NOISE-RESILIENT VARIATIONAL HYBRID QUANTUM-CLASSICAL OPTIMIZATION

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Variational method

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  - $|\psi(\theta^{opt})\rangle$ best approximation of the ground state

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Variational method and Hybrid approach

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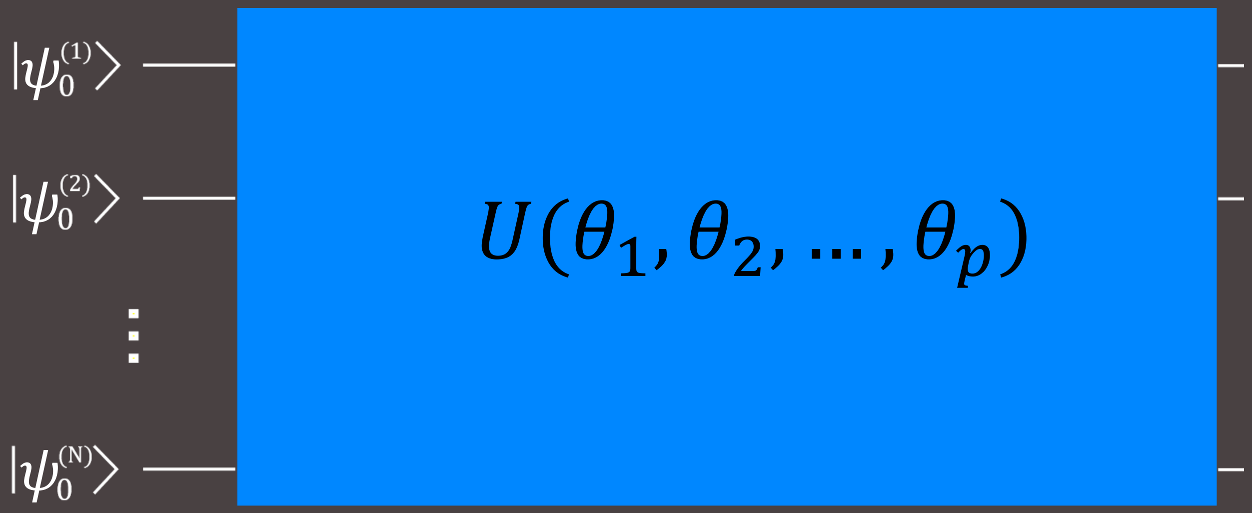

Quantum

- $|\psi(\theta)\rangle = U(\theta)|\psi_0\rangle$

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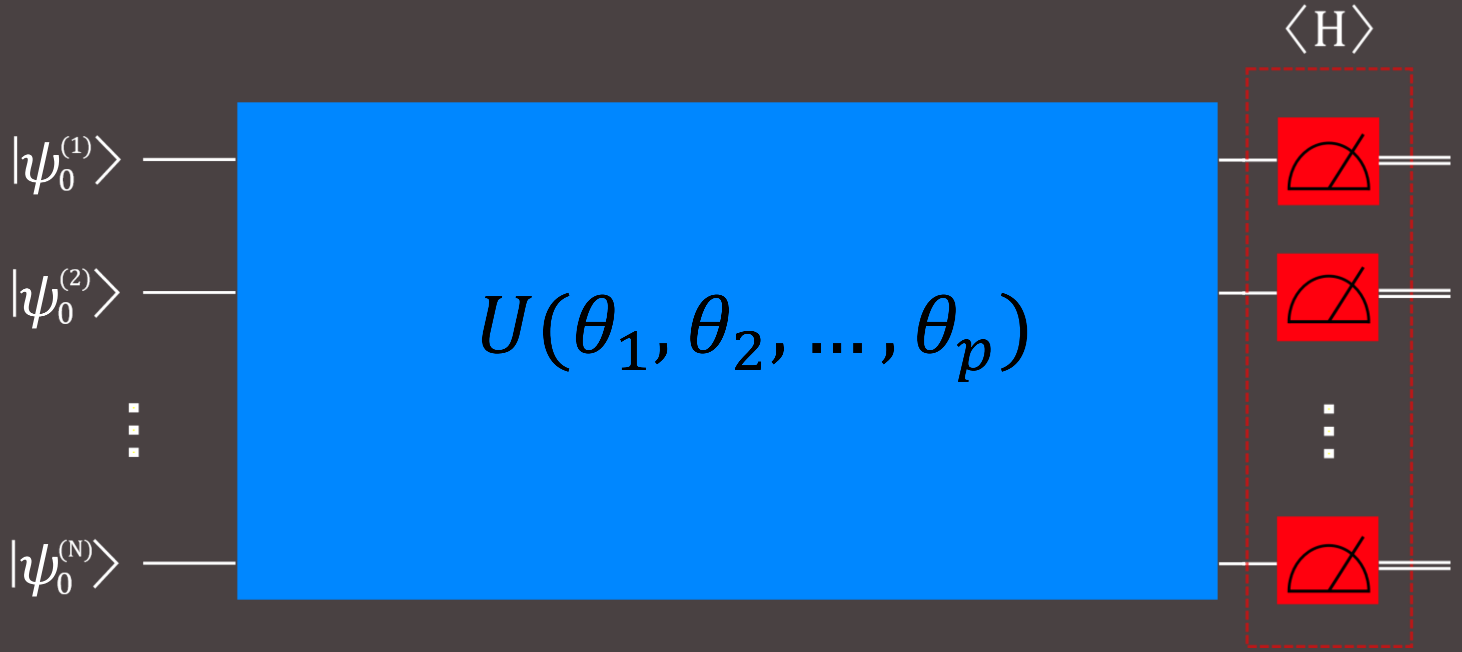

