Quantum correlations in a gravitational classical-channel model

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Abstract

In models where gravity is mediated by a quantum field, gravitational interaction between two masses is accompanied by generation of entanglement. Gravitational interaction can yet be reproduced also by gravitational decoherence models where interaction is treated as a classical channel with no entanglement generation [1,2]. Here, we show that, despite the absence of entanglement, such a classical model entails creation of quantum correlations in the form of discord. Starting from a (fully classical) two-mode coherent state of two masses, we show that discord is generated and eventually vanishes without any thermal noise, while it approaches a stationary value when the two masses are in contact with local thermal baths.

Model

Two suspended masses $m_1 = m_2 \equiv m$ at distance L moving harmonically around their equilibrium positions, with free Hamiltonian ($\hbar = 1$)

$$H_0 = \sum_{i=1}^{2} \frac{\hat{p}_i^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}_i^2$$

Small deviations from equilibria due to gravitational interaction → expansion of classical potential:

$$V = -\frac{Gm^2}{L + (x_2 - x_1)} \approx -\frac{Gm^2}{L}\left(1 + \frac{(x_2 - x_1)}{L} + \frac{(x_2 - x_1)^2}{L^2}\right) \tag{1}$$

Quantum gravitational interaction modeled in at least two ways:

▶ Setting, as in [3],

$$H_{\text{grav}} = -\frac{Gm^2}{L}\left(\underbrace{1}_{\text{const. offset}} + \underbrace{\frac{(\hat{x}_2 - \hat{x}_1)}{L}}_{\text{new eq. pos.}} + \underbrace{\frac{(\hat{x}_2 - \hat{x}_1)^2}{L^2}}_{\text{true grav. int. and freq. shift}}\right) \tag{2}$$

the total Hamiltonian then reads

$$H = \sum_{i=1}^{2} \frac{\hat{p}_i^2}{2m} + \frac{1}{2} m\Omega^2 \hat{x}_i^2 + K\hat{x}_1\hat{x}_2 \tag{3}$$

where $K = 2Gm^2/L^3$ and $\Omega^2 = \omega^2 - K/m$.

Upshot: generation of entanglement → quantum character of gravitational interaction.

▶ Goal: obtain an Hamiltonian as in (3) **without entanglement** → model based on a *feedback mechanism* proposed in [1] (KTM model). Two main ingredients:

– Measurement of position (**LO**): $\hat{x}_i \to x_i = \langle \hat{x}_i \rangle$ with information gain rate λ

– Feedback dynamics (**CC**): Gravitational Hamiltonian: $H_{\text{fb}} = K\,(x_1\hat{x}_2 + x_2\hat{x}_1)$.

→ unconditional dynamics for the two masses:

$$\dot{\rho} = -i[H, \rho] + \left(\lambda^2 + \frac{K^2}{4\lambda}\right) \sum_{i=1}^{2} \mathcal{D}[\hat{x}_i]\rho \tag{4}$$

where $\mathcal{D}[\hat{A}]\rho = \hat{A}\rho\hat{A}^\dagger - \frac{1}{2}\{\hat{A}^\dagger\hat{A}, \rho\}$ and H as in (3), while the second noise term represents *gravitational decoherence*. The latter is minimal if $\lambda = K/2$, as assumed hereafter.

→ quantum description of gravity without entanglement (by construction).

→ …However, entanglement is not the only (useful) form of quantum correlation, and here we focus on the calculation of quantum discord in the KTM model with and without thermal noise.

Gaussian states and correlations

Master equation (4) (Gaussian preserving) implies a Lyapunov equation for the covariance matrix:

$$\dot{\sigma} = Y\sigma + \sigma Y^T + 4D \tag{5}$$

where Y and D are matrices depending on model parameters.

