Joint measurement of non-classical correlations

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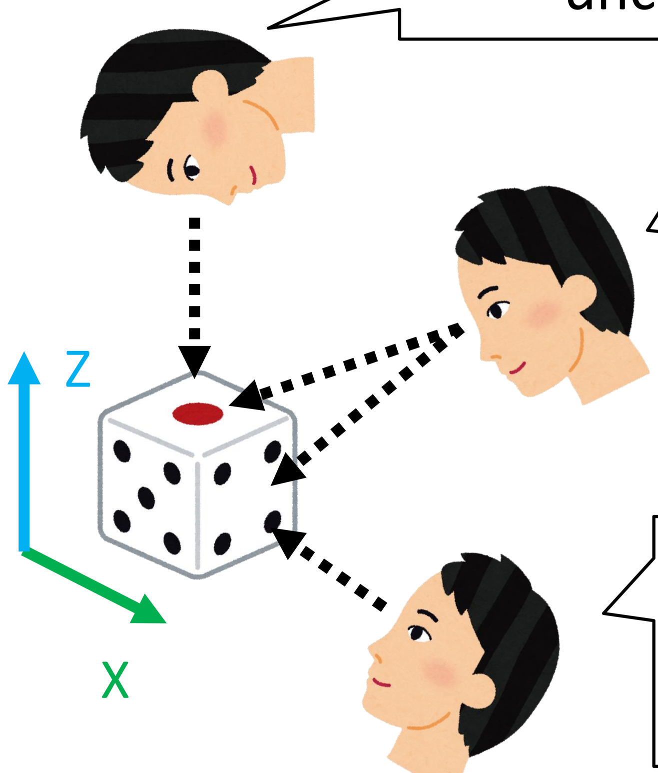
Uncertainty trade-off of joint measurement

Concept

The Z side is clear but the other sides are completely unclear (precise measurement of Z side).

Both of the X and Z sides are a little unclear (measurement uncertainty) but I can recognize both of them simultaneously (joint measurement of X and Z sides.)

The X side is clear and the other sides are completely unclear (precise measurement of X side).

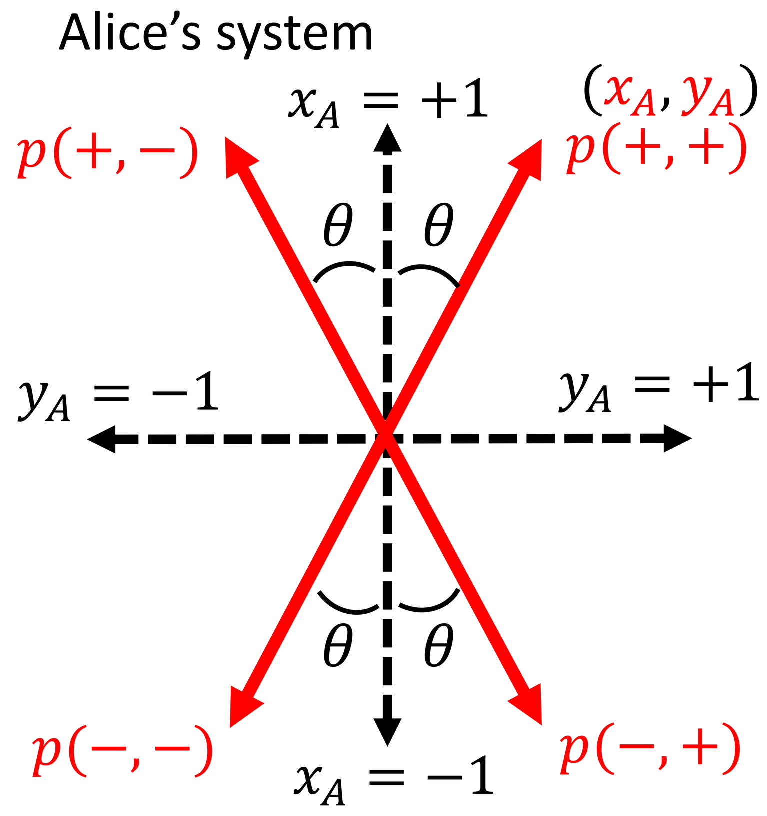


Measurement resolutions of X and Z are limited by an uncertainty trade-off[1-3] depending on the implementation of the measurement.