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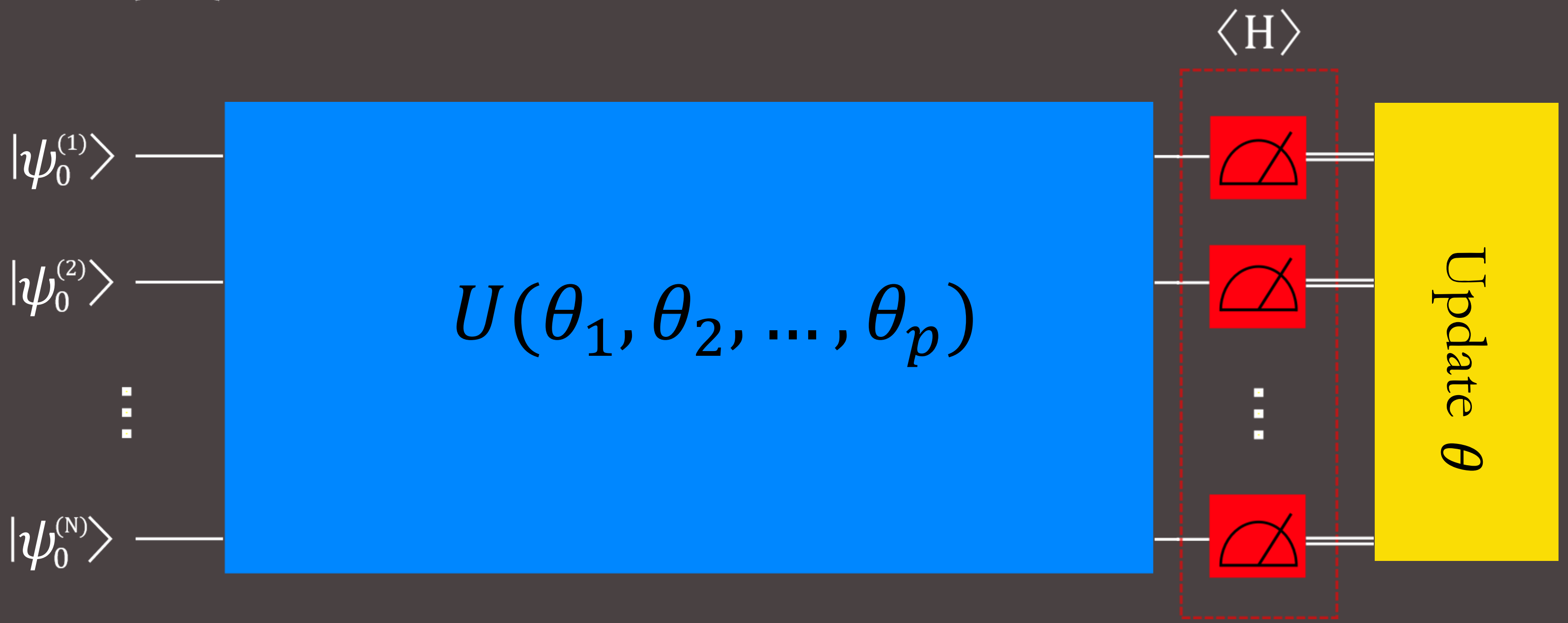

