Detection of virtual photons in superconducting architectures

J. Rajendran[1,2], **A. Ridolfo**[1,2], **L. Giannelli**[1,2], **E. Paladino**[1,2,3], **& G. Falci**[1,2,3]

[1]Dipartimento di Fisica e Astronomia "Ettore Majorana", Universitá di Catania

[2]INFN Sezione di Catania, Via Santa Sofia 64, Catania I-95125, Italy

[3]CNR-IMM MATIS Catania, Via Santa Sofia 64, Catania I-95125, Italy

mail to: gfalci@dmfci.unict.it

Micro/nano-fabrication techniques have recently allowed producing quantum devices displaying ultra-strong coupling (USC) of radiation and matter. In this non-perturbative regime, distinctive phenomena emerge, as the highly entangled ground state involving many-photon states of the field. These ground-state virtual photons (VPs) cannot be revealed by standard photodetection since the ground-state does not decay. Several works [1–6] have proposed that VPs in USC solid-state devices can be converted to real photons using architectures with three-level artificial atoms (AAs). In this enlarged Hilbert space, appropriate classical control drives may induce leakage from the Rabi subspace which marks the presence of VPs. The works [1–6] consider different control protocols, namely, AC continuous pumping of photons with dynamical Casimir effect (DCE) [1], spontaneous emission pumping in Λ scheme [2], Raman oscillations with two-tone pulses [3] and STIRAP in Λ [4,5] and Vee [5] scheme, and stimulated emission, also implemented by a two-tone control field in Λ scheme [6]. Notwithstanding having been emphasis of many investigations, such a population of virtual excitations remains still unobserved. The aim is of this work is to understand if experimental progress can be made in the detection of ground state photons with controlled three-level atom structures. To this end, we study a model integrating the various conversion protocols as mentioned above [1–6] and the detection steps, and with optimal control theory (OCT) to find optimal conditions for the efficient detection of VPs. Further, open systems effects have to be accounted for, since they may impact in particular on the fully coherent protocols [3–6].

Theoretical description

Hamiltonian of a three level atom interacting with a single optical mode ($\hbar = 1$):

$$H_{\rm sys} = H_0 + \lambda(a + a^\dagger)(|e\rangle\langle g| + |g\rangle\langle e|) \tag{1}$$

where $H_0 = \omega_c a^\dagger a + \epsilon|e\rangle\langle e| + \epsilon'|u\rangle\langle u|$, and ϵ, ϵ' are the energies of the atomic levels, ω_c is the optical mode frequency, and λ is the coupling rate between light mode and transitions $|e\rangle \to |g\rangle$.

