Variational counterdiabatic driving of the ferromagnetic p-spin model

G. Passarelli[1,2,†], V. Cataudella[1,2], R. Fazio[1,3], P. Lucignano[1]

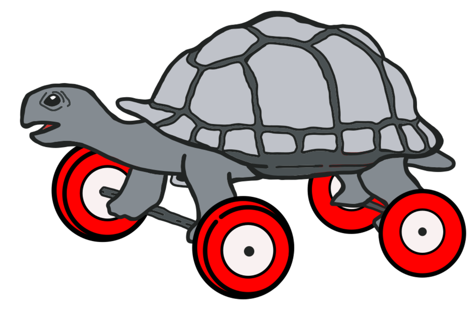
[1] Dipartimento di Fisica "E. Pancini", Università di Napoli Federico II

[2] CNR-SPIN Napoli [3] Abdus Salam ICTP, Trieste, Italy

† gianluca.passarelli@unina.it

Abstract

In adiabatic quantum computation (AQC), or quantum annealing (QA), the goal is to find the ground state of a many-body Hamiltonian H_p, starting from the simple ground state of a transverse field potential H_{TF}. A fast evolution with the time-dependent Hamiltonian $H_0(s) = (1-s)\, H_{\mathrm{TF}} + s\, H_p$ renders the system robust against decoherence and thermal noise but can cause diabatic transition towards excited states. Adding a counterdiabatic (CD) potential H_{CD} to the original Hamiltonian can suppress Landau-Zener excitations and drive the system towards the correct target state, improving the efficiency of QA. The variational approach to CD driving can help us build approximate CD operators that are more viable experimentally than the exact CD potential.



Exact counterdiabatic driving

$$H_{\mathrm{CD}}(t) = \dot{s}(t) \sum_{n\neq m} \frac{\langle E_n(s)|\partial_s H_0(s)|E_m(s)\rangle}{E_n(s) - E_m(s)} |E_n(s)\rangle\langle E_m(s)|$$

- Nonlocal
- Needs the exact spectrum in advance
- Poorly defined around Quantum Phase Transitions
- Useless for practical purposes!

M. V. Berry, J. Phys. A: Math. Theor. 42 (2009) 365303

Variational approach

$$H_{\mathrm{CD}}(t) = \dot{s}(t) A_s(t)$$

$$[\mathrm{i}\, \partial_s H_0 - [A_s, H_0], H_0] = 0 \qquad G_s(A_s) = \partial_s H_0 + \mathrm{i}\,[A_s, H_0]$$

$$\mathcal{S}(A_s) = \mathrm{tr}\left[G_s^2(A_s)\right] \qquad \frac{\delta \mathcal{S}(A_s)}{\delta A_s} = 0$$

Local ansatz

$$A_s(t) = \sum_{k=1}^{l} \alpha_k(t) \mathcal{L}_k$$

$$\mathcal{L}_k \in \{\{\sigma_i^\alpha\}, \{\sigma_i^\alpha \sigma_j^\beta\}, \dots\}$$

$$\alpha, \beta \in \{x, y, z\} \quad i, j \in \{1, \dots, N\}$$

D. Sels, A. Polkovnikov, PNAS May 16, 2017 114 (20)

Nested commutator (NC)

$$A_s^{(l)}(t) = \mathrm{i} \sum_{k=1}^{l} \alpha_k(t) \mathcal{O}_{2k-1}(t)$$

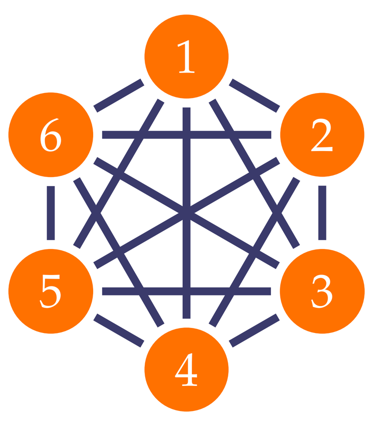
$$\mathcal{O}_k = \begin{cases} \partial_s H_0 & k = 0 \\ [H_0, \mathcal{O}_{k-1}] & k > 0 \end{cases}$$

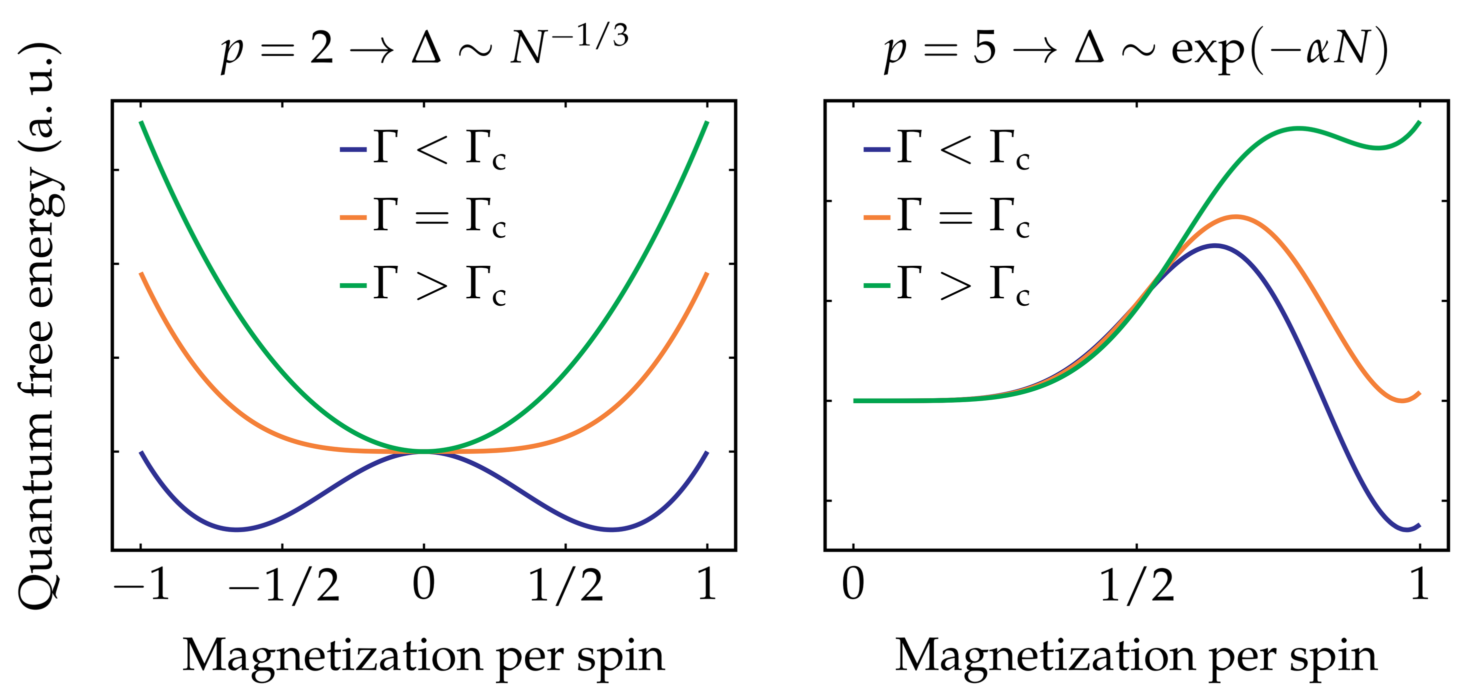
P. W. Claeys *et al.*, Phys. Rev. Lett. 123, 090602 (2019)

Application: ferromagnetic p-spin model

$$H_{\mathrm{TF}} = -\Gamma \sum_{i=1}^{N} \sigma_i^x$$

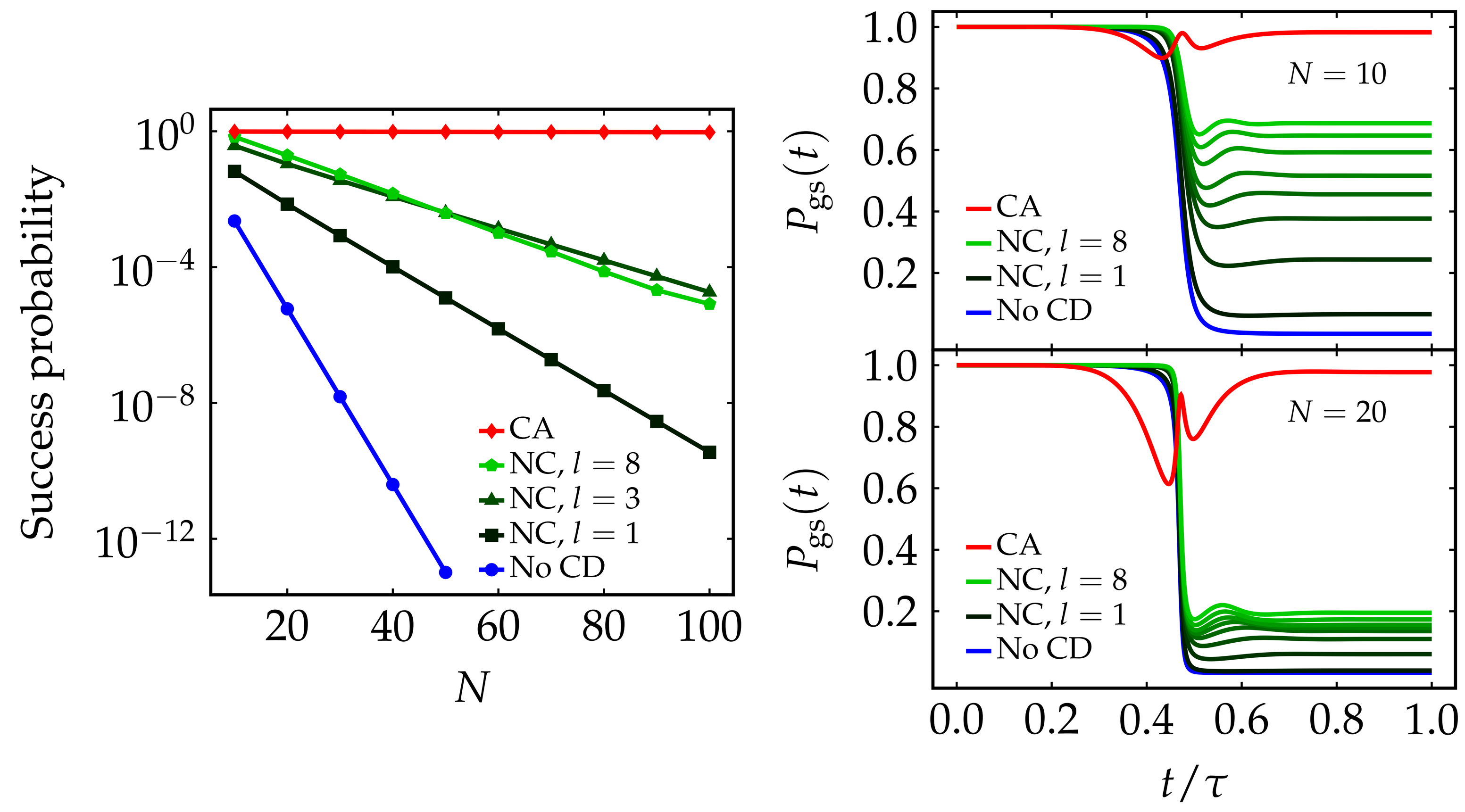
$$H_p = -JN \left(\frac{1}{N} \sum_{i=1}^{N} \sigma_i^z \right)^p \qquad p \geq 2$$





Results for $p = 3$: the cyclic ansatz (CA)

$$\mathcal{L}_1 = S_y \qquad \mathcal{L}_2 = S_y^3 \qquad \mathcal{L}_3 = S_x S_y S_z + S_z S_y S_x$$

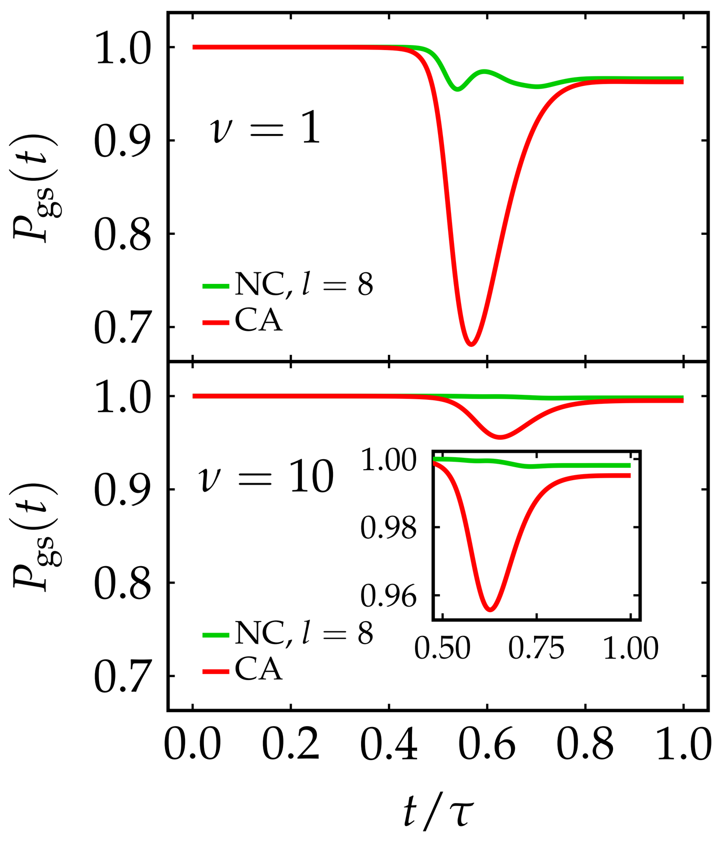


Finite-range and random models

Finite-range p-spin model

$$H_p = -\frac{1}{N^{p-1}} \sum_{i_1,\dots,i_p} \frac{J}{d^\nu_{i_1,\dots,i_p}} \sigma_{i_1}^z \cdots \sigma_{i_p}^z$$

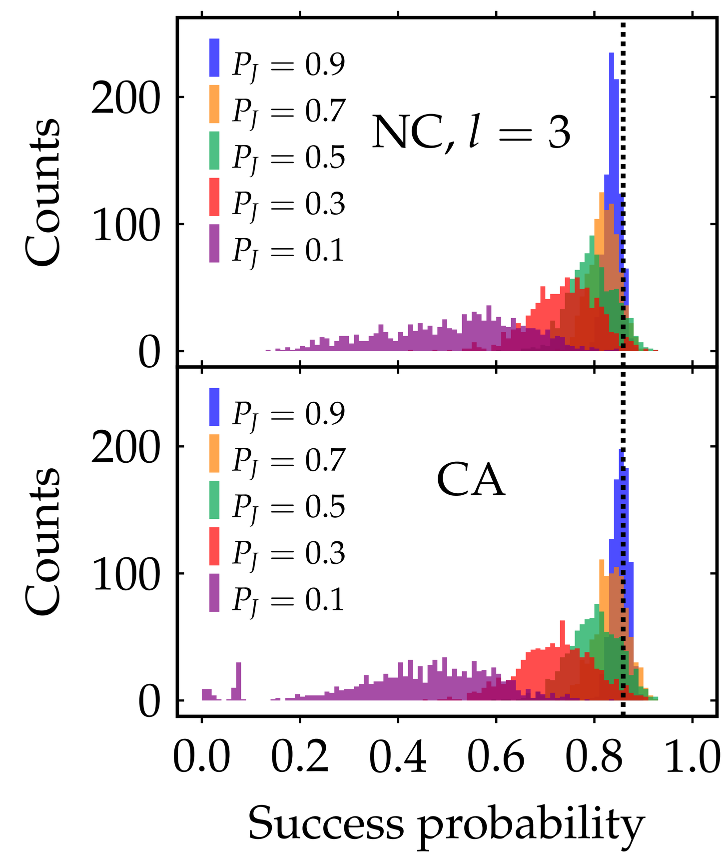
$$\nu \begin{cases} = 0 & \text{infinite-range} \\ > 0 & \text{finite-range} \end{cases}$$



Random model

$$J \to K = \begin{cases} J & \text{probability } P_J \\ 0 & \text{probability } 1 - P_J \end{cases}$$

$P_J = 1$: standard model (dashed line)



G. Passarelli *et al.*, Phys. Rev. Research 2, 013283 (2020)

Conclusions

- The NC ansatz can improve the annealing performance of the p-spin model
- Its efficiency is bound to decrease in the thermodynamic limit
- For $p = 3$, the cyclic ansatz is a viable alternative
- The CA yields optimal fidelity independently of the system size
- Analyzing finite-range and random models can provide some insights into the efficiency of the CA