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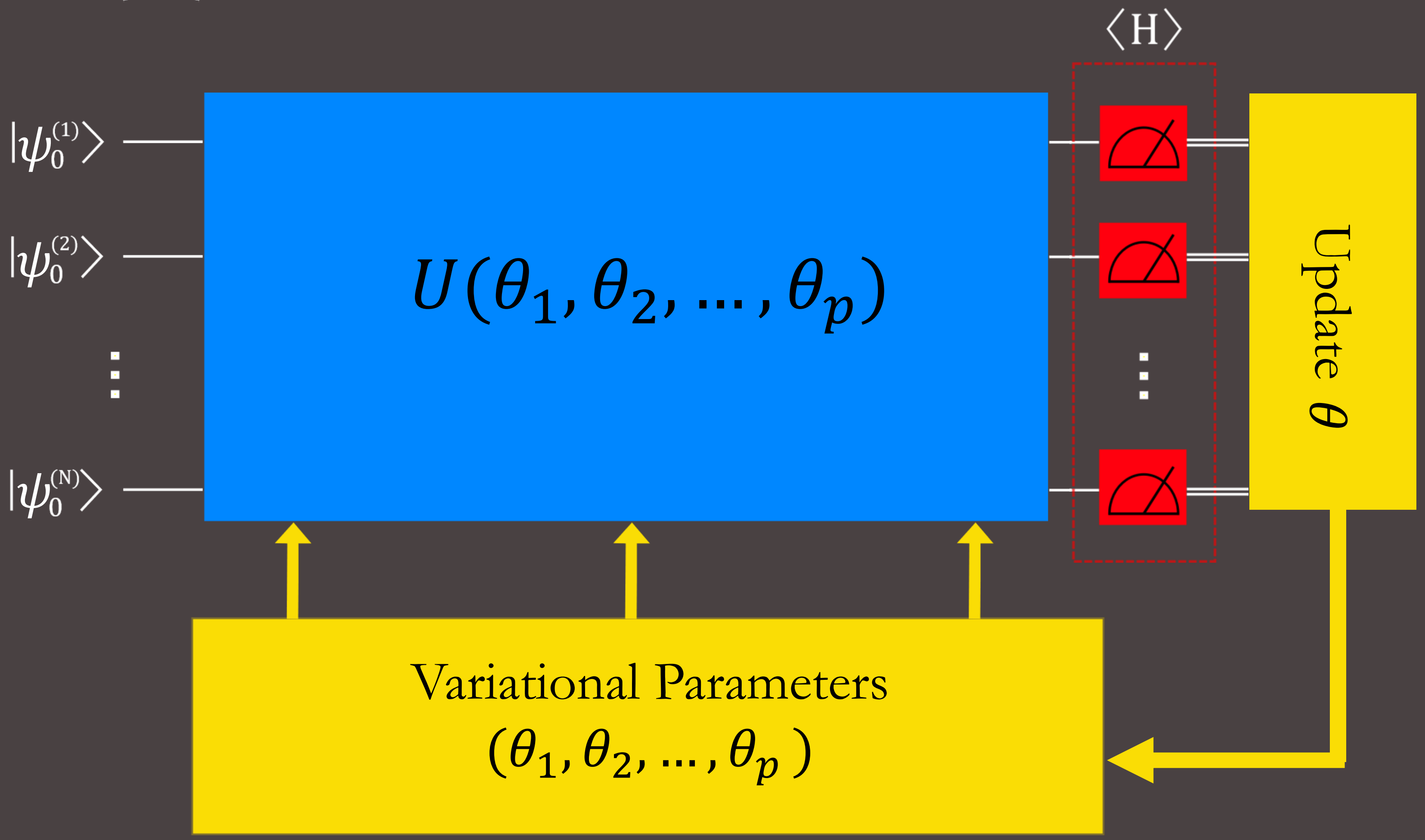

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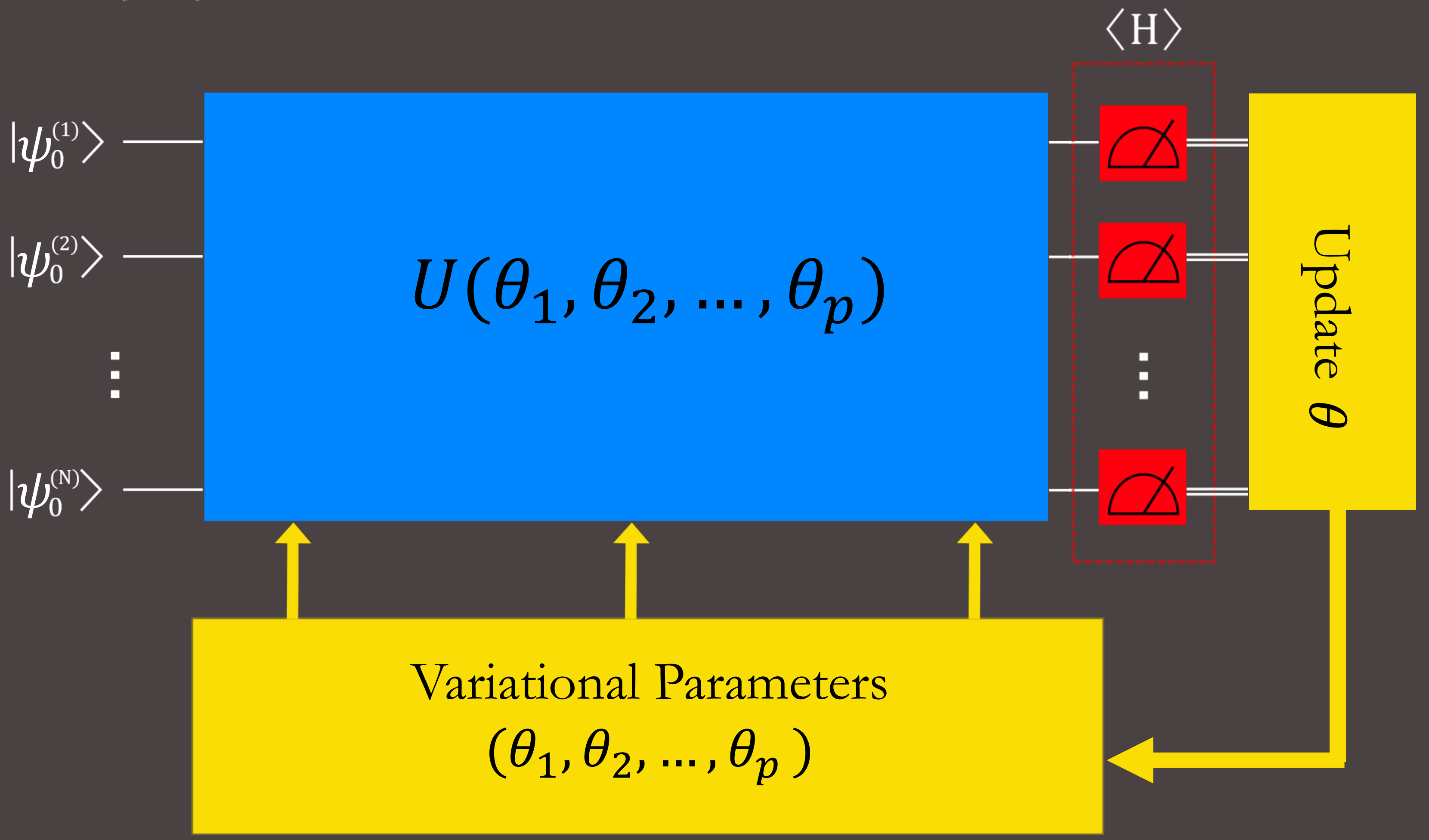