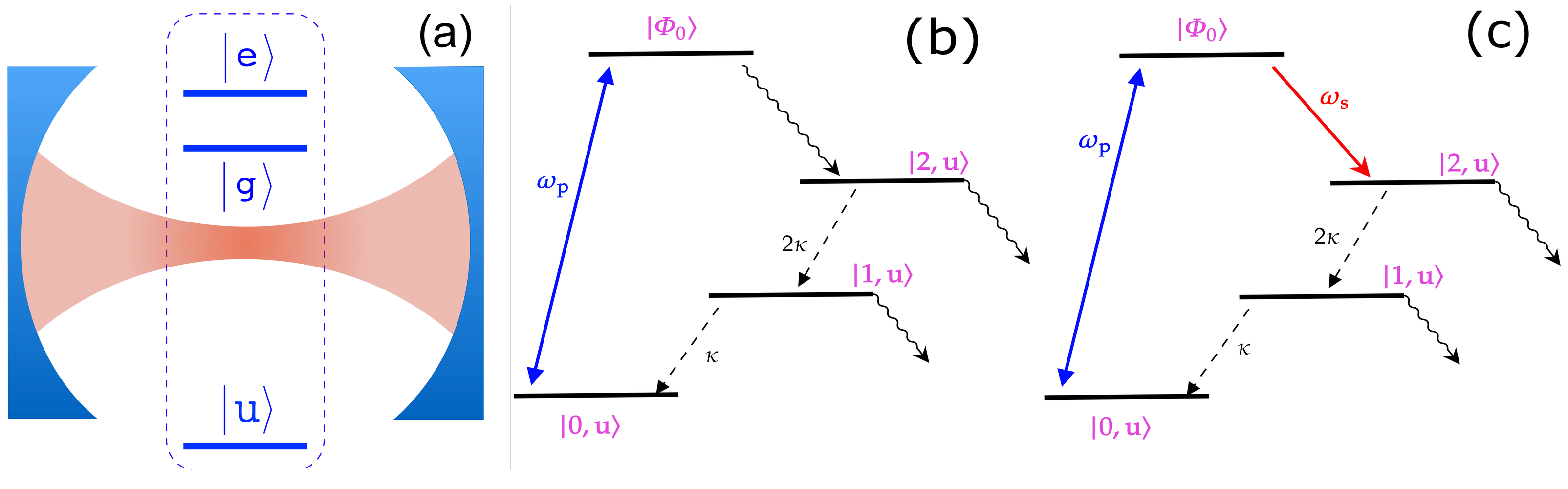

Figure 1: (a) Schematic representation of a three level atom interacting with an optically confined mode. (b) SEP protocol: a coherent drive (in continuous wave) at frequency ω_p resonant with the transition $|0,u\rangle \to |\Phi_0\rangle$, brings excitations to $|\Phi_o\rangle$ state, and by means of spontaneous emission, $|2,u\rangle$ gets populated. (c) STIRAP protocol: a counterintuitive temporal sequence of pulses resonant with transitions $|\Phi_0\rangle \to |2,u\rangle$ and $|0,u\rangle \to |\Phi_0\rangle$, namely ω_s and ω_p, is able to populate state $|2,0\rangle$ almost completely, by exploiting the evolution of the dark state $|D\rangle = \frac{\Omega_s(t)|0u\rangle - \Omega_p(t)|2u\rangle}{\sqrt{|\Omega_s^2|+|\Omega_p^2|}}$.

As $\lambda \sim \omega_{\rm c}$, the presence of counter-rotating terms in the light-matter interaction opens a channel for the dynamical detection of virtual photons in the dressed ground state. By expanding the dressed $|\Phi_0\rangle$ state on the H_0 basis, we get $|\Phi_0\rangle = \sum_{n=0}^{\infty} c_{0,2n}(\lambda)|2n,g\rangle + d_{0,2n}(\lambda)|2n+1,e\rangle$, and then, the dipole matrix $\langle 2,u|\sigma_{u,g}|\Phi_0\rangle \neq 0$, with $\sigma_{u,g} = |u\rangle\langle g|$. Thus, $|\Phi_0\rangle$ contains a finite number of virtual photons that can be detected if these latter can decay through the optical mode.

The above photon pair production can be amplified with a coherent control $H_{\rm C} = W(t)(|u\rangle\langle g| + |g\rangle\langle u|)$; where $W(t) = \sum_{k=p,s} \mathcal{W}_k(t)\cos(\omega_k t)$ This protocol yields non-vanishing coherent population transfer $|0u\rangle \to |2u\rangle$ only if $\langle\Phi_0| H_{\rm C} |2u\rangle \neq 0$

Projecting $H_{\rm sys} + H_{\rm C}$ into the relevant levels with the projection $P = |0u\rangle\langle 0u| + |1u\rangle\langle 1u| + |2u\rangle\langle 2u| + |\Phi_0\rangle\langle\Phi_0|$, we obtain

$$H_4 = \omega_c |1u\rangle\langle 1u| + 2\omega_c |2u\rangle\langle 2u| + E_0 |\Phi_0\rangle\langle\Phi_0| + W(t)\big[c_{00}(\lambda)\,|0u\rangle\langle\Phi_0| + c_{02}(\lambda)\,|2u\rangle\langle\Phi_0| + \text{h.c.}\big],$$

here $\omega_p = E_0 - \delta_p$, $\omega_s = E_0 - 2\omega_c - \delta_s$, and also we project the operators a and σ also into the 4-level space and that will be used in the further discussion. Now applying the rotating wave approximation (RWA), we obtain

