MID exist for both the ferromagnetic and antiferromagnetic cases and reach zero only at the critical point $J = 0$. This shows that both the quantum discord and measurement-induced disturbance are more suitable to capture the quantum correlations in thermal states.

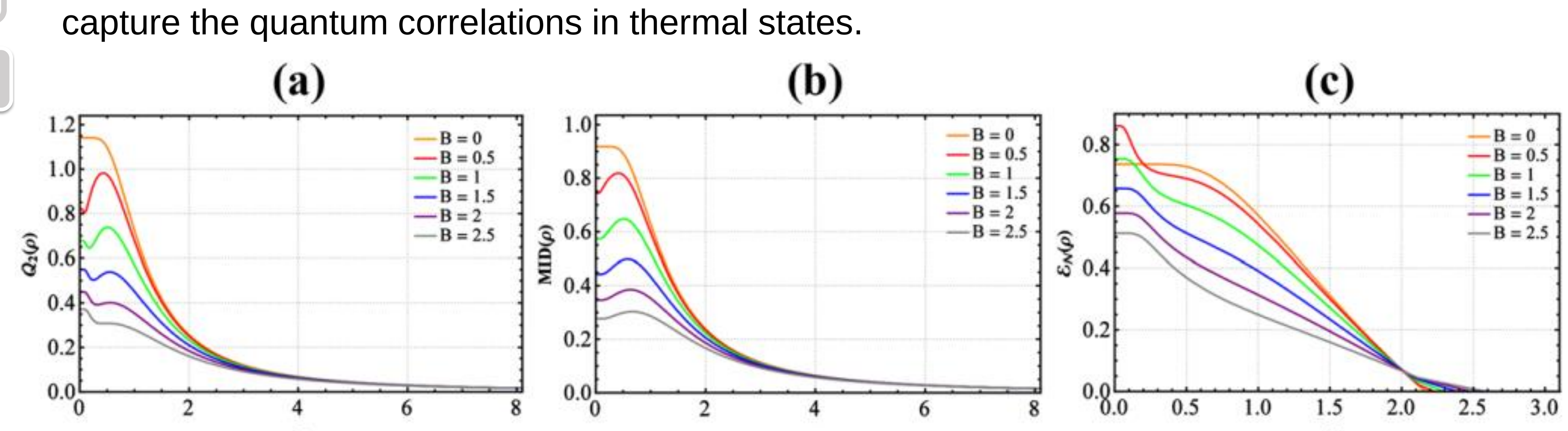

Fig. 2. Quantum discord $Q_2(\rho)$,measurement-induced disturbance MID(ρ) and logarithmic negativity $\mathcal{E}_{\mathcal{N}}(\rho)$as function of the temperature T for different values of the external magnetic field B when $J = 1$.

Fig. 2 shows that the thermal quantum correlations reach the maximum value at absolute zero temperature and decrease with the increasing values of T as B increases. This behavior is caused by the superpositions of the ground states with the excited states. Furthermore, both QD and MID have almost the same behavior, except that the maximum of QD is larger than that of MID at $B = 0$. They tend to zero for high values of the temperature (Fig.2a and Fig.2b). Besides, the logarithmic negativity disappears at critical temperature T, which can be tuned by the external magnetic field to increase the value of the critical point to avoid the disappearance of the logarithmic negativity and subsequently the intricacy.

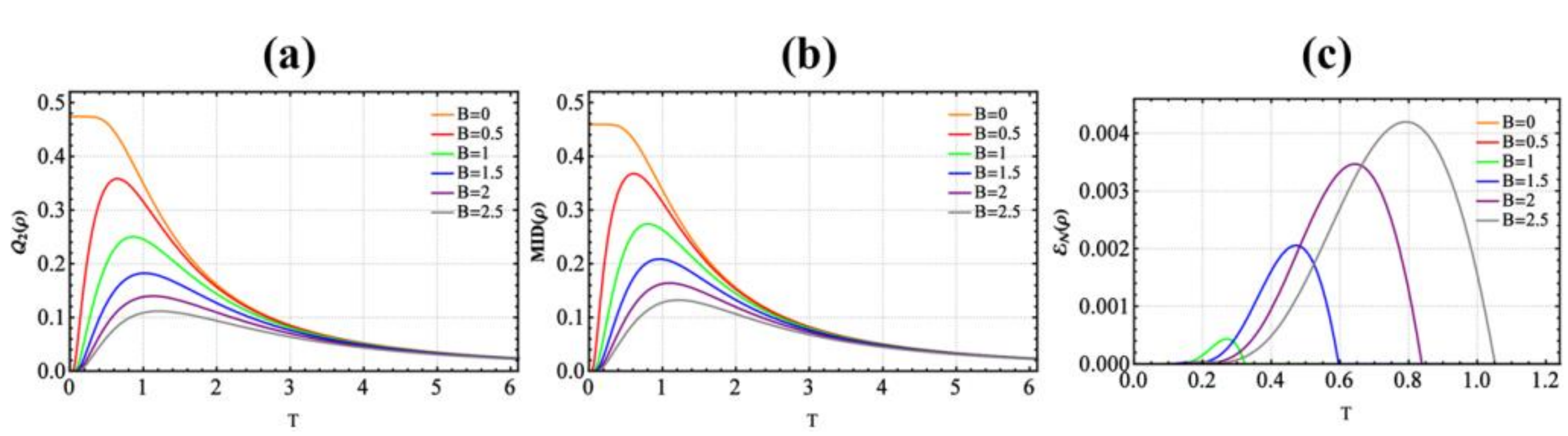

Fig. 3. Quantum discord $Q_2(\rho)$,measurement-induced disturbance MID(ρ) and logarithmic negativity $\mathcal{E}_{\mathcal{N}}(\rho)$as function of the temperature T for different values of the external magnetic field B when $J = -1$.

From **Fig.3**, it is clearly seen that the thermal correlation measures have different behavior compared to the antiferromagnetic case (**Fig.2**). **Fig.3a** and **Fig.3b** show that QD and MID are maximal for $T = 0$ and $B = 0$. We notice that in this situation the thermal entanglement is zero (**Fig.3c**). When $B > 0$ no thermal quantum correlations occur at the temperature T which is very near to zero, even for thermal quantum entanglement. Furthermore, the lower temperature enhances thermal quantum correlations. The logarithmic negativity varies from zero to a finite value and disappears at critical temperature T_c. The minimal value of QD and MID achieved for low temperature near zero for any magnetic field $B > 0$ with a tendency to be zero when the temperature becomes higher.