YIQIS 2020

Young IQIS 2020 - Young Italian Quantum Information Science Conference

Aperiodic space-inhomogeneous quantum walks: Localization properties, energy spectra, and enhancement of entanglement

A. R. C. Buarque and *W. S. Dias

Physics Institute, Universidade Federal de Alagoas, 57072-900 Maceió, Alagoas, Brazil

*wandearley@fis.ufal.br

Introduction

Sometimes described as natural quantum analog of the classical random walk, discrete-time quantum walks (DTQWs) have received an outstanding attention. Coherent superposition and quantum interference make DTQWs usually faster than their classic counterpart, and therefore an interesting and versatile tool for the realization of quantum algorithms, quantum computation and quantum simulations [1–3]. Some ingredients have been explored in order to discover their potential, applicability and feasibility, as well as their shortcoming.

In general, disorder (noise) induces deviations from quadratic spreading of the wave packet. A localized behavior has been described for systems with spatial inhomoneities [4–6]. Here, we explore a more general description, in which a lack of homogeneity is described by an aperiodic distribution of quantum gates along the lattice. We consider a quantum walker (qubit) moving in an infinite lattice of interconnected sites. The state of the walker after t steps is given by unitary transformation $|\psi(t)\rangle = \sum_n [a(n,t)|\uparrow\rangle\otimes|n\rangle + b(n,t)|\downarrow\rangle\otimes|n\rangle] = \hat{U}^t|\psi(0)\rangle$,

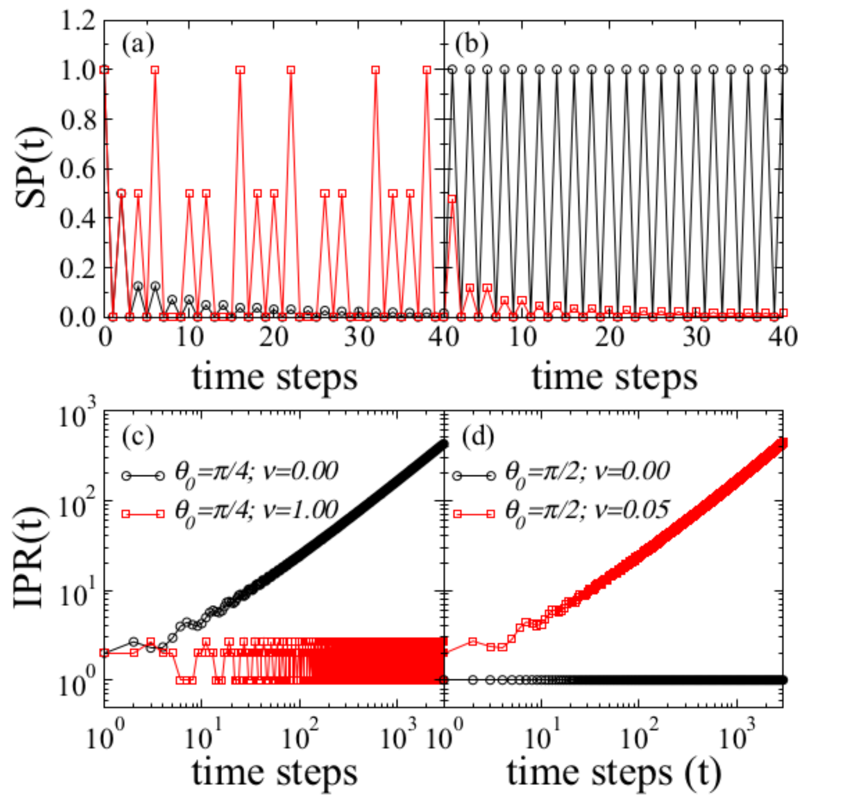
with $\hat{U}^t = \prod_{i=1}^{t} \hat{S}\cdot(\hat{C}_i \otimes \mathbb{I}_p)$ and $\sum_n[|a(n,t)|^2 + |b(n,t)|^2] = 1$. The displacement operator and the quantum gates are

$$\hat{S} = \sum_n \left(|\uparrow\rangle\langle\uparrow| \otimes |n+1\rangle\langle n| + |\downarrow\rangle\langle\downarrow| \otimes |n-1\rangle\langle n|\right),$$

$$\hat{C}_n = \cos(\theta_0 n^\nu)|\uparrow\rangle\langle\uparrow| + \sin(\theta_0 n^\nu)|\uparrow\rangle\langle\downarrow| + \sin(\theta_0 n^\nu)|\downarrow\rangle\langle\uparrow| - \cos(\theta_0 n^\nu)|\downarrow\rangle\langle\downarrow|.$$

ν is a tunable parameter that controls the aperiodicity degree on the rotation angle $\theta_0 = [0, 2\pi]$.

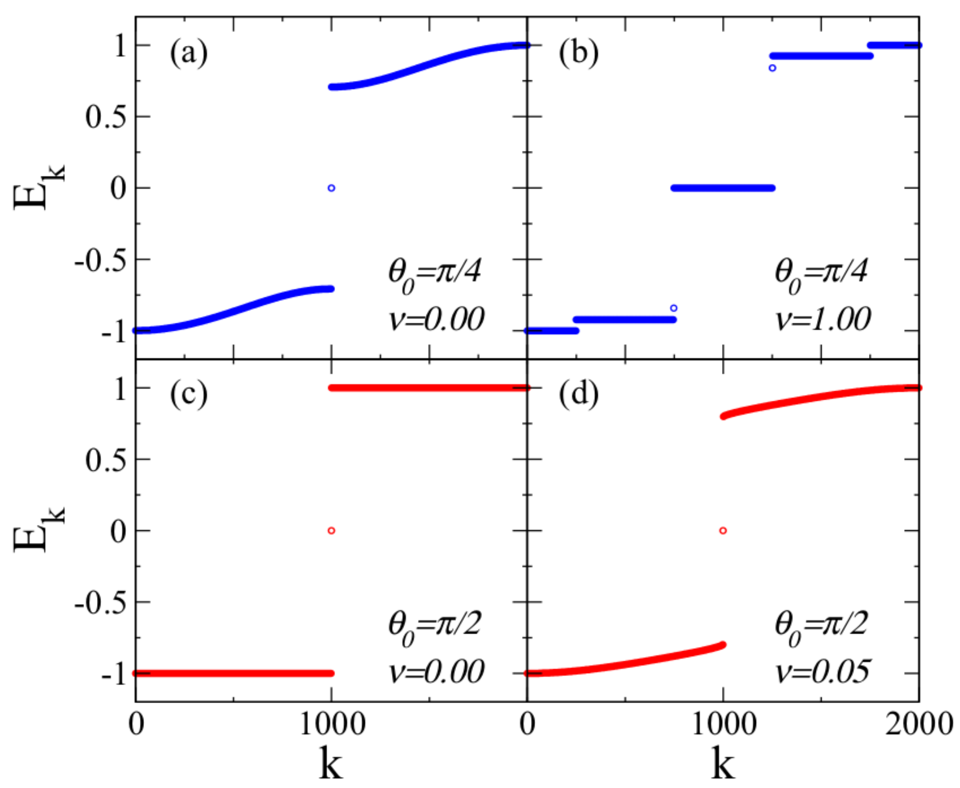
Transport Properties and Energy Spectra



Time evolution of survival probability (SP) and the inverse participation ratio (IPR) of a quantum walker with initial state $|\psi(0)\rangle = |\uparrow\rangle \otimes |N/2\rangle$: (a-c) $\theta_0 = \pi/4$ with $\nu = 0.00$ and $\nu = 1.00$; (b-d) $\theta_0 = \pi/2$ with $\nu = 0.00$ and $\nu = 0.05$. Specific spatial inhomogeneities on well-known quantum coins (Hadamard and Pauli-X) induce opposite dynamics those shown by homogeneous systems, with transitions between delocalized and strong-localized regimes ruled by aperiodic inhomogeneity.