$$\tilde H_4 = \delta\,|2u\rangle\langle 2u| + \delta_p\,|\Phi_0\rangle\langle\Phi_0| + [\frac{\Omega_p}{2}|0u\rangle\langle\Phi_0| + \frac{\Omega_s}{2}|2u\rangle\langle\Phi_0| + \text{h.c.}]$$

$$\dot\rho = i[\rho, \tilde H_4] + \mathcal{L}_{\rm cav}[\rho] + \mathcal{L}_{\rm atom}[\rho] \tag{2}$$

where cavity dissipator is $\mathcal{L}_{\rm cav}[\rho] = \frac{\kappa}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$
and atomic dissipator is $\mathcal{L}_{\rm atom}[\rho] = \frac{\gamma}{2}(2\sigma_{ug}\rho\sigma_{ug}^\dagger - \sigma_{gg}\rho - \rho\sigma_{gg}) + (2\sigma_{ge}\rho\sigma_{ge}^\dagger - \sigma_{ee}\rho - \rho\sigma_{ee})$
$\sigma_{ug} = |u\rangle\langle g|$, $\sigma_{gg} = |g\rangle\langle g|$, $\sigma_{ge} = |g\rangle\langle e|$, $\sigma_{ee} = |e\rangle\langle e|$ in the projected subspace.

In the dynamical detection of USC with STIRAP (see Ref.([4, 6, 7])), the system does not reach a steady state, in general we expect to create the target state, wait until the system decays reaching its equilibrium (for instance the ground state or a thermal state for a finite temperature), and then we start it again. The smaller is the time for a cycle, the larger is the number of iterations per second. The cycle time, $t_p + t_m$, where t_p is the duration of STIRAP and t_m is a subsequent interval when the system may relax. Number of photon detected per cycle is

$$\langle n\rangle_{\rm cycle} = \int_0^{t_p+t_m} \langle a^\dagger(t)a(t)\rangle_{\rm out}\,dt \tag{3}$$

and then, the detected photon rate (per second) is $\langle n\rangle_{\rm out} = \langle n\rangle_{\rm cycle}/(t_p + t_m)$.

SEP protocol

A coherent drive at frequency $\omega_{\rm p}$ (see Fig.1(b)), $H_{\rm drive} = \Omega_0\cos[\omega_{\rm p}t](|u\rangle\langle g| + |g\rangle\langle u|)$, generates a finite excitation in $|\Phi_0\rangle$ that can be transferred in $|2,u\rangle$, and possibly detected by ordinary photodetection if the light mode has a decay: this is the core of the SEP. Then, we calculate the dynamics of the interacting open quantum system with a Master Equation (ME) approach, developed on the eigenvectors of $H_{\rm sys}$ and finally, with a generalized input-output theory suitable for USC regime [2,7], we calculate the rate of detected photons. The SEP hamiltonian in RWA obtained from $\tilde H_4$,

$$\tilde H_4^{\rm SEP} = \delta\,|2u\rangle\langle 2u| + \delta_p\,|\Phi_0\rangle\langle\Phi_0| + \frac{c_{00}(\lambda)\Omega_p}{2}\big[\,|0u\rangle\langle\Phi_0| + |2u\rangle\langle\Phi_0| + \text{h.c.}\big]$$

from the solution of master equation at steady state we can obtain

$$\langle n\rangle_{\rm out} = \frac{4|c_{00}(\lambda)|^2\Omega_0^2\kappa\gamma_s}{|c_{00}(\lambda)|^2\Omega_0^2(4\kappa + 2\gamma_s) + 2\kappa(\gamma_p + \gamma_s)^2}$$

where the γ_s and γ_p are the rate corresponding to the $|\Phi_0\rangle \to |2u\rangle$ and $|\Phi_0\rangle \to |0u\rangle$ and $\gamma_s = \gamma|c_{02}(\lambda)|^2$, $\gamma_p = \gamma|c_{00}(\lambda)|^2$

In case of strong drive, i.e, $\Omega_0 \gg \gamma$ and $\kappa > \gamma$, we obtain $\langle n\rangle_{\rm out} = \gamma|c_{02}(\lambda)|^2$ where as for smaller Ω_0, the detected photon rate becomes,

$$\langle n\rangle_{\rm out} = \frac{2|c_{00}(\lambda)|^2|c_{02}(\lambda)|^2\Omega_0^2}{\gamma(|c_{00}(\lambda)|^2 + |c_{02}(\lambda)|^2)^2} \simeq \frac{2|c_{02}(\lambda)|^2\Omega_0^2}{\gamma}$$

From this analytical expression we can compare the efficiencies of STIRAP and SEP.

Results

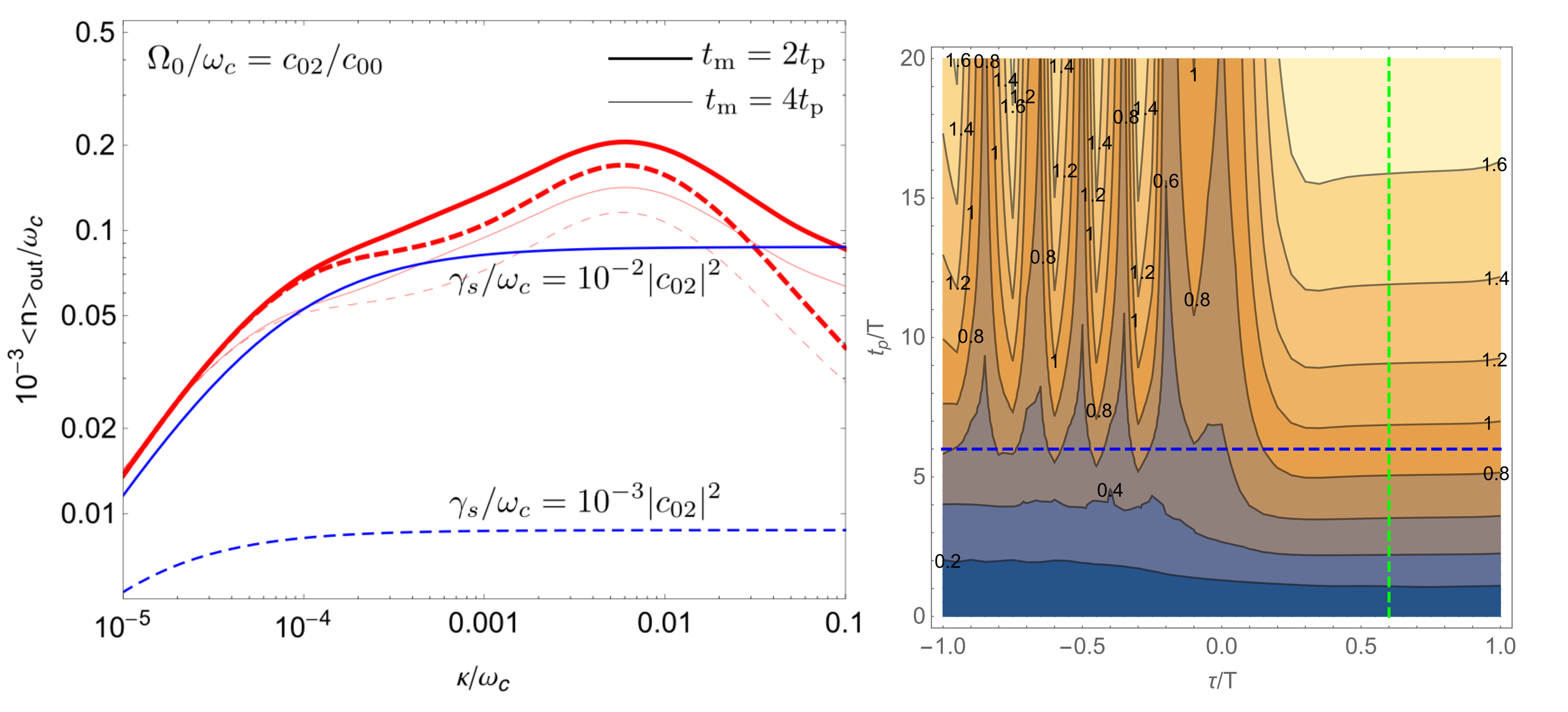

Figure 2: Left: Efficiency of STIRAP(red lines) and SEP(blue lines) is shown in terms of detected photon flux as a function of the damping rate of the lossy cavity for reported values of γ_s (dashed and continuous). Thin lines refers to a measurement time that is double with respect to that of thick ones (t_m in legend). Here the STIRAP reports much higher yield of photocurrent with respect to SEP protocol. In right figure, $\langle n\rangle_{\rm cycle}$ vs t_p and τ is reported. The blue dotted line represents $t_p = 6T$ and green for $\tau = 0.6T$ for $\Omega_0 T = 20$. Here we can see that the counter-intuitive pulse($\tau > 0$) as that of STIRAP gives us a robust and efficient photon production

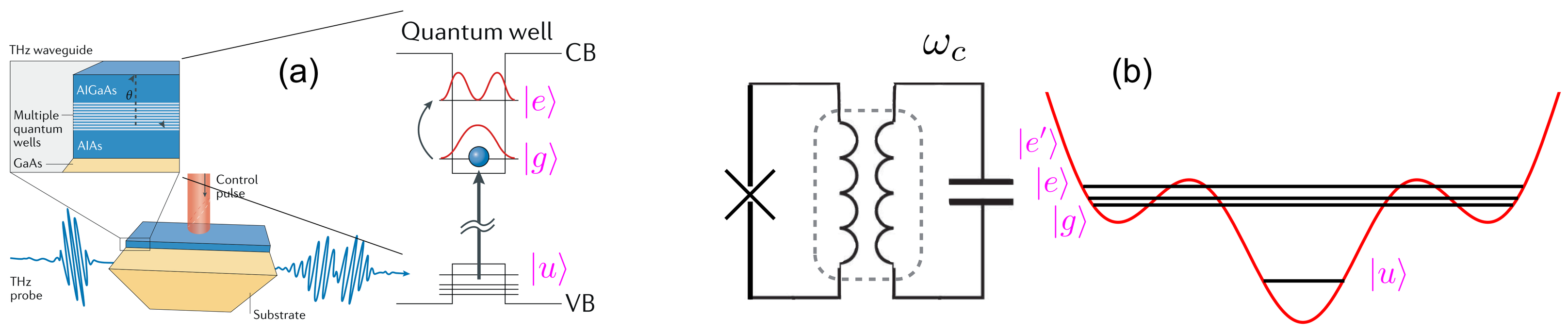

Figure 3: Possible implementation of the proposed schemes in specific ultrastrongly coupled systems, that exhibit the peculiar dark state evolution, exploiting semiconductors or superconductors architectures: (a) intersubband polaritons (from Ref. [8]) (b) superconducting phase qubit.

Conclusions

- We demonstrated that STIRAP is a robust protocol able to dynamically detect unambiguously the virtual photons generated by the USC light-matter interaction, by means of ordinary photodetection. This protocol is very promising, reliable and suitable for superconducting circuits coupled to artificial atoms or semiconductor nanostructures.
- Our next goal is to find the optimal values of parameters, namely λ and κ, that yields the highest efficiency in photodetection. A repumping effect can originate as κ increases, effectively leading to repopulating the ground state during the STIRAP protocol which in effect can lead to producing more than two photons. Further study of this repumping effect in different protocol will be explored.

References

[1] Carusotto *et al.*, Phys. Rev. A **85**, 023805 (2012)
[2] R. Stassi *et al.*, Phys. Rev. Lett. **110**, 243601 (2013).
[3] J.F. Huang and C.K. Law Phys. Rev. A **89**, 033827 (2014)
[4] G. Falci *et al.*., Fortschr. Phys **65**, 1600077 (2017)
[5] Di Stefano *et al.*, New J. Phys. **19**, 053010 (2017)
[6] G. Falci *et al.*, Sci. Rep. **9**, 9249 (2019)
[7] A. Ridolfo *et al.*, Phys. Rev. Lett. **109**, 193602 (2012).
[8] A.F. Kockum *et al.*, Nat. Rev. Phys. **1**, 19 (2019).
[9] Forn-Díaz *et al.*, Rev. Mod. Phys. **91**, 025005