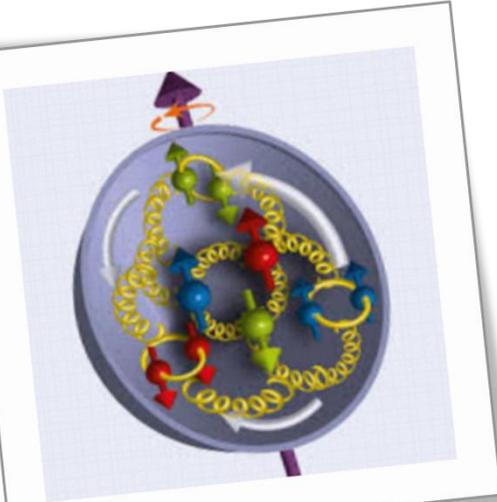
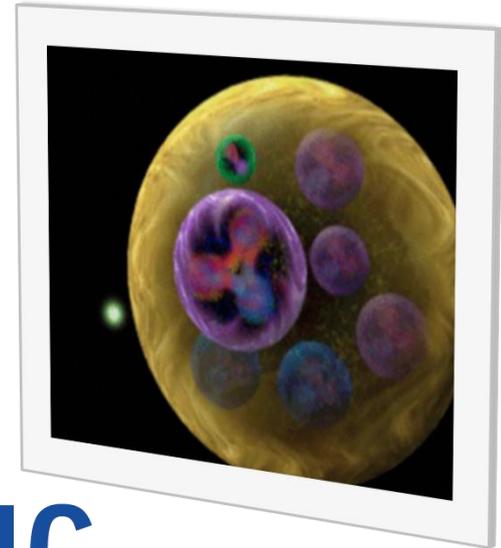


STRUTTURA 3D DI ADRONI LEGATI E LIBERI

Matteo Rinaldi

Perugia, Dipartimento di Fisica e Geologia
30/05/2020



Non siamo soli (anche se a distanza di sicurezza)



Il Gruppo di Fisica teorica Nucleare a Perugia:

Professore Associato: Sergio Scopetta
RTD-A: Matteo Rinaldi
Dottoranda: Sara Fucini

Il gruppo fa parte:
- NINPHA, commissione IV dell' INFN
- del progetto europeo STRONG2020

Colleghi con i quali abbiamo collaborato e ancora collaboriamo:

Roma: Giovanni Salmè, Emanuele Pace e A. Del Dotto

Valencia (Spagna): Santiago Noguera & Vicente Vento

Dubna (Russia): Leonid Kaptari

Liegi (Belgio), Haifa (Israele): Federico Alberto Ceccopieri

Mexico: Aurore Courtoy

Orsay Parigi (Francia): Raphael Duprè; Samuel Wallon, J. P. Lansberg

Trento: Marco Claudio Traini, F. Pederiva

Trieste: Daniele Treleani

Mainz (Germania): Tomas Kasemets

Pisa: Michele Viviani

Argonne NL, Chicago (USA): Kawtar Hafidi e Whitney Armstrong

Buenos Aires (Argentina): Daniel Gomez Dumm e Norberto Scoccola

Varsavia (Polonia): Lech Szimanowsky

Indice

- ◉ Introduzione
- ◉ Le distribuzioni partoniche generalizzate di nucleoni e nuclei leggeri
- ◉ Le distribuzioni partoniche doppie di protoni e mesoni
- ◉ Glueballs studiate con modelli olografici
- ◉ Conclusioni: prospettive e aggiornamenti