Mutual Information: $\mathcal{I} = S(\rho_1) + S(\rho_2) - S(\rho)$

▶ Total amount of correlations

Classical Correlations: $\mathcal{C}_{21} = S(\rho_1) - \min_{\hat{\Pi}_{1_k}} \sum_k p_k S(\rho_{1|k})$

▶ Classically equivalent to mutual information,

▶ In general asymmetric in $1 \leftrightarrow 2$ (not this case)

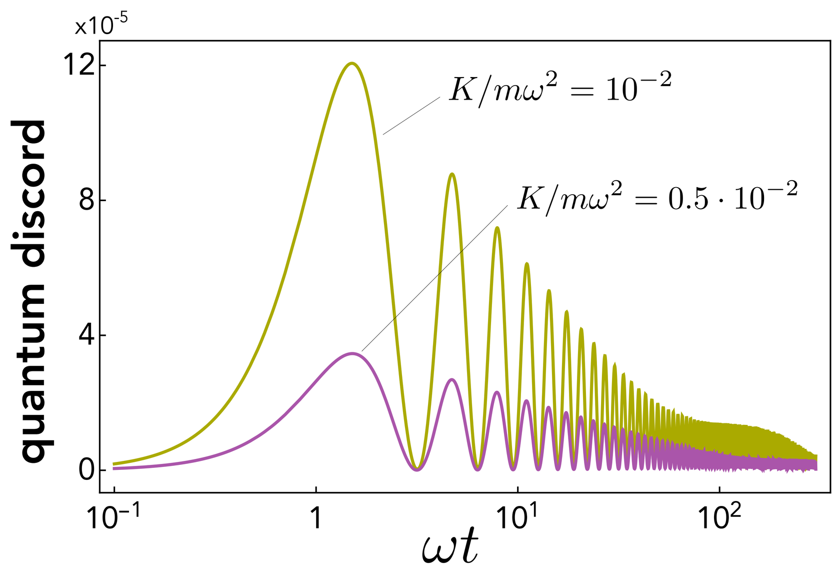
Quantum Discord: $\mathcal{D}_{21} = \mathcal{I} - \mathcal{C}_{21}$

▶ **Pure quantum** correlations,

▶ **Gaussian** discord has a closed analytical formula and is **optimal**

▶ Gaussian states: separable $\Rightarrow 0 < \mathcal{D} < 1$, entangled $\Rightarrow \mathcal{D} > 1$

Quantum Discord in KTM model

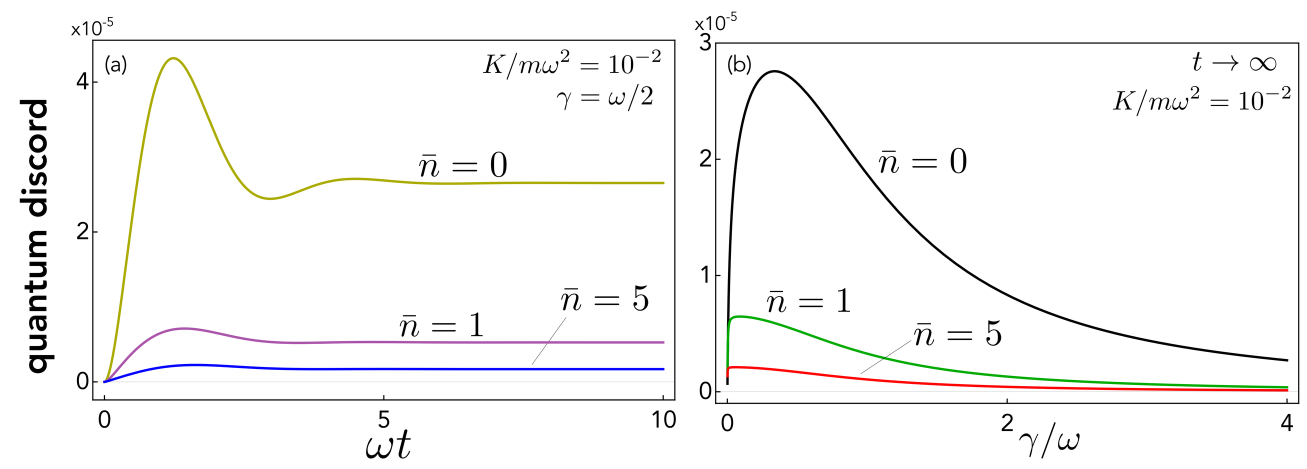


Initial **product** state $\rho_0 = |\alpha_1\rangle\langle\alpha_1| \otimes |\alpha_2\rangle\langle\alpha_2|$. Quantum discord against time (in units of ω^{-1}). The $K/m\omega^2$ parameter measures the strength of gravitational interaction compared to the harmonic potential. The ability to improve this ratio would be crucial to create a larger amount of quantum discord, which eventually vanishes. In this sense, KTM model is *asymptotically* classical.

