A family of multipartite separability criteria based on correlation tensor

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Abstract

A family of separability criteria based on correlation matrix (tensor) is provided. Interestingly, it unifies several criteria known e.g. realignment criterion, de Vicente criterion and derived recently separability criteria based on SIC POVMs. These new criteria are linear in the density operator and hence one may find new classes of entanglement witnesses and positive maps. Interestingly, there is a natural generalization to multipartite scenario using multipartite correlation matrix.

Definition: Trace norm of a real matrix \( A \) is defined as \( \| A \|_1 = \text{Tr} \sqrt{AA^\dagger} \). For positive \( A \): \( \| A \|_1 = \text{Tr} A \), in general: sum of singular values.

Separability criteria by Correlator tensor

Let \( \{ C_i \}_{i=1}^{d^2} \) be an orthonormal hermitian basis of \( \mathcal{B}(\mathbb{C}^{d^2}) \)

Definition: We define the correlation tensor of \( \rho \) as:

\[
C(\rho)_{i_1 \ldots i_n} = \text{Tr} \left( \rho \rho^{(1)}_{i_1} \cdots \rho^{(n)}_{i_n} \right)
\]

(1)

if \( n = 2 \) it is \( d^2 \times d^2 \) matrix of coordinates of \( \rho \).

Known criteria

- Realignment criterion:
  \[ \| C(\rho - \rho_A \otimes \rho_B) \|_1 \leq \sqrt{1 - \text{Tr} \rho_A^2} \sqrt{1 - \text{Tr} \rho_B^2} \]

- De Vicente criterion:
  \[ \| \hat{C}(\rho) \|_1 \leq \sqrt{1 - \frac{d}{2}} \sqrt{1 - \frac{d}{2}}, \] where \( \hat{C} \) is obtained removing the first column and the first row from \( C \).

- ESIC criterion:
  \[ \| \hat{C}(\rho) \|_1 \leq \sqrt{\frac{d^2 - 1}{d^2}} \left( 1 - \frac{d}{2} \right), \]

where \( \hat{C}_{ij} = \text{Tr} (\rho P_i \otimes Q_j), \{ P_i \}_{i=1}^{d^2} \) and \( \{ Q_j \}_{j=1}^{d^2} \) are SIC-POVMs.

XY-criteria

Main idea

Our criterion, redefine

\[
C_{x,y} = \text{diag}(x, 1, \ldots, 1) C \text{diag}(y, 1, \ldots, 1)
\]

then for \( \rho \) (bipartite) separable

\[
\| C_{x,y}(\rho) \|_1 \leq \sqrt{\frac{d_A - 1 + x^2}{d_A}} \sqrt{1 - \frac{d_B - 1 + y^2}{d_B}}
\]

(2)

As special cases we have:
- \( x = y = 0 \) de Vincenete
- \( x = y = 1 \) realignment
- \( x = y = 2 \) ESIC (any SIC-POVMs is required!)

For appropriately chosen \( \rho \in \mathcal{B}(\mathbb{C}^2 \otimes \mathbb{C}^2) \):

References