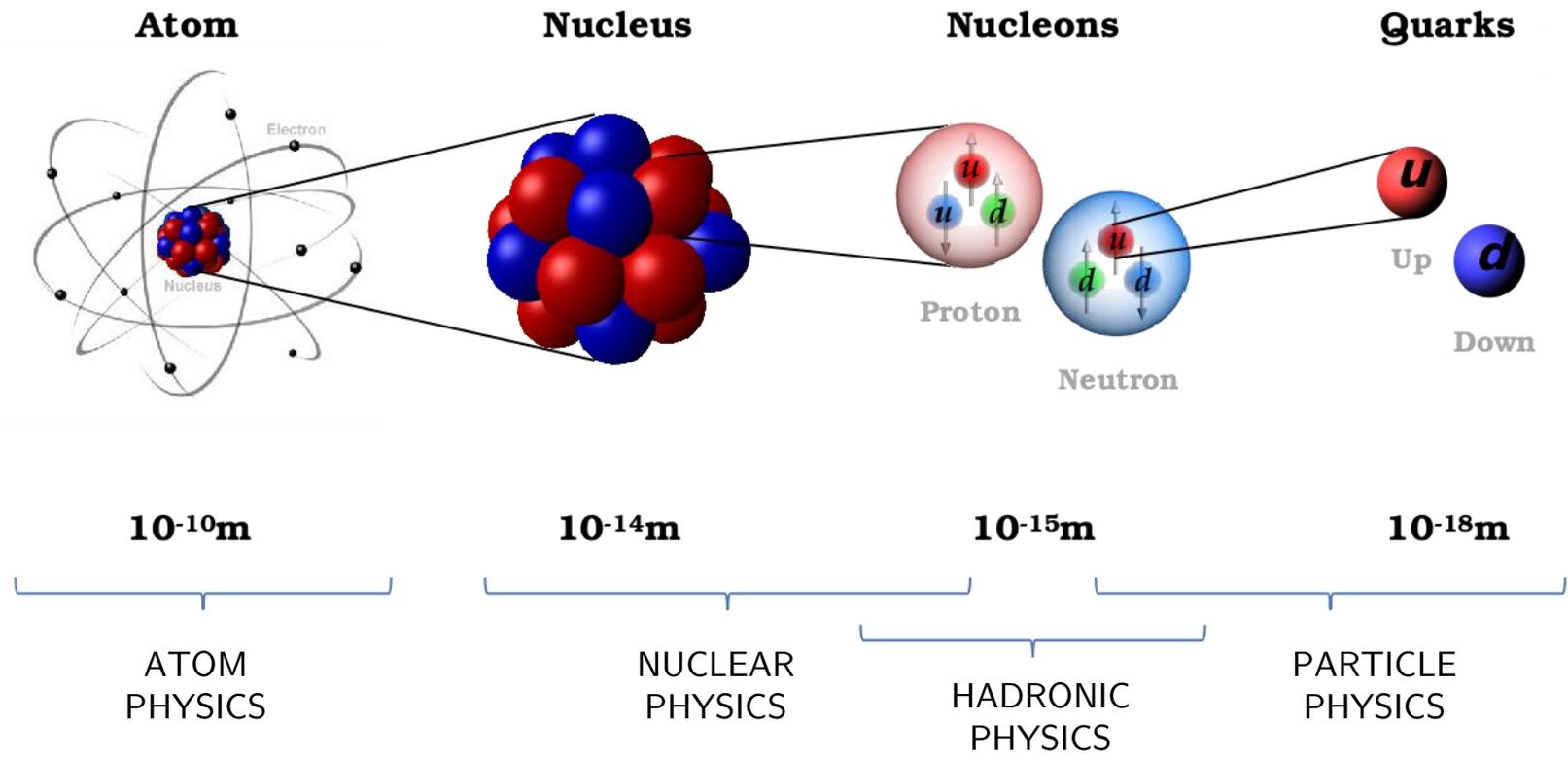
ARGOMENTI PRINCIPALI
DEI MIEI LAVORI

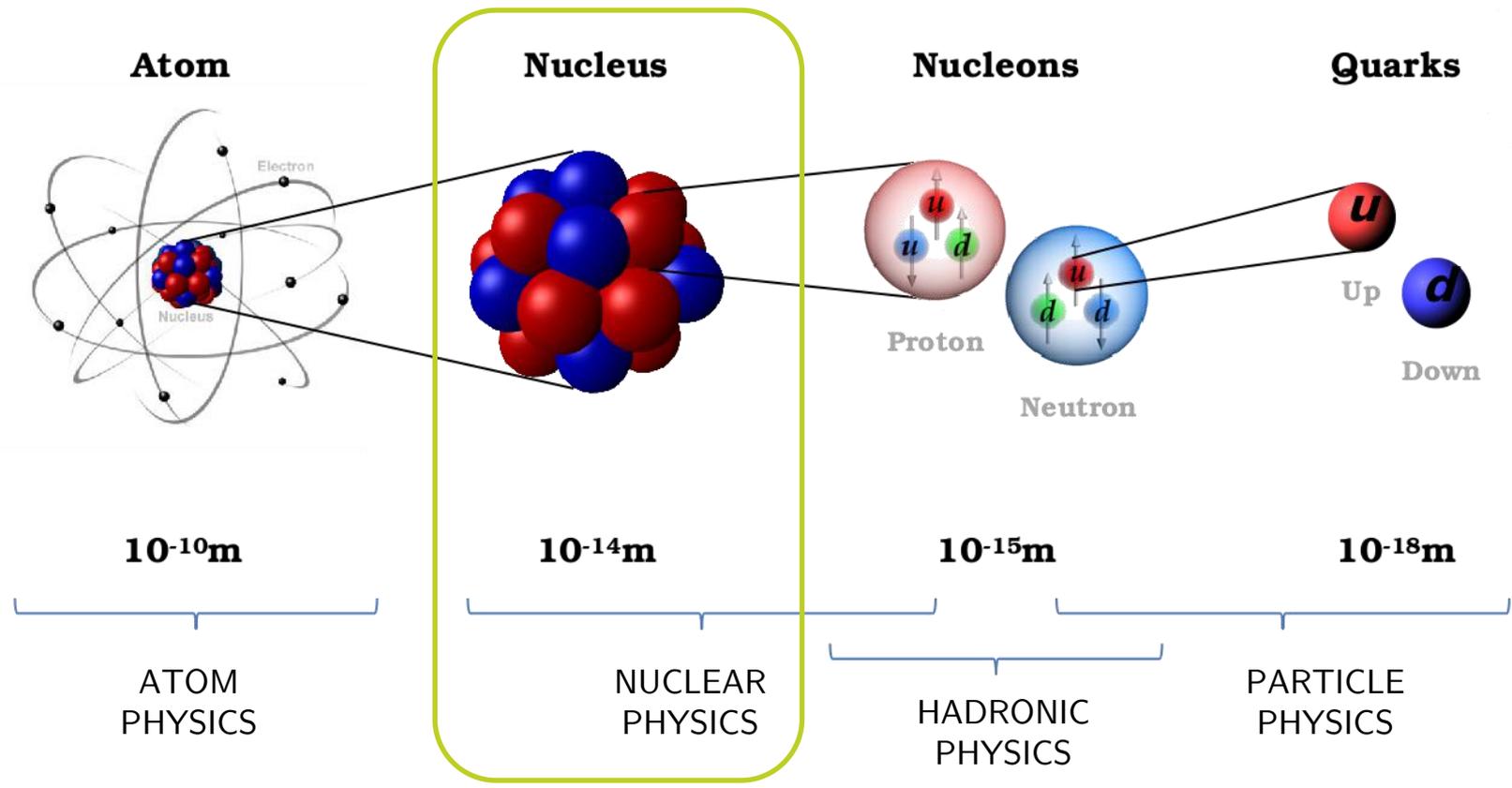


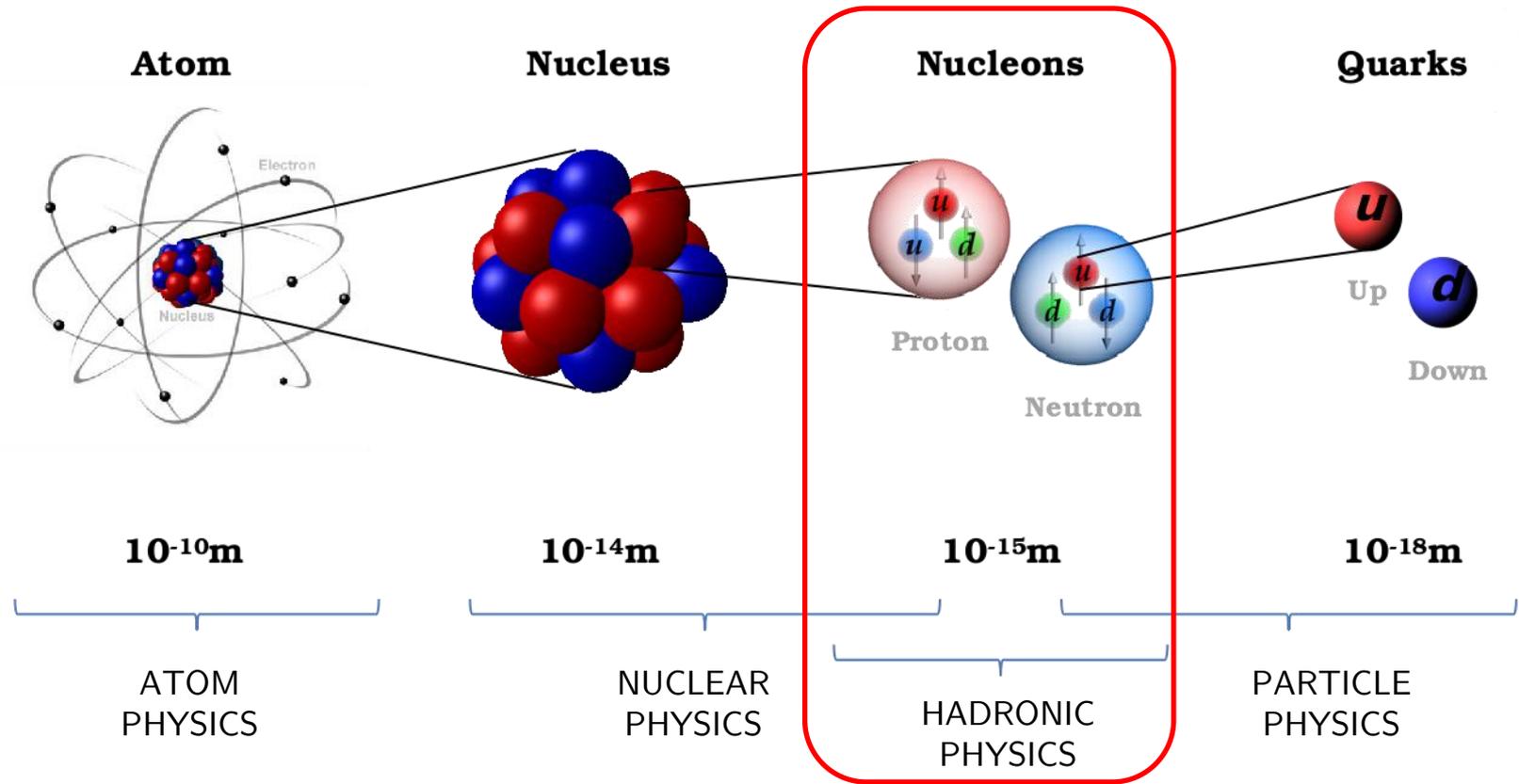


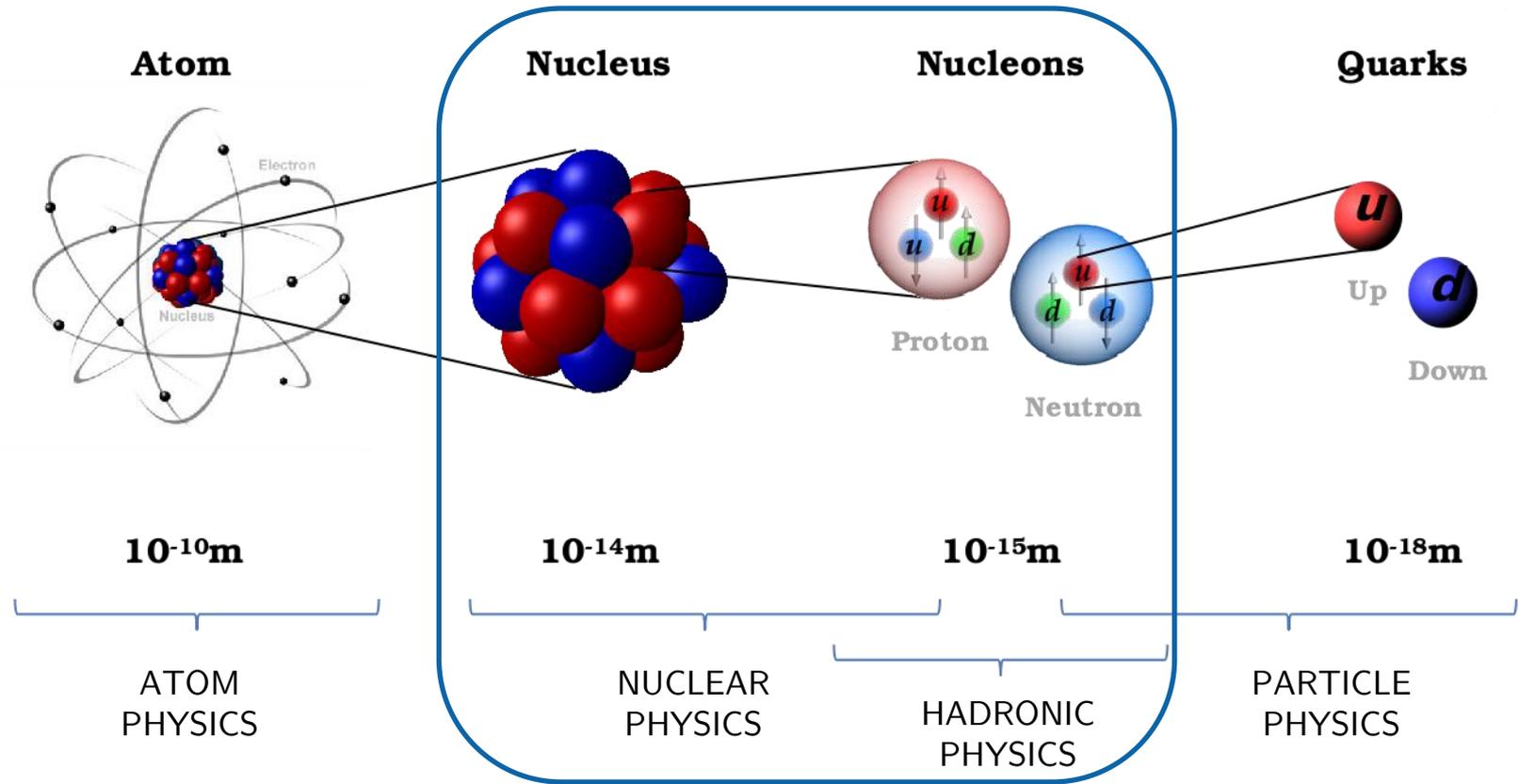
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INTRODUZIONE

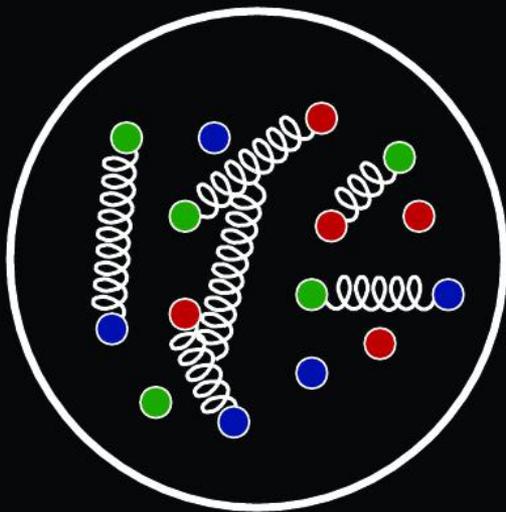








Obiettivo a lungo termine:
Capire la QCD e il confinamento



A. Bacchetta/INFN12

1 day, 13 November 12

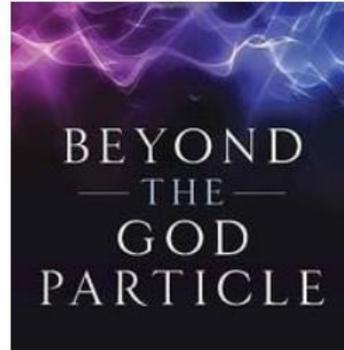
- ※ Massa di protoni & neutroni (nucleoni)
- ※ SPIN
- ※ Come cambia la struttura quando protoni e neutroni sono legati per formare i nuclei?

LA MASSA DEGLI ADRONI?

3

Origin of Mass

- LHC has NOT found the “God Particle” because the Higgs boson is NOT the origin of mass
 - Higgs-boson only produces a little bit of mass
 - Higgs-generated mass-scales explain neither the proton’s mass nor the pion’s (*near-*)masslessness
 - Hence LHC has, as yet, taught us very little about the origin, structure and nature of the nuclei whose existence support the Cosmos
- Strong interaction sector of the Standard Model, *i.e.* Quantum ChromoDynamics (QCD), is the key to understanding the origin, existence and properties of (almost) all known matter



STANDARD MODEL

$$\mathcal{L}_{SM} = \mathcal{L}_{Dirac} + \mathcal{L}_{mass} + \mathcal{L}_{gauge} + \mathcal{L}_{gauge/\psi} .$$

Here,

$$\mathcal{L}_{Dirac} = i\bar{e}_L^i \partial e_L^i + i\bar{\nu}_L^i \partial \nu_L^i + i\bar{e}_R^i \partial e_R^i + i\bar{u}_L^i \partial u_L^i + i\bar{d}_L^i \partial d_L^i + i\bar{u}_R^i \partial u_R^i + i\bar{d}_R^i \partial d_R^i ;$$

$$\mathcal{L}_{mass} = -v \left(\lambda_e^i \bar{e}_L^i e_R^i + \lambda_u^i \bar{u}_L^i u_R^i + \lambda_d^i \bar{d}_L^i d_R^i + \text{h.c.} \right) - M_W^2 W_\mu^+ W^{-\mu} - \frac{M_Z^2}{2 \cos^2 \theta_W} Z_\mu Z^\mu ;$$

$$\mathcal{L}_{gauge} = -\frac{1}{4} (G_{\mu\nu}^a)^2 - \frac{1}{2} W_\mu^+ W^{-\mu} - \frac{1}{4} Z_\mu Z^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{WZA} ,$$

where

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_3 f^{abc} A_\mu^b A_\nu^c \\ W_\mu^\pm &= \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \\ Z_\mu &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu , \end{aligned}$$

and

$$\begin{aligned} \mathcal{L}_{WZA} &= ig_2 \cos \theta_W \left[(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu Z^\nu + W_\mu^+ W^{-\mu} Z^\nu - W_\mu^- W^{+\mu} Z^\nu \right] \\ &+ ie \left[(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu A^\nu + W_\mu^+ W^{-\mu} A^\nu - W_\mu^- W^{+\mu} A^\nu \right] \\ &+ g_2^2 \cos^2 \theta_W (W_\mu^+ W_\nu^- Z^\mu Z^\nu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu) \\ &+ g_2^2 (W_\mu^+ W_\nu^- A^\mu A^\nu - W_\mu^+ W^{-\mu} A_\nu A^\nu) \\ &+ g_2 e \cos \theta_W [W_\mu^+ W_\nu^- (Z^\mu A^\nu + Z^\nu A^\mu) - 2W_\mu^+ W^{-\mu} Z_\nu A^\nu] \\ &+ \frac{1}{2} g_2^2 (W_\mu^+ W_\nu^-) (W^{+\mu} W^{-\nu} - W^{+\nu} W^{-\mu}) ; \end{aligned}$$

and

$$\mathcal{L}_{gauge/\psi} = -g_3 A_\mu^a J_{(3)}^{\mu a} - g_2 (W_\mu^+ J_{W^+}^\mu + W_\mu^- J_{W^-}^\mu + Z_\mu J_Z^\mu) - e A_\mu J_A^\mu ,$$

where

$$\begin{aligned} J_{(3)}^{\mu a} &= \bar{u}^i \gamma^\mu T_{(3)}^a u^i + \bar{d}^i \gamma^\mu T_{(3)}^a d^i \\ J_{W^+}^\mu &= \frac{1}{\sqrt{2}} (p_L^i \gamma^\mu e_L^i + V^{ij} \bar{u}_L^i \gamma^\mu d_L^j) \\ J_{W^-}^\mu &= (J_{W^+}^\mu)^* \\ J_Z^\mu &= \frac{1}{\cos \theta_W} \left[\frac{1}{2} p_L^i \gamma^\mu \nu_L^i + \left(-\frac{1}{2} + \sin^2 \theta_W \right) \bar{e}_L^i \gamma^\mu e_L^i + (\sin^2 \theta_W) \bar{e}_R^i \gamma^\mu e_R^i \right. \\ &+ \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \bar{u}_L^i \gamma^\mu u_L^i + \left(-\frac{2}{3} \sin^2 \theta_W \right) \bar{u}_R^i \gamma^\mu u_R^i \\ &+ \left. \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) \bar{d}_L^i \gamma^\mu d_L^i + \left(\frac{1}{3} \sin^2 \theta_W \right) \bar{d}_R^i \gamma^\mu d_R^i \right] \\ J_A^\mu &= (-1) \bar{e}^i \gamma^\mu e^i + \left(\frac{2}{3} \right) \bar{u}^i \gamma^\mu u^i + \left(-\frac{1}{3} \right) \bar{d}^i \gamma^\mu d^i . \end{aligned}$$

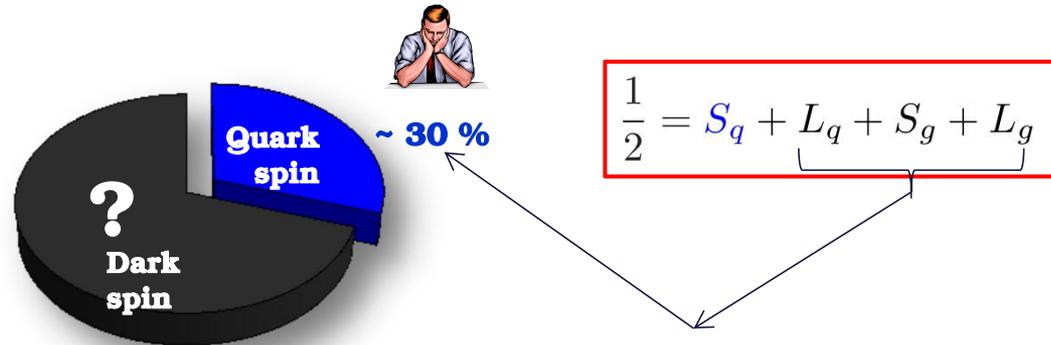
Espressione molto compatta di alcune interazioni fondamentali che governano la composizione della maggior parte della materia nell'universo!

Questo “piccolissimo termine”, che descrive la self-interaction tra i bosoni di gauge della QCD (gluoni) è responsabile per quasi il 98% della materia visibile!

Tuttavia, la QCD nel regime non perturbativo non è risolvibile. Dopo quasi 50 anni dalla scoperta dei quarks siamo ancora agli inizi per comprendere come dai quark e gluoni si “costruiscono” pioni, protoni, neutroni e nuclei!

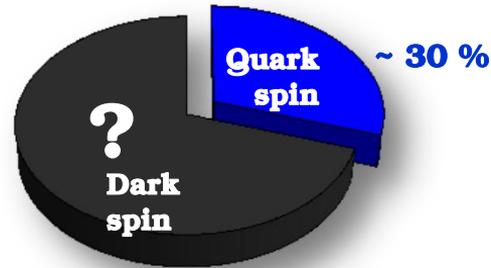
Per esempio, infatti, ancora oggi non è chiaro come lo SPIN del protone si ottiene a partire dai suoi costituenti (SPIN CRISIS)

LO SPIN DEL PROTONE: UN PUZZLE IRRISOLTO



Dai modelli ci si aspettava
che questi contributi fossero piccoli

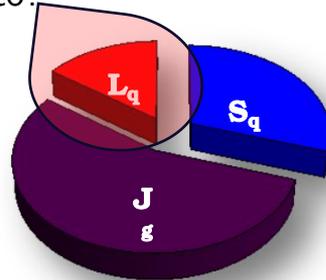
LO SPIN DEL PROTONE: UN PUZZLE IRRISOLTO



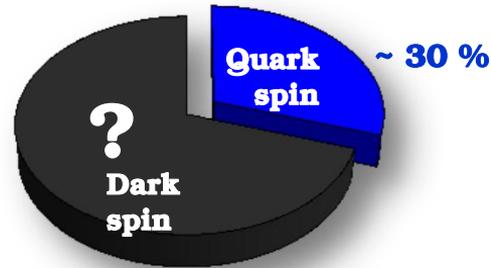
$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

L_q = Orbital Angular Momentum (OAM)

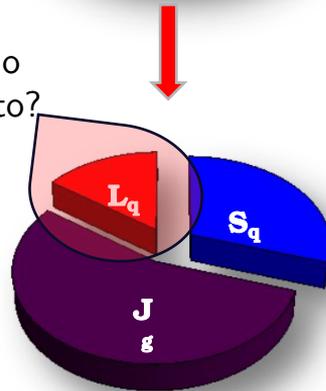
Come misuriamo
questo contributo?



