Two-fermion molecules in a one-dimensional harmonic trap

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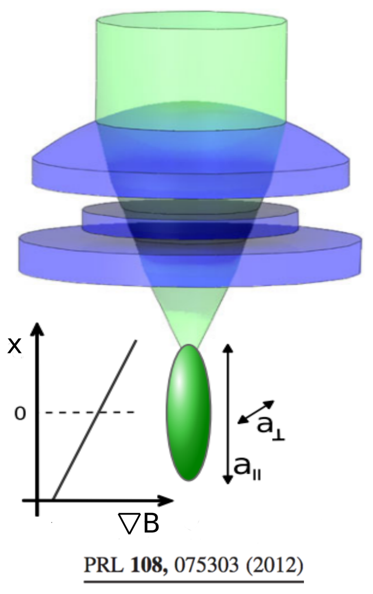
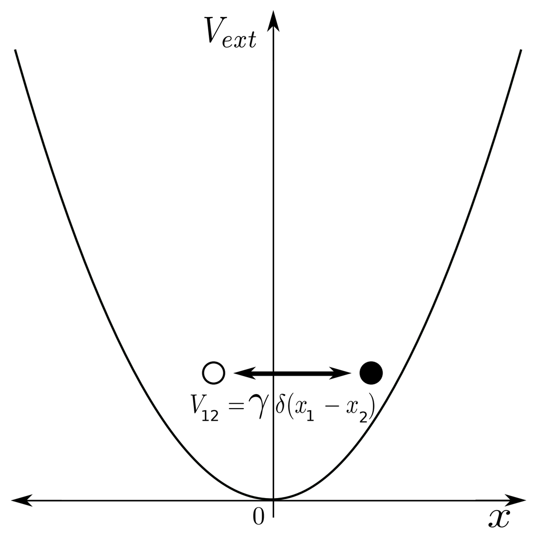
Abstract

Low-particle systems constitute a direct link between the physics of one or two bodies and the physics of many bodies. We study a system consisting of independent molecules formed by two distinguishable fermions that interact via a contact potential in an elongated harmonic trap (this was already experimentally achieved), so the system can effectively be considered as 1D. We focus on the entanglement between the constituents of the molecule. By using an ansatz of composite bosons for the ground state, we analyze properties such as the condensate fraction, the occupancy numbers of the Schmidt modes, and the density profile of a particle both, when adding molecules to the system and when the interaction parameter between the constituent fermions is modified. In the strongly attractive regime, the pair is maximally entangled and the molecules behave like ideal bosons (BECs). On the other hand, in the strongly repulsive regime, the system exhibits a finite but not maximum amount of entanglement. The fermionization phenomenon is also studied in this regime. Our analysis paves the way to the exploration of 1D Fermi gases with an arbitrary number of pairs across the full interaction range.

Interacting fermions in a optical trap





The system consists in N pairs of two distinguishable fermions interacting via a contact potential in an elongated optical trap. It can be treated as a one-dimensional system.

