Certification of incomaptible measurements and entangled subspaces using quantum steering

Shubhayan Sarkar*, Debashis Saha, Remigiusz Augusiak

Center for Theoretical Physics, Polish Academy of Sciences, Aleja Lotnikow 32/46, 02-668 Warsaw, Poland

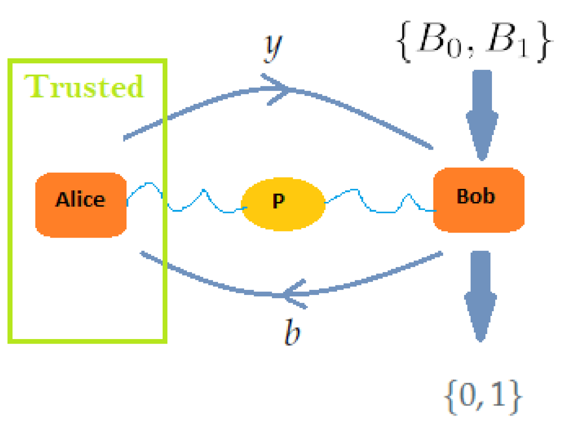
*sarkar@cft.edu.pl

Abstract

- It remains a highly nontrivial problem to propose a device independent scheme which could certify arbitrary pairs of incompatible measurements.
- We propose a one-sided device independent protocol which could certify any set of d-outcome projective measurements which do not share any common eigenvector along with entangled subspaces.
- Then we characterise the class of measurements which be used to certify the maximally entangled state of local dimension d.
- We derive a new analytical technique to find robustness bounds of our certification scheme.

Quantum steering

- Alice and Bob share a state ρ_{AB}.
- In the simplest scenario [1, 2], Bob can choose among 2 measurements denoted by B_y where $y \in \{1,2\}$ each of which results in 2 outcomes labeled by $b \in \{0,1\}$.
- Conditioned on Bob's measurement choices, different unnormalised states are prepared at Alice's side denoted by $\sigma_b^y \in \mathcal{H}_A$.
- Alice is trusted and can always perform a full tomography on the received system.



Certification using quantum steering

- In the steering scenario, untrusted preparation device and Bob's measurement devices can be certified.
- Unlike the Bell scenario, we cannot characterise Bob's full measurement rather the part of the measurement which is projected onto the support of Bob's local state upto some local isometry.
- For this, a steering inequality needs to be maximally violated by a state and some measurements. The first steering inequality written in the observable picture reads as [1, 3],

$$\mathcal{B}_2 = \langle \sigma_z \otimes B_1 \rangle + \langle \sigma_x \otimes B_2 \rangle \leq \sqrt{2} \tag{1}$$

can be maximally violated (upto local unitaries on Bob's side) by only,

$$|\psi_{AB}\rangle = \mathbb{1} \otimes U_B \left[\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \right] \tag{2}$$

and

$$U_B \overline{B}_1 U_B^\dagger = \sigma_z \ , \quad U_B \overline{B}_2 U_B^\dagger = \sigma_x$$

where $\overline{B}_i$ are projected onto the support of Bob's local state. Certification using the above steering inequality was recently shown in [4, 5].

Steering Inequality

- We propose the following steering inequality

$$\mathcal{B}_d = \sum_{i=1}^{n} \sum_{k=1}^{d-1} \langle A_i^k \otimes B_i^k \rangle \leq \beta_L \tag{3}$$

- The maximal quantum value of the steering inequality (3) is its algebraic value, $\beta_Q = N(d-1)$ that can be realized by the maximally entangled state of local dimension d given by,

$$|\phi_d^+\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |ii\rangle \tag{4}$$

and Bob's measurements satisfy the relation $B_i = A_i^*$.

- The classical bound of the steering inequality is given by,

$$\beta_L = \max_{\rho_A} \sum_{i=1}^{n} \sum_{k=1}^{d-1} \left| \left\langle \hat{A}_i^k \right\rangle_{\rho_A} \right|. \tag{5}$$

- The steering inequality (3) is non-trivial ($\beta_L < \beta_Q$) whenever there does not exist any quantum state ρ_A which is a simultaneous eigenstate of every Alice's observable $A_i's$.