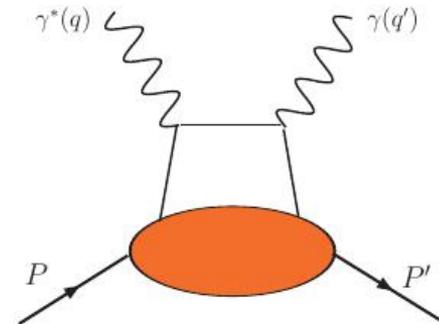
LO SPIN DEL PROTONE: UN PUZZLE IRRISOLTO



Come misuriamo questo contributo?



$$\frac{1}{2} = S_q + L_q + S_g + L_g$$



Deeply Virtual
Compton Scattering
DVCS

Protoni e Neutroni in 3D

6

IL NOSTRO APPROCCIO:

Ottenere delle
immagini della
struttura di protoni
e neutroni!



Protoni e Neutroni in 3D

Consideriamo la diffusione profondamente anelastica (DIS): $A(e, e')X$, se il bersaglio A ha spin $J_A = 1/2$, nel sistema del laboratorio (LAB) allora $q = (\nu, 0, 0, -q)$, Nel limite di Bjorken, $Q^2 = -q^2$, $\nu \rightarrow \infty$, allora il rapporto Q^2 / ν finito

$$\frac{d^2\sigma}{d\Omega dE'} \propto F_2(x) \simeq \sum_q e_q^2 x f_q(x)$$

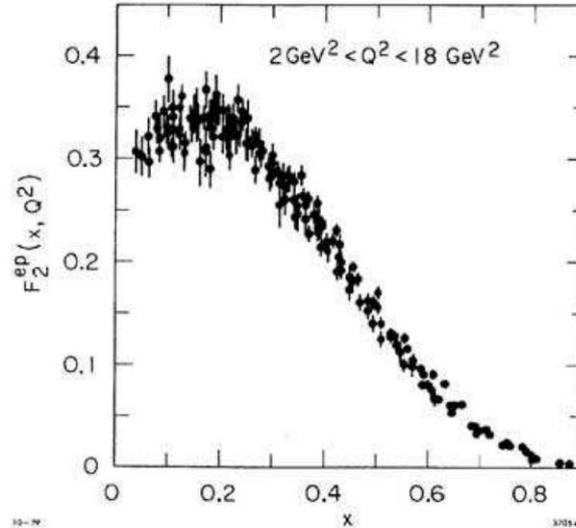
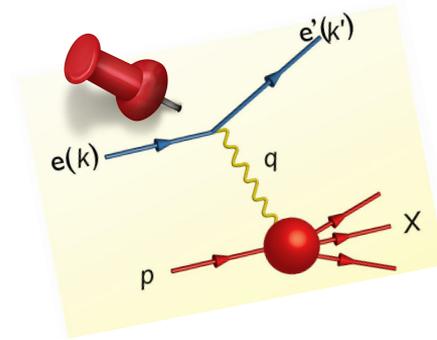
$F_2(x)$ = funzione di struttura

$f_q(x)$ = distribuzione partonica (PDF)

$x = \frac{Q^2}{2P_A \cdot q}$ è uno scalare:

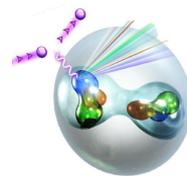
 $x = \frac{Q^2}{2M_A \nu}$ (LAB);

 x = frazione di momento del bersaglio portata dal quark. nell' *Infinite Momentum Frame* (IMF) ($p_z \rightarrow \infty$)



In generale, F_2 dipende da Q^2 . Nel limite di Bjorken, F_2 scala in x : **diffusione incoerente su costituenti puntiformi, i partoni** (Al LO in QCD, solo i quark contribuiscono ad F_2).

Un problema aperto: effetto EMC



8

Consideriamo un processo DIS su un nucleo A (EMC coll., CERN 1983) e studiamo il rapporto tra le sezioni d'urto per un nucleone legato in un nucleo e per un nucleone libero. Si vide:



Se il rapporto fosse 1, allora il nucleone libero e legato sarebbero UGUALI.

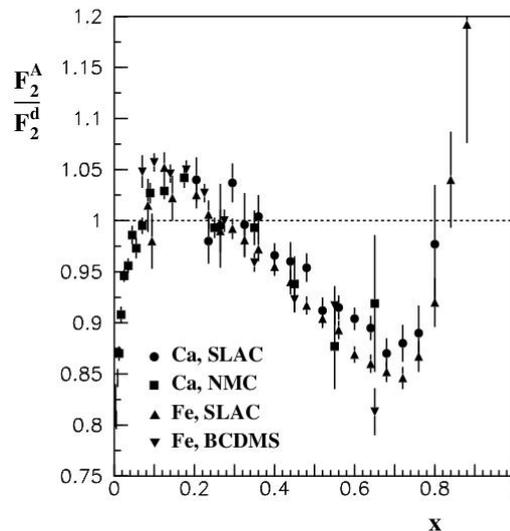
 **Il rapporto....non è 1**

Dopo tanti anni ancora non sappiamo perché.



Abbiamo tante ipotesi e per avere una risposta chiara servono nuovi esperimenti, come il DVCS

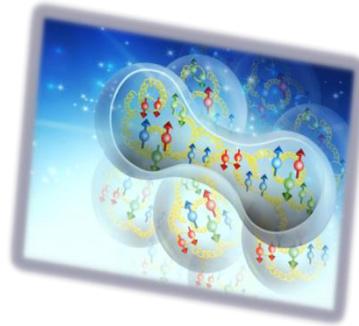
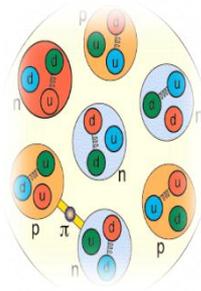
 Misure più difficili e descrizioni teoriche più complesse



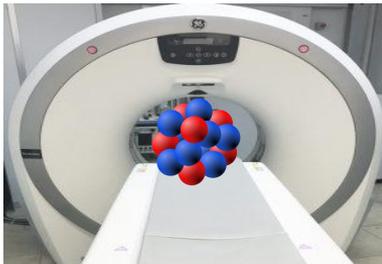
Effetto EMC: come ne usciamo?

9

Per rispondere al problema dell'effetto EMC, dobbiamo arrivare, essenzialmente, a capire a quale dei due spaccati i nuclei assomigliano:



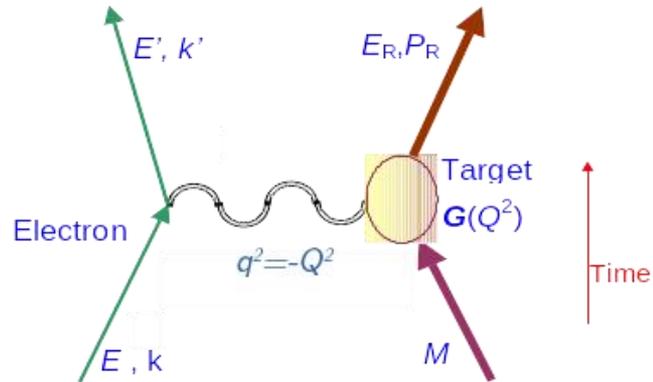
Per rispondere serve fargli una **TOMOGRAFIA**.



Si può fare! Possiamo studiare processi come: Deeply Virtual Compton Scattering (DVCS) e ottenere info riguardo le distribuzioni partoniche generalizzate (GPDs). Difficili misure ed analisi ma oggi possibile in vari laboratori!



ELASTIC processes

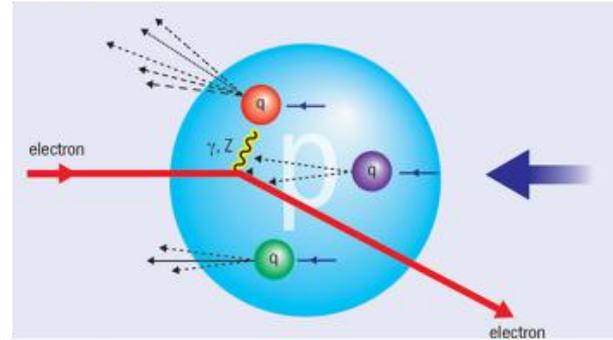


$$\langle N(P') | J_{EM}^\mu(0) | N(P) \rangle = \bar{u}(P') \left[\gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2M} F_2(Q^2) \right] u(P)$$

DIRAC
Form Factor

PAULI
Form Factor

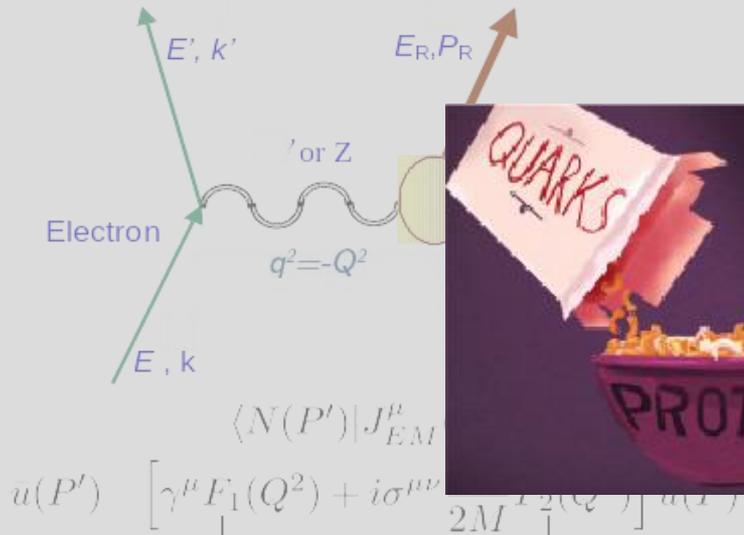
DEEP INELASTIC processes



$$f(x) = \int dz^- e^{ixP^+z^-} \langle N(P) | \bar{q} \left(\frac{-z^-}{2} \right) \gamma^+ q \left(\frac{z^-}{2} \right) | N(P) \rangle$$

Parton Distribution Function (PDF)
 $x =$ Longitudinal momentum fraction
carried by the parton

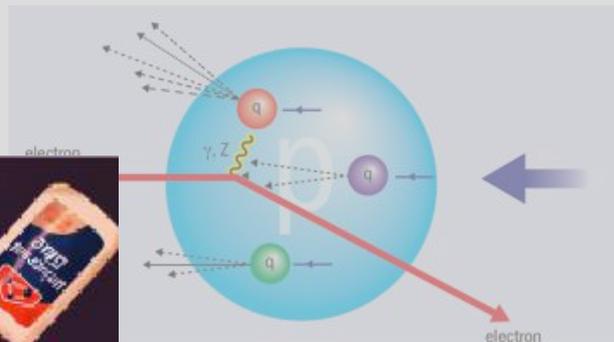
ELASTIC processes



DIRAC
Form Factor

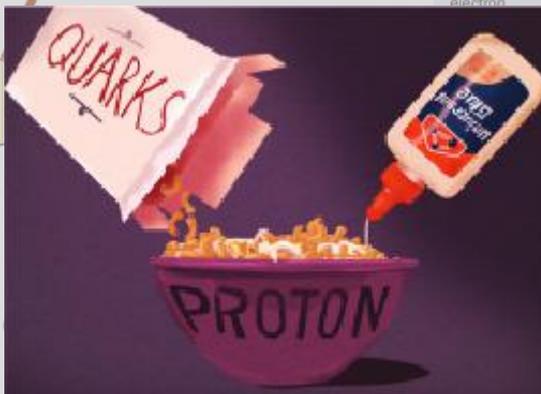
PAULI
Form Factor

DEEP INELASTIC processes

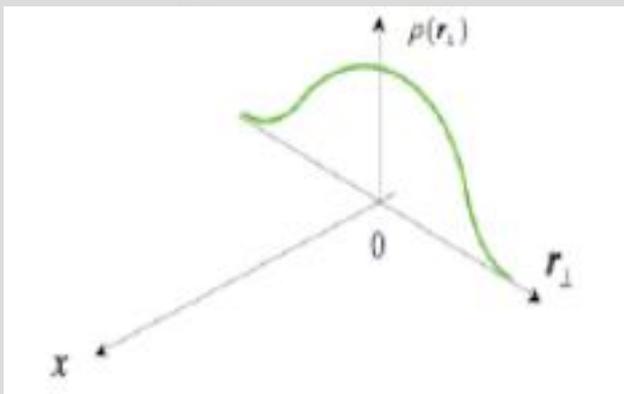


$$f(x) = \langle N(P) | \bar{q} \left(\frac{-z^-}{2} \right) \gamma^+ q \left(\frac{z^-}{2} \right) | N(P) \rangle$$

Parton Distribution Function (PDF)
 $x =$ Longitudinal momentum fraction carried by the parton

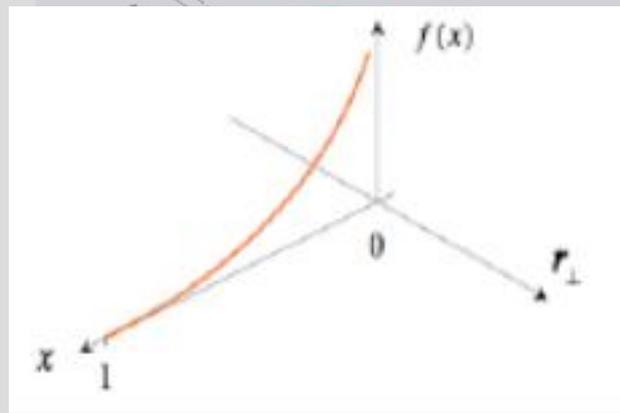


ELASTIC processes



$$\langle N(P') | J_{EM}^\mu(0) | N(P) \rangle = u(P') \left[\gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{Q^2} F_2(Q^2) \right] u(P)$$

DEEP INELASTIC processes



$$\int dz^- e^{ixP^+z^-} \langle N(P) | \bar{q} \left(\frac{-z^-}{2} \right) \gamma^+ q \left(\frac{z^-}{2} \right) | N(P) \rangle$$

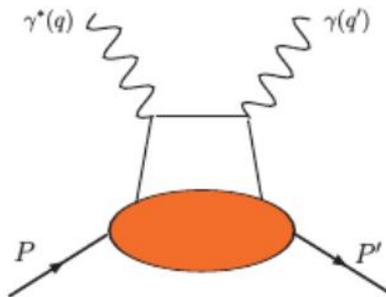
1-DIMENSIONAL INFORMATION!

DIRAC
Form Factor

PAULI
Form Factor

$x =$ Longitudinal momentum fraction
carried by the parton

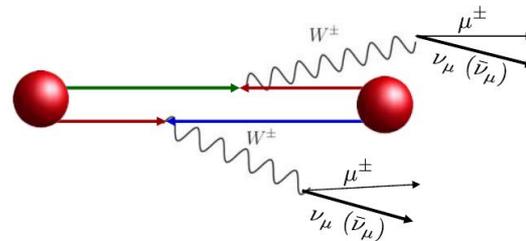
DEEPLY VIRTUAL COMPTON SCATTERING



$$\int dz^- e^{ixP^+z^-} \langle N(P') | \bar{q} \left(\frac{-z}{2} \right) \gamma^+ q \left(\frac{z}{2} \right) | N(P) \rangle = \bar{u}(P') \left[\gamma^t H(x, \xi, \Delta^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2M} E(x, \xi, \Delta^2) \right] u(P)$$

GENERALIZED PARTON DISTRIBUTION FUNCTIONS (GPDs)

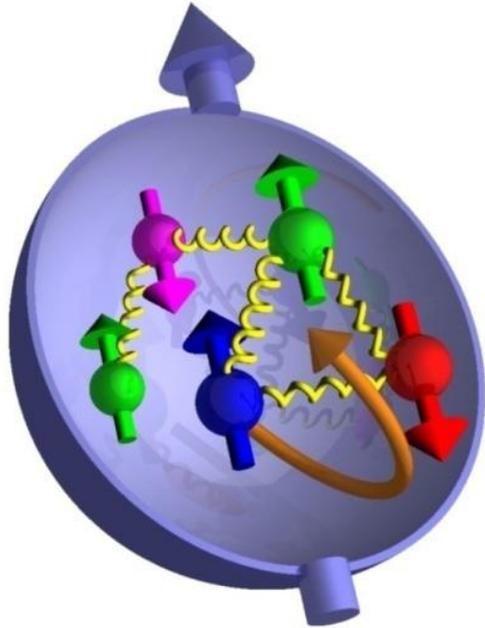
Double Parton Scattering



$$d\sigma = \frac{1}{S} \sum_{i,j,k,l} \hat{\sigma}_{ij}(x_1, x_3, \mu_A) \hat{\sigma}_{kl}(x_2, x_4, \mu_B) \int d^2z_\perp \mathbf{F}_{ik}(x_1, x_2, z_\perp, \mu_A, \mu_B) \mathbf{F}_{jl}(x_3, x_4, z_\perp, \mu_A, \mu_B)$$

dPDF
 Momentum fraction carried by the parton inside the hadron
 Transverse distance between the two partons
 Momentum scale

dPDF= double parton distribution functions



Da molti processi si ottengono un gran numero di distribuzioni, ognuna che fornisce informazioni diverse sulla struttura non-perturbativa di un adrone

Lo zoo delle distribuzioni

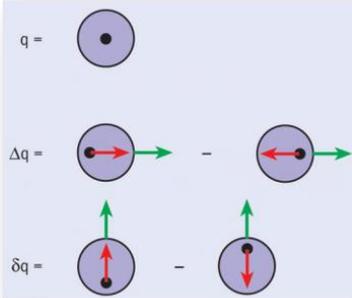
12

$$\frac{1}{2P^+} \langle p^+, \vec{0}_\perp, \Lambda' | \bar{\psi}(0) \Gamma \psi(0) | p^+, \vec{0}_\perp, \Lambda \rangle$$

Depends on :

$\Lambda, \Lambda', \Gamma$ • **Polarization**

Vector
Parton number



Axial
Parton helicity

Tensor
Parton
transversity

•
Charges

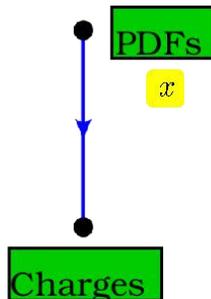
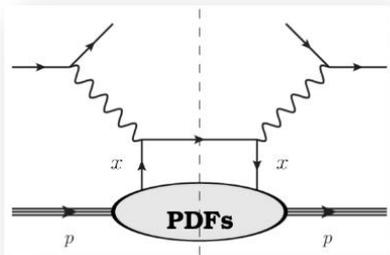
Lo zoo delle distribuzioni

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik^+ z^-} \langle p^+, \vec{0}_\perp, \Lambda' | \bar{\psi}(-\frac{z^-}{2}) \Gamma \mathcal{W} \psi(\frac{z^-}{2}) | p^+, \vec{0}_\perp, \Lambda \rangle$$

Depends on :

- $\Lambda, \Lambda', \Gamma$ • **Polarization**
- $x = \frac{k^+}{P^+}$ • **Longitudinal momentum (fraction)**

DIS



$\rightarrow \int dx$

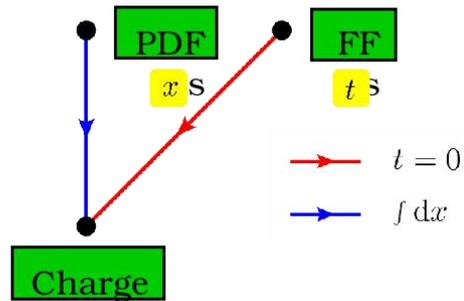
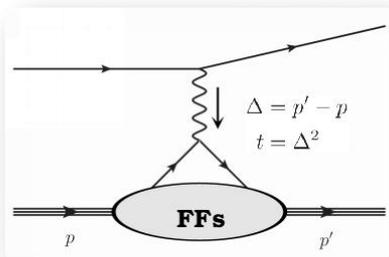
Lo zoo delle distribuzioni

$$\frac{1}{2P^+} \langle p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(0) \Gamma \psi(0) | p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle$$

Depends on :

- $\Lambda, \Lambda', \Gamma$ • **Polarization**
- Δ • **Longitudinal momentum (fraction)**
- **Momentum transfer**

Elastic scattering



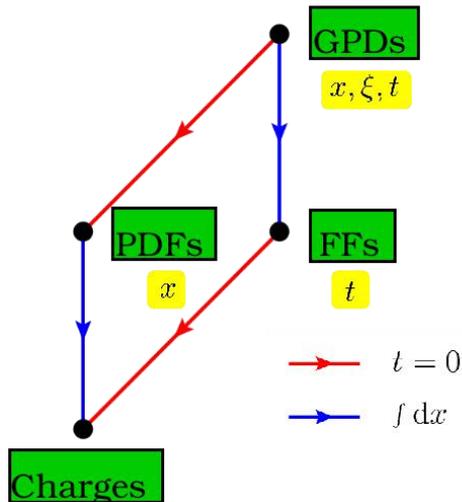
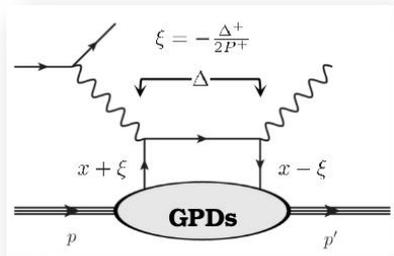
Lo zoo delle distribuzioni

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik^+ z^-} \langle p'^+, -\frac{\Delta_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z^-}{2}) \Gamma \mathcal{W} \psi(\frac{z^-}{2}) | p^+, \frac{\Delta_\perp}{2}, \Lambda \rangle$$

Depends on :

- $\Lambda, \Lambda', \Gamma$ • **Polarization**
- $x = \frac{k^+}{P^+}$ • **Longitudinal momentum (fraction)**
- Δ • **Momentum transfer**

DVCS



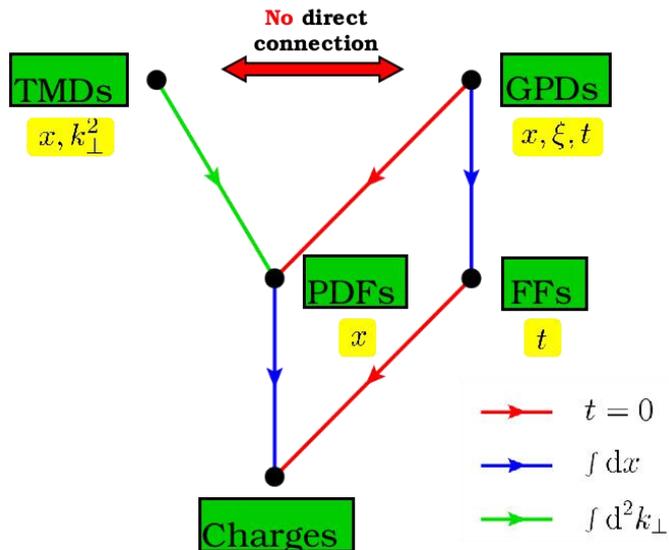
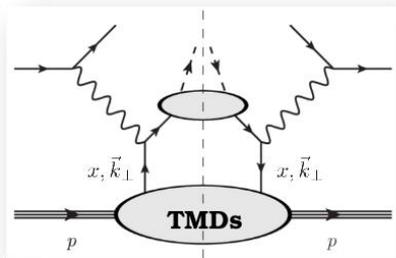
Lo zoo delle distribuzioni

$$\frac{1}{2} \int \frac{dz^- d^2z_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p^+, \vec{0}_\perp, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, \vec{0}_\perp, \Lambda \rangle \Big|_{z^+=0}$$

Depends on :

- $\Lambda, \Lambda', \Gamma$ • **Polarization**
- $x = \frac{k^+}{P^+}$ • **Longitudinal momentum (fraction)**
- \vec{k}_\perp • **Momentum transfer**
- **Transverse momentum**

SIDIS



Lo zoo delle distribuzioni

$$\frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p'^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle \Big|_{z^+=0}$$

Depends on :

- Polarization
- Longitudinal momentum (fraction)
- Momentum transfer
- Transverse momentum

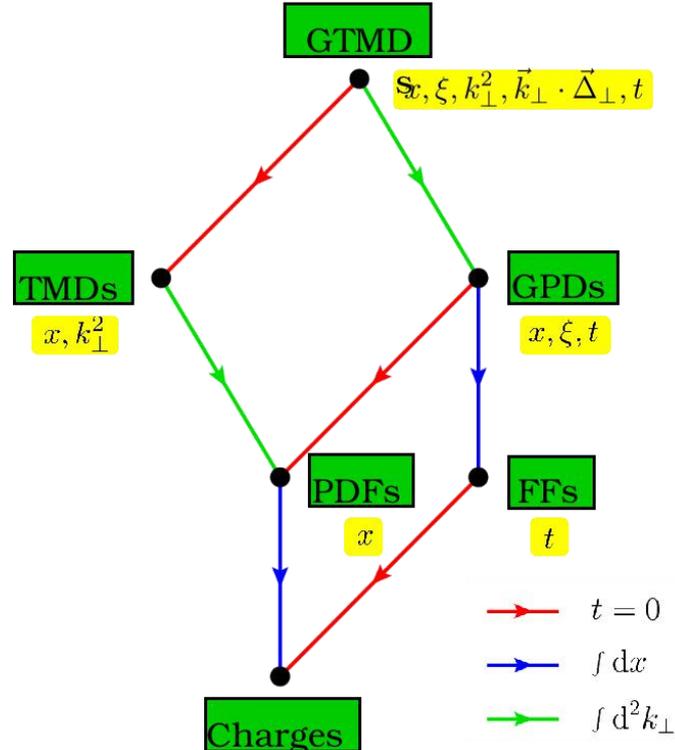
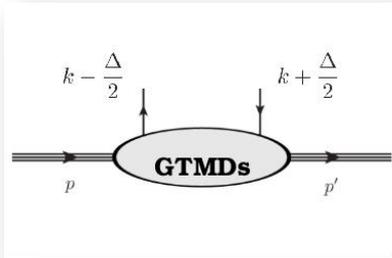
$\Lambda, \Lambda', \Gamma$

$x = \frac{k^+}{P^+}$

Δ

\vec{k}_\perp

???



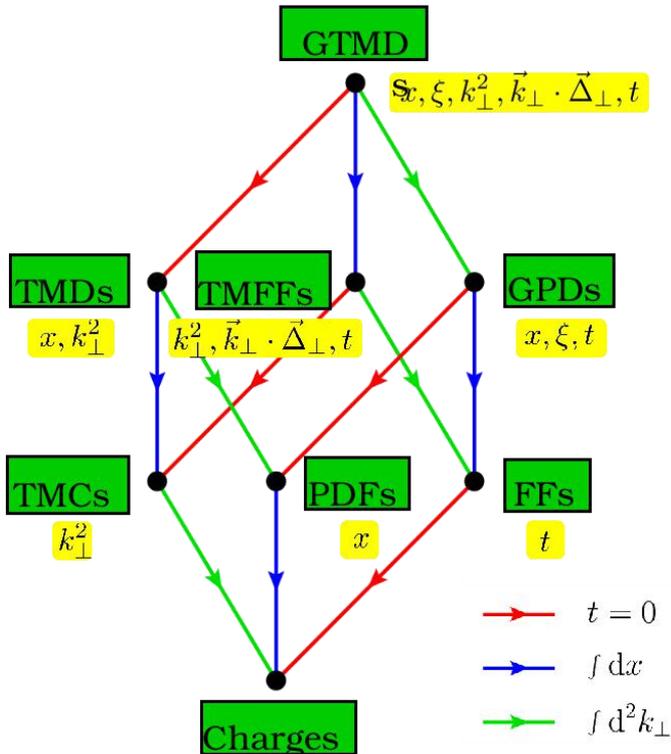
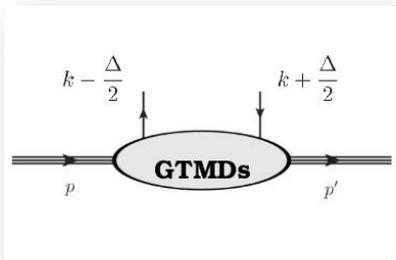
Lo zoo delle distribuzioni

$$\frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p'^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle \Big|_{z^+=0}$$

Depends on :

- $\Lambda, \Lambda', \Gamma$ • **Polarization**
- $x = \frac{k^+}{P^+}$ • **Longitudinal momentum (fraction)**
- Δ • **Momentum transfer**
- \vec{k}_\perp • **Transverse momentum**

???