Quantum Discord in KTM model with thermal noise

Here we consider the dynamics given by eq. (4), starting from an initial product state as before, plus local couplings to thermal baths at the same temperature:

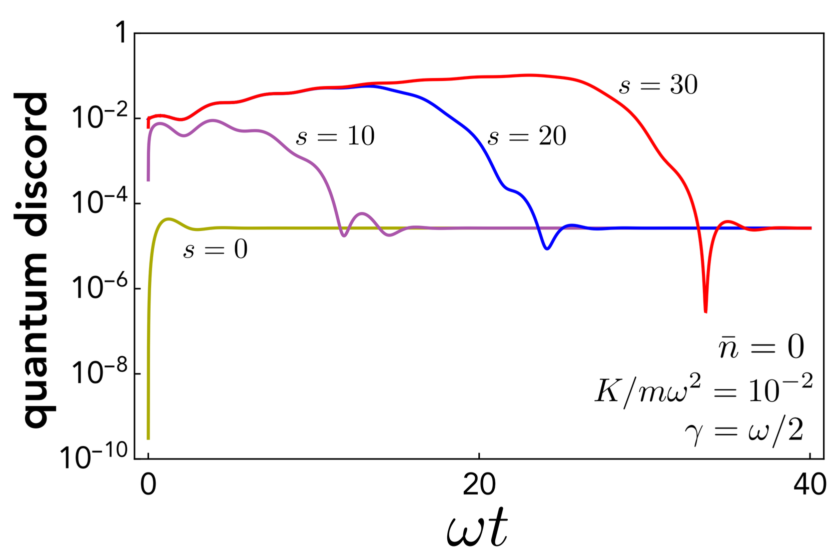
$$\dot{\rho} = \text{KTM} + 2\gamma(\bar{n} + 1) \sum_{i=1}^{2} \mathcal{D}[a_i]\rho + 2\gamma\bar{n} \sum_{i=1}^{2} \mathcal{D}[a_i^\dagger]\rho \tag{6}$$



(a) Quantum discord against time (in units of ω^{-1}) with mean photon number $\bar{n} = 0, 1, 5$ and dissipation rate $\gamma = 1/2$ (in units of ω). The presence of local thermal environments allows quantum discord to reach a, although small, stationary value. This value increase as the ratio $K/m\omega^2$ approaches 1/2.

(b) Asymptotic quantum discord against the dissipation rate γ (in units of ω). For a fixed value of the ratio $K/m\omega^2$ and of $\bar{n}$, there is an optimal value of γ/ω for which the amount of quantum discord is maximal. This value is higher as $K/m\omega^2$ approaches 1/2.

Starting with a product of single mode squeezed states, with squeezing parameter s, a higher value of quantum discord can be reached in the transient:



Conclusions

▶ The KTM model [1], despite being constructed to avoid entanglement, generates quantum discord.

▶ Quantum discord eventually vanishes in KTM model.

▶ Considering a more realistic setup, with both masses in contact with local thermal baths at the same temperature, discord reaches a finite value

▶ For a fixed interaction strength and bath temperature, there is an optimal dissipation rate for which asymptotic quantum discord is maximal.

References

[1] D. Kafri, J. M. Taylor, and G. J. Milburn, New J. Phys. 16, 065020 (2014)

[2] J. L. Gaona-Reyes, M. Carlesso and A. Bassi, arXiv:2007.11980

[3] Krisnanda, Tanjung, et al. "Observable quantum entanglement due to gravity." npj Quantum Information 6.1 (2020): 1-6.