A Quantum model for rf-SQUIDs based metamaterials enabling 3WM and 4WM Travelling Wave Parametric Amplification

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ABSTRACT

In superconducting quantum computing a qubit state can be inferred through the measurement of low-power microwave fields. In this context, the large bandwidth and ultralow-noise amplifier given by a Travelling Wave Josephson Parametric Amplifier (TWPA) plays an essential role. In our work we derive a quantum model for an rf-SQUID (rf-Superconducting Quantum Interference Device) based TWPA, enabling the amplification of the input signal through a three-wave or a four-wave mixing process. These two different working regimes can be properly selected by changing the external bias conditions, represented by a DC current or a magnetic flux. Within the model, we derive an analytical expression for the gain and the squeezing of a given input signal at the single-photon level. Furthermore, we investigate the time evolution inside the device of the probability distribution of the photonic population, in the cases of two-mode Fock and coherent input states.

THREE-WAVE and FOUR-WAVE MIXING

Three-wave mixing (3WM) and four-wave mixing (4WM) are energy-conservative intermodulation phenomena that occur in natural or artificial non-linear media.

![Schematic of 3WM and 4WM mixing](image)

The Hamiltonian of the system can be calculated as the sum of the energies stored in each component of the circuit presented in Fig. 1.

\[ H(x) = \frac{1}{2a} \int (2\gamma_0 |\psi(t)|^2 - \cos(\Delta\Phi(t)/\psi_0) + \frac{\gamma}{2} \Delta\Phi(t)^2 + \frac{\gamma}{2} \Delta\Phi(t)^2)^2 + \frac{\gamma}{2} N \Delta\Phi(t)^2 \) dx

where \( \gamma_0 = \gamma_0/2 \) is the reduced magnetic flux quantum, with \( \gamma_0 \) the Planck constant and \( \omega \) the elementary charge.

Exploiting a mode decomposition of the electromagnetic field, the Hamiltonian can be expressed in the second quantization framework as [2]:

\[ \hat{H} = \hbar \omega_0 + \sum_{\alpha} \sum_{\mathbf{k}} \hbar \omega_{\alpha \mathbf{k}} a_{\alpha \mathbf{k}}^\dagger a_{\alpha \mathbf{k}} + \sum_{\alpha} \sum_{\mathbf{k}} \hbar \omega_{\alpha \mathbf{k}} a_{\alpha \mathbf{k}}^\dagger a_{\alpha \mathbf{k}} + \frac{\hbar \alpha_0}{2} \sum_{\mathbf{k}} \left( a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + a_{\mathbf{k}} a_{\mathbf{k}}^\dagger \right)

\]

**KIEV CONCEPT:** Due to the non-centrosymmetric nonlinearity of a flux-biased rf-SQUID, the amplifier can work both as a three-wave or a four-wave mixer. The two regimes can be selected by properly tuning the external bias conditions.


GAIN and SQUEEZING SPECTRUM

![Schematic of gain and squeezing spectrum](image)

**BIMODAL FOCK and COHERENT STATES EVOLUTION**

![Schematic of bimodal Fock and coherent states evolution](image)

**BIBLIOGRAPHY**

Luca Fasolo graduated in Physics (110/110 cum laude) in July 2019 at the University of Torino, with a dissertation entitled “Superconducting parametric amplifier for microwave photons quantum metrology”. From November 2019 he is a PhD student in Metrology at the Politecnico of Torino, with a scholarship financed by INRIM (Istituto Nazionale di Ricerca Metrologica). His research activities are focused on the fabrication, electrical characterization and theoretical modelling of superconducting devices operating in the microwave regime.

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