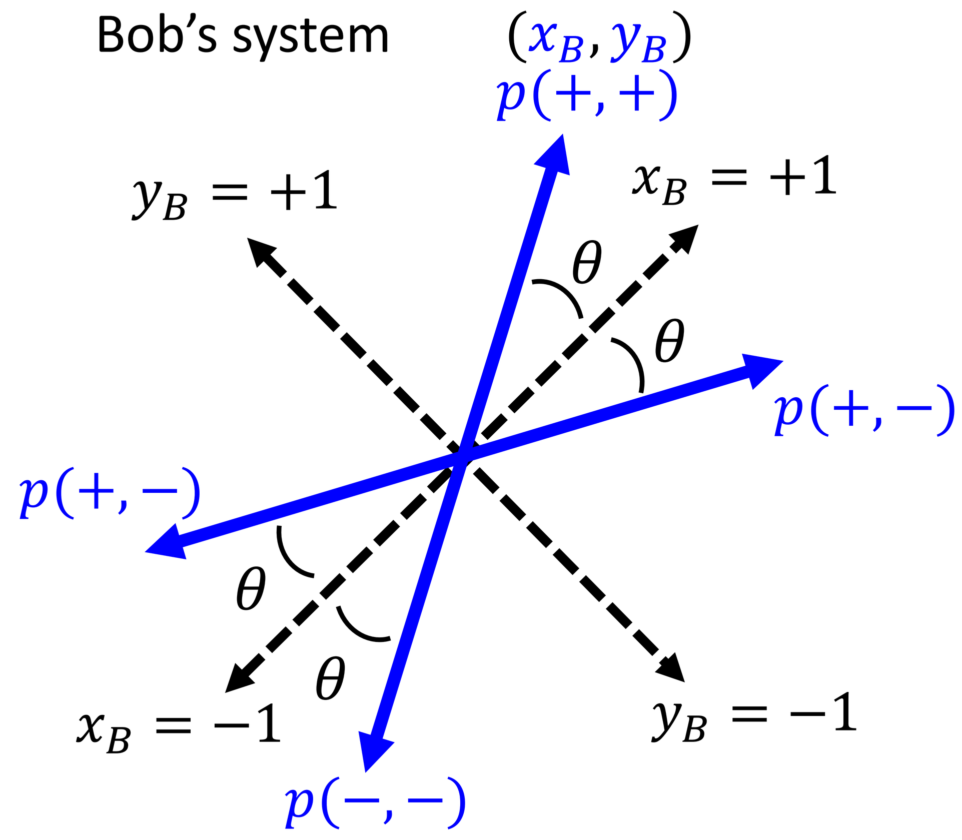
Realization of the joint measurements using polarizer settings

Effective polarization directions on the Bloch sphere

Alice's system



Bob's system



$\hat{X}_A$: Horizontal-Vertical linear polarization
$\hat{Y}_A$: Diagonal-Antidiagonal linear polarization

