Generic aspects of the resource theory of coherence

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Basic ingredients of the Resource Theory

A fixed, particular basis $\{|i\rangle\}_{i=1}^{n}$ of the Hilbert space is chosen (based on physical or practical considerations) and coherence in this basis is studied as a resource.

Free states

States ρ which are diagonal in the "incoherent" basis $\{|i\rangle\}_{i=1}^{n}$:

$$\rho = \sum_{i=1}^{n} p_i \, |i\rangle\langle i| \, .$$

Allowed operations

Transformations which do not generate coherent states starting from incoherent ones.

Many questions of interest can be asked for a certain resource theory. Here, we will focus on state conversions among pure random states.

States conversions with Incoherent Operations

For pure states, IO conversions are ruled by majorization relations.
Given $|\psi\rangle = \sum_{i=1}^{n} \psi_j |i\rangle$, we denote with:

$$\Delta(\psi) := (|\psi_1|^2, \dots, |\psi_n|^2)$$

Theorem: Deterministic conversion among pure states

A pure state $|\psi\rangle$ can be transformed into $|\psi'\rangle$ under IO if and only if $\Delta(\psi) \prec \Delta(\psi')$

Theorem: Maximal success probability for a stochastic conversion

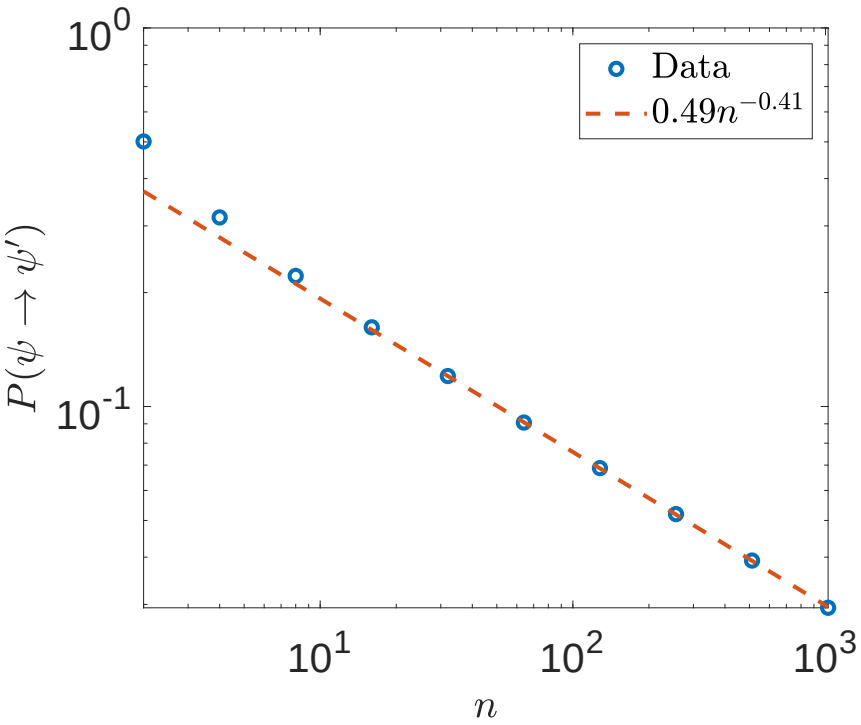
For two pure states $|\psi\rangle$ and $|\psi'\rangle$, the maximal conversion probability under IO is given by

$$\Pi(\Delta(\psi), \Delta(\psi')) = \min_{1 \leqslant k \leqslant n} \frac{\sum_{j=1}^{k} \Delta(\psi)_j^{\downarrow}}{\sum_{j=1}^{k} \Delta(\psi')_j^{\downarrow}}$$

Deterministic conversion among random pure states

Question[a]**:** is it true that "most" pairs of pure n-dimensional quantum states are *not* IO-intercovertible?
Answer: Yes.

[a] The analogous question for entanglement conversion was posed by Nielsen in [PRL **83**, 436 (2000)] and addressed recently in [Cunden, Facchi, Florio, Gramegna, J. Phys. A **53** 175303 (2020)]



Specifically, we prove:

Theorem: Most pairs are not IO-interconvertible

Let $|\psi\rangle$ and $|\psi'\rangle$ be independent random (Haar uniform) pure states on $\mathbb{C}^n$. Then

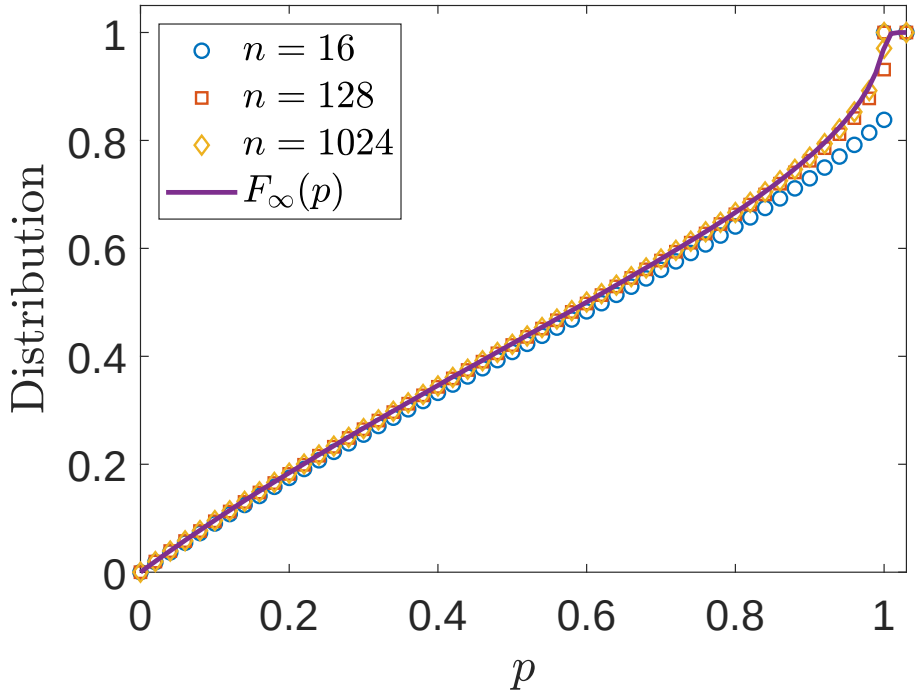
$$\lim_{n\to\infty} P(|\psi\rangle \xrightarrow{\text{IO}} |\psi'\rangle) = 0$$

Distribution of the maximal success probability of IO-conversion

We investigated numerically the distribution of the maximal success probability of IO-conversion, i.e. the function

$$F_n(p) := P(\Pi(\Delta(\psi), \Delta(\psi')) \leqslant p)$$

for $|\psi\rangle, |\psi'\rangle$ sampled uniformly and independently in $\mathbb{C}^n$.



Numerical evidence shows convergence towards a limit distribution $F_n(p) \to F_\infty(p)$ as $n \to \infty$.

Thanks for the attention