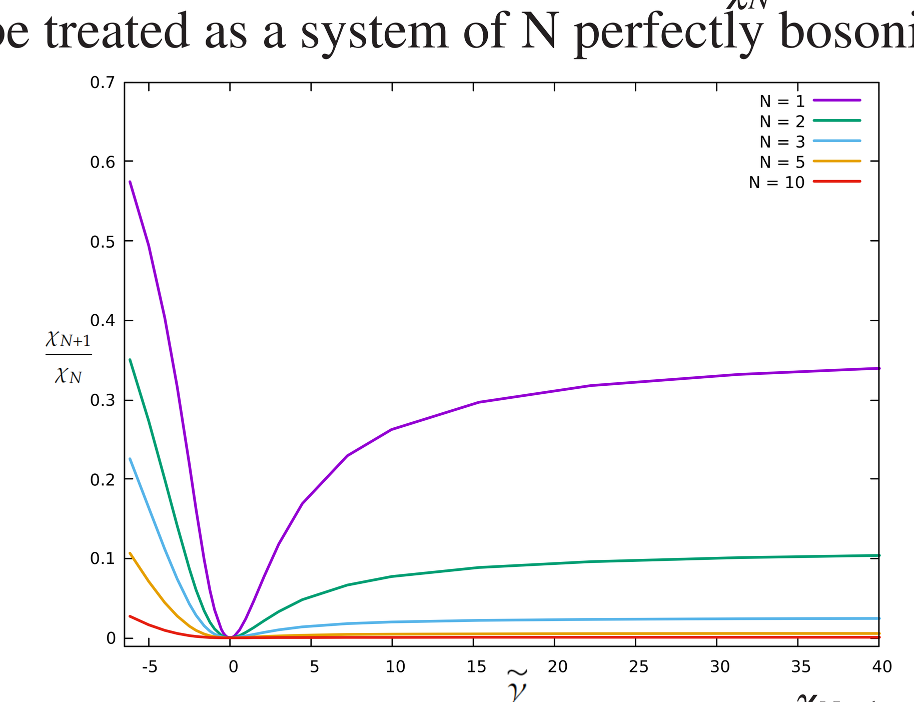
Coboson Ansatz and bosonic behavior

The heart of the composite boson formalism is an ansatz based on the successive application of the operator $\hat{c}^\dagger$ that creates a molecule in the ground state[1]. In terms of the Schmidt decomposition we have:

$$\hat{c}^\dagger = \sum_j \sqrt{\lambda_j}\,\hat{a}_j^\dagger \hat{b}_j^\dagger$$

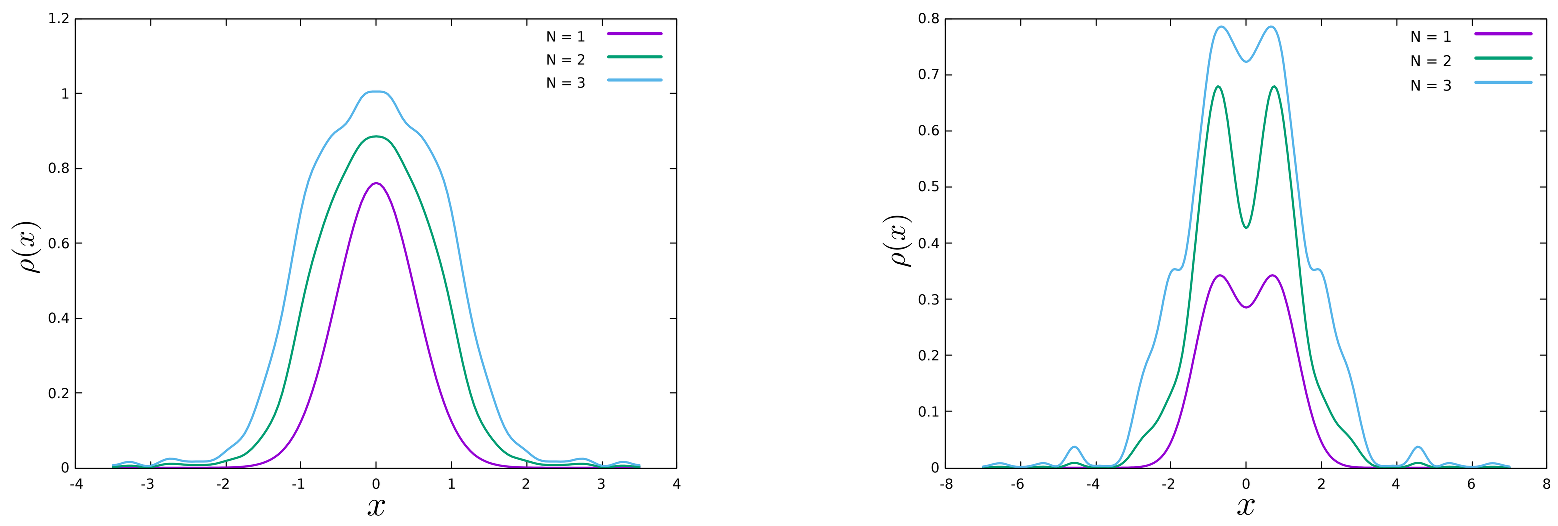
$$|N\rangle = \frac{\left(\hat{c}^\dagger\right)^N}{\sqrt{N!\,\chi_N}}\,|0\rangle\,, \quad \text{with} \;\; \chi_N = N! \sum_{p_N > \ldots > p_1} \lambda_{p_N} \ldots \lambda_{p_1}\,.$$

The bosonic behavior can be readed from the ratio $\frac{\chi_{N+1}}{\chi_N}$. When this ratio tends to 1 the system can be treated as a system of N perfectly bosonic particles.