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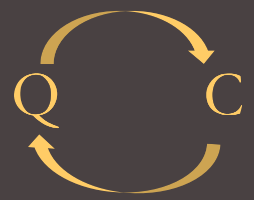
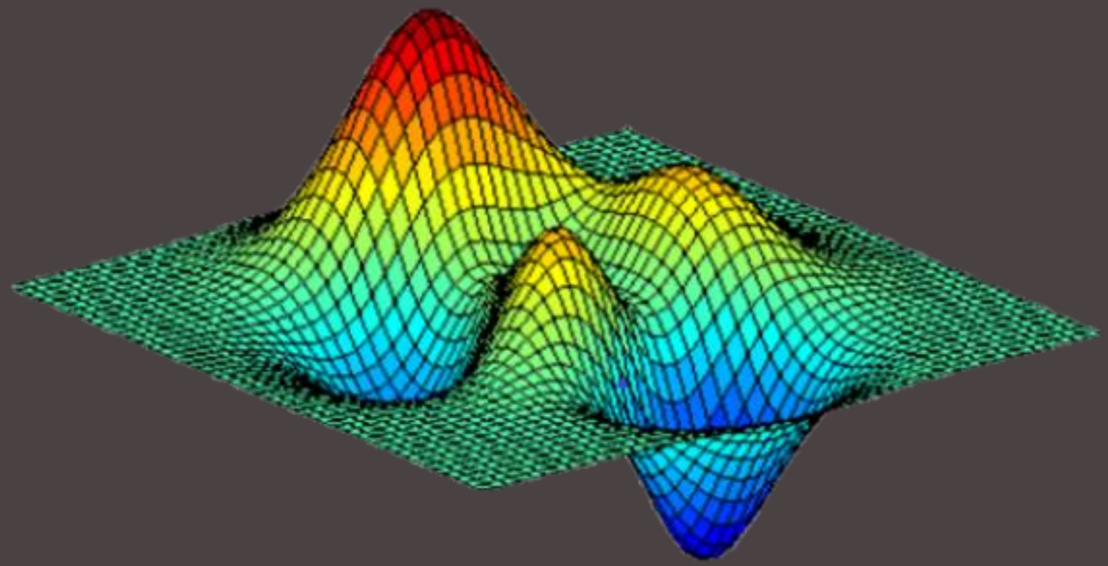


Find θ^{opt}!

Role of $\theta' s$ and optimization landscape

- θ define a parameter space

$$C(\theta) \rightarrow \text{landscape}$$



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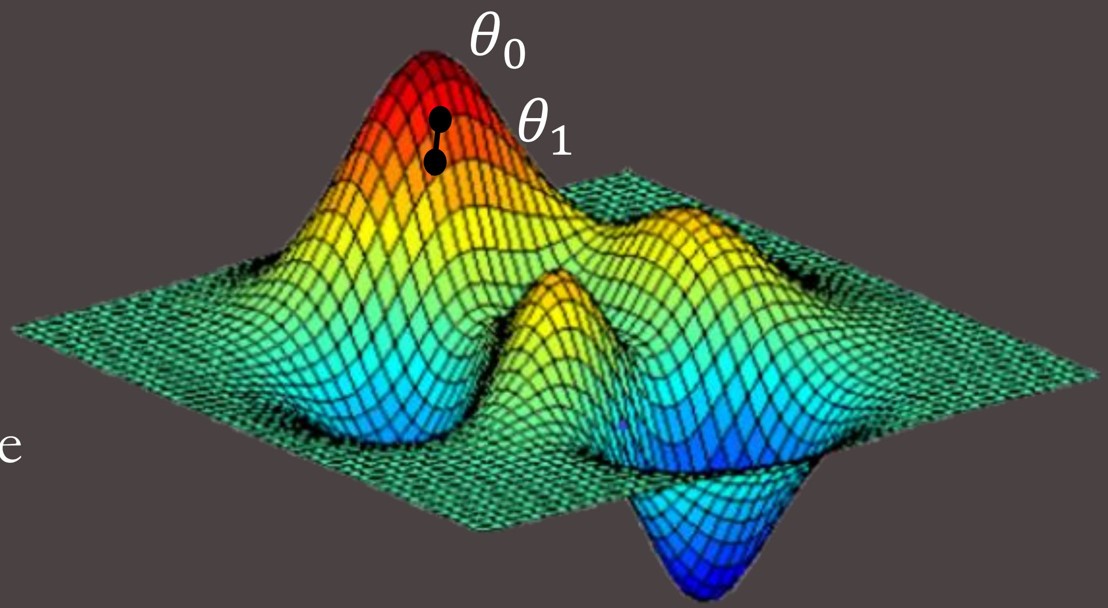

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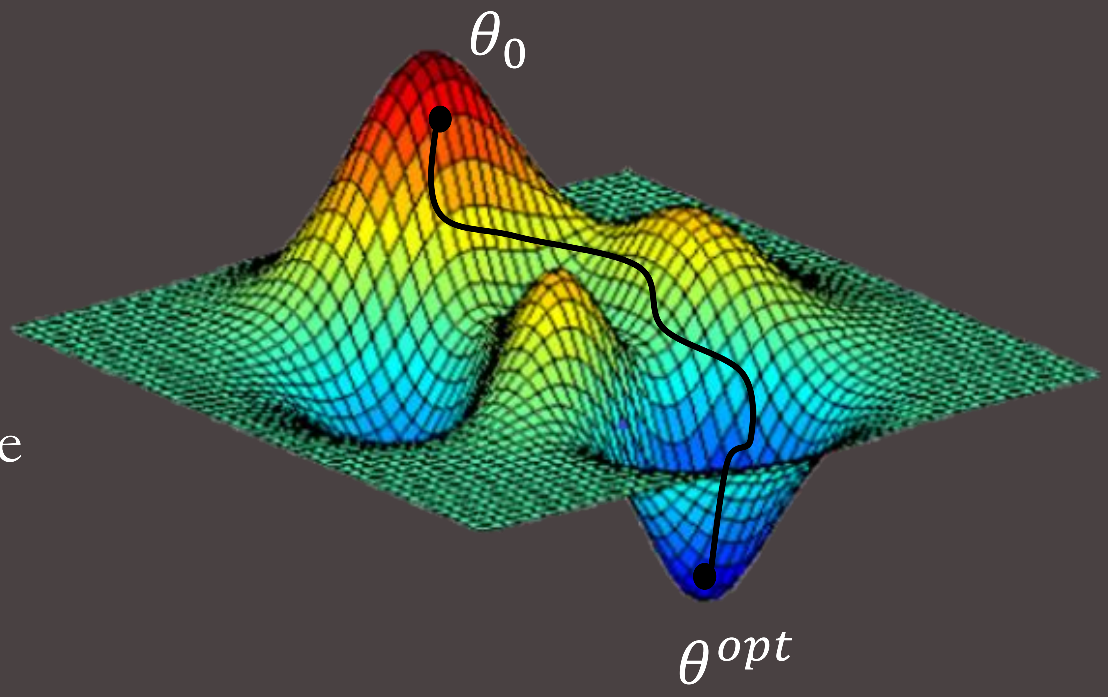

Role of $\theta's$ and optimization landscape

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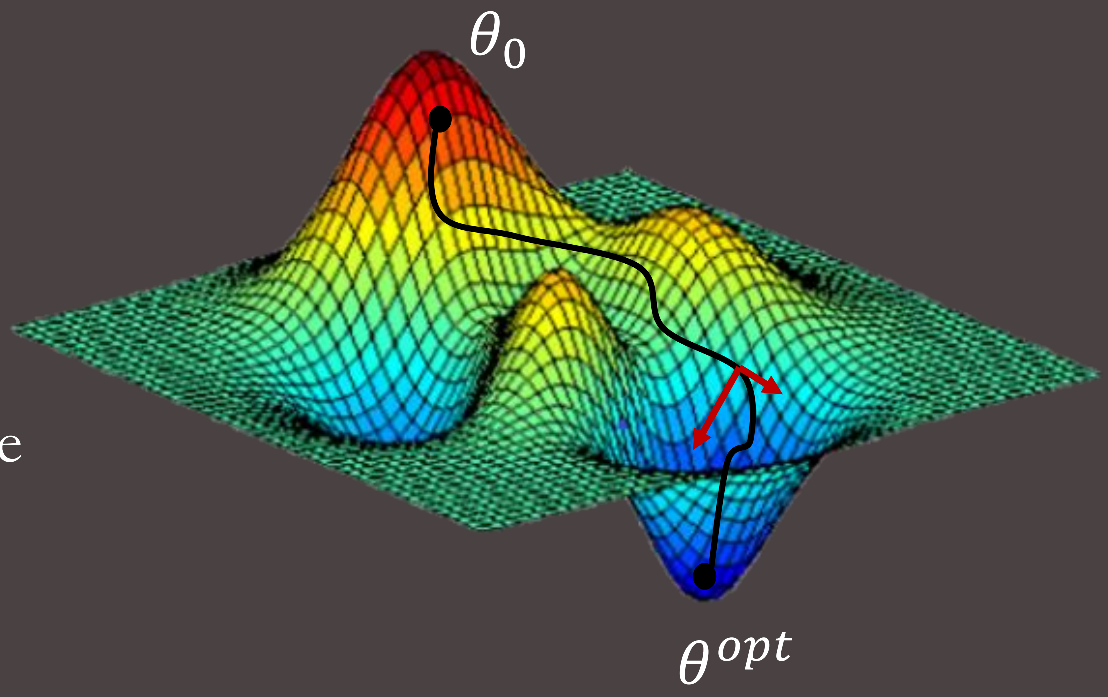
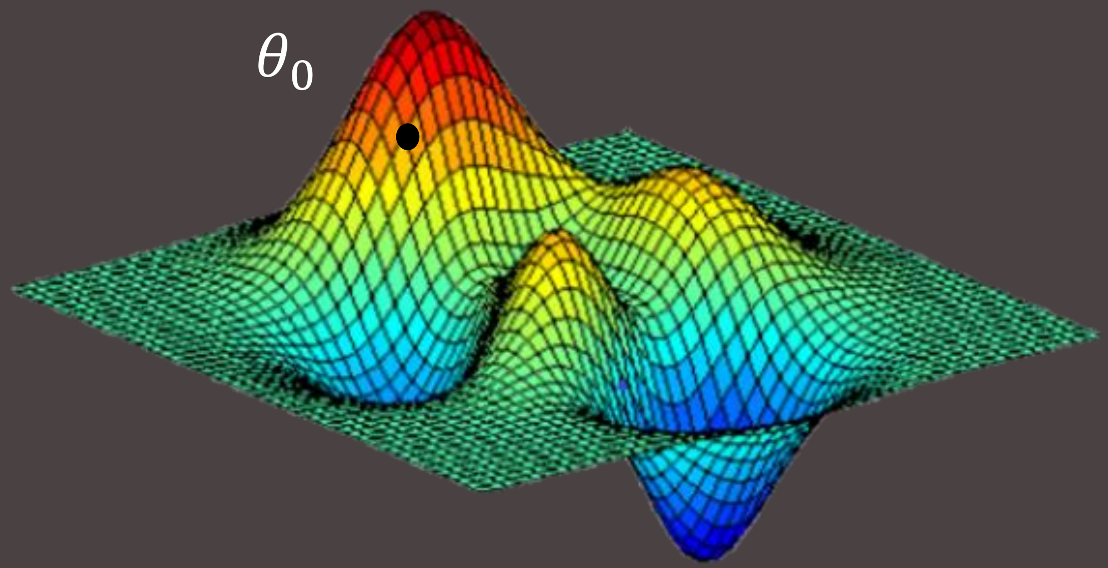
- At each optimization step we move a little
- The algorithm define a *path*…
- And the gradient is algorithm's compass!



Stochastic Optimization and Noise

- Path full of traps



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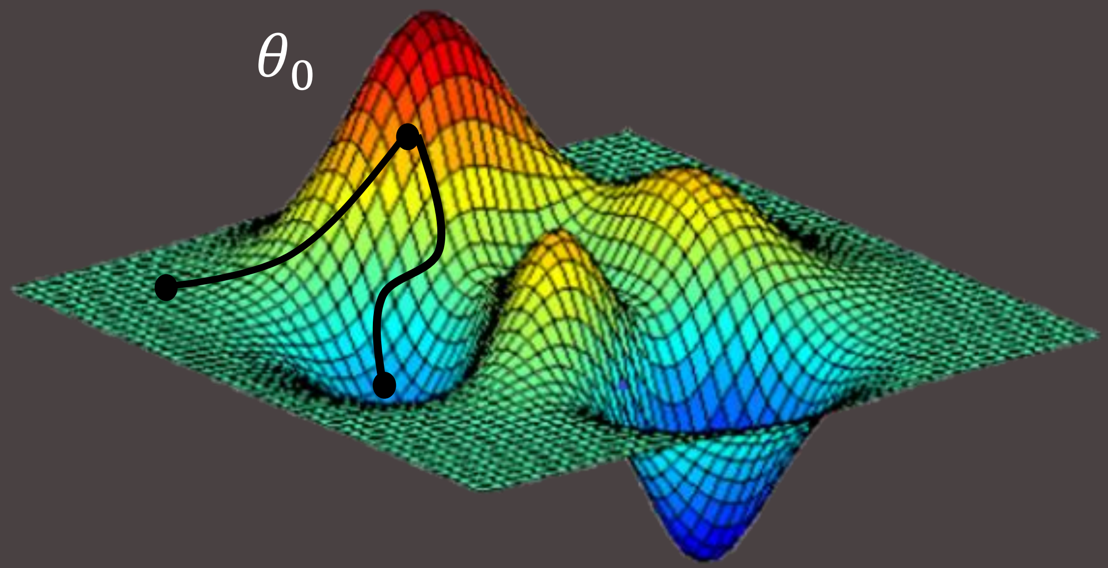
$$\nabla_\theta C = 0$$



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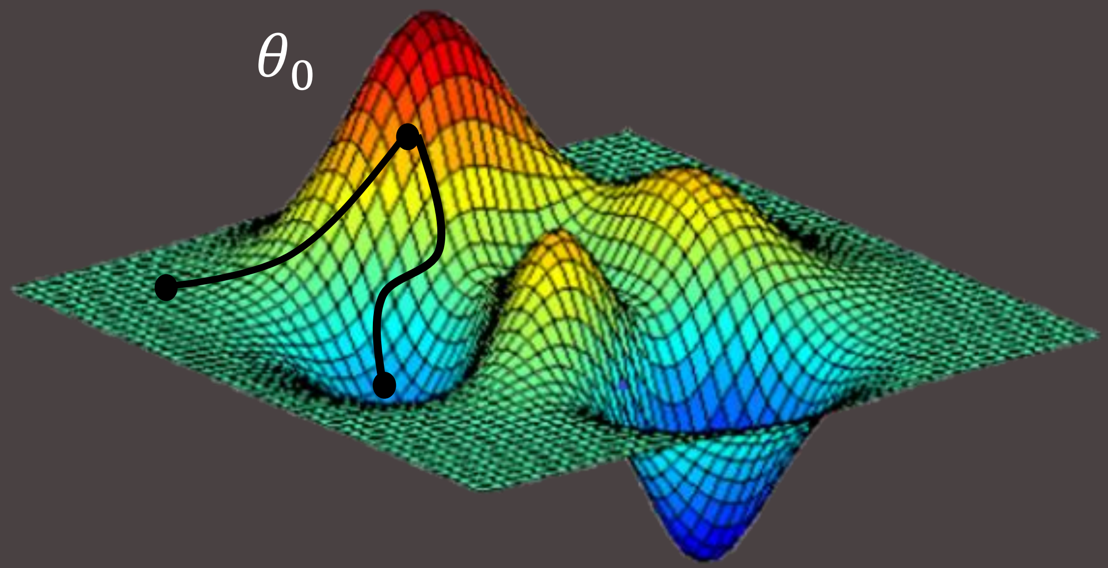

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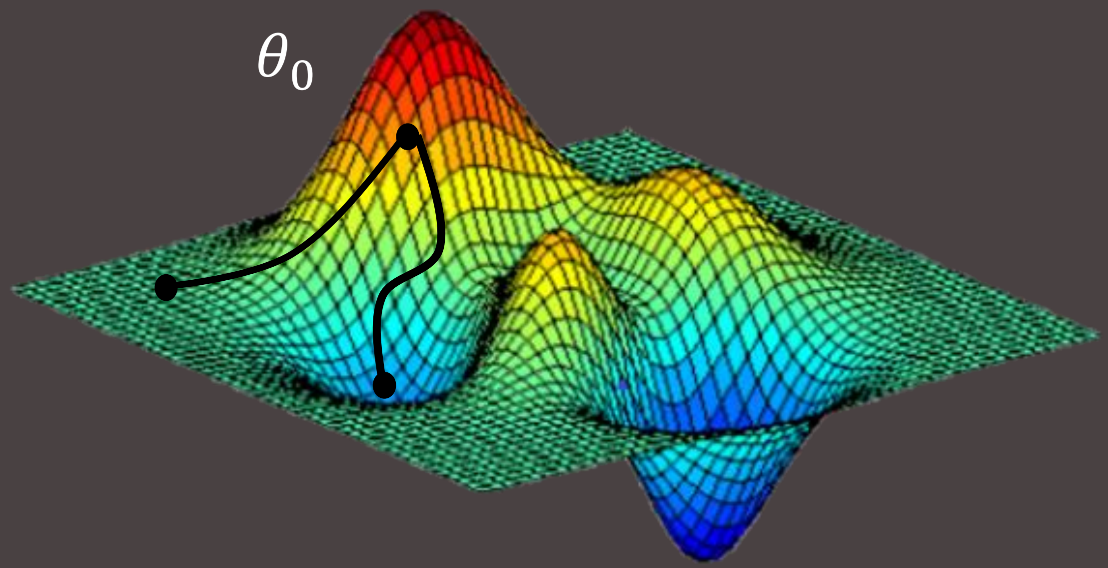

