Nonclassical steering and the Gaussian steering triangoloids

Massimo Frigerio [1],
Claudio Destri [1], Stefano Olivares [2], Matteo G. A. Paris [2]

September 30, 2020

[1]Università degli Studi di Milano-Bicocca

[2]Università degli Studi di Milano

YIQIS 2020

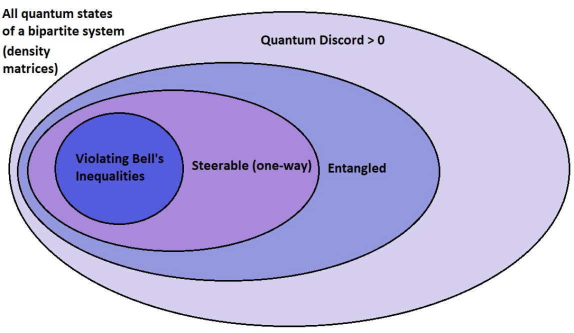
Overview on quantum steering

Overview on quantum steering

- Introduced by Schrödinger in 1935 to discuss EPR argument
- Rigorously defined for generic mixed states only in 2007 (*Wiseman et al.* [3])
- General definition based on one-party entanglement verification
- Finds applications in Quantum Key Distribution (QKD)

Overview on quantum steering

- Intermediate between entanglement and Bell's inequality violation, in the hierarchy of **quantum correlations**



Nonclassicality

Nonclassicality according to the Glauber P-function

$$\hat{\boldsymbol{\rho}} = \int_{\mathbb{C}^n} \mathrm{d}^{2n}\alpha \; P[\hat{\boldsymbol{\rho}}](\alpha) \, |\alpha\rangle\langle\alpha|$$

- **Most physical** definition of nonclassicality
- **Nonclassical** whenever the P-function is not a valid probability distribution
- Necessary for photon antibunching and sub-Poissonian statistics

Nonclassical steering with TMSTs

TMST states

- Two mode squeezed thermal states

$$\hat{\boldsymbol{\rho}}_{AB} := \hat{\mathcal{S}}^{(2)}(r)\left[\hat{\boldsymbol{\nu}}_{th}(N_A) \otimes \hat{\boldsymbol{\nu}}_{th}(N_B)\right]\hat{\mathcal{S}}^{(2)}(r)^{\dagger}$$

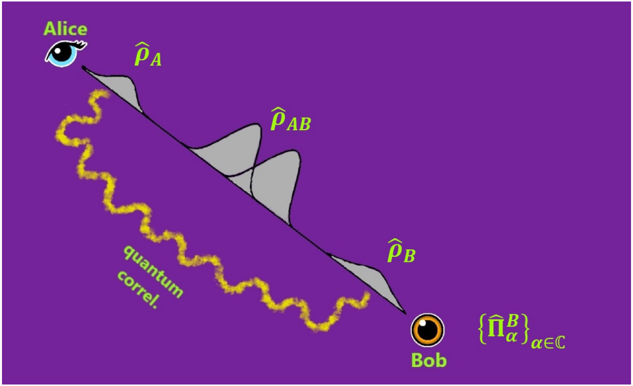
$$\hat{\boldsymbol{\nu}}_{th}(N) = \frac{1}{1+N}\sum_{n=0}^{\infty}\left(\frac{N}{1+N}\right)^{n} |n\rangle\langle n|$$

$$\hat{\mathcal{S}}^{(2)}(r) := e^{r(\hat{a}^{\dagger}\hat{b}^{\dagger} - \hat{a}\hat{b})} \quad , \qquad r \in \mathbb{R}$$

- They can be **separable** or **entangled**: interesting enough for quantum correlations

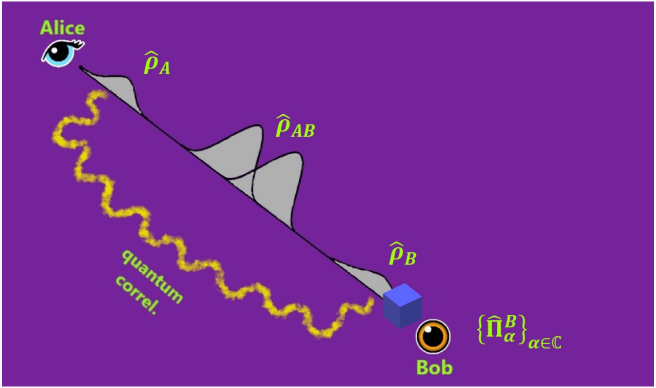
- $\hat{\boldsymbol{\rho}}_A = \mathrm{Tr}_B[\hat{\boldsymbol{\rho}}_{AB}]$ and $\hat{\boldsymbol{\rho}}_B = \mathrm{Tr}_A[\hat{\boldsymbol{\rho}}_{AB}]$ are **always classical**

Nonclassical steering



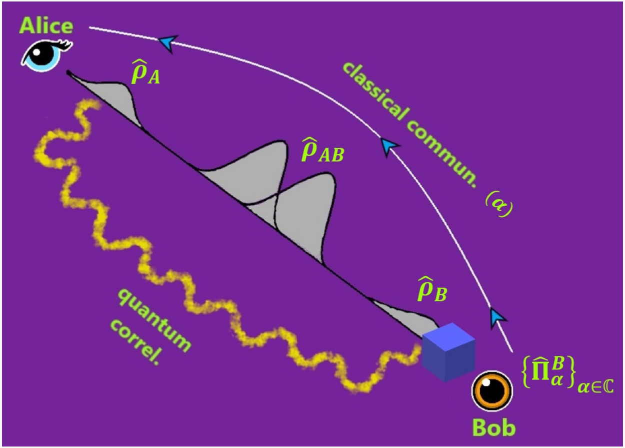
Nonclassical steering

$$\hat{\Pi}_\alpha = \frac{1}{\pi}\,\hat{D}(\alpha)\hat{\rho}_M\hat{D}^\dagger(\alpha)\ , \qquad p_\alpha = \mathrm{Tr}_{AB}\left[\hat{\rho}_{AB}\left(\mathbb{I}_A \otimes \hat{\Pi}_\alpha\right)\right]$$



Nonclassical steering

$$\hat{\rho}_A \quad \to \quad \hat{\rho}_A^{(\alpha)} \;=\; \mathrm{Tr}_B[\hat{\rho}_{AB}^{(\alpha)}] \;=\; \frac{1}{p_\alpha}\mathrm{Tr}_B\left[\hat{\rho}_{AB}\left(\mathbb{I}_A \otimes \hat{\Pi}_\alpha\right)\right]$$



Gaussian steering triangoloids

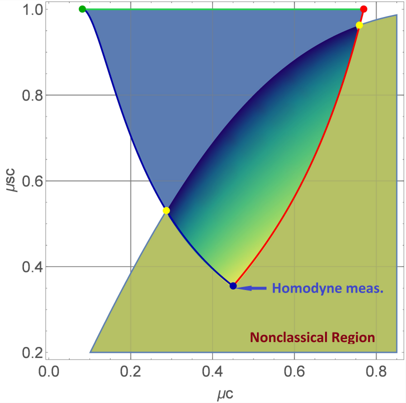


Figure 1: Gaussian triangoloid of a TMST, in parameter's space of conditional CM. μ_c is the conditional purity, μ_{sc} is related to conditional squeezing.

Nonclassical steering with TMST states

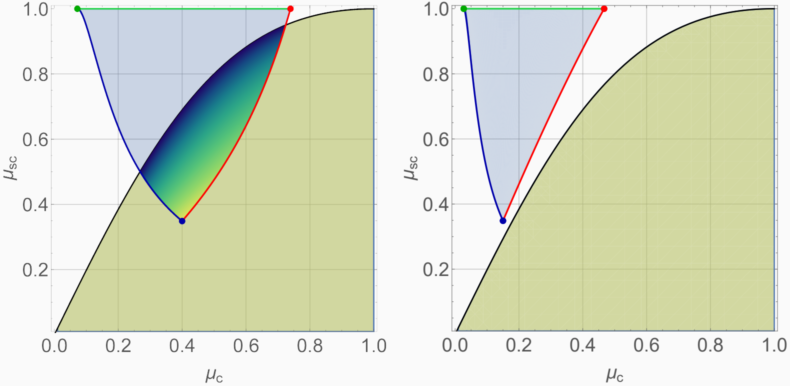


Figure 2: Triangoloids for TMST states with $N_A = N_B = 0.75$, $r = 1.2$ (left) and with $N_A = N_B = 2.8$, $r = 1.2$ (right)

Nonclassical steering with TMST states

- Necessary and sufficient condition (clearly asymmetric):

$$\sinh^2 r \; > \; \frac{N_A\,(1+2N_B)}{1+N_A+N_B}$$

- Entanglement is necessary but not sufficient

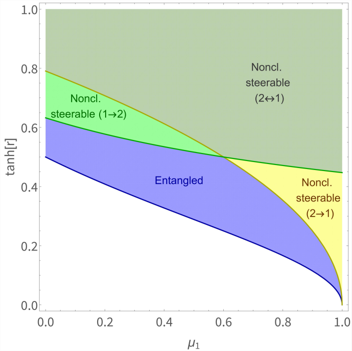


Figure 3: Slice of parameter's space of TMST states

Nonclassical steering with generic two-mode Gaussian state

Weak and strong nonclassical steering

$\hat{\rho}_{AB}$ is:

- **Weakly nonclassically steerable (WNS)** if it is possible to generate nonclassical conditional state with some Gaussian measurement

  Does not imply entanglement

  Possible with arbitrarily low (but nonzero) Gaussian Quantum Discord

- **Strongly nonclassically steerable (SNS)** if *any* quadrature measurement on Bob's mode will prepare a nonclassical conditional state of Alice's mode

  Implies EPR steerability, thus also entanglement

Back to TMSTs

- For **TMSTs**, weak and strong nonclassical steering are equivalent
- They also coincide with **EPR steering**
- This fact has a powerful implication: **nonclassicality** and **entanglement** are strongly related, at least for TMST states.

Conclusion

New quantum correlations

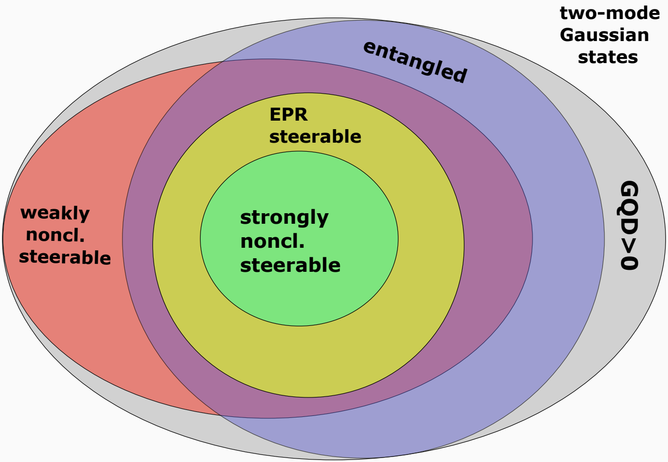


Figure 4: Quantum correlations for two-mode Gaussian states

M. Frigerio, S. Olivares, and M. G. A. Paris.
Nonclassical steering and the Gaussian steering triangoloids.
preprint, arXiv:2006.11912 [quant-ph].

R. Uola, A. C. S. Costa, H. C. Nguyen, and O. Gühne.
***Rev. Mod. Phys.*, 92:015001, 2020.**

H. M. Wiseman, S. J. Jones, and A. C. Doherty.
***Phys. Rev. Lett.*, 98:140402, 2007.**

Questions?

Backup slide 1: WNSN and SNS conditions

WNS
necessary and sufficient condition:

$$a - \frac{c^2}{b} \; < \; \frac{1}{2}\,, \qquad c = \max\{|c_1|, |c_2|\}\,.$$

SNS
necessary and sufficient condition:

$$a - \frac{c'^2}{b} \; < \; \frac{1}{2}\,, \qquad c' = \min\{|c_1|, |c_2|\}$$

EPR steerability
necessary and sufficient condition:

$$\left(a - \frac{c_1^2}{b}\right)\left(a - \frac{c_2^2}{b}\right) \; < \; \frac{1}{4}$$

Backup slides: Gaussian quantum states

CV quantum systems

- **Continuous-variable** (CV) quantum system of n **modes**
- Hilbert space: $\mathrm{L}^2(\mathbb{R}^n)$

$$[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk}\,,$$

$$\hat{x}_j = \frac{\hat{a}_j + \hat{a}_j^\dagger}{\sqrt{2}}\,, \qquad \hat{p}_j = \frac{\hat{a}_j - \hat{a}_j^\dagger}{i\sqrt{2}}\,, \qquad \hat{\boldsymbol{R}} = \begin{pmatrix} \hat{x}_1 \\ \hat{p}_1 \\ \vdots \\ \hat{x}_n \\ \hat{p}_n \end{pmatrix}$$

CV quantum systems

- $\hat{\boldsymbol{\rho}} \geq 0, \qquad \mathrm{Tr}[\hat{\boldsymbol{\rho}}] = 1$

- Characteristic and **Wigner** functions:

$$\chi\left[\hat{\boldsymbol{\rho}}\right](\Lambda) \;=\; \mathrm{Tr}\left[\hat{\boldsymbol{\rho}} \exp\left(-i\Lambda^T \hat{\mathbf{R}}\right)\right]$$

$$W[\hat{\boldsymbol{\rho}}](X) \;=\; \int_{\mathbb{R}^{2n}} \frac{\mathrm{d}^{2n}\Lambda}{(2\pi^2)^n}\, \chi\left[\hat{\boldsymbol{\rho}}\right](\Lambda)\, e^{i\Lambda^T X}$$

Gaussian quantum states

- $\hat{\rho}$ is **Gaussian** when:

$$W[\hat{\rho}](X) \;=\; \frac{1}{\pi^n \sqrt{\det[\boldsymbol{\sigma}]}} \exp\left[-\frac{1}{2}\left(X - \langle \hat{\boldsymbol{R}} \rangle\right)^T \boldsymbol{\sigma}^{-1} \left(X - \langle \hat{\boldsymbol{R}} \rangle\right)\right]$$

- **First-moments vector**:

$$\langle \hat{\boldsymbol{R}} \rangle \;=\; \mathrm{Tr}[\hat{\rho}\hat{\boldsymbol{R}}]$$

- **covariance matrix (CM)**:

$$[\boldsymbol{\sigma}]_{jk} = \frac{1}{2}\langle \hat{R}_j\hat{R}_k + \hat{R}_k\hat{R}_j\rangle \;-\; \langle \hat{R}_k\rangle\langle \hat{R}_k\rangle$$

Gaussian quantum states

- Similarly, a POVM $\{\hat{\Pi}_\alpha\}_{\alpha\in\mathbb{C}^n}$:

$$\forall\alpha \in \mathbb{C}^n :\; \hat{\Pi}_\alpha \geq 0\,, \qquad\qquad \int_{\mathbb{C}^n} \mathrm{d}^{2n}\alpha\; \hat{\Pi}_\alpha \;=\; \mathbb{I}$$

  is Gaussian if all $\hat{\Pi}_\alpha$ have Gaussian Wigner functions

- Uncertainty Relations:

$$\boldsymbol{\sigma} \;+\; \frac{i}{2}\Omega \;\geq\; 0$$

Backup slides: Nonclassicality

P-nonclassicality

- It can be quantified by **nonclassical depth**:

$$\mathfrak{T}[\hat{\rho}] = \frac{1 - s_{max}}{2}$$

- It has a **resource** character

- Number states ($n > 0$), Schrödinger cat's states and squeezed vacuum states are all nonclassical

- Coherent states are only classical pure states. Thermal states are also classical

Backup slide: covariance matrices

Conditional covariance matrix

- CM of initial state $\hat{\boldsymbol{\rho}}_{AB}$:

$$\boldsymbol{\sigma} = \begin{pmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{C}^T & \mathbf{B} \end{pmatrix}$$

- CM of POVM:

$$\boldsymbol{\sigma}_M = \frac{1}{2\mu\mu_s} \begin{pmatrix} 1 + \sqrt{1 - {\mu_s}^2}\cos\phi & -\sqrt{1 - {\mu_s}^2}\sin\phi \\ -\sqrt{1 - {\mu_s}^2}\sin\phi & 1 - \sqrt{1 - {\mu_s}^2}\cos\phi \end{pmatrix}$$

- CM of $\hat{\boldsymbol{\rho}}_A^{(\alpha)}$ (conditional CM) does not depend on outcome α:

$$\boldsymbol{\sigma}_A^c = \mathbf{A} - \mathbf{C}^T (\mathbf{B} + \boldsymbol{\sigma}_M)^{-1} \mathbf{C}$$

States in canonical form

$$\boldsymbol{\sigma} = \begin{pmatrix} a & 0 & c_1 & 0 \\ 0 & a & 0 & c_2 \\ c_1 & 0 & b & 0 \\ 0 & c_2 & 0 & b \end{pmatrix}$$

- Simplest generalization of TMSTs. $\hat{\boldsymbol{\rho}}_A$ and $\hat{\boldsymbol{\rho}}_B$ are still always classical

Backup slides: Noisy propagation of twin-beam states

TWB states

- Two-mode squeezed vacuum states:

$$|r\rangle\rangle = e^{r(\hat{a}^\dagger \hat{b}^\dagger - \hat{a}\hat{b})} |0\rangle_A \otimes |0\rangle_B$$

- Mode A interacts with a Markovian, purely thermal environment with average number of photons N_{th} and damping rate Γ

- The TWB state evolves into a generic TMST state

Triangoloid for TWB

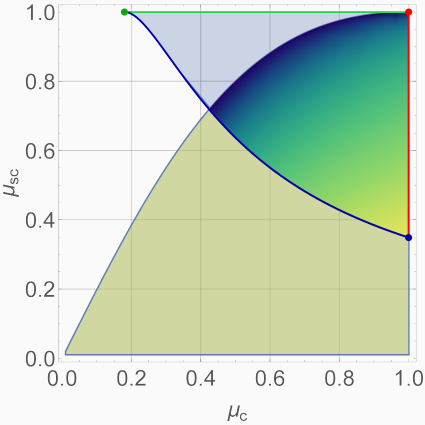


Figure 5: Triangoloid for TWB state with $r = 1.2$.

Noisy evolution of TWB triangoloid

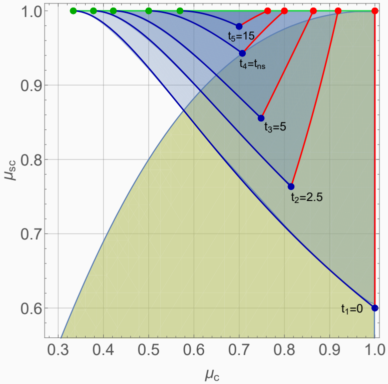


Figure 6: Snapshots of triangoloid evolution from initial TWB ($r = 1.2$)

Noisy evolution of TWB triangoloid

- Maximum propagation time for nonclassical steering:

$$t_{\text{ns}} \quad = \quad \frac{1}{\Gamma}\,\log\left[1+\frac{N_s}{N_{th}(1+2N_s)}\right]\,, \qquad\qquad (N_s=\sinh^2 r)$$

- Disentangling time:

$$t_{\text{ent}} \quad = \quad \frac{1}{\Gamma}\,\log\left(1+\frac{1}{N_{th}}\right)$$