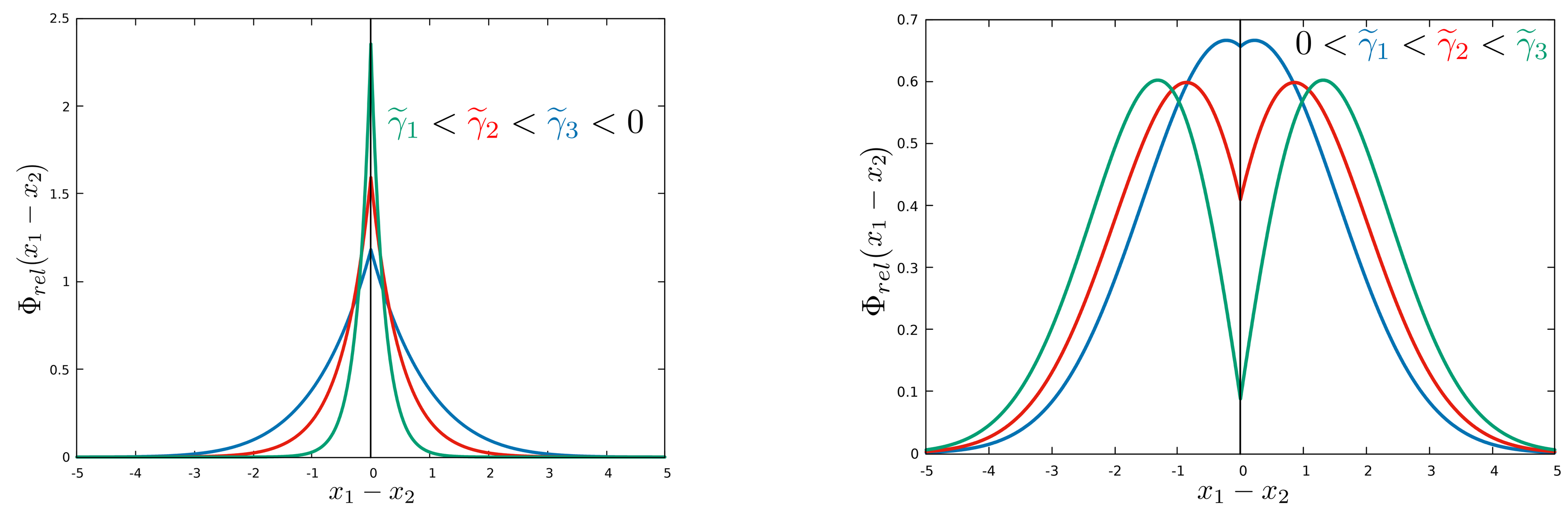
These are the ratios between the normalization factors $\frac{\chi_{N+1}}{\chi_N}$ in the whole interaction range. All the ratios $\frac{\chi_{N+1}}{\chi_N} \longrightarrow 1$ when $\tilde{\gamma} \longrightarrow -\infty$.

One-particle density profile



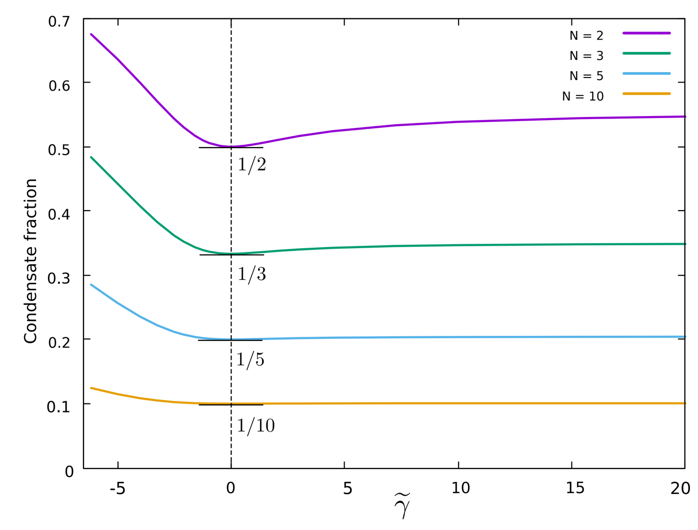
The function $\langle N|\hat{\Psi}_a^\dagger(x)\hat{\Psi}_a(x)|N\rangle$, with $\hat{\Psi}_a^\dagger(x)$ the creation operator over the coordinate space of a particle for instance a, represents the type-a particle density inside the trap when we have N molecules or pairs. In these figures we show these density profiles for representative states in the strongly attractive(left) and repulsive(right) regimes.

One-molecule relative wave function



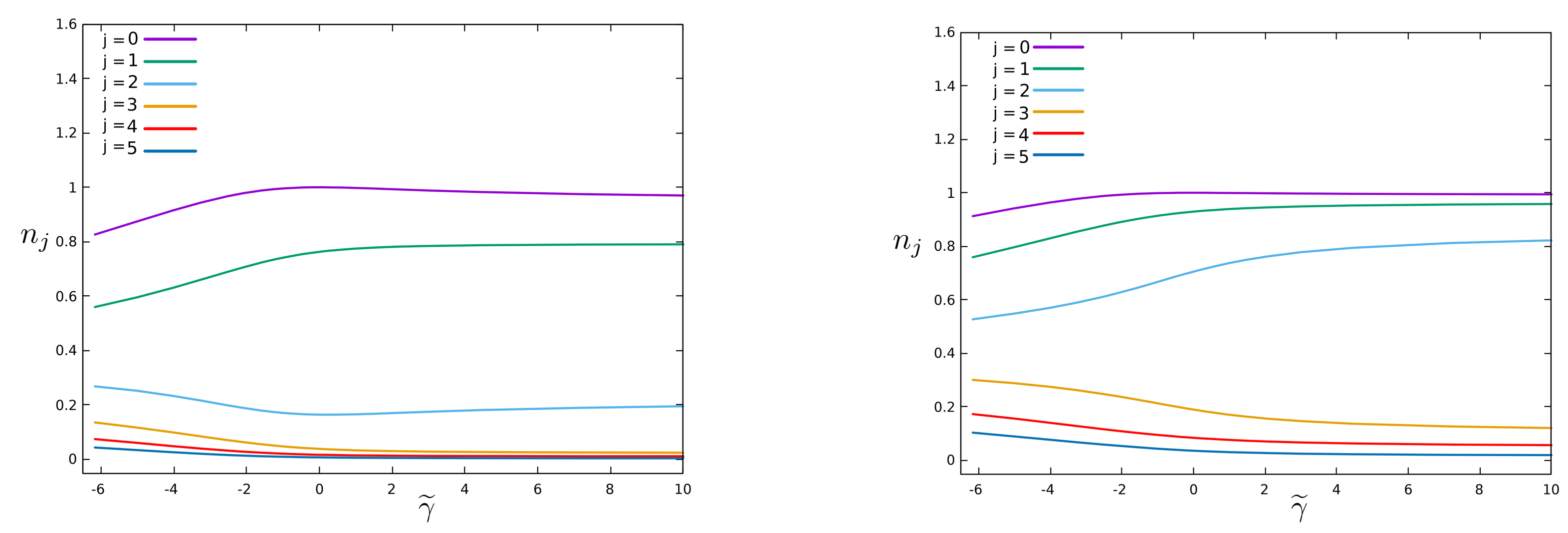
In these figures we show the relative wave-function in the attractive(left) and repulsive(right) interacting regimes for different values of the interaction parameter. In the strong attractive regime the fermions are strongly bounded and behave as molecules. Here $\tilde{\gamma} = \gamma\sqrt{\frac{m}{\omega\hbar^3}}$.

Condensate fraction



The condensate fraction is given by[2] $\frac{\langle N|\,\hat{c}^\dagger\hat{c}\,|N\rangle}{N} = \frac{1}{N} + \left(1 - \frac{1}{N}\right)\frac{\chi_{N+1}}{\chi_N}$, and represents the number of molecules in the ground state. When the system is maximally entangled (i.e., on the strongly attractive regime), the condensate fraction is equal to 1 (BECs). On the other hand, near the zero interaction regime, the Pauli exclusion principle only allows one molecule in the ground state.

Occupation numbers



We compute the Schmidt modes occupation numbers $n_j(N) = \langle N|\,\hat{a}_j^\dagger\hat{b}_j^\dagger\hat{a}_j\hat{b}_j\,|N\rangle$. Notice that near the zero interaction regime the first N modes are not totally occupied. This is due to the fact that the Schmidt coefficients ratios $\frac{\lambda_{j+1}}{\lambda_j}$ are $\neq 0$ in this limit.

Conclusions

We deeply analyze the ground state entanglement of the two-fermion system for the whole range of the interaction. With the results obtained from this analysis + an ansatz of composite bosons strongly based on the solutions of a pair, we build the state of N pairs and study for few pairs some of their properties: the condensate fraction, the occupation numbers, and the density profiles. In the strongly attractive regime, the molecules behave like perfect bosons while the exchange correlations modify this behavior when the interaction goes to zero. We also note that different properties of the system are obtained when reaching the noninteracting regime from the interacting one, or when a noninteracting system is directly prepared.

References

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