Entanglement of indistinguishable particles

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Entanglement of distinguishable particles

$|\Psi\rangle \in \mathcal{H} \otimes \mathcal{H}$ entangled

$$\Leftrightarrow |\Psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

Distinguishable particles

Nice properties !

- Equivalent to correlations [Werner, Phys. Rev. A **40**, 4277, (1989)]

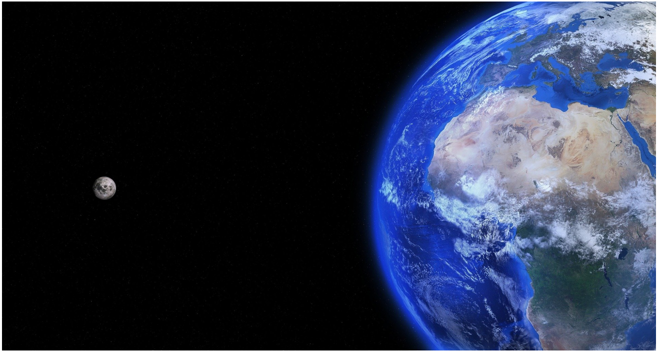
$$\langle\psi|AB|\psi\rangle \neq \langle\psi|A|\psi\rangle\langle\psi|B|\psi\rangle,$$

  of particle **local** observables

$$A = O_1 \otimes I, \quad B = I \otimes O_2$$

- Measure of amount of entanglement → resource theory [Chitambar et. al., Comm. Math, Phys. **328**, 303 (2014)]
- Requires interaction in the past. **Non-local** operators.

2 Identical Hydrogen atoms



$$|\Psi^{id}\rangle = |0_M\rangle|1_E\rangle + |1_E\rangle|0_M\rangle$$

Identical particle entanglement?

Naively: $|\Psi^{id}\rangle \neq |\psi\rangle \otimes |\psi\rangle$ entangled

Does indistinguishability create entanglement?

NO! Symmetrized operators.
$A \otimes I \rightarrow A \otimes I + I \otimes A$

Several definitions in literature.

Want:

- Local operators
  - Correlations between subsystems. Subsystems defined through observables
- Resource theory
- Effective distinguishability

Naive definition (and others) fail criteria.

[Benatti et. al., Phys. Rep. **878**, 1, (2020); Johann, Marzolino (submitted for publication)]

→ Entanglement between modes.

[Zanardi, Phys. Rev. Lett. **87**, 077901 (2001)]

Mode Entanglement

Identical particle operators: 2nd quantization $a^\dagger_{E,0}|vac\rangle = |0_E\rangle$

Group $\mathcal{A}_E = \{a^\dagger_{E,\sigma}, a_{E,\sigma}\}$, $\mathcal{A}_M = \{a^\dagger_{M,\sigma}, a_{M,\sigma}\}$

Local operators: A_E, A_M, $A_E\, A_M$

Def: Mode-Entanglement $\Leftrightarrow$ correlations between local operators.

$$\langle a^\dagger_{E,0} a_{E,0}\; a^\dagger_{M,1} a_{M,1}\rangle \neq \langle a^\dagger_{E,0} a_{E,0}\rangle\langle a^\dagger_{M,1} a_{M,1}\rangle$$

$$\rightarrow |\psi_{sep}\rangle \;=\; \sum_{\sigma_1} c_{\sigma_1}\, a^\dagger_{E,\sigma_1} \sum_{\sigma_2} c_{\sigma_2}\, a^\dagger_{M,\sigma_2}|vac\rangle$$

or

$$|\psi_{sep}\rangle \;=\; \sum_{\sigma_1,\sigma_2} a^\dagger_{E,\sigma_1}\, a^\dagger_{E,\sigma_2}|vac\rangle$$

[Benatti et. al., Phys. Rep. **878**, 1, (2020)]

Mode Entanglement

- Locality
- Entanglement as resource $\rightarrow$ LOCC theory
- Effective distinguishability:

$$\begin{aligned} |\Psi^{id}\rangle &= |0_M\rangle|1_E\rangle + |1_E\rangle|0_M\rangle \\ &= a^\dagger_{M,0} a^\dagger_{E,1}|vac\rangle \Leftrightarrow |0\rangle|1\rangle \ \text{Separable!} \end{aligned}$$

[Benatti et. al. (2020); Marzolino, Buchleitner, Phys. Rev. A **91** 032316 (2015); Cunden et. al., Int. J. Quantum Inform. **12**, 1461001 (2014)]

Conclusions

- Naive def. has several problems
- No canonical definition
- Natural requirements lead to mode entanglement

Other Definitions

Entanglement II.:

Separable states are

$$\mathcal{S}|\psi_1\rangle|\psi_2\rangle,$$

with

$$|\psi_1\rangle = |\psi_2\rangle \text{ or } \langle\psi_1|\psi_2\rangle = 0$$

[Ghirardi, Marinatto, Weber, J. Stat. Phys. **108**, 49 (2002); Eckert et. al., Ann. Phys. **299**, 88]

Other definitions

Entanglement III.:

$|\psi\rangle$ entangled wrt. subspace $\mathcal{K} \in \mathcal{H}$ iff

$$X_1 = \frac{1}{\sum_{\psi_k \in \mathcal{K}} \langle\psi| a^\dagger_{\psi_k} a_{\psi_k} |\psi\rangle} \sum_{\psi_k \in \mathcal{K}} a_{\psi_k} |\psi\rangle\langle\psi| a^\dagger_{\psi_k}$$

has non vanishing von-Neumann entropy

$$S(X_1) = -\mathrm{tr} X_1 \log(X_1)$$

[Lo Franco, Compagno, Sci. Rep. **6**, 20603 (2016)]

Entanglement measure

Schmidt decomposition for bosons:

$$|\psi\rangle = \sum_{i,j} \beta_{ij} b_i^\dagger b_j^\dagger |vac\rangle$$
$$= \sqrt{2} \sum_k \mathrm{B}_k {b'_k}^\dagger {b'_k}^\dagger |vac\rangle$$

Von Neumann entropy

$$S = -\sum_k 2|\mathrm{B}_k|^2 \log(2|\mathrm{B}_k|^2)$$

[Paskauskas, You, Phys. Rev. A **64**, 042310, (2001)]