$$\mathrm{SP(t)} = \sum_{\alpha=\uparrow,\downarrow} |\langle n| \otimes \langle\alpha|\psi(t)\rangle|^2 \Big|_{n=n_0}$$

$$\mathrm{IPR(t)} = \frac{1}{\sum_n |\psi_n(t)|^4}.$$



Energy spectrum $E_k = -i log(\lambda_k)$, with λ_k being the eigenvalues of the operator $\hat{U}$ for same configurations used in fig. 1. Allied to the presence of the internal degree of freedom, both systems exhibit two main bands in absence of inhomogeneity: Hadamard coins shows a continuous energy spectrum inside the two main bands, while Pauli-X coins shows a flat degenerate energy spectrum into both bands. The inhomogeneity for $\theta_0 = \pi/4$ open new gaps in the main energy bands, besides imposing a degeneracy on them. On the other hand, the inhomogeneity breaks the high degeneracy level within two subbands for $\theta_0 = \pi/2$.

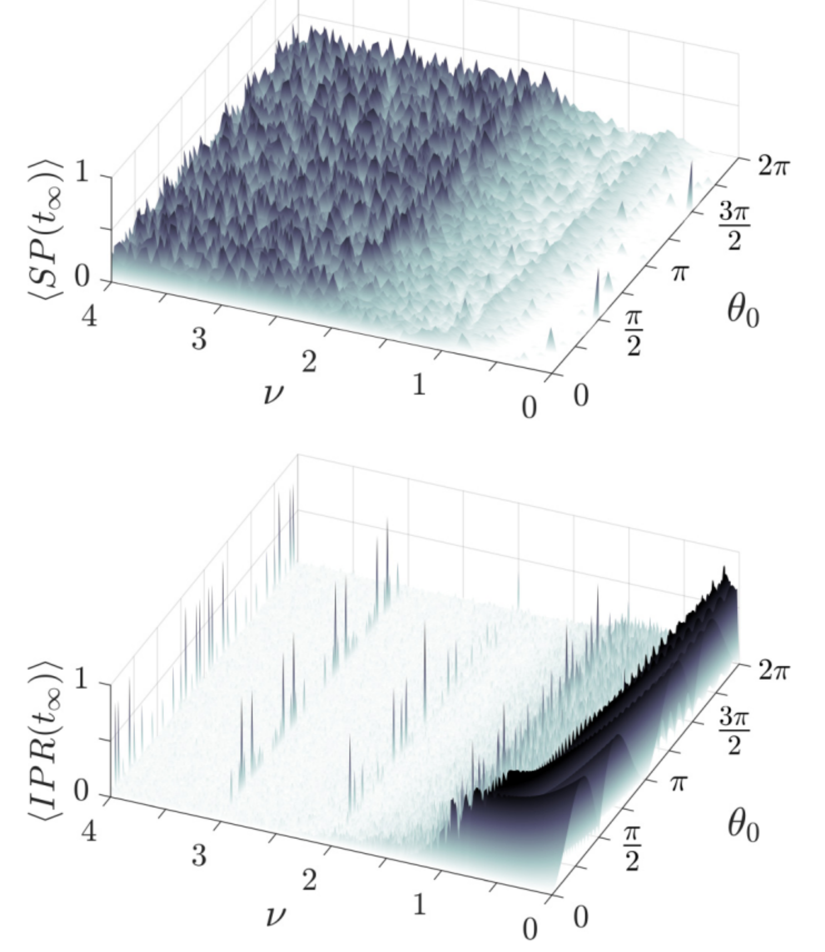


Diagram of long-time average of the survival probability ($\langle \mathrm{SP}(t_\infty)\rangle$) and the inversal participation ratio ($\langle \mathrm{IPR}(t_\infty)\rangle$) versus ν versus θ_0. Delocalized quantum walks are predominant for $\nu < 0.5$, while the localized nature is predominant with increasing ν, except for $\nu \in \mathbb{Z}^*$. The latter, which may be associated with the translational symmetry of inhomogeneity, appear to be a relevant aspect about the spread of wave function along the lattice.

Entanglement Properties

By considering the changes on the spreading behavior of the quantum walker described previously, we investigate the entanglement between the coin state and the particle position. We computed the von Neumann entropy of the reduced density matrix

$$S_E(t) = -Tr\left[\rho_c(t)\log_2 \rho_c(t)\right] \quad (1)$$

where ρ_c is

$$\rho_c(t) = \sum \langle m|\rho|m\rangle = \sum_m \begin{bmatrix} \alpha(t) & \gamma(t) \\ \gamma^*(t) & \beta(t) \end{bmatrix}, \quad (2)$$

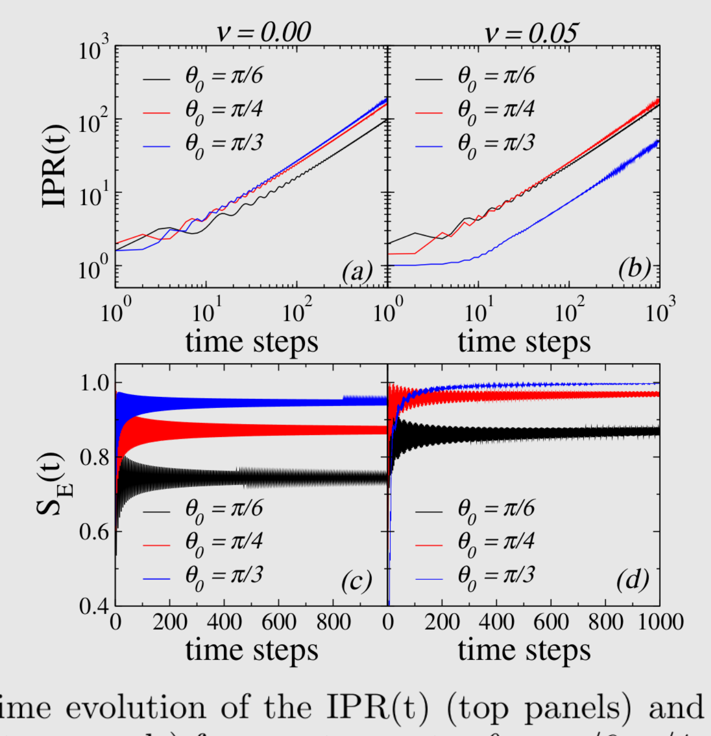
with $\alpha(t) = \sum_m |a_m(t)|^2$, $\beta(t) = \sum_m |b_m(t)|^2$, $\gamma(t) = a_m(t)b_m^*(t)$ and the probability distribution $|\psi_n(t)|^2 = \alpha(t) + \beta(t) = 1$. By diagonalizing $\rho_c(t)$ we obtain

$$S_E\left[\rho_c(t)\right] = -\lambda_+ \log_2 \lambda_+ - \lambda_- \log_2 \lambda_-, \quad (3)$$

where $\lambda_\pm$ are the eigenvalues of matrix ρ_c,

$$\lambda_\pm = \frac{1}{2}\left\{1 \pm \sqrt{1 - 4\left[\alpha(t)\beta(t) - |\gamma(t)|^2\right]}\right\}. \quad (4)$$

Thus, $S_E(t) \in [0, 1]$, with separable states (not entangled) giving $S_E = 0$ and maximally entangled states providing $S_E = 1$.



Time evolution of the IPR(t) (top panels) and $S_E(t)$ (bottom panels) for quantum gates $\theta_0 = \pi/6, \pi/4, \pi/3$ in the (a,c) absence and (b,d) presence of inhomogeneity. While IPR(t) exhibits a delocalized character for both regimes, we observe an increasing of entanglement power induced by inhomogeneity, even for a configuration with strong entanglement in the homogeneous configuration ($\theta_0 = \pi/3$).

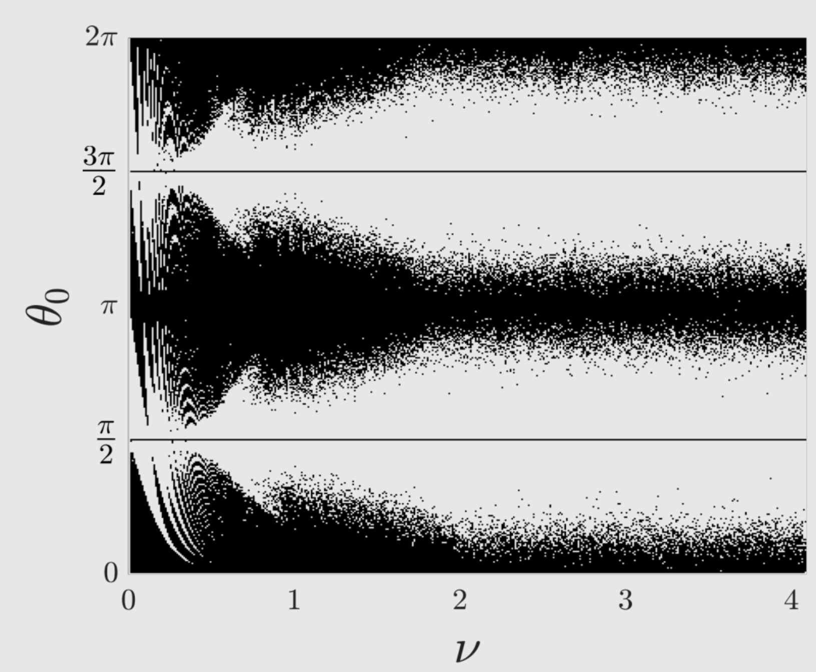


Diagram θ_0 versus ν reveals the enhancement in the ability to entangle internal (spin/polarization) and external (position) degrees of freedom with respect to homogeneous distribution of quantum coins. Black points means an increase of long-time average of entanglement, while white points denote a weakening. The increase in entanglement is predominant for initial values of ν. As ν grows, the increasing is restricted to values close to $\theta_0 = 0, \pi$ and 2π, as well as $\theta_0 = \pi/2$ and $3\pi/2$.

Conclusions and References

- With an adjustable distribution ruled by a single parameter ν we show the existence of delocalized and localized quantum walks, as well as the proper adjusting of the aperiodicity in order to developing both.

- With the energy spectra analysis, we observe the early stage the inhomogeneity leading to vanishing gap between two main bands, which justifies the the delocalized behavior observed for $\nu < 0.5$.

- With increase of ν arise gaps and flat-bands on the energy spectra, which justifies the suppression of transport detected for $\nu > 0.5$. For ν high enough the energy spectra resembles that described by the 1d Anderson model.

The translational symmetry on the inhomogeneity, recovered for $\nu \in \mathbb{Z}^*$, shows to be a relevant aspect, which favors delocalized quantum walks.

- For the coin-position entanglement, taking as reference the homogeneous distribution (disorder-free) of quantum coins, we identified many settings in which an enhancement in the ability to entangle is observed. Thus, this behavior brings new informations about the role played by aperiodicity on the coin-poisition entanglement for static inhomogeneous systems, reported before as almost always reducing the entanglement when comparing with the homogeneous case [6].

[1] Aharonov, Y., Davidovich, L., and Zagury, N. *Phys. Rev. A* **48**, 1687 (1993).
[2] Childs, A. M. *Phys. Rev. Lett.* **102**, 180501 (2009).
[3] Shenvi, N., Kempe, J., and Whaley, K. B. *Phys. Rev. A* **67**, 052307 (2003).
[4] Schreiber, A., Cassemiro, K. N., Potoček, V., Gábris, A., Jex, I., and Silberhorn, C. *Physical Review Letters* **106**, 180403 (2011).
[5] Ghosh, J. *Phys. Rev. A* **89**, 022309 (2014).
[6] Vieira, R., Amorim, E. P. M., and Rigolin, G. *Physical Review A* **89**, 042307 (2014).

Acknowledgments: CNPq FAPEAL CAPES

Detailed description:
A. R. C. Buarque and W. S. Dias, Physical Review E, 100, 032106 (2019)