Introduction

Aperiodic space-inhomogeneous quantum walks: Localization properties, energy spectra, and enhancement of entanglement

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Introduction

Sometimes described as natural quantum analog of the classical random walk, discrete-time quantum walks (DTQWs) have received an outstanding attention. Coherent superposition and quantum interference make DTQWs usually faster than their classical counterpart, and therefore an interesting and versatile tool for the realization of quantum algorithms, quantum computation and quantum simulations [1-3]. Some ingredients have been explored in order to discover their potential, applicability and feasibility, as well as their shortcoming.

In general, disorder (noise) induces deviations from quadratic spreading of the wave packet. A localized behavior has been described for systems with spatial homogeneities [4-6]. Here, we explore a more general description, in which a lack of homogeneity is described by an aperiodical quantum walk.

We consider a quantum walker (qubit) moving in an infinite lattice of interconnected sites. The state of the walker after t steps is given by unitary transformation

\[ |\psi(t)\rangle = \sum_{n} c(n, t)|n\rangle = U^t|\psi(0)\rangle, \]

with \( U = \prod_{i=1}^{n} S \left( C_{i} + i\nu \right) \) and \( \sum_{n}|n, t\rangle\langle n| = 1 \). The displacement operator and the quantum gates are

\[ S = \sum_{x, y} \left| \langle x + 1|n + 1\rangle + \langle x - 1|n + 1\rangle \right|, \]

where \( C_{i} = \cos(\theta_{0}n_{i}) |\langle x + 1|n + 1\rangle + \sin(\theta_{0}n_{i}) |\langle x - 1|n + 1\rangle \rangle, \]

\( \nu \) is a tunable parameter that controls the aperiodicity degree on the rotation angle \( \theta_{0} = \nu \cdot 2\pi \).

Transport Properties and Energy Spectra

By considering the changes on the spreading behavior of the quantum walker described previously, we investigate the entanglement between the coin state and the particle position. We computed the von Neumann entropy of the reduced density matrix

\[ S_{E}(\rho) = -\text{Tr}[\rho \log \rho], \]

where \( \rho_{x} \) is

\[ \rho_{x} = S_{E}(\rho) = -\sum n \langle n | \rho | n \rangle \log \langle n | \rho | n \rangle, \]

with \( \langle n | \rho | n \rangle = \sum |n, m\rangle \langle n, m| \langle n | \rho | n \rangle = \sum a_{n}(\rho_{0})^{2}, \langle n | \rho | n \rangle = \sum a_{n}(\rho_{0})^{2}, \langle n | \rho | n \rangle = \sum a_{n}(\rho_{0})^{2}, \rho_{x} = \sum a_{n}(\rho_{0})^{2}, \rho_{x} = \sum a_{n}(\rho_{0})^{2}, \]

and the probability distribution \( \gamma(t) = \alpha(t) + \beta(t) = 1 \). By diagonalizing \( \rho_{x} \) we obtain

\[ S_{E}(\rho_{x}) = -\lambda_{0} \log \lambda_{0} - \lambda_{1} \log \lambda_{1} - \lambda_{2} \log \lambda_{2}, \]

where \( \lambda_{0}, \lambda_{1}, \lambda_{2} \) are the eigenvalues of matrix \( \rho_{x} \),

\[ \lambda_{0} = \left| \frac{1}{2} \pm \sqrt{1 - 4 \text{Re}(\alpha(t))\left(1 - |\gamma(t)|^{2}\right)} \right|, \]

Thus, \( S_{E}(\rho_{x}) \in [0, 1] \), with separable states (not entangled) giving \( S_{E} = 0 \) and maximally entangled states providing \( S_{E} = 1 \).

Conclusions and References

The translational symmetry on the inhomogeneity, recovered for \( \nu \in Z^{2} \), shows a relevant aspect, which favors the aperiodic behavior observed for \( \nu < 0.5 \).

- With an adjustable distribution ruled by a single parameter, we show the existence of delocalized and localized quantum walks, as well as the proper adjusting of the aperiodicity in order to developing both.

- With the energy spectra analysis, we observe the existence of a band gap, leading to vanishing gap between two main bands, which justifies the delocalization behavior observed for \( \nu < 0.5 \).

- For the coin-position entanglement, taking as reference the homogeneous distribution (disorder-free) of quantum coins, we studied many settings in which an enhancement in the ability to entangle is observed. Thus, this behavior brings new informations about the role played by aperiodicity on the coin-position entanglement for static inhomogeneous systems, reported before as almost always reducing the entanglement when comparing with the homogeneous case [6].


Acknowledgments:

Detailed description: