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A Quantum model for rf-SQUIDs based metamaterials enabling 3WM and 4WM Travelling Wave Parametric Amplification

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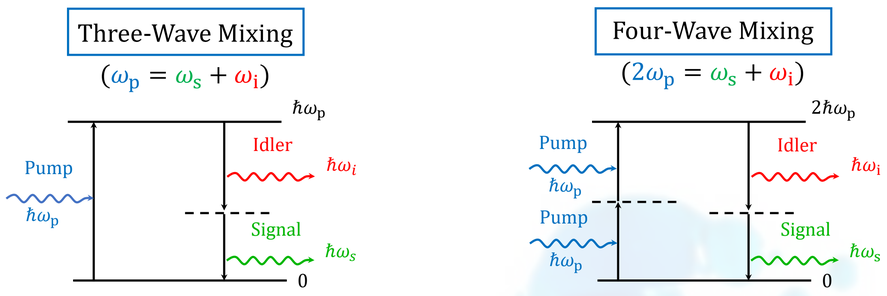
ABSTRACT

In superconducting quantum computing a qubit state can be inferred through the measurement of low-power microwave fields. In this context, the large bandwidth and ultralow-noise amplification given by a Travelling Wave Josephson Parametric Amplifier (TWJPA) plays an essential role. In our work we derive a quantum model for an rf-SQUID (rf-Superconducting QUantum Interference Device) based TWJPA, enabling the amplification of the input signal through a three-wave or a four-wave mixing process. These two different working regimes can be properly selected by changing the external bias conditions, represented by a DC current or a magnetic flux. Within the model, we derive an analytical expression for the gain and the squeezing of a given input signal at the single-photon level. Furthermore, we investigate the time evolution inside the device of the probability distribution of the photonic population, in the cases of two-mode Fock and coherent input states.

1 THREE-WAVE and FOUR-WAVE MIXING

Three-wave mixing (3WM) and four-wave mixing (4WM) are energy-conservative **intermodulation phenomena** that occur in natural or artificial **non-linear media**.



Due to the bosonic nature of photons, the creation of a signal photon at a certain frequency can be stimulated by the injection in the medium of other signal photons (**stimulated emission**). This peculiarity enables the non-linear medium to act as an **amplifier**.

2 NON-LINEAR MEDIUM FOR MICROWAVE PHOTONS

For **microwave photons** (1-12 GHz) a non-linear medium can be engineered embedding in a superconducting coplanar wave guide a repetition of non-linear elementary cells (meta-atoms). In these systems the non-linearity arises from the behavior of the **Josephson junctions** ($I(\varphi) = I_c \sin(\varphi)$).

We analyzed the case proposed in [1], in which the meta-atoms are based on a **rf-SQUID** (a superconducting loop containing a geometric inductance L_g and a Josephson junction), capacitively coupled to ground through C_g.

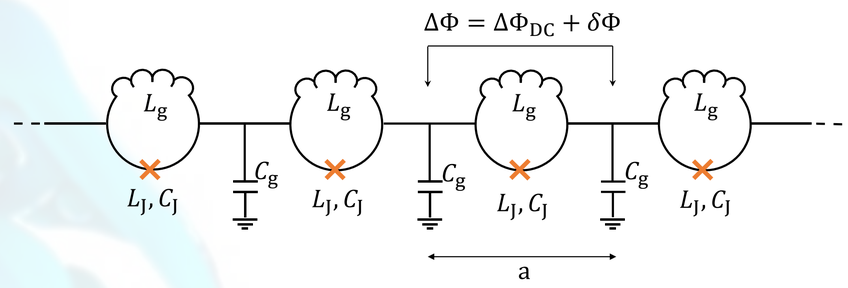


FIG 1: Representation of the electrical equivalent of a repetition of four elementary cell (periodicity a). Each cell is composed by a geometrical inductance L_g, a Josephson junction (with an associated capacitance C_J and inductance L_J) and a coupling capacitance C_g. The difference in the magnetic flux $\Delta\Phi$ between two consecutive nodes is the sum of two different contribution: $\Delta\Phi_{DC}$ is due to the bias condition (an external magnetic flux or a DC current) while $\delta\Phi$ is generated by the propagating microwave fields.

3 THE QUANTUM MODEL

The Hamiltonian of the system can be calculated as the sum of the energies stored in each component of the circuit presented in FIG 1.

$$H(t) = \frac{1}{2a}\int_0^{aN}\left[2I_c\varphi_0\left(1-\cos\left(\frac{\Delta\Phi(z,t)}{\varphi_0}\right)\right) + \frac{1}{L_g}\Delta\Phi(z,t) + C_J\left(\frac{\partial}{\partial t}[\Delta\Phi(z,t)]\right)^2 + C_g V_{C_g}^2(z,t)\right]dz$$

where $\varphi_0 = \Phi_0/2\pi = \hbar/2e$ is the reduced magnetic flux quantum, with $\hbar$ the reduced Planck constant and e the elementary charge.

Exploiting a mode decomposition of the electromagnetic field, the Hamiltonian can be expressed in the **second quantization framework** as [2]:

$$\hat{H} = \hbar\chi_0 + \quad \longrightarrow \text{Magnetic energy stored in the rf-SQUID}$$

$$+\sum_n \hbar\chi_1^{(n)}\left(\hat{a}_n^\dagger\hat{a}_n + \frac{1}{2}\right) + \quad \longrightarrow \text{Free field energy of the travelling modes}$$

$$+\sum_{n,l,m} \hbar\chi_3^{(n,l,m)}\left\{\hat{a}+\hat{a}^\dagger\right\}_{n,l,m}\delta_{\Delta\omega_{n,l,m},0} + \quad \longrightarrow \text{Coupling parameter associated to the three-wave mixing process}$$

$$+\sum_{n,l,m,s} \hbar\chi_4^{(n,l,m,s)}\left\{\hat{a}+\hat{a}^\dagger\right\}_{n,l,m,s}\delta_{\Delta\omega_{n,l,m,s},0} \quad \longrightarrow \text{Coupling parameter associated to the four-wave mixing process}$$

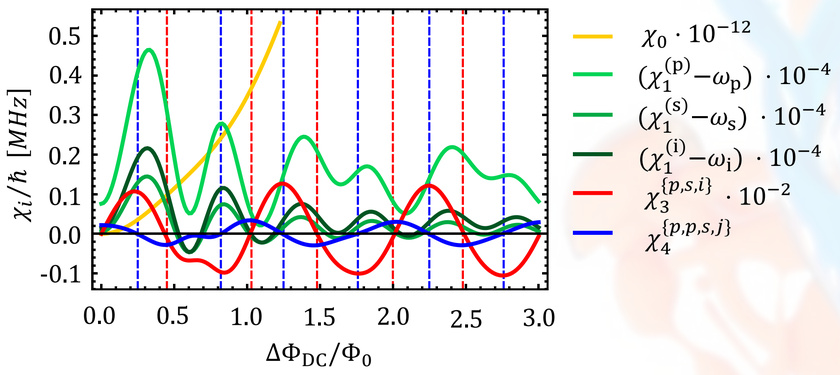


FIG 2: Hamiltonian coupling parameters χ_i as a function of the normalized magnetic flux bias ($\Delta\Phi_{DC}/\Phi_0$). The blue vertical dotted lines identify the flux biases at which the amplifier works as a pure three-wave mixer, while the red vertical dotted lines identify the flux biases at which the amplifier works as a pure four-wave mixer. The circuit parameters used to perform the numerical evaluation are: $I_c = 5$ μA, $C_g = 14$ fF, $L_g = 53$ pH, $C_J = 60$ fF, a = 60 μm, $N = 900$, $\omega_p/2\pi = 12$ GHz, $\omega_s/2\pi = 5$ GHz, $\omega_i/2\pi = 7$ GHz, and $\omega_j/2\pi = 19$ GHz.

KEY CONCEPT: Due to the non-centrosymmetric nonlinearity of a flux-biased rf-SQUID, the amplifier can work both as a three-wave or a four-wave mixer. The two regimes can be selected by properly tuning the external bias conditions.

4 GAIN and SQUEEZING SPECTRUM

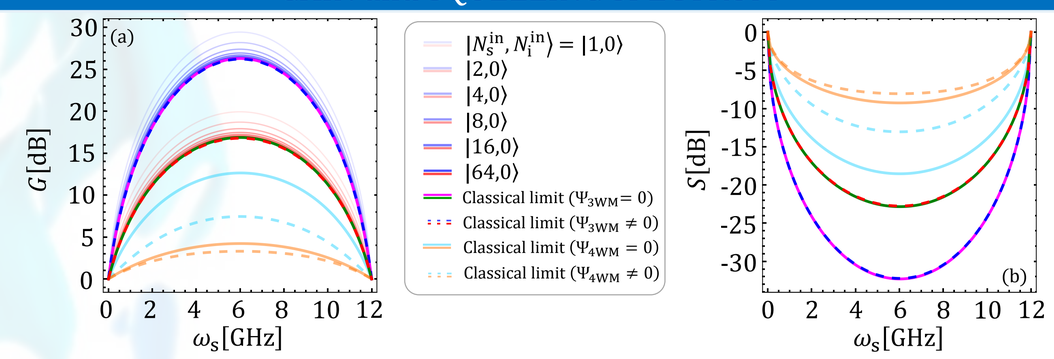


FIG 3: (a) Gain and (b) Squeezing spectra of a TWJPA under the undepleted pump approximation in the 4WM (light blue/orange) and 3WM (blue/red/purple/green) regimes for different input states. Blue, light blue, and purple curves are obtained for $P_p = -58$ dBm ($I_p/I_c = 0.9$) while red, orange and green curves are obtained for $P_p = -55$ dBm ($I_p/I_c = 1.2$).

5 BIMODAL FOCK and COHERENT STATES EVOLUTION

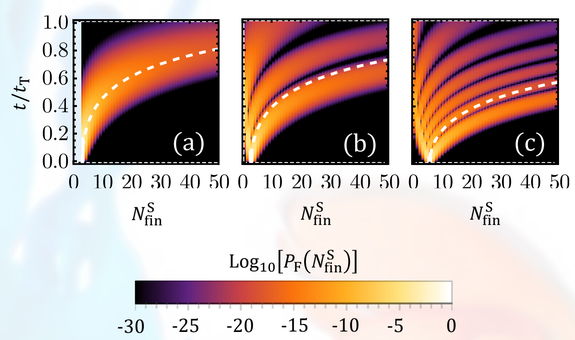


FIG 4: Time evolution inside the medium, from its input port ($t = 0$) to the output port ($t = t_T$) of the probability distribution P_F to find N_{fin}^S signal photons for three different initial **bimodal Fock states** (a) $|3\rangle_s\,|0\rangle_i$, (b) $|3\rangle_s\,|2\rangle_i$, and (c) $|6\rangle_s\,|6\rangle_i$. The dashed white lines represent the time evolution of the expectation value $\langle\hat{n}_s\rangle$.

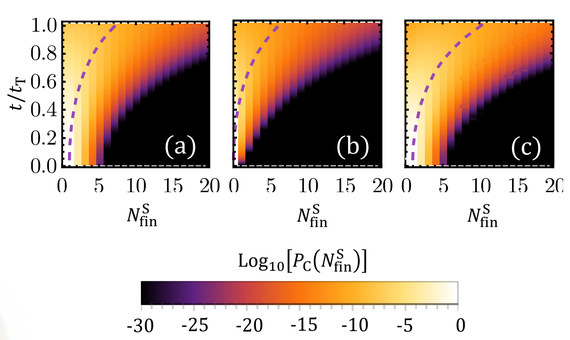


FIG 5: Time evolution inside the medium, from its input port ($t = 0$) to the output port ($t = t_T$) of the probability distribution P_C to find N_{fin}^S signal photons for three different initial **bimodal coherent states** $|\alpha\rangle_s\,|\beta\rangle_i$. (a) $|1\rangle_s\,|0\rangle_i$, (b) $|0\rangle_s\,|1\rangle_i$, and (c) $|1\rangle_s\,|1\rangle_i$. The dashed purple lines represent the time evolution of the expectation value $\langle\hat{n}_s\rangle$.

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BIOGRAPHY

Luca Fasolo graduated in Physics (110/110 cum laude) in July 2019 at the University of Torino, with a dissertation entitled "Superconductive parametric amplifier for microwave photons quantum metrology".

From November 2019 he is a **PhD student in Metrology**, at the Politecnico of Torino, with a scholarship financed by INRiM (Istituto Nazionale di Ricerca Metrologica). His research activities are focused on the fabrication, electrical characterization and theoretical modelling of superconducting devices operating in the microwave regime.

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OUR WORK