Suppressing decoherence in quantum computers with unitary operations

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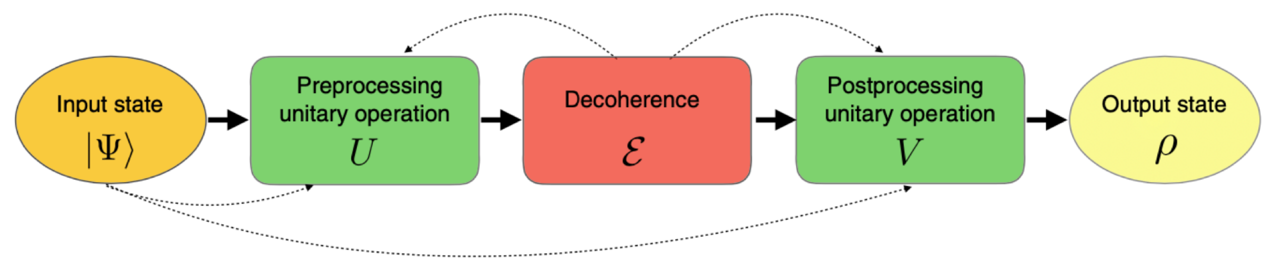
INTRODUCTION

Decoherence is a fundamental obstacle to the implementation of large-scale and low-noise quantum computing devices[1]. In the present work, we investigate the role of the fidelity of finite-dimensional quantum systems in the context of their robustness to decoherence. We suggest an approach for suppressing errors by employing. We consider the realization of our approach for the basic decoherence models, which include single-qubit depolarizing, dephasing, and amplitude damping channels. We demonstrate that for the case of depolarization channels there is a general relation between linear entropies of quantum states and fidelities of the quantum state after the action of the depolarizing channel on a particular subsystem of quantum states. We prove the general relation between linear entropies of quantum states for depolarization channels and illustrate it for qubit systems and we consider a generalization of the suggested approach for qudit ensembles.

ERROR SUPPRESSION SCHEME



Error suppression scheme based on pre-processing and post-processing unitary operations, which are designed specifically for the input state and decoherence channel in order to maximize the fidelity of the output state.

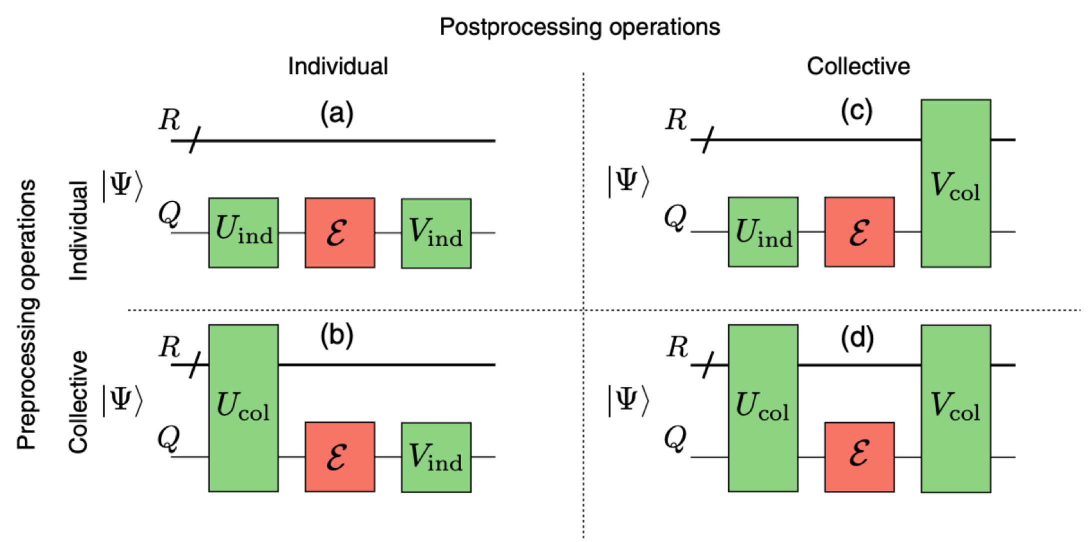
Let the whole system be initialized in a pure state written in the following form:

$$|\Psi\rangle_{1,\dots,n} = \sum_{x\in\{0,1\}^n} c_x\, |x\rangle_{1,\dots,n}$$

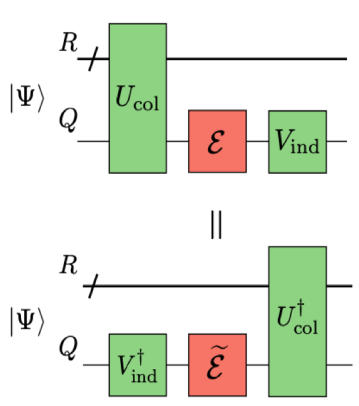
As the main target characteristic, we consider the fidelity regarding input and output states, which is given by the following expression:

$$F := \langle\Psi|_{1,\dots,n}\rho^{(k)}_{1,\dots,n}|\Psi\rangle_{1,\dots,n}$$

SCHEMES FOR PROTECTING FROM DECOHERENCE



We use a pre-processing procedure for preparing a given known quantum state of the system in a specific form. Next, we use a post-processing operation, which follows the action of a decoherence channel. These operations can be realized as unitary operators, and their particular form can be efficiently constructed on the basis of prior knowledge of the state under the protection and decoherence channel.



'Both individual' scheme:

$$U_{\text{ind}} = \sum_{i=0,1} |\varphi_i\rangle\langle\psi_i|, \qquad V_{\text{ind}} = \sum_{i=0,1} |\psi_i\rangle\langle\chi_i|.$$

'Individual-then-collective' & *'collective-then-individual'* scheme:

$$V^{\text{opt}}_{\text{col}} = |\Psi\rangle\langle\chi_0| + [\dots],\ V_{\text{col}} \leftrightarrow U^{\dagger}_{\text{col}},\ U_{\text{ind}} \leftrightarrow V^{\dagger}_{\text{ind}}.$$

'Both collective' scheme:

$$U_{\text{col}}|\Psi\rangle_{\text{QR}} = |\Upsilon\rangle_{\text{Q}} \otimes |\Xi\rangle_{\text{R}},$$

$$|\Upsilon^{\text{opt}}\rangle_Q := \arg\min_{|\Upsilon\rangle} S_{\min}(\boldsymbol{\varepsilon}[|\Upsilon\rangle\langle\Upsilon|]),$$

$$U_{\text{col}} = |\Upsilon^{\text{opt}}\rangle \otimes |\Xi\rangle\langle\Psi| + [\dots],\ V_{\text{col}} = |\Psi\rangle\langle\Theta| \otimes \langle\Xi| + [\dots].$$

DECOHERENCE CHANNELS

Basic single-qubit decoherence channels [2]

depolarizing channel:

$$\boldsymbol{\varepsilon}[\rho] = (1-p)\rho + p\frac{\mathbf{1}}{2}tr(\rho)$$

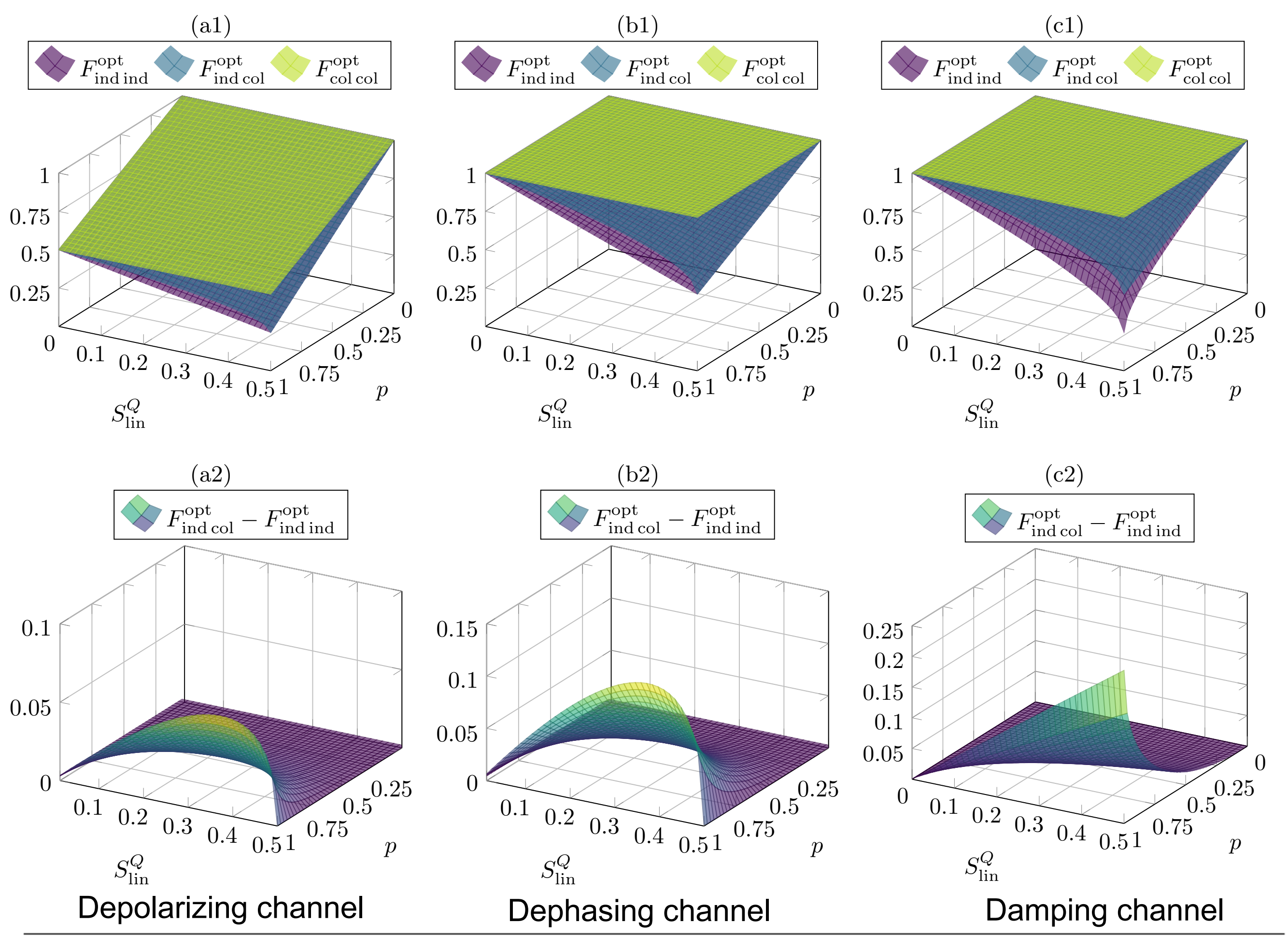
dephasing channel:

$$\boldsymbol{\varepsilon}[\rho] = (1-p)\rho + p(\rho_{00}|0\rangle\langle 0| + \rho_{11}|0\rangle\langle 0|)$$

damping channel:

$$\boldsymbol{\varepsilon}[\rho] = \sum_k A_k\rho A_k, \qquad A_1 = \begin{bmatrix}1 & 0\\ 0 & \sqrt{1-p}\end{bmatrix}, \qquad A_2 = \begin{bmatrix}0 & \sqrt{p}\\ 0 & 0\end{bmatrix}.$$

APPLICATIONS FOR THE BASIC DECOHERENCE CHANNELS



	'Both individual'	'individual-then-collective' & 'collective-then-individual'	'Both collective'
Depolarizing	$1-p(1+S^Q_{\text{lin}})/2$	$1/2-p/4+\sqrt{(p-2)^2-2p(4-3p)S^Q_{\text{lin}}}/4$	$1-p/2$
Dephasing	$1-pS^Q_{\text{lin}}$	$1/2+\sqrt{1-2p(2-p)S^Q_{\text{lin}}}/2$	1
Amplitude damping	$\sqrt{1-p}S^Q_{\text{lin}}+(1-p/2)(1-S^Q_{\text{lin}})+p\sqrt{1-2S^Q_{\text{lin}}}/2$	$1-p\left(1-\sqrt{1-2S^Q_{\text{lin}}}\right)/2$	1

CONCLUSION

We have suggested the method for improving fidelity based on the class of state-dependent operations protecting the system from decoherence. We have shown that two schemes of this class, namely 'individual-then-collective' and 'collective-then-individual' provide the same maximal achievable levels of the fidelity. For basic channels we have seen that for given decoherence strength the maximal fidelities in all the schemes except 'both collective' is expressed with the linear entropy of the qubit under decoherence. In particular, we have obtained that the larger is the linear entropy the lower is the fidelity. This feature provides an answer on the question which qubit from the whole n-qubit system is the most vulnerable and which qubit is the most robust in the sense of preserving the whole n-qubit state after qubit decoherence. For the qubit case, the reduced state with the lowest linear entropy is automatically the state with the lowest von Neumann and min-entropies. This is, however, not the case for higher dimensions.

ACKNOWLEDGEMENTS

The work was supported by Grant of the President of the Russian Federation (project MK-923.2019.2)

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