Meter sensitivity in quantum measurements

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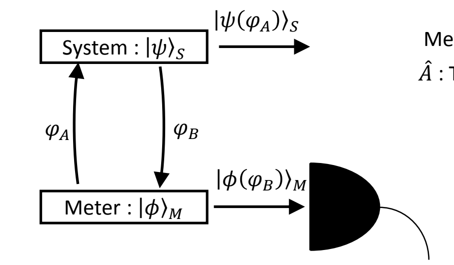
Introduction

As the level of control over individual quantum systems improves, the precise nature of the relation between measurement resolution and back-action has became an important area of research [1-3]. We analyze the effects of a quantum interaction on a general meter system and show that the sensitivity of the meter depends on the uncertainty of its back-action.

Analysis of the measurement interaction



$|\psi\rangle_S$: Input state
$|\phi\rangle_M$: meter initialization
φ_A back action (B)
φ_B meter response (A)

Outcome : m

Measurement interaction : $\hat{U}_{SM} = exp\left(-\frac{i}{\hbar}\hat{A}\otimes\hat{B}\right)$

$\hat{A}$: Target Observable in the system $\hat{B}$: Generator of the meter response

Meter Response: $|\phi(\varphi_B)\rangle_M = exp\left(-\frac{i}{\hbar}\varphi_B\hat{B}\right)|\phi\rangle_M$

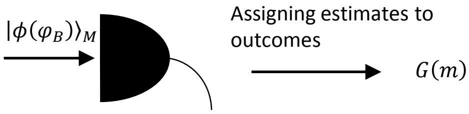
Back-action: $|\psi(\varphi_A)\rangle_S = exp\left(-\frac{i}{\hbar}\varphi_A\hat{A}\right)|\psi\rangle_S$

Measurement resolution is achieved by estimating the strength of the conditional unitary acting on the meter.

Result of the analysis:

The meter sensitivity F_M is upper bounded by the uncertainty of the cause of the back-action, ΔB^2.

Estimation of the meter response



Outcome : m

$P(m|\varphi_B) = |\langle m|\phi(\varphi_B)\rangle_M|^2$

Estimating φ_B from $G(m)$ based on probabilities $P(m|\varphi_B)$:

$$\varphi_B^{est.}(m) = \frac{G(m)}{\frac{d\langle\hat{G}\rangle}{d\varphi_B}}$$

Sensitivity of the meter (inverse error of the estimate):

$$\frac{1}{\delta^2_{\varphi_B}} = \frac{\left(\frac{d\langle\hat{G}\rangle}{d\varphi_B}\right)^2}{\Delta G^2}$$

The sensititvity of the optimal estimate is given by the Fisher Information (Cramer-Rao bound):

$$\boxed{F_M = \sum_m \frac{1}{P(m|\varphi_B)}\left(\frac{d}{d\varphi_A}P(m|\varphi_B)\right)^2}$$

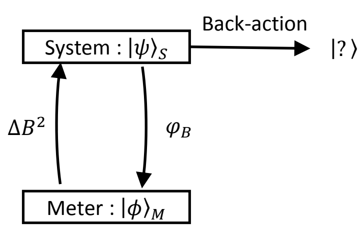
The Fisher information F_M determines the maximal sensitivity of the meter

Meter sensitivity and back- action

The quantum Cramer-Rao bound originates from the non-commutativity of estimator G and generator B,

$$\Delta B^2 \Delta G^2 \geq \frac{\hbar^2}{4}\left(\frac{d\langle\hat{G}\rangle}{d\varphi_B}\right)^2$$

$$\boxed{F_M \leq \frac{4}{\hbar^2}\Delta B^2}$$



The meter sensitivity F_M requires an uncertainty of ΔB^2 in the back-action!

1. M. Ozawa, Phys. Rev. A 67, 042105 (2003).
2. P. Busch, P. Lahti, and R. F. Werner, Rev. Mod. Phys. 86, 1261 (2014).
3. K. Patekar and H. F. Hofmann, New J. Phys. 21, 103006 (2019).