Two-fermion molecules in a one-dimensional harmonic trap

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Abstract

Low-particle systems constitute a direct link between the physics of one or two bodies and the physics of many bodies. We study a system consisting of independent molecules formed by two distinguishable fermions that interact via a contact potential in an elongated harmonic trap (this was already experimentally achieved), so the system can effectively be considered as 1D. We focus on the entanglement between the constituents of the molecule. By using an ansatz of composite bosons for the ground state, we analyze properties such as the condensate fraction, the occupancy numbers of the Schmidt modes, and the density profile of a particle both, when adding molecules to the system and when the interaction parameter between the constituent fermions is modified. In the strongly attractive regime, the pair is maximally entangled and the molecules behave like ideal bosons (BECs). On the other hand, in the strongly repulsive regime, the system exhibits a finite but not maximum amount of entanglement. The fermionization phenomenon is also studied in this regime. Our analysis paves the way to the exploration of 1D Fermi gases with an arbitrary number of pairs across the full interaction range.

Interacting fermions in an optical trap

The system consists in \(N\) pairs of two distinguishable fermions interacting via a contact potential in an elongated optical trap. It can be treated as a one-dimensional system.

Coboson Ansatz and bosonic behavior

The heart of the composite boson formalism is an ansatz based on the successive application of the operator \(\hat{\epsilon}^\dagger\) that creates a molecule in the ground state\(^1\). In terms of the Schmidt decomposition we have:

\[
|N\rangle = \left(\frac{e^N}{\sqrt{N!}}\right) |0\rangle, \quad \text{with} \quad \chi_N = N! \sum_{j_1 > \cdots > j_N} \hat{\lambda}_{j_1} \cdots \hat{\lambda}_{j_N}.
\]

The bosonic behavior can be readed from the ratio \(\frac{\chi_N}{\chi_N}\). When this ratio tends to 1 the system can be treated as a system of \(N\) perfectly bosonic particles.

These are the ratios between the normalization factors \(\frac{\chi_N}{\chi_N}\) in the whole interaction range. All the ratios \(\frac{\chi_{N+1}}{\chi_N}\) \(\rightarrow\) 1 when \(\gamma \rightarrow -\infty\).

One-particle density profile

The function \(\langle x|\hat{\Psi}_a^\dagger(x)|\hat{\Psi}_a(x)|N\rangle\), with \(\hat{\Psi}_a(x)\) the creation operator over the coordinate space of a particle for instance \(a\), represents the type-\(a\) particle density inside the trap when we have \(N\) molecules or pairs. In these figures we show these density profiles for representative states in the strongly attractive(left) and repulsive(right) regimes.

References