Witnessing non-Markovian effects of quantum processes through Hilbert-Schmidt speed

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ABSTRACT

Non-Markovian effects can speed up the dynamics of quantum systems. We introduce a witness for characterizing the non-Markovianity through the Hilbert-Schmidt speed (HSS). This witness has the advantage of not requiring diagonalization of the evolved density matrix. The proposed witness being sensitive to system-environment information backflows, in agreement with the BLP witness.

HILBERT-SCHMIDT SPEED

Consider the family of distance measures

$$[d_\alpha(p,q)]^\alpha = \frac{1}{2}\sum_x |p_x^{1/\alpha} - q_x^{1/\alpha}|^\alpha \qquad (1)$$

$\alpha \geq 1, p, q$ are probability distributions.

The classical statistical speed is given by

$$s_\alpha[p(\varphi_0)] = \frac{d}{d\varphi} d_\alpha(p(\varphi+\varphi_0), p(\varphi_0)) \qquad (2)$$

Considering now a given pair of quantum states ρ and σ, one can extend these classical notions to the quantum case by taking $p_x = Tr\{E_x\rho\}$, and $q_x = Tr\{E_x\sigma\}$ as the measurement probabilities associated with the positive-operator valued measure (POVM) defined by the set of $E_x \geq 0$ [1].

Extending S to the quantum case, one then obtains the quantum statistical speed as [2]

$$S_\alpha[\rho(\varphi)] = max_{\{E_x\}} s_\alpha[\rho(\varphi)] = \left(\frac{1}{2}\, Tr\left|\frac{d\rho}{d\varphi}\right|^\alpha\right)^{1/\alpha} \qquad (3)$$

In the special case when α = 2, the quantum statistical speed is given by the Hilbert-Schmidt speed (HSS) [2]

$$HSS(\rho_\varphi) = \sqrt{Tr[\left(\frac{d\rho_\varphi}{d\varphi}\right)^2]} \qquad (4)$$

It does not require the diagonalization of $\frac{d\rho_\varphi}{d\varphi}$.

NON-MARKOVIANITY WITNESS BASED ON HSS

Here we aim at exploiting a convenient quantum statistical speed [2] as a figure of merit of the non-Markovian character of quantum evolutions, which avoids diagonalization of the system density matrix. The initial state for a quantum system with n dimensional Hilbert space is

$$|\psi_{0>} = \frac{1}{\sqrt{n}}(e^{i\varphi}|\psi_{1>} + \dots + |\psi_{n>}) \qquad (5)$$

φ : unknown phase shift

$\{|\psi_{i>,\, i=1,\dots,n}\}$: complete and orthonormal basis for H [3]

the HSS-based witness of non-Markovianity as

$$\chi(t) := \frac{dHSS(\rho_\varphi(t))}{dt} > 0 \qquad (6)$$

were ρ_φ (t) denotes the evolved state of the system. We show that the proposed HSS-based non-Markovianity witness detects memory effects, in agreement with the well-established trace-distance-based witness, being sensitive to system-environment information backflows.

The non-Markovianity effect of the system dynamics can be identified through another well-known perspective proposed by Breuer, Laine, and Piilo (BLP), namely, the distinguishability of two evolving quantum states of the same system[4]. This distinguishability is quantified by the trace distance, a commonly useful witness for two arbitrary states ρ_1, ρ_2 is defined as $D(\rho_1,\rho_2) = \frac{1}{2}Tr|\rho_1 - \rho_2|$, that is contractive under CPTP maps. The BLP witness is given by

$$\sigma(t) := \frac{dD(\rho_1,\rho_2)}{dt} \qquad (7)$$

It's nonmonotonicity (σ>0) as a witness of non-Markovianity due to system-environment backflows of information.

Examples

We used the HSS witness for a single qubit system which is subject to phase covariant noise [5] and Pauli channel [6]. As a result we found the HSS-based witness is sensitive to system-environment information backflows, in perfect agreement with the BLP measure. Here we describe explicitly the time behavior of χ(t) for Two-qubit system and V-type three-level open quantum system.

1- Two-qubit system:

Qubits A , B independently Interact with a leaky cavity with a Lorentzian spectral density for the cavity modes, in zero temperature.

The initial sate for the system is:

$$|\psi_0> = \frac{1}{2}(e^{i\varphi}|11> + |10> + |01> + |00>) \qquad (8)$$

The evolved reduced density matrix is:

$$\rho_s(t) = \begin{pmatrix} \rho_{11}(t)p(t) & \rho_{10}(t)\sqrt{p(t)} \\ \rho_{01}(t)\sqrt{p(t)} & 1-\rho_{11}(t)p(t) \end{pmatrix} \qquad (9)$$

where P = P(t) ∈[0, 1] is the coherence characteristic function [7].

$$A)\ \chi(t) := \frac{dHSS(\rho_\varphi(t))}{dt} = \frac{dHSS(\rho_\varphi(t))}{dp}\frac{dp}{dt} > 0 \Rightarrow \frac{dp}{dt} > 0 \qquad (10)$$

$$B)\ \sigma(t) := \frac{dD(\rho_1,\rho_2)}{dt} = \frac{dD}{dp}\frac{dp}{dt} > 0 \Rightarrow \frac{dp}{dt} > 0$$

We hence obtain $\chi(t) > 0 \Leftrightarrow \sigma(t) > 0$. [3]

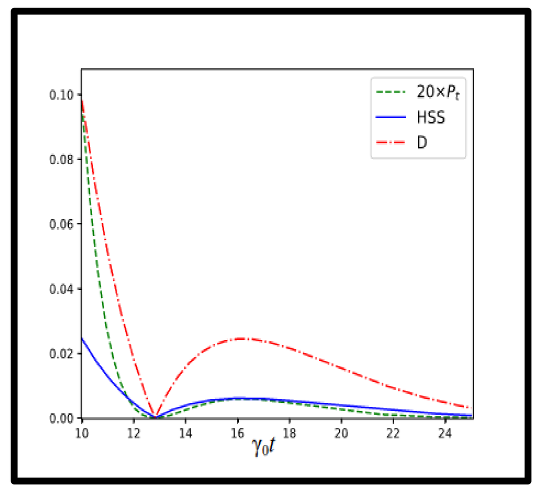

Fig 1. Dynamics of HSS (blue solid line), D (red dot-dashed line), and coherence characteristic function P(t) as a function of the $\gamma_0 t$ for the two-qubit system in the strong-coupling regime, with $\lambda = 1.25\gamma_0$

2- V-type three-level open quantum system

a V-type qutrit interacting with a dissipative reservoir [8], the initial state is:

$$|\psi_0> = \frac{1}{\sqrt{3}}(e^{i\varphi}|\tilde{2}> + |\tilde{1}> + |\tilde{0}>) \qquad (11)$$

The evolved reduced density matrix is:

$$\rho(t) = \sum_{i=1}^{3} \kappa_i\rho(0)\kappa_i^+ \quad \text{with} \quad \kappa_1 = \begin{pmatrix} G_+(t) & 0 & 0 \\ 0 & G_-(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\kappa_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sqrt{1-|G_+(t)|^2} & 0 & 0 \end{pmatrix}, \kappa_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{1-|G_-(t)|^2} & 0 \end{pmatrix}$$

$$G_\pm(t) = e^{-\lambda t/2}\left[cosh\left(\frac{d_\pm t}{2}\right) + \frac{\lambda}{d_\pm} sinh\left(\frac{d_\pm t}{2}\right)\right] \qquad (12)$$

Where $d_\pm = [\lambda^2 - 2\lambda\gamma(1 \pm |\theta|)]^{1/2}$, λ is the spectral width of the reservoir, γ is the relaxation rate of the two upper levels to the ground state, and ϑ depends on the relative angle between two dipole moment elements associated with the transitions

|2> → |0 >and |1 >→ |0>.

The HSS and BLP witnesses are given by [3]:

$$HSS(\rho_\varphi(t)) = \frac{1}{3}|G_+(t)|\sqrt{|G_-(t)|^2 + 1} \qquad (13)$$

$$D = |G_+(t)|$$

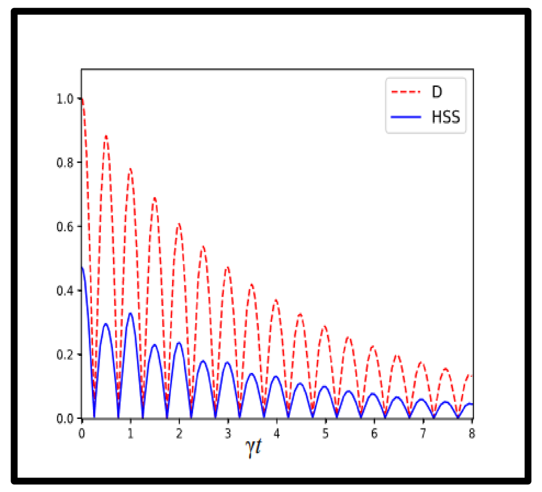

Fig 2. Dynamics of the HSS (blue solid line) and trace distance D (red dashed line) as a function of the γ t for the V-type three-level atom, with $\lambda = 5 \times 10^{-3}\gamma$

CONCLUSION

*We introduce a witness for characterizing the non-Markovianity of quantum evolutions through HSS.
*the proposed witness is as efficient as BLP witness in detecting the non-Markovianity.
*Our study supplies a useful alternative tool to detect non-Markovianity based on the concept of quantum statistical speed detecting system-environment backflows of information.
*the HSS-based witness does not require diagonalization of the reduced system density matrix.
*faithful non-Markovianity witness for all the quantum system with dim n=2,3.
*As a prospect, these results stimulate the investigation for systems of higher dimension to assess the extent of validity.

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