Persistence of Topological Phases in Non-Hermitian Quantum Walks

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Abstract

Discrete-time quantum walks (DTQWs) are known to exhibit exotic topological states and phases. Physical realization of quantum walks in a noisy environment may destroy these phases. We investigate the behavior of topological states in quantum walks in the presence of a lossy environment. The environmental effects in the quantum walk dynamics are addressed using the non-Hermitian Hamiltonian approach. We show that the topological phases of the quantum walks are robust against moderate losses. The topological order in one-dimensional (1D) split-step quantum walk persists as long as the Hamiltonian is $\mathcal{PT}$-symmetric. Although the topological nature persists in two-dimensional (2D) quantum walks as well, the $\mathcal{PT}$-symmetry has no role to play there. Furthermore, we observe the noise-induced topological phase transition in two-dimensional quantum walks.

1D DTQW

- A DTQW consists of a quantum walker over a 1D lattice whose evolution consists of two operation spin flip operator $R(\theta)$ and conditional translational operator T such that time evolution operation reads

$$U(\theta) = TR(\theta) \quad (1)$$

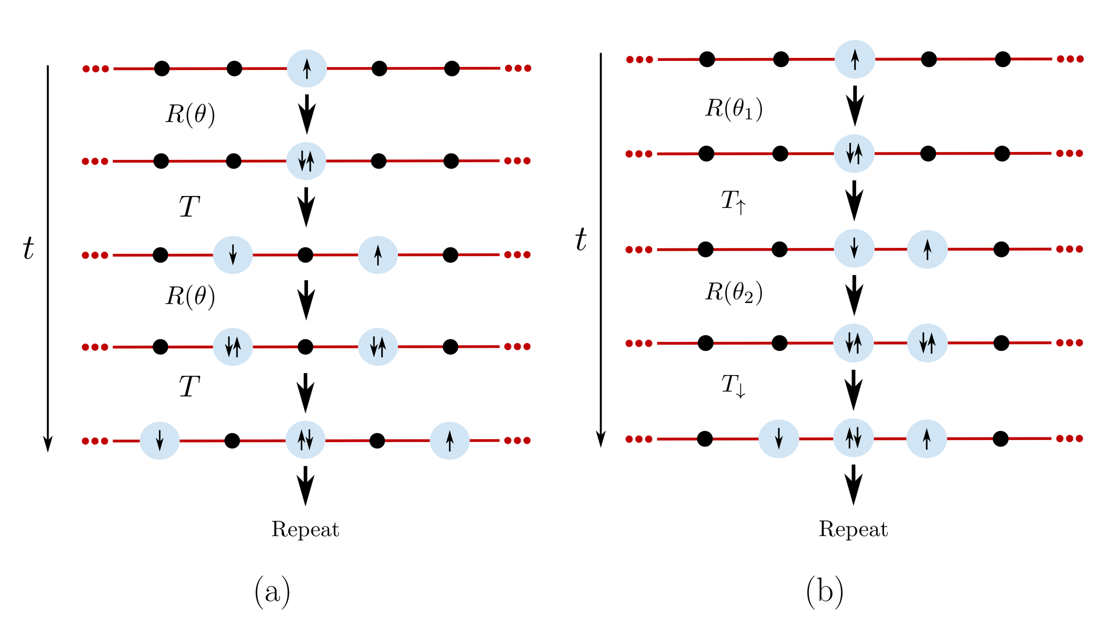

Figure 1: Schematic protocol for (a) 1D DTQW and (b) for 1D SSQW.

- A more enriched class of 1D DTQW is split-step quantum walk (SSQW), which involves splitting the conditional shift operator T into left-shift ($T_\downarrow$) and right-shift ($T_\uparrow$) operators, separated by an additional coin toss $R(\theta_2)$ [2] such that

$$U_{\rm ss}(\theta_1,\theta_2) = T_\downarrow R(\theta_2) T_\uparrow R(\theta_1)$$

2D DTQW

- A 2D DTQW on a triangular lattice consists of three conditional shift operators separated by coin-flip operations such that

$$U_{\rm 2D}(\theta_1,\theta_2) = T_{xy}R(\theta_1)T_yR(\theta_1)T_yR(\theta_2)T_xR(\theta_1)T_x.$$

- An equivalent 2D DTQW on a square lattice and using cyclic property we can rewrite time evolution operator given as

$$U_{\rm 2D}(\theta_1,\theta_2) = T_yR(\theta_1)T_yR(\theta_2)T_xR(\theta_1)T_x.$$

Non-Hermitian QW

- We extend 1D SSQW to non-hermitian domain by introducing scaling operator $G = e^{\gamma\sigma_z}$ [4]

$$U^{\rm NU}_{\rm ss}(\theta_1,\theta_2,\gamma) = T_\downarrow G_\gamma R(\theta_2) T_\uparrow G_\gamma^{-1} R(\theta_1).$$

- We go to momentum (quasi) basis by performing Fourier transform and write corresponding generator, $H_{\rm NU}(\theta_1,\theta_2,\gamma)$ [2] which reads

$$H_{\rm NU}(\theta_1,\theta_2,\gamma) = \oplus_k E(k)\,\hat{\mathbf{n}}(k)\cdot\sigma,$$

with quasi-energy $E(k)$ and Bloch vector $\hat{\mathbf{n}}(k)$.

- For $\gamma \neq 0$, G_γ as well as U becomes non-unitary and the corresponding generator non-hermitian.
- In 2D, we introduce loss and gain in one of the direction (say x) which results in non-hermitian dynamics such that

$$U^{\rm NU}_{\rm 2D}(\theta_1,\theta_2,\gamma) = T_yR(\theta_1)T_yR(\theta_2)G_\gamma T_xR(\theta_1)G_\gamma^{-1}T_x.$$

Topological Phases in QWs

- 1D SSQW is known to exhibit topological phases which are characterized using winding number, $W = 0, 1$.
- In 2D DTQW topological phases are characterized by Chern number C. 2D DTQW supports topological phases with $C = \pm 1, 0$.

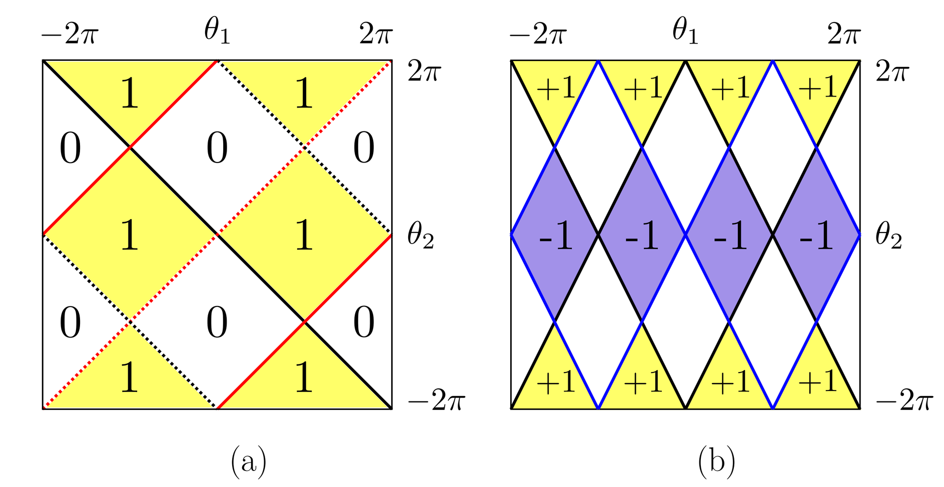

Figure 2: (Color online) (a) Different topological phases realized in 1D SSQW as a function of θ_1 and θ_2. Here, black and red lines represent closing of energy band at $k = 0$ and $k = \pi$, respectively, and solid and dotted lines demonstrate the closing at $E = 0$ and $E = \pi$, respectively. (b) Topological phases which exist in 2D DTQW for different values of θ_1 and θ_2. Here, blue and black lines show the closing of energy gap at $E = 0$ and $E = \pi$, respectively.

Results

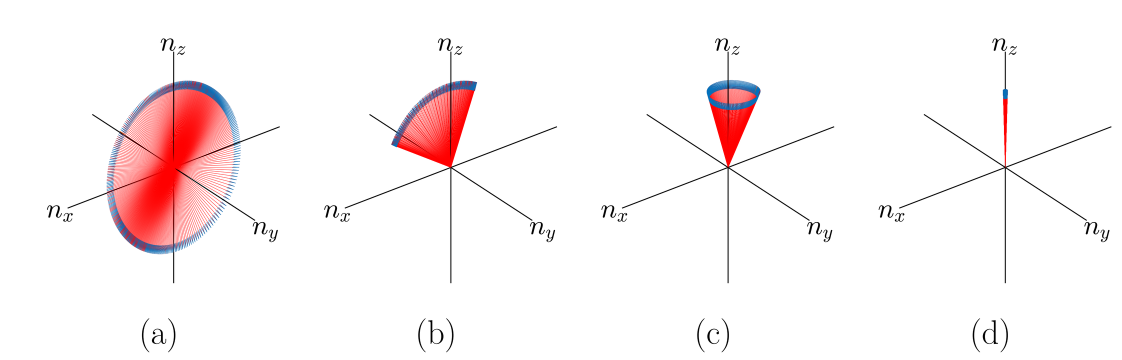

Figure 3: (Color online) Winding of the Bloch vector around the origin with the lattice size, $N = 100$ (a) $\theta_1 = -3\pi/8,\ \theta_2 = \pi/8,\ \gamma = 0.25$ (b) $\theta_1 = -3\pi/8,\ \theta_2 = 5\pi/8,\ \gamma = 0.25$ (c) $\theta_1 = -3\pi/8,\ \theta_2 = \pi/8, \gamma = 1.8$ (d) $\theta_1 = -3\pi/8,\ \theta_2 = \pi/8, \gamma = 3.0$.

- In 1D SSQW, the topological phases are unaffected even when the system is non-Hermitian [1] (i.e., $\gamma \neq 0$), as far as the system possesses a real spectrum following the $\mathcal{PT}$-symmetry [5].
- The topological nature of the system vanishes asymptotically as we cross the exceptional point γ_c, which can be seen in Fig. 4(a), 4(b).
- In 2D DTQW, the persistence of the Chern number has been observed C as well until the scaling factor γ reaches a critical value.
- We cannot associate any symmetry breaking with the point where the topological phase transition happens due to the absence of the symmetry in 2D DTQW.
- In 2D DTQW we observe sharp transitionand it further shows noise-induced topological phase.

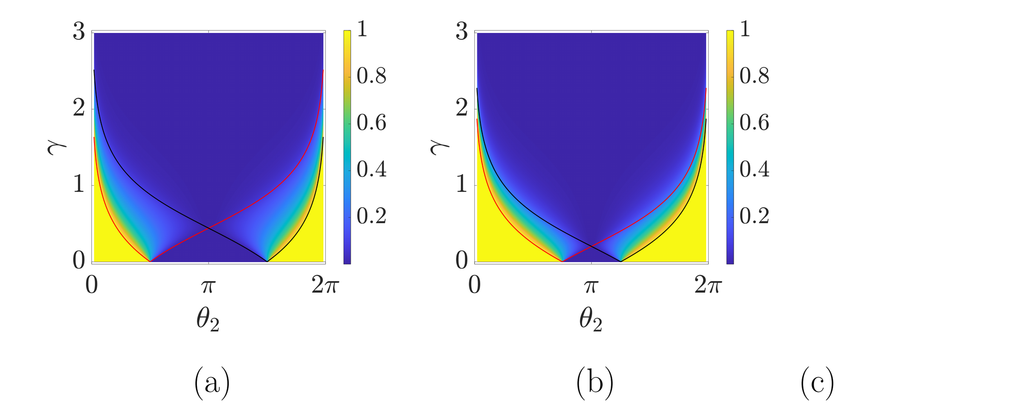

Figure 4: (Color online) Winding number for lower energy band W_- as a function of γ and θ_2, and (a) $\theta_1 = -\pi/2$ (b) $\theta_1 = -3\pi/4$ (c) $\theta_1 = -\pi$. The system size is taken to be $N = 100$. The red and black lines in all of the panels represent γ_c for $(k, E) = (0, 0)$ and $(k, E) = (\pi, 0)$, respectively.

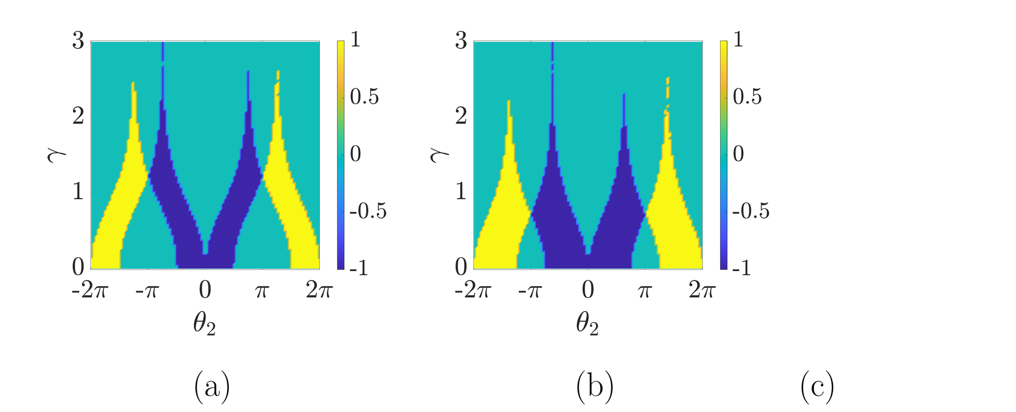

Figure 5: (Color online) Effect of γ on Chern number is plotted with varying θ_2 for (a) $\theta_1 = \pi/4$ (b) $\theta_1 = 3\pi/8$ (c) $\theta_1 = 3\pi/2$. The system size is taken to be 50.

Conclusion

- The topological phases of the quantum walks are robust against moderate losses.
- The topological order in 1D SSQW persists as long as the Hamiltonian is $\mathcal{PT}$ – symmetry.
- The topological nature persists in 2D DTQW as well, although, $\mathcal{PT}$ – symmetry does not play any role there.
- We observe noise-induced topological phase transition in 2D DTQW, which was absent in 1D systems.

References

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