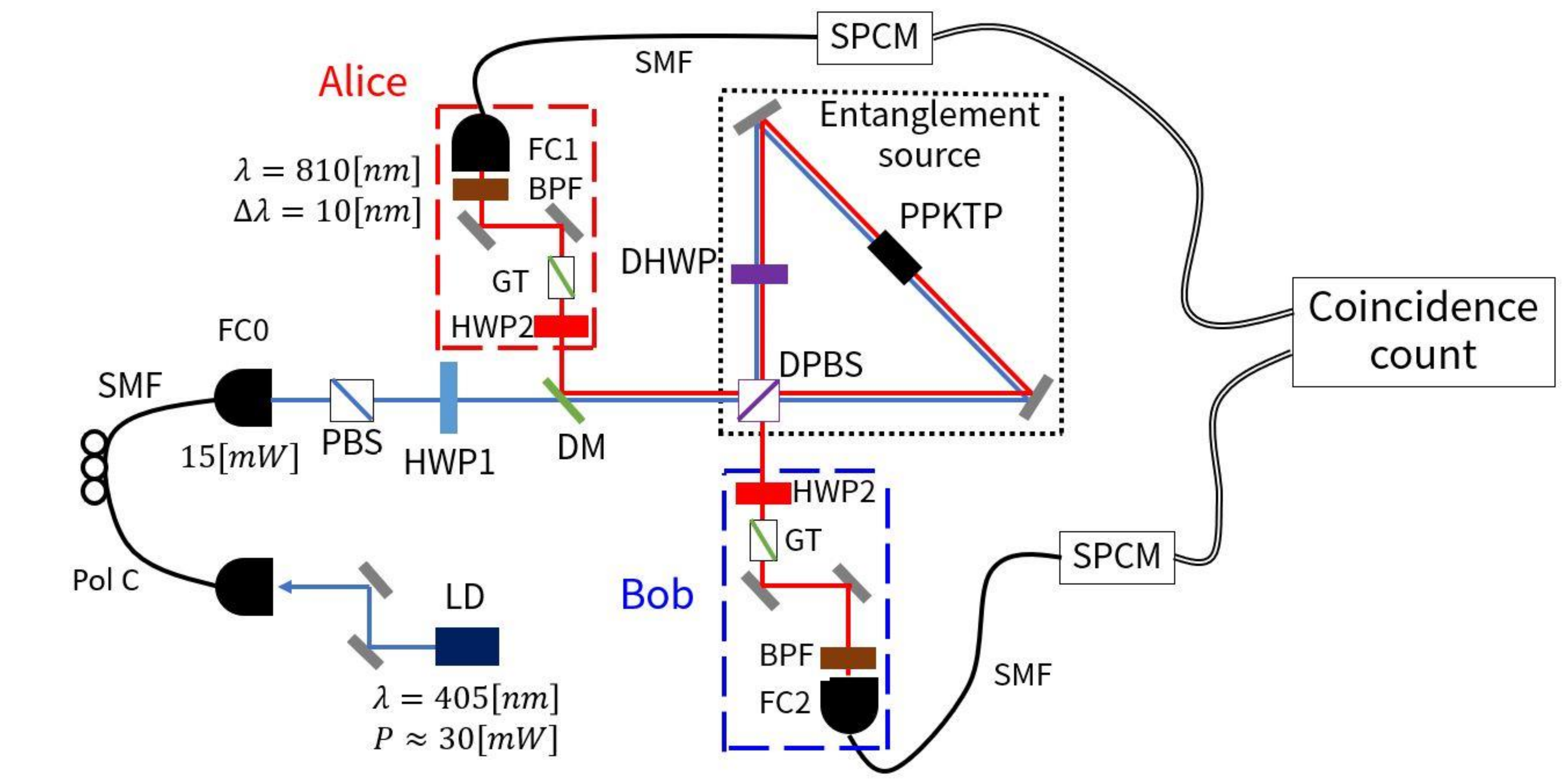
θ describes the uncertainty trade-off between $\hat{X}$ and $\hat{Y}$ in both systems.

Conclusion

The uncertainty trade-off can be optimized to achieve zero probability results at the Cirel'son bound.

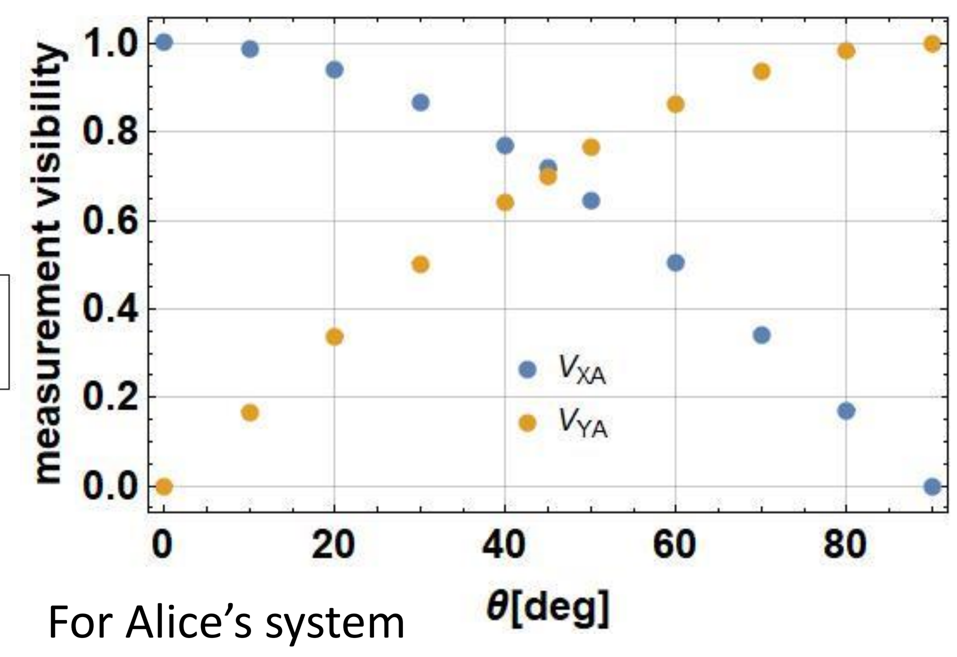
Experiment

Experimental setup



LD : Laser Diode, Pol C : polarization controller, SMF : Single mode fiber, FC : fiber coupler, PBS : Polarizing beam splitter, HWP : Half wave plate, DM : Dichroic mirror, GT : Glan-Taylor prism, BPF : Band pass filter, SPCM : Single photon counting module PerkinElmer (SPCM-AQR-14-FC13237-1) and EXCELITAS (SPCM-AQRH-14-FC24360)

Result of measurement visibility



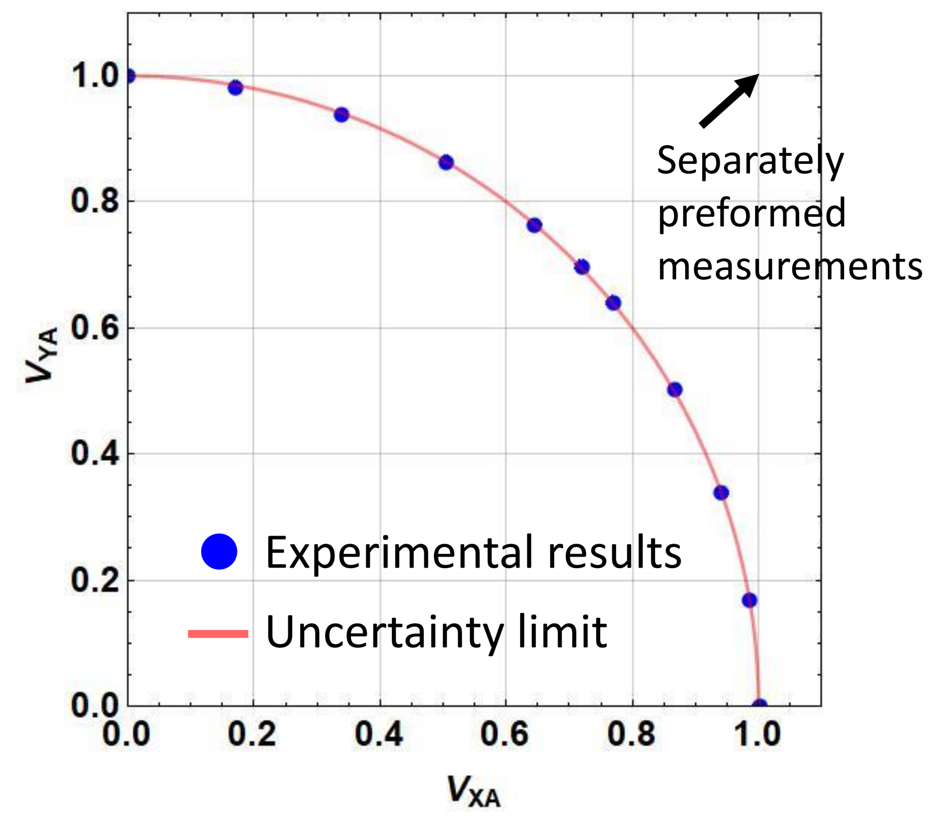
For Alice's system

$$V_\alpha \equiv \frac{\langle\alpha\rangle_{joint}}{\langle\alpha\rangle_{precise}} \quad (\alpha = X_A, Y_A).$$

V_α : measurement visibility of α.
$\langle\alpha\rangle_{joint}$: Average of α with joint measurement.
$\langle\alpha\rangle_{precise}$: Average of α with precise measurement.

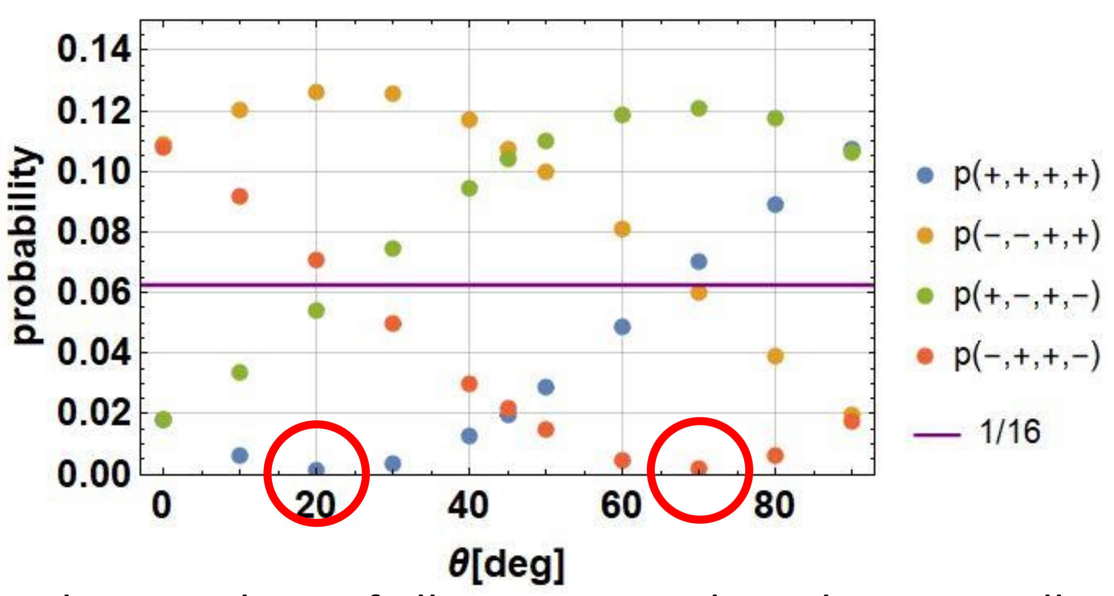
Likewise for Bob's system.

Result of uncertainty trade-off



Positivity of joint measurement outcomes prevents joint measurements beyond the uncertainty limit. Separate measurement can be precise, but no joint probability distribution $p(x_A, y_A)$ is obtained.

Results of joint probabilities $p(x_A, y_A, x_B, y_B)$ for a joint measurement of correlations violating Bell's inequalities



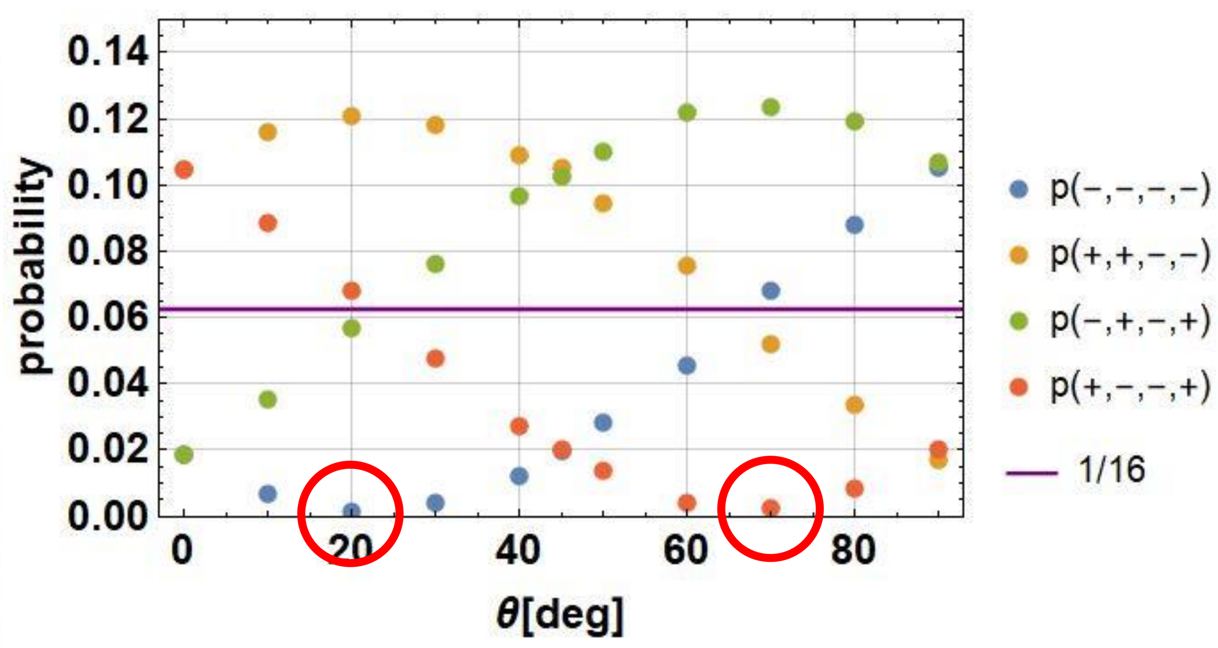
The error bars of all experimental results are smaller than the size of the dots. The two results marked ○ are close to zero. If the non-classical correlation was increased beyond the Cirel'son bound, the differences between $p(+,+,+,+)$ and $p(-,-,+,+)$ and between $p(+,-,+,-)$ and $p(-,+,+,-)$ would have to increase, which is impossible if the lower probabilities are already at zero. Therefore, the zero probability points are experimental evidence that the Cirel'son bound is defined by the uncertainty limited resolution of local measurements [4].

Outcomes of Alice's system / Outcomes of Bob's system

$p(x_A, y_A, x_B, y_B)$	$x_A = +1$, $y_A = +1$	$x_A = +1$, $y_A = -1$	$x_A = -1$, $y_A = +1$	$x_A = -1$, $y_A = -1$
$x_B = +1$, $y_B = +1$	$P(+,+,+,+)$	$P(+,-,+,+)$	$P(-,+,+,+)$	$P(-,-,+,+)$
$x_B = +1$, $y_B = -1$	$P(+,+,+,-)$	$P(+,-,+,-)$	$P(-,+,+,-)$	$P(-,-,+,-)$
$x_B = -1$, $y_B = +1$	$P(+,+,-,+)$	$P(+,-,-,+)$	$P(-,+,-,+)$	$P(-,-,-,+)$
$x_B = -1$, $y_B = -1$	$P(+,+,-,-)$	$P(+,-,-,-)$	$P(-,+,-,-)$	$P(-,-,-,-)$

$$b \equiv x_A x_B + x_A y_B + y_A x_B - y_A y_B$$

Values of Bell correlations for the 16 possible outcomes of joint measurements. Outcomes shown in yellow have a value of $b = +2$, outcomes shown in green have a value of $b = -2$. Since a Bell inequality violation corresponds to an average b-value below -2, low probabilities of yellow results are a characteristic signature of non-classical correlations.



Same graph as the picture on the left for a different set of outcomes. $p(-,-,-,-)$ and $p(+,-,-,+)$ are found to be close to zero at uncertainty trade-offs of 20 degrees and 70 degrees respectively. The purple line marks the probability of 1/16 obtained when the outcomes are completely random. The average value of $p(-,-,-,-)$ and $p(+,+,-,-)$ and the average value of $p(-,+,-,+)$ and $p(+,-,-,+)$ are equal to 1/16 at all measurement resolutions.

[1] B. G. Englert, Phys. Rev. Lett. 77, 2154, (1996).
[2] E. Andersson, S. M. Barnett, and A. Aspect. Phys. Rev. A 72. 042104. (2005).
[3] M. Iinuma, Y. Suzuki, G. Taguchi, Y. Kadoya, and H. F. Hofmann, New J. Phys. 13, 033041, (2011).
[4] H. F. Hofmann, Phys. Rev. A Vol. 100, 012123, (2019).