Certification in the steering scenario

Theorem 1. *Assume that the steering inequality (3) is maximally violated by a state $|\psi\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$ and observables A_i, B_i ($i \in \{1, 2, \ldots, n\}$) acting on, respectively $\mathcal{H}_A$ and $\mathcal{H}_B$ such that the observable set A_i on the trusted side are projective measurements. Then, the following statement hold true for any d, that there exist local unitary transformations on Bob's side, $U_B : \mathbb{C}_d \otimes \mathcal{H}_{B_{aux}} \to \mathbb{C}_d \otimes \mathcal{H}_{B_{aux}}$, such that*

$$\forall i, \quad U_B \overline{B}_i U_B^\dagger = A_i^* \tag{6}$$

where $\overline{B}_i$ is B_i projected onto the support of Bob's state $\rho_B \in \mathbb{C}_d$ and the state which maximally violate the steering inequality (3) is given as,

$$U_B |\psi\rangle_{AB} = P \otimes \mathbb{1} \left(\frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle|i\rangle \right) \tag{7}$$

where P is the family parameter matrix such that $P = \sum_{i=0}^{d-1} \tilde{\lambda}_i |i\rangle\langle i|$ conditioned as, $\sum_{i=0}^{d-1} \tilde{\lambda}_i^2 = d$, $\tilde{\lambda}_i \neq 0$ and $[P, A_i] = 0$ for all $i \in \{1, 2, \ldots, n\}$.

Certification in the weakened steering scenario

- In the steering scenario as described above, the assumption of local tomography is required which imposes that the measurements of Alice are full-rank. This assumption can be dropped.
- In the weakened steering scenario, only the part of the measurement which is projected onto the support of Bob's state can be certified.

$$\forall i, \quad U_B \overline{B}_i U_B^\dagger = \overline{A}_i^* \tag{8}$$

where $\overline{A}_i$ and $\overline{B}_i$ are projections of A_i and B_i onto the support of Alice's and Bob's state ρ_A and $\rho_B \in \mathbb{C}_{d'}$ respectively where $d' \leq d$.

- However, unlike the steering scenario where the measurements are not performed by Alice but inferred from the local tomography of the state, in the weakened steering scenario the measurements are needed to be performed by Alice.
- We consider a set of measurements $A_i's$ such that they share the following property: if $[P, A_i] = 0$ for all i then $P = \mathbb{1}$. We name such measurements as "strongly incompatible measurements". Interestingly, if the trusted side performs strongly incompatible measurements, even in the weakened steering scenario it could be certified that the only state which maximally violates the steering inequality (3) is $|\phi_d^+\rangle$.

Robustness

- The trusted side performs strongly incompatible observables $A_1 = X_d$ and $A_2 = Z_d$. Then,

$$\left\| (1 \otimes U_B)(1 \otimes B_i^k)|\psi\rangle - 1 \otimes \widetilde{B}_i^k |\phi_d^+\rangle \right\| \leq \sqrt{2\varepsilon} + 2\sqrt{d}\sqrt[4]{2\varepsilon} \tag{9}$$

and

$$\left\| B_i^k - \widetilde{B}_i^k \right\|_2^2 \leq \sqrt{d}\sqrt{2\varepsilon} \left(1 + 4\sqrt{d}\sqrt{2\varepsilon} \right) \tag{10}$$

with $k = 0, \ldots, d-1$, where $\widetilde{B}_i$ are the ideal observables of Bob.

Conclusions

- We propose the first, to the best of our knowledge a certification scheme which could verify arbitrary incompatible measurements for any dimension d.
- We found a steering inequality which is violated by $d-$outcome projective measurements which do not share a common eigenvector.
- We weaken the steering scenario and showed that for strongly incompatible measurements on the trusted side, the state which self-tests is the maximally entangled state of local dimension d.
- We computed the robustness for a large class of strongly incompatible measurements which makes our certification scheme much more applicable in the practical scenario.

References

[1] H. M. Wiseman, S. J. Jones, and A. C. Doherty Phys. Rev. Lett. **98**, 140402 (2007).
[2] M. T. Quintino, T. Vértesi, D. Cavalcanti, R. Augusiak, M. Demianowicz, A. Acín, and N. Brunner, Phys. Rev. A **92**, 032107(2015).
[3] E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid, Phys. Rev. A **80**, 032112 (2009).
[4] I. Šupić, M. J. Hoban, New J. Phys. **18**, 075006(2016)
[5] A. Gheorghiu, P. Wallden and E. Kashefi, New J. Phys. **19**, 023043(2017)