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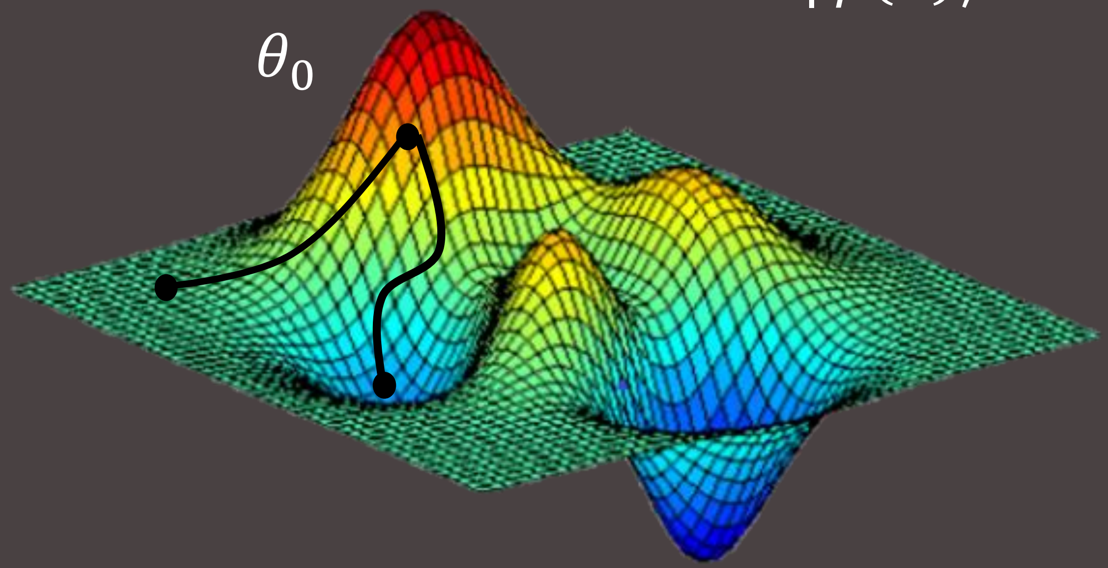
$\theta \Leftrightarrow |\psi(\theta)\rangle$



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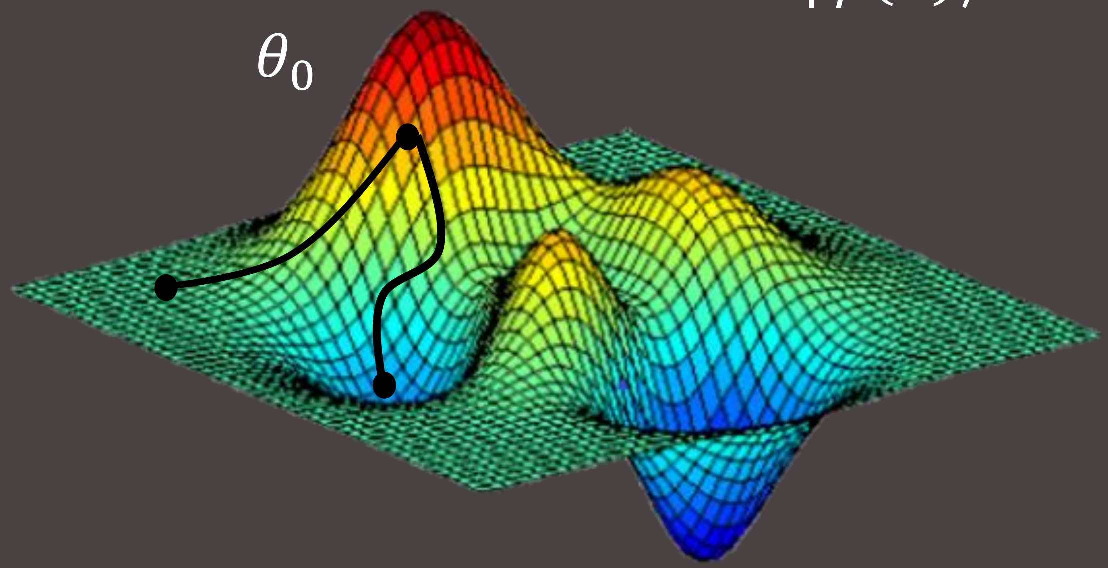

Stochastic outcomes & Noise → useful?

Convergence
Role of Noise and outcomes } geometrical point of view?

Quantum Fisher Information bound

$$C_{noisy}\left(\theta^{[1:I]}\right) - C(\theta^{opt}) \leq Err(\theta^{opt}, \vartheta^{opt}) + \frac{R\sqrt{p}\|H\|_\infty}{\sqrt{I}} \max_{j,\theta} \sqrt{\mathrm{QFI}_j(\theta)}$$

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Accuracy after I iterations

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Accuracy after I iterations

Increases with noise strength

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Accuracy after I iterations

Increases with noise strength

Decreases with noise strength

Tradeoff → Noise can help to explore parameter space AND to bound the error

Contact me at

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Or

Visit our group site

https://qtif.weebly.com/

(Work in progress,

Come back soon!)

THANK YOU