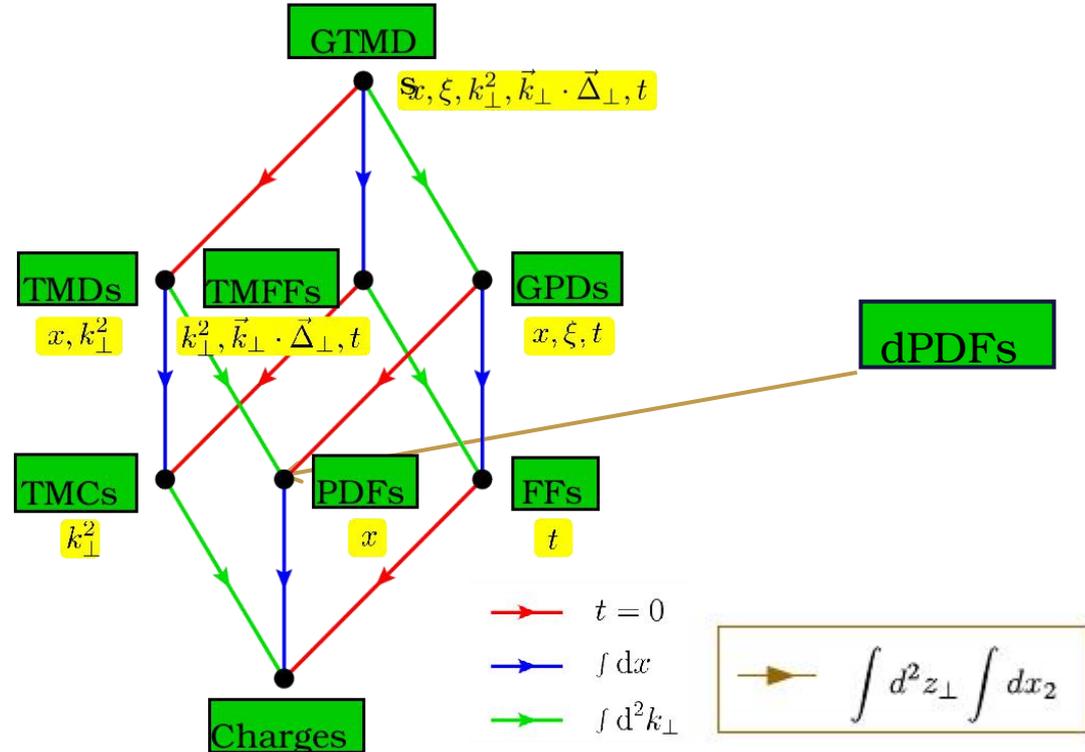
- $t = 0$
- $\int dx$
- $\int d^2 k_\perp$

Lo zoo delle distribuzioni

$$\frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle \Big|_{z^+=0}$$

Depends on :

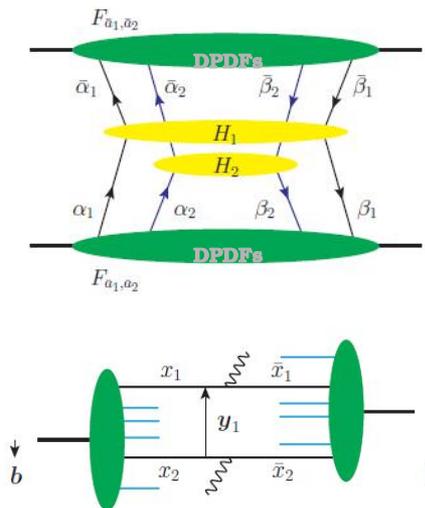
- $\Lambda, \Lambda', \Gamma$ • **Polarization**
- $x = \frac{k^+}{P^+}$ • **Longitudinal momentum (fraction)**
- Δ • **Momentum transfer**
- \vec{k}_\perp • **Transverse momentum**
- Z_\perp • **Transverse distance**



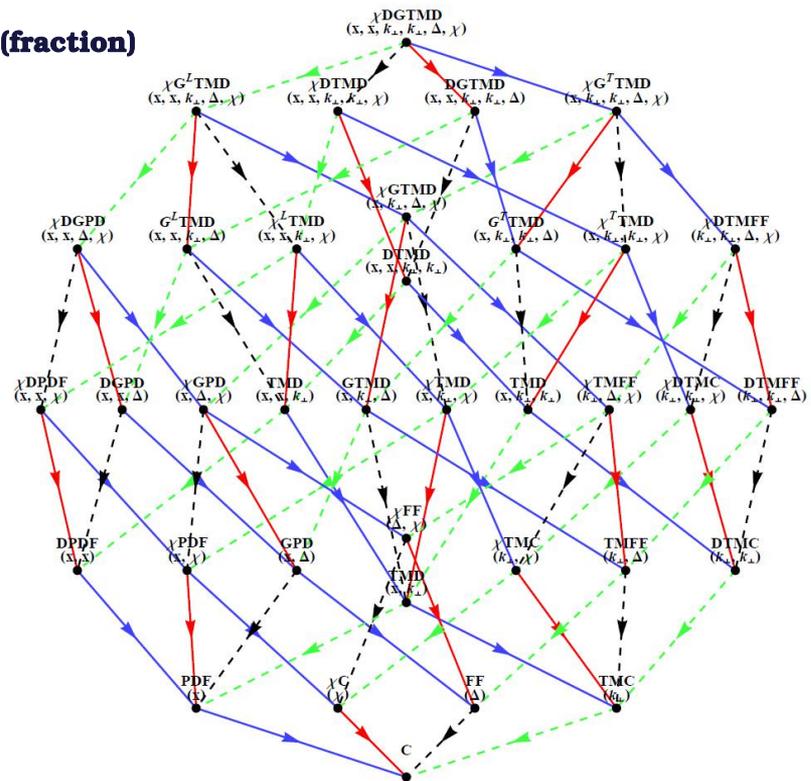
Lo zoo delle distribuzioni

Depends on :

- $\Lambda, \Lambda', \Gamma$ • **Polarization**
- $x = \frac{k^+}{P^+}$ • **Longitudinal momentum (fraction)**
- Δ • **Momentum transfer**
- \vec{k}_\perp • **Transverse momentum**
- \vec{y}_\perp • **Inter-parton distance**



[Diehl, Ostermeier, Schäfer (2012)]

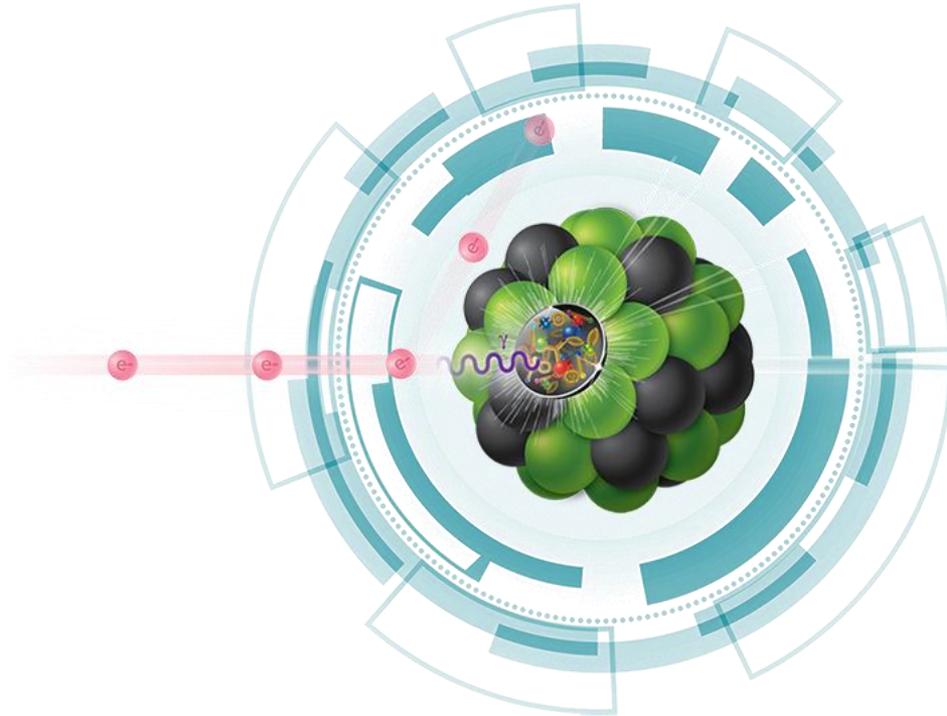


[Thürman, Master thesis (2012)]

Electron Ion Collider (EIC)

13

“A machine that will unlock the secrets of the strongest force in Nature”



Electron Ion Collider (EIC)

“A machine that will unlock the secrets of the strongest force in Nature”



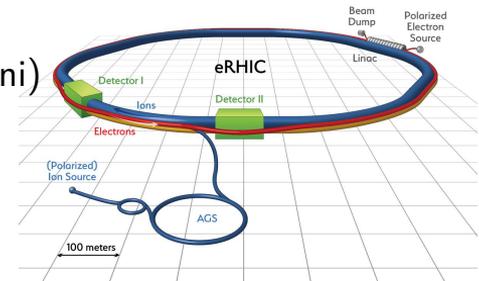
Nel prossimo decennio, l'unico acceleratore attivo negli USA sarà l' EIC. Servirà per capire la QCD nella sua anima non perturbativa: adronizzazione, confinamento... passi fondamentali per la ricerca di nuova Fisica!

Oltre 2 miliardi di dollari di investimento

1055 users, 216 istituzioni (PG rappresentata da me, Sergio Scopetta e Sara Fucini)

La partecipazione italiana è la più consistente in Europa. Inoltre, l' EIC è considerato un esempio di iniziativa extra-Europea supportata anche dal CERN.

L'EIC è il luogo naturale dove si faranno esperimenti che riguardano la linea di ricerca del gruppo teorico nucleare di Perugia. In quest'ultimo periodo sono aumentati notevolmente inviti e richieste di calcoli. È il momento per proporre misure. Il nostro gruppo è coinvolto nella stesura dello “Yellow report” dove si raccolgono queste idee! DOBBIAMO CONTINUARE A LAVORARE IN QUESTA DIREZIONE e ci sono molti temi caldi da studiare!





2

LE GPDs DI NUCLEONI E NUCLEI LEGGERI

Lavori ed articoli prodotti:

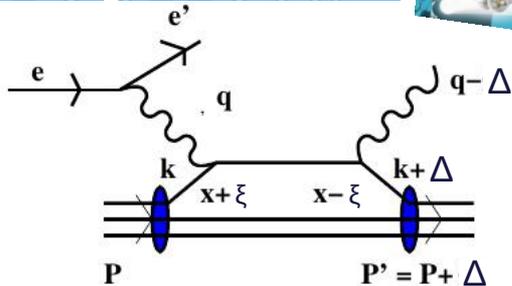
- Tesi di laurea magistrale
- M. R. and S. Scopetta, PRC 87 (2013) no.3, 035208
- M. R. and S. Scopetta, PRC 85 (2012) no.3, 062201
- M. R., PLB 771 (2017), 563-567

GPDs definizione e proprietà



For a $J = \frac{1}{2}$ target, in a hard-exclusive process, ($Q^2, \nu \rightarrow \infty$, Q^2/ν finite) such as DVCS:

- * $\Delta = P' - P$, $q^\mu = (q_0, \vec{q})$, and $\bar{P} = (P + P')^\mu / 2$
- * $x = k^+ / P^+$; $\xi = \text{"skewness"} = -\Delta^+ / (2\bar{P}^+)$



GPDs $H_q(x, \xi, \Delta^2)$ and $E_q(x, \xi, \Delta^2)$ are introduced:

$$F_{H',H}^{\mu,q}(x, \Delta^2, \xi) = \int \frac{dz^-}{2\pi} e^{ixz^- - P^+ z^-} \langle P' H' | \bar{\psi}_q \left(-\frac{z}{2} \right) \gamma^\mu \psi_q \left(\frac{z}{2} \right) | P H \rangle |_{z^+ = z_\perp = 0} =$$

$$= \frac{1}{2P^+} \left[H^q(x, \Delta^2, \xi) \bar{U}' \gamma^\mu U + E^q(x, \Delta^2, \xi) \bar{U}' \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} U \right] + \dots$$

- $P' = P$, i.e., $\Delta^2 = \xi = 0$, one recovers the usual PDFs:
 $H^q(x, 0, 0) = q(x)$; $E^q(x, 0, 0)$ unknown

- the x -integration yields Form Factors (F.F.)

$$\int dx \tilde{G}_M^q(x, \Delta^2, \xi) = G_M^q(\Delta^2) \text{ where here: } \tilde{G}_M^q = H^q + E^q$$

Ji Sum Rule

$$\langle J_{q,g}^z \rangle = \frac{1}{2} \int_{-1}^1 dx x [H^{q,g}(x, 0, 0) + E^{q,g}(x, 0, 0)]$$

\Rightarrow

Access

to

OAM

GPDs Nucleari: ^3He



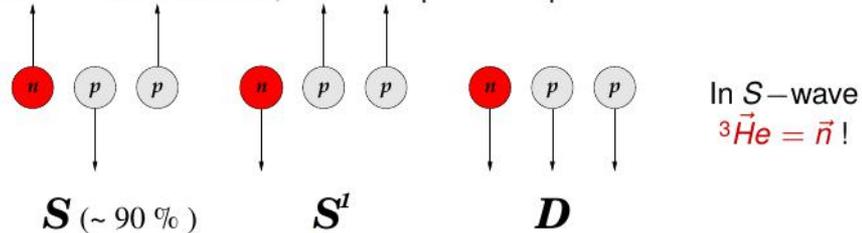
16

Nuclear targets are crucial to obtain:

- * *the nuclear short range structure, at quark level*, can be accessed and the reaction mechanism of DIS off nuclei, e.g. the *validity of I.A.* and the relevance of *effects beyond it* (non nucleonic degrees of freedom, nucleon modifications...) can be investigated... origin of the EMC effect...;
- * information on **the Neutron**.

To this aim ^3He is an ideal nucleus (**and we are proposing an experiment at JLab**):

- ^3He is **theoretically well known**. Even a **relativistic treatment** may be implemented;
- ^3He has been used extensively as an **effective neutron target**, especially to unveil the **spin content** of the **free neutron**, due to its peculiar spin structure



To what extent for **OAM** and \tilde{G}_M^q ? The answer here.

- To this aim, ^3He is a **unique** target:
 - * ^2H has a very small GPD E_q , crucial to access **OAM**;
 - * ^4He is scalar and has no E_q GPD;
 - * **heavier targets do not allow refined theoretical treatments.**

GPDs Nucleari: ^3He

coherent DVCS in I.A.

(^3He does not break-up $\Delta^2 \ll M^2, \xi^2 \ll 1$):

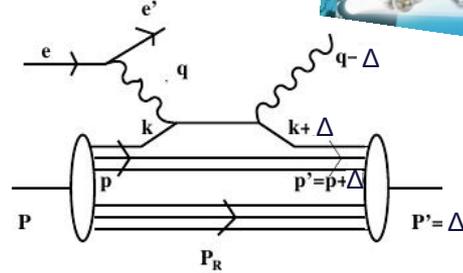
In a symmetric frame $\vec{P} = (P' + P)/2, a^\pm = a^0 \pm a^3$

$$k^+ = (x + \xi)\vec{P} = (x' + \xi')\vec{P}, \quad \xi' = -\frac{\Delta^+}{2\vec{P}}$$

$$k'^+ = (x - \xi)\vec{P} = (x' - \xi')\vec{P}, \quad x' = \frac{\xi'}{\xi}x$$

Impulse Approximation

- * the nucleus, A, is described by A - 1 interacting nucleons and an **off-shell** free nucleon, a kinematical condition;
- * the virtual photon interacts **only** with the **off-shell** nucleon;



$$F_{S,S'}^{3,q,\mu}(x, \Delta^2, \xi) = \int \frac{dz^-}{4\pi} e^{ix\vec{P}^+z^-} {}_3\langle P' S' | \prod_{\beta} |\alpha_{\beta}\rangle \langle \alpha_{\beta}| \hat{O}_q^{\mu} \prod_{\beta'} |\alpha'_{\beta'}\rangle \langle \alpha'_{\beta'}| | PS \rangle_3 |_{z^+=0, z_{\perp}=0} \cdot$$

where here: $|\alpha_{\beta}\rangle = |\vec{P}_R S_R\rangle |\vec{t} s_t\rangle |\vec{p} s\rangle \Rightarrow \beta = \vec{P}_R, S_R, \vec{t}, s_t, \vec{p}, s$

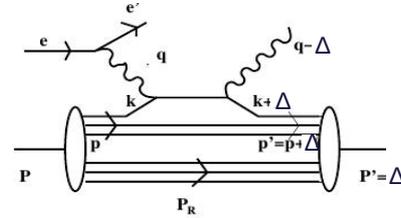
2-body state in CM

2-body intrinsic state

Nucleon state

$$\langle \vec{P}_R S_R | \langle \vec{t} s_t | \langle \vec{p} s | \vec{P} S \rangle = \langle \vec{p} s, \vec{t} s_t | \vec{P} S \rangle (2\pi)^3 \delta^3(\vec{P} - \vec{P}_R - \vec{p}) \delta_{S,S_R,s,s_t}$$

$$\hat{O}_q^{\mu} = \bar{\Psi}_q(0, -\frac{z^-}{2}, 0) \gamma^{\mu} \Psi_q(0, \frac{z^-}{2}, 0) \quad \text{“One body operator acting on the nucleon state”}$$



H_q^A , with the correct behavior, can be obtained in terms of H_q^N :

$$H_q^A(x, \xi, \Delta^2) = \sum_N \int dE \int d\vec{p} \sum_S \sum_s P_{SS,ss}^N(\vec{p}, \vec{p}', E) \frac{\xi'}{\xi} H_q^N(x', \Delta^2, \xi')$$

Nucleonic input: GPDs
Radyushkin PRD 61, 074027 (2000)

and $\tilde{G}_M^{3,q}$ in terms of $\tilde{G}_M^{N,q}$ (M. R. S. Scopetta PRC 85, 062201(R) (2012)):

$$\tilde{G}_M^{3,q}(x, \Delta^2, \xi) = \sum_N \int dE \int d\vec{p} [P_{+-,+}^N - P_{-+,-}^N](\vec{p}, \vec{p}', E) \frac{\xi'}{\xi} \tilde{G}_M^{N,q}(x', \Delta^2, \xi'),$$

where $P_{SS,ss}^N(\vec{p}, \vec{p}', E)$ is the one-body, spin-dependent, off-diagonal spectral function for the nucleon N in the nucleus,

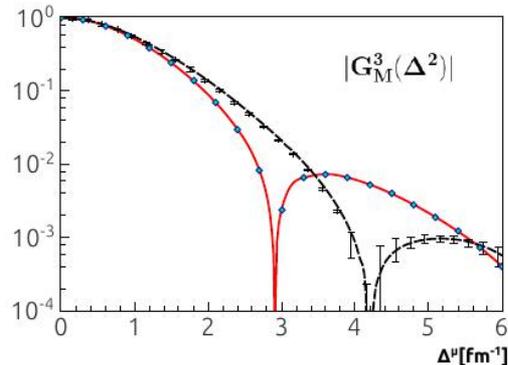
$$P_{SS',ss'}^N(\vec{p}, \vec{p}', E) = \frac{M\sqrt{ME}}{2(2\pi)^6} \int d\Omega_t \sum_{s_t} \langle \vec{p}' s' | \vec{p} s' | \vec{s}_t \rangle^N \langle \vec{p} s, \vec{s}_t | \vec{P} S \rangle^N$$

Nuclear input: Overlap
A. Kievsky et. al, PRC 56, 64 (1997)

For $\tilde{G}_M^{3,q}$ the only possible check is the magnetic F.F.:

$$\sum_q \int dx \tilde{G}_M^{3,q}(x, \Delta^2, \xi) = G_M^3(\Delta^2); \Delta^\mu = \sqrt{-\Delta^2}$$

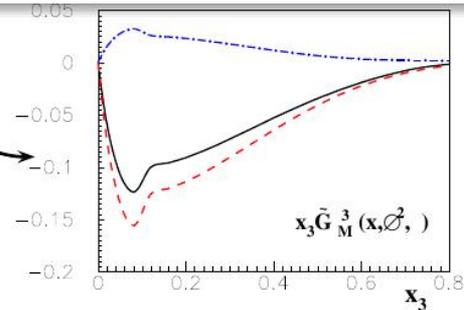
- * in perfect agreement with previous IA, Av18 calculations (L.E. Marcucci et al. PRC 58 (1998))
- * in good agreement with data in the region relevant to the coherent process, $-\Delta^2 \ll 0.15 \text{ GeV}^2$
- * To have agreement at higher Δ^2 , effects beyond IA are necessary: not important for the coherent channel!



Results of numerical evaluation of GPDs show that in the forward limit the **neutron** contribution largely dominates ^3He and the **proton** one is almost negligible. (M. R, S.Scopetta PRC 85, 062201(R) (2012))

Beyond the forward limit, as shown: $\Delta^2 = -0.1 \text{ GeV}^2, \xi = 0.1$

The **neutron** contribution to ^3He still dominates
The **proton** contribution to ^3He gets sizable



It is theoretically possible to separate the flavor contributions in the same case:
For the **u** flavor, the neutron contribution (dashed) to ^3He (full) is less important than for the **d** flavor:

Understandable, sketching the formula:

$$\tilde{G}_M^{3,q} \approx P_p^3 \otimes \tilde{G}_M^{p,q} + P_n^3 \otimes \tilde{G}_M^{n,q},$$

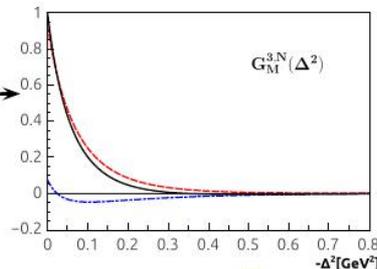
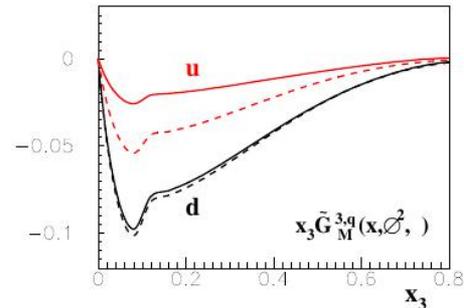
where $P_{p(n)}^3$ describes the proton (neutron) dynamics in ^3He .

As already explained, due to the spin structure of ^3He , $P_n^3 \gg P_p^3 \rightarrow$ neutron dominates in the forward limit.

For this purpose it is useful to show the **proton** and the **neutron** contribution to the ^3He F.F.

With increasing Δ^2 , for the **u** flavor, $\tilde{G}_M^{p,u} \gg \tilde{G}_M^{n,u} \rightarrow$ the proton contribution grows.

Not for **d**! Besides, 1/2 of the **d** content of ^3He comes from the neutron, only 1/5 of the **u** one comes from it.



GPDs Nucleari: ${}^3\text{He}$

The convolution formula can be written as

$$\tilde{G}_M^{3,q}(x, \Delta^2, \xi) = \sum_N \int_{x_3} \frac{M_A}{M} \frac{dz}{z} g_N^3(z, \Delta^2, \xi) \tilde{G}_M^{N,q} \left(\frac{x}{z}, \Delta^2, \frac{\xi}{z} \right),$$

where $g_N^3(z, \Delta^2, \xi)$ is a "light cone off-forward momentum distribution":

$$g_N^3(z, \Delta^2, \xi) = \int dE \int d\vec{p} \tilde{P}_N^3(\vec{p}, \vec{p}', E) \delta \left(z + \frac{M_A}{M} (\xi - p^+ / \bar{P}^+) \right)$$

Where:

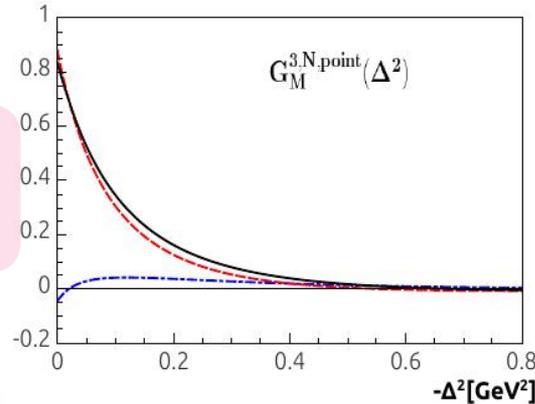
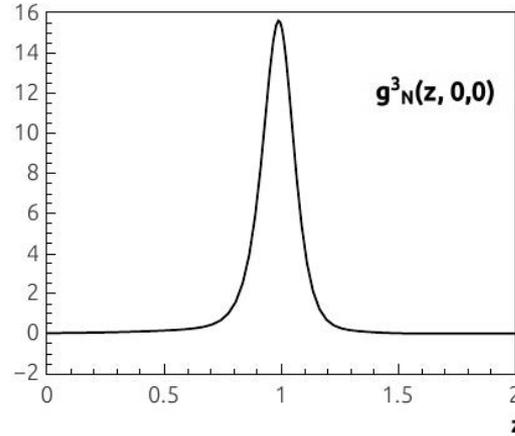
$$\tilde{P}_N^3(\vec{p}, \vec{p}', E) = P_{+-,-,+}^N(\vec{p}, \vec{p}', E) - P_{+,-,-,+}^N(\vec{p}, \vec{p}', E)$$

which, close to the forward limit, is strongly peaked around $z = 1$, so that:

$$\begin{aligned} \tilde{G}_M^{3,q}(x, \Delta^2, \xi) &\underset{\text{low } \Delta^2}{\simeq} \sum_N \tilde{G}_M^{N,q}(x, \Delta^2, \xi) \int_0^{\frac{M_A}{M}} dz g_N^3(z, \Delta^2, \xi) \\ &= G_M^{3,p,\text{point}}(\Delta^2) \tilde{G}_M^p(x, \Delta^2, \xi) + G_M^{3,n,\text{point}}(\Delta^2) \tilde{G}_M^n(x, \Delta^2, \xi), \end{aligned}$$

where the magnetic point like ff has been introduced

$$G_M^{3,N,\text{point}}(\Delta^2) = \int dE \int d\vec{p} \tilde{P}_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) = \int_0^{\frac{M_A}{M}} dz g_N^3(z, \Delta^2, \xi).$$



The Validity of the approximated formula:

full: IA calculation, $\tilde{G}_M^3(x, \Delta^2, \xi)$ and **proton** and **neutron** contributions to it, at $\Delta^2 = -0.1 \text{ GeV}^2$, $\xi = 0.1$;

dashed: same quantities, with the approximated formula:

$$\tilde{G}_M^{3,q}(x, \Delta^2, \xi) \simeq G_M^{3,p,\text{point}}(\Delta^2) \tilde{G}_M^p(x, \Delta^2, \xi) + G_M^{3,n,\text{point}}(\Delta^2) \tilde{G}_M^n(x, \Delta^2, \xi)$$

Impressive agreement!

The **only Nuclear Physics ingredient** in the approximated formula is the **magnetic point like ff**, which is under good theoretical control.

