Generic aspects of the resource theory of coherence

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Basic ingredients of the Resource Theory

A fixed, particular basis \( \{ |i\rangle \}_{i=1}^{n} \) of the Hilbert space is chosen (based on physical or practical considerations) and coherence in this basis is studied as a resource.

**Free states**

States \( \rho \) which are diagonal in the “incoherent” basis \( \{ |i\rangle \}_{i=1}^{n} \):

\[
\rho = \sum_{i=1}^{n} p_i |i\rangle\langle i| .
\]

**Allowed operations**

Transformations which do not generate coherent states starting from incoherent ones.

Many questions of interest can be asked for a certain resource theory. Here, we will focus on state conversions among pure random states.
States conversions with Incoherent Operations

For pure states, IO conversions are ruled by majorization relations. Given \( |\psi\rangle = \sum_{i=1}^{n} \psi_j |i\rangle \), we denote with:

\[
\Delta(\psi) := (|\psi_1|^2, \ldots, |\psi_n|^2)
\]

**Theorem: Deterministic conversion among pure states**

A pure state \( |\psi\rangle \) can be transformed into \( |\psi'\rangle \) under IO if and only if

\[
\Delta(\psi) \prec \Delta(\psi')
\]

**Theorem: Maximal success probability for a stochastic conversion**

For two pure states \( |\psi\rangle \) and \( |\psi'\rangle \), the maximal conversion probability under IO is given by

\[
\Pi(\Delta(\psi), \Delta(\psi')) = \min_{1\leq k \leq n} \frac{\sum_{j=1}^{k} \Delta(\psi)_j}{\sum_{j=1}^{k} \Delta(\psi')_j}
\]
Deterministic conversion among random pure states

**Question**: is it true that “most” pairs of pure \( n \)-dimensional quantum states are *not* IO-intercovertible?

**Answer**: Yes.

\(^a\)The analogous question for entanglement conversion was posed by Nielsen in [PRL 83, 436 (2000)] and addressed recently in [Cunden, Facchi, Florio, Gramegna, J. Phys. A 53 175303 (2020)]

Specifically, we prove:

**Theorem**: Most pairs are not IO-interconvertible

Let \( |\psi\rangle \) and \( |\psi'\rangle \) be independent random (Haar uniform) pure states on \( \mathbb{C}^n \). Then

\[
\lim_{n \to \infty} P(|\psi\rangle \xrightarrow{\text{IO}} |\psi'\rangle) = 0
\]
We investigated numerically the distribution of the maximal success probability of IO-conversion, i.e. the function

\[ F_n(p) := P(\Pi(\Delta(\psi), \Delta(\psi'))) \leq p) \]

for \(|\psi\rangle, |\psi'\rangle\) sampled uniformly and independently in \(\mathbb{C}^n\).

Numerical evidence shows convergence towards a limit distribution \(F_n(p) \rightarrow F_\infty(p)\) as \(n \rightarrow \infty\).
Thanks for the attention