The approximated relation can now be solved to extract the neutron contribution:

$$\tilde{G}_M^{n,\text{extr}}(x, \Delta^2, \xi) \simeq \frac{1}{G_M^{3,n,\text{point}}(\Delta^2)} \left\{ \tilde{G}_M^3(x, \Delta^2, \xi) - G_M^{3,p,\text{point}}(\Delta^2) \tilde{G}_M^p(x, \Delta^2, \xi) \right\}$$

from data for $\tilde{G}_M^3(x, \Delta^2, \xi)$ and $\tilde{G}_M^p(x, \Delta^2, \xi)$, using as theoretical ingredients the **magnetic point**

full : the neutron model for $\tilde{G}_M^n(x, \Delta^2, \xi)$ and the different flavor contributions to it;

dashed: the neutron extracted using the IA calculation for $\tilde{G}_M^3(x, \Delta^2, \xi)$

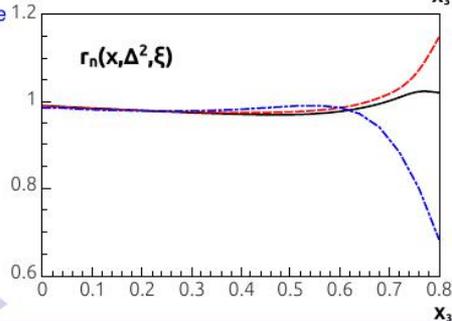
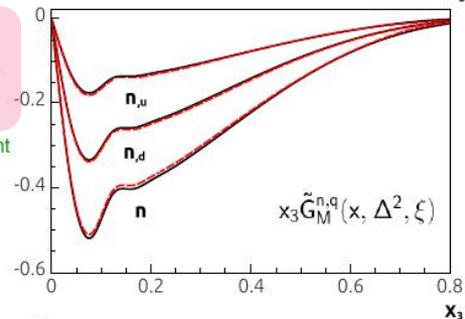
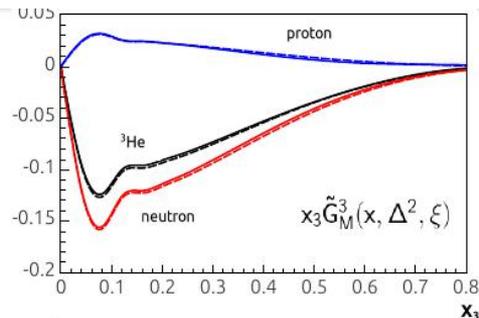
and the model used in it for $\tilde{G}_M^p(x, \Delta^2, \xi)$ together with the **magnetic point like ffs**.

The validity of the extraction procedure is emphasized showing the following ratio, which would be one if the procedure were perfect:

$$r_n(x, \Delta^2, \xi) = \frac{\tilde{G}_M^{n,\text{extr}}(x, \Delta^2, \xi)}{\tilde{G}_M^n(x, \Delta^2, \xi)}$$

full: forward limit; **dashed:** $\Delta^2 = -0.1 \text{ GeV}^2$, $\xi = 0$; **dot-dashed:** $\Delta^2 = -0.1 \text{ GeV}^2$, $\xi = 0.1$.

at $x_3 < 0.7$, in all the kinematical range relevant for coherent DVCS at JLab, the error in the extraction is a few percents.





3

LE dPDFs DI PROTONI E MESONI

Lavori ed articoli prodotti:

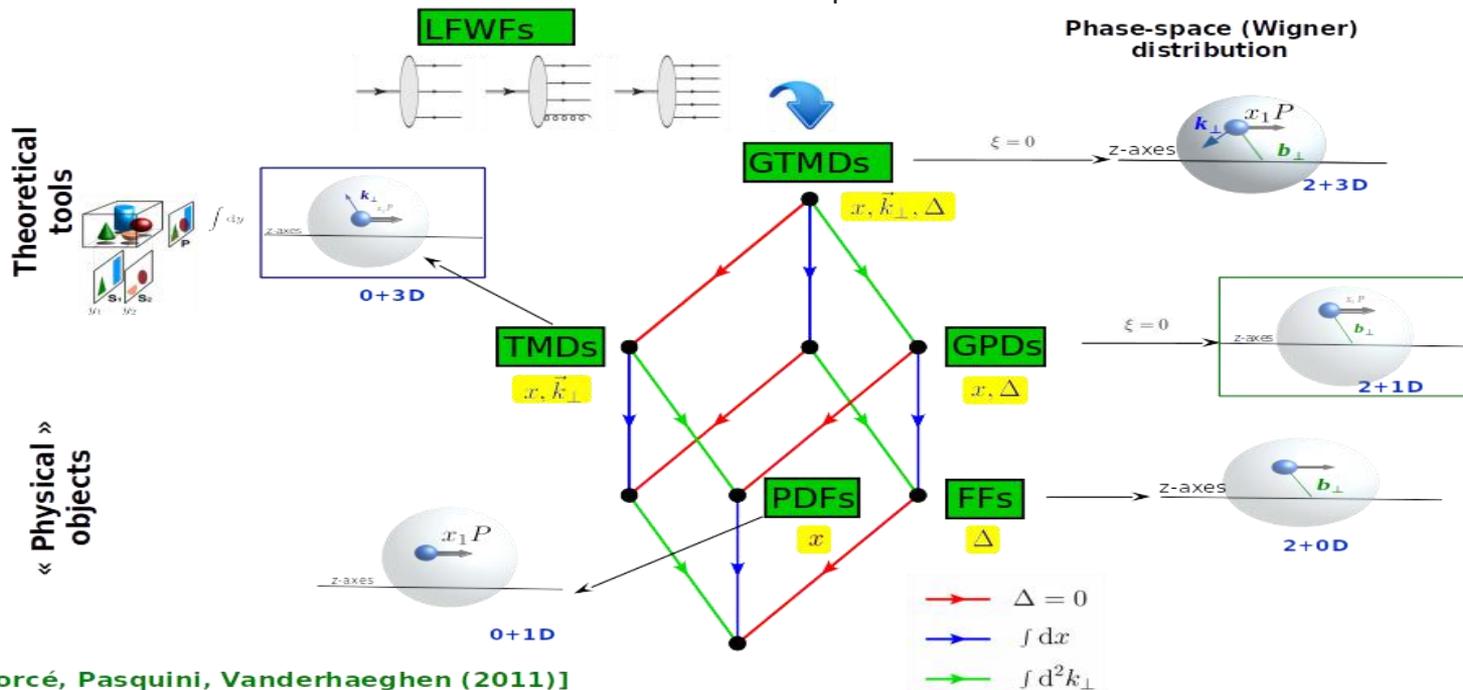
- M. R., S. Scopetta and V. Vento, PRD 87 (2013) 114021
- M. R., S. Scopetta, M. Traini and V. Vento, JHEP 12 (2014) 028
- M. R., S. Scopetta, M. Traini and V. Vento, JHEP 10 (2016) 063
- F. A. Ceccopieri, M. R. and S. Scopetta, PRD 95 (2017), no.11, 114030
- M. R., S. Scopetta, M. Traini and V. Vento, PLB 752 (2016), 40-45
- M. Traini, M. R., S. Scopetta and V. Vento, PLB 768 (2017), 270-273
- M. R., F. A. Ceccopieri, PRD 95 (2017), no.3, 034040
- M. R., F. A. Ceccopieri, PRD 97 (2018), no.7, 071501
- M. R., S. Scopetta, M. Traini and V. Vento, EPJC 78 (2018), no.9, 781
- M. R., F. A. Ceccopieri, JHEP 09 (2019) 097
- M. R., arXiv:2003.09400, sottomesso a EPJC, richieste correzioni minori

Voglio menzionare che l'interesse per questo argomento è nato grazie alla collaborazione con CMS, rappresentato a Perugia dal professore Livio Fanò, del Dipartimento di Fisica e Geologia, Università degli Studi di Perugia.

Grazie a questa collaborazione il sottoscritto è entrato nella comunità che studia le Multi Parton Interactions, facendo anche parte dei conveners di un Workshop tenutosi a Perugia nel 2018.

Double Parton Distribution Functions

The 3D structure of a strongly interacting system (e.g. nucleon, nucleus..) could be accessed through different processes (e.g. SIDIS, DVCS ...), measuring different kind of parton distributions, providing different kind of information. The parton distribution puzzle is:



[Lorcé, Pasquini, Vanderhaeghen (2011)]

Double Parton Distribution Functions

23

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All these distributions are ONE-BODY functions!

How can we access new information as two particle correlations?



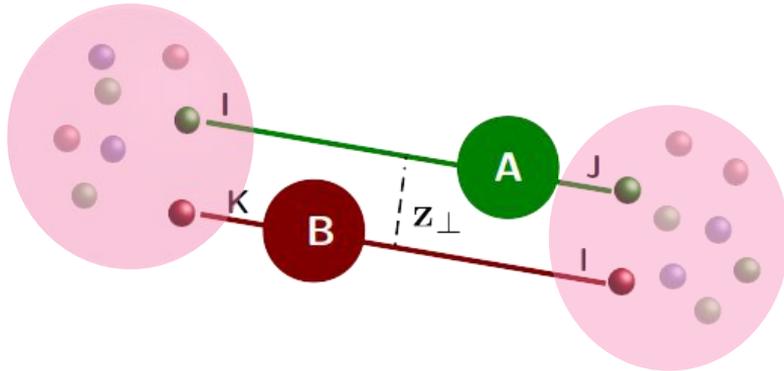
Theoretical tools

« Physical » objects

[Lorcé, Pasquini, Vanderhaeghen (2011)]

Risposta: Multi Parton Interactions

Multiparton interaction (MPI) can contribute to the, pp and pA, cross section @ the LHC:



The cross section for a double parton scattering (DPS) event can be written in the following way:
N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982)

$$d\sigma = \frac{1}{S} \sum_{i,j,k,l} \hat{\sigma}_{ij}(x_1, x_3, \mu_A) \hat{\sigma}_{kl}(x_2, x_4, \mu_B) \int d^2z_{\perp} \underbrace{F_{ik}(x_1, x_2, z_{\perp}, \mu_A, \mu_B)}_{\text{dPDF}} \underbrace{F_{jl}(x_3, x_4, z_{\perp}, \mu_A, \mu_B)}_{\text{Momentum scales}}$$

Momentum fractions carried by the parton inside the proton

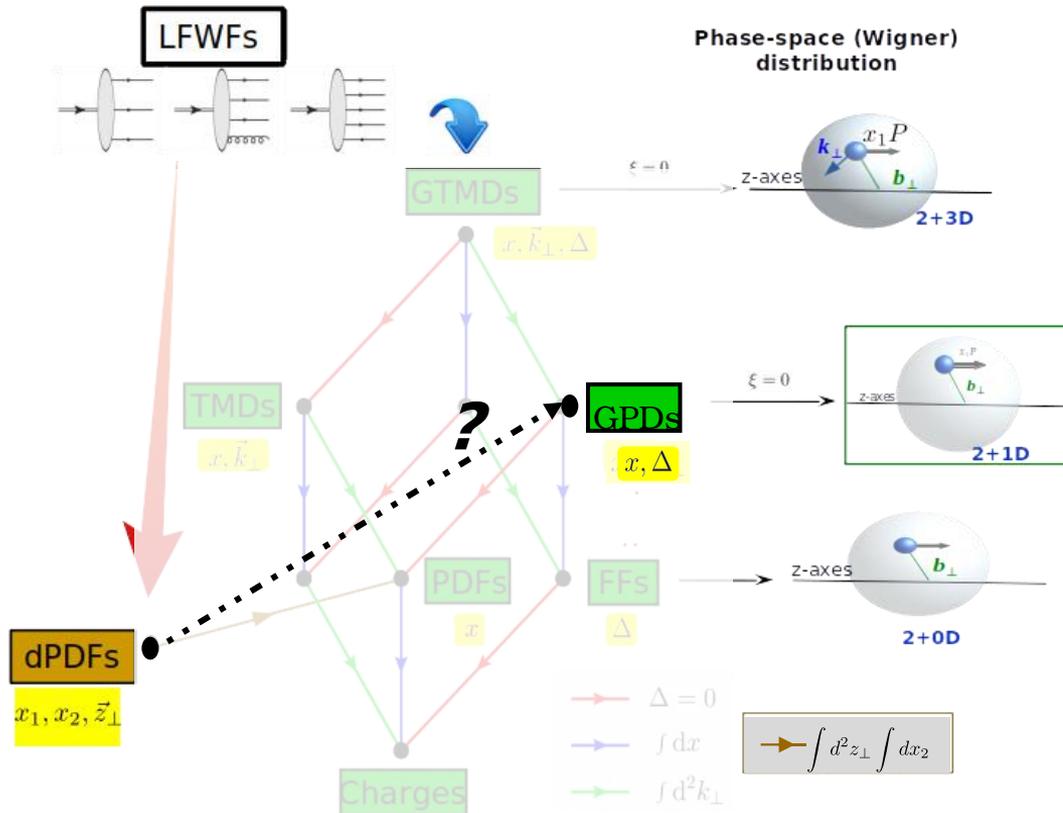
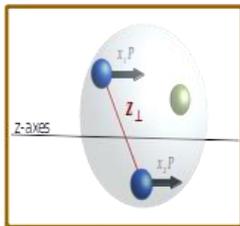
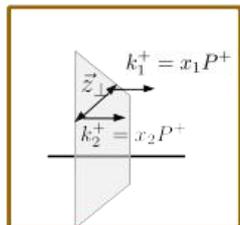
Transverse distance between partons

DPS processes are important for fundamental studies, e.g. the background for the research of new physics and to grasp information on the 3D PARTONIC STRUCTURE OF THE PROTON

Double Parton Distribution Functions

1-body

2-body



Double Parton Distribution Functions

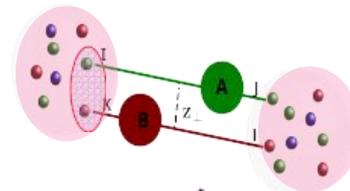
@ LHC kinematics it is often used a factorized form of the dPDFs: $(\mathbf{x}_1, \mathbf{x}_2) - \mathbf{z}_\perp$ factorization:

$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu) \quad \text{and} \quad x_1, x_2 \quad \text{factorization:}$$

* Here and in the following:
 $\mu = \mu_A = \mu_B$

$$\underbrace{F_{ij}(x_1, x_2, \mu)}_{\substack{\text{dPDF (2-Body)} \\ \text{unknown}}} = \underbrace{q_i(x_1, \mu)}_{\text{PDF (1-Body)}} \underbrace{q_j(x_2, \mu)}_{\text{PDF (1-Body)}} \theta(1 - x_1 - x_2) (1 - x_1 - x_2)^n f(x_1, x_2, \mu) \rightarrow \text{To fulfill sum rules}$$

NO DYNAMICAL CORRELATIONS?



In this scenario, dynamical parton correlations inside the proton are neglected \rightarrow NO NEW INFORMATION!

BUT:

- Correlations are present
- dPDFs are non perturbative in QCD and DPCs cannot be directly evaluated within QCD

HOW CAN WE BE SURE OF THE ACCURACY OF SUCH APPROXIMATION



WHAT CAN WE LEARN ABOUT dPDFs AND THE PROTON STRUCTURE?

• **CONSTITUENT QUARK MODELS (CQMs)**

Effective potential and particles strongly bound and **correlated**
 Predictions are related to low energy scales and valence region

• **RELATIVISTIC IMPLEMENTATION: LIGHT-FRONT APPROACH**

$$F_{ij}(x_1, x_2, \mathbf{k}_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^3 d\tilde{\mathbf{k}}_i \delta\left(\sum_{i=1}^3 \tilde{\mathbf{k}}_i\right) \Phi^*(\{\tilde{\mathbf{k}}_i\}, \mathbf{k}_\perp) \Phi(\{\tilde{\mathbf{k}}_i\}, -\mathbf{k}_\perp) \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right)$$

LF wave function
(we need a CQMs)

GOOD SUPPORT

$$M_0^2 = \sum_{i=1}^3 \frac{m_i^2 + \tilde{k}_{i\perp}^2}{x_i}$$

Free proton mass

• **pQCD EVOLUTION OF dPDFs**
 Conjugate to Z_\perp

i) dPDF evaluated at the initial scale of the model

pQCD evolution of dPDFs

ii) dPDF evaluated at high generic scale. We can compare with experimental analyses.

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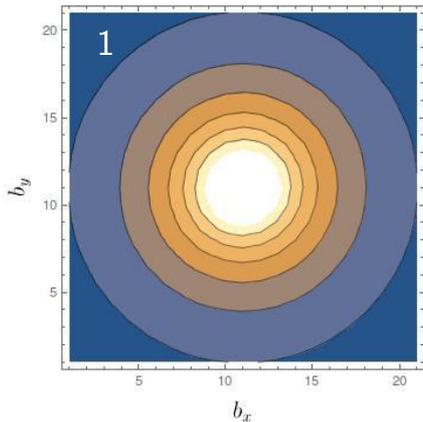
pQCD evolution of dPDFs

ii) dPDF evaluated at high generic scale. We can compare with experimental analyses.

M. R. and F. A. Ceccopieri, JHEP 1909 (2019) 097

Since, in coordinates space, dPDFs get a number density interpretation, in principle one can calculate the mean distance between partons!

For example, for 2 gluons perturbatively generated:

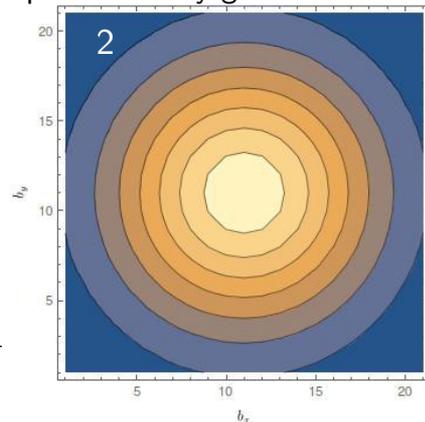


1) HP model

2) HO model

$$x_1 = 10^{-4} \text{ and } x_2 = 10^{-2}$$

$$\vec{d}_\perp = \vec{b}_\perp = \vec{z}_\perp$$



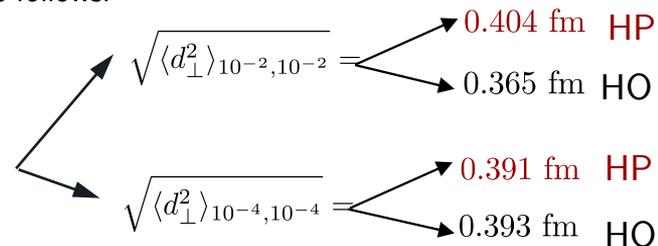
M. Traini *et al*, Nucl. Phys. A 656, 400-420 (1999), non relativistic Hyper-Central CQM (potential by M. Ferraris *et al*, PLB 364 (1995)) (HP)

The harmonic oscillator (HO)

One can also define the mean transverse distance $(x_1 - x_2)$ distribution as follows:

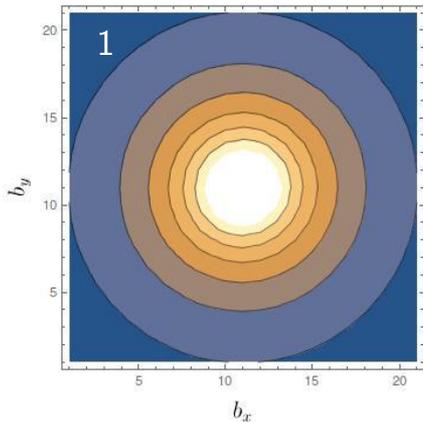
$$\langle d_\perp^2 \rangle_{x_1, x_2}^{ij} = \frac{\int d^2 b_\perp b_\perp^2 F_{ij}(x_1, x_2, b_\perp, Q^2 = M_W^2)}{\int d^2 b_\perp F_{ij}(x_1, x_2, b_\perp, Q^2 = M_W^2)}$$

For example, for 2 gluons and two different models, one gets:



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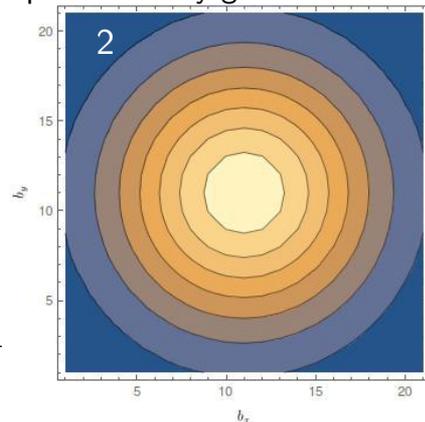


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Are two slow partons closer (in \perp plane) than two fast partons?

The **dPDF** is formally defined through the Light-cone correlator:

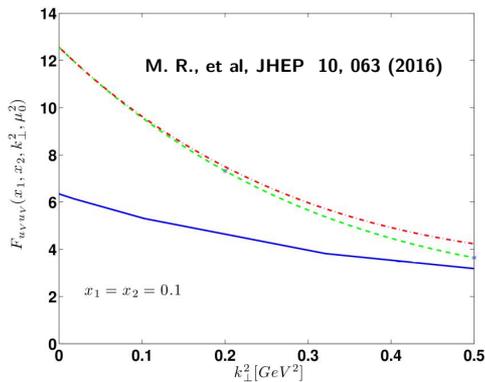
$$F_{12}(x_1, x_2, \vec{z}_\perp) \propto \left(\sum_X \right) \int dz^- \left[\prod_{i=1}^2 dl_i^- e^{ix_i l_i^- p^+} \right] \langle p | O(z, l_1) | X \rangle \langle X | O(0, l_2) | p \rangle \Big|_{\substack{\vec{l}_{1\perp} = \vec{l}_{2\perp} = 0 \\ l_1^+ = l_2^+ = z^+ = 0}}$$

Approximated by the proton state!

$$\int \frac{dp'^+ dp'^\perp}{p'^+} |p'\rangle \langle p'|$$

GPDS

$$F_{12}(x_1, x_2, \vec{k}_\perp) \sim f(x_1, 0, \vec{k}_\perp) f(x_2, 0, \vec{k}_\perp)$$



..... dPDF = GPD x GPD

— dPDF

Violation due to:

- ✓ Correlation between (x_1, x_2)
 - M. R., et al, PRD 87, 114021 (2013)
 - M. R., et al, JHEP 12, 028 (2014)
 - M. R., et al, JHEP 10, 063 (2016)
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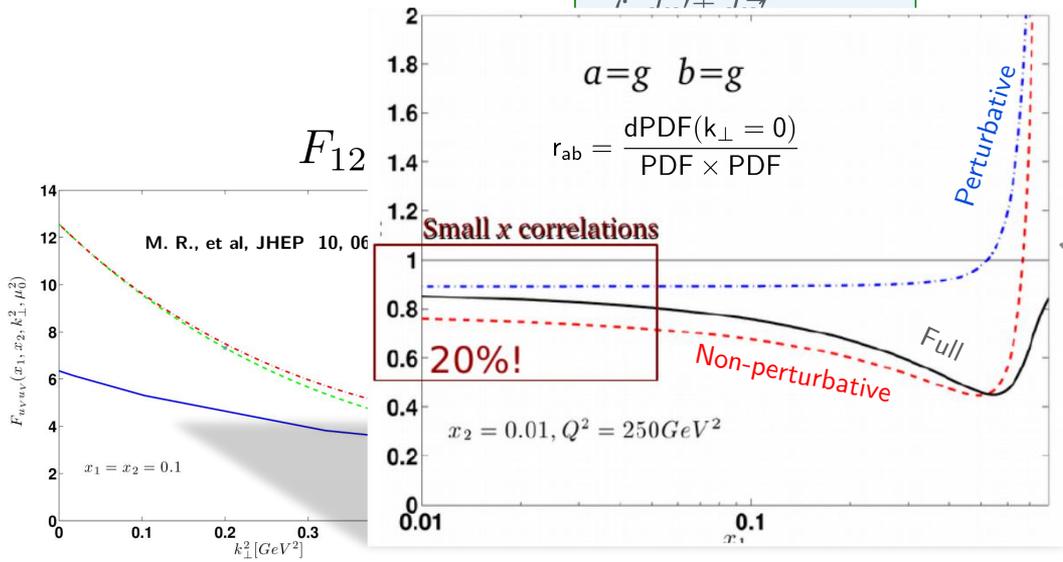
COSA OTTENUTO: EFFETTI DELLE CORRELAZIONI

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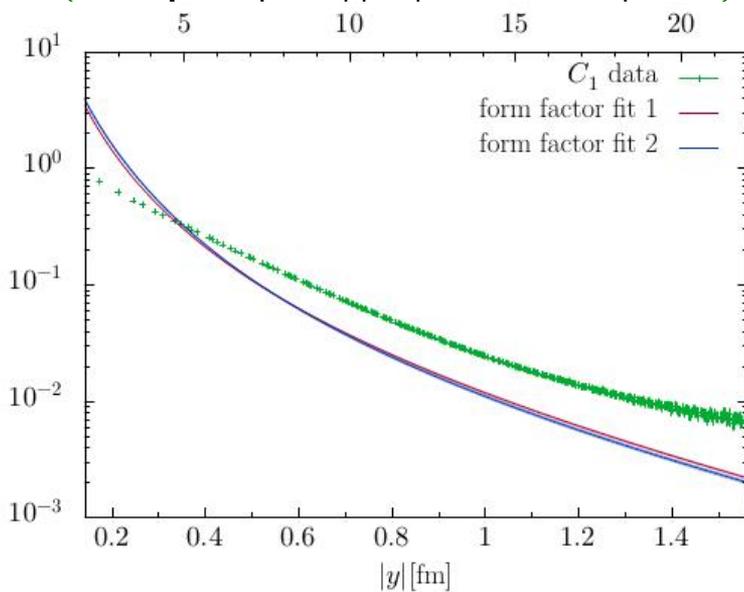
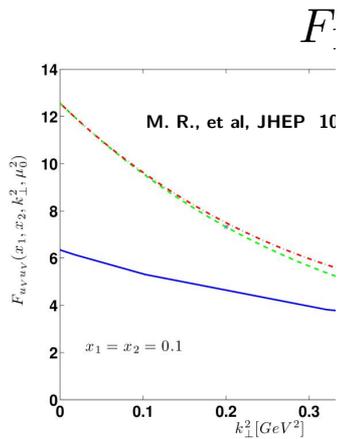
$$\vec{k}_\perp$$

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dDs

Similar violation observed in lattice analysis for the pion.

G. S. Bali et al, JHEP 12 (2018) 061

A fundamental tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called “effective X-section”.

This object can be defined through a “pocket formula”:

$$\sigma_{eff} = \frac{m}{2} \frac{\sigma_A^{pp'} \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$

Sensitive to correlations → σ_{eff}

Combinatorial factor ← $\frac{m}{2}$

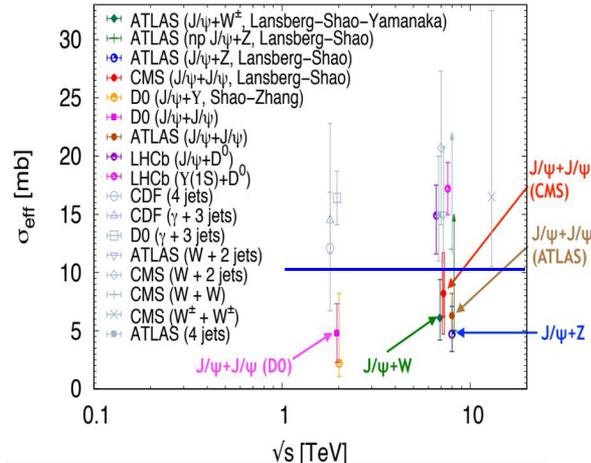
Differential cross section for a DPS event: $pp' \rightarrow A + B + X$ ← σ_{double}^{pp}

Differential cross section for the process: $pp' \rightarrow A(B) + X$ ← $\sigma_B^{pp'}$

....EXPERIMENTAL STATUS:

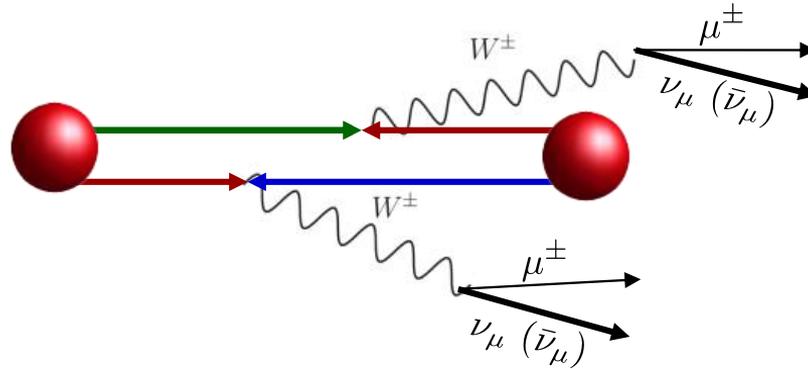
- Difficult extraction, approved analysis for the same
- the model dependent extraction of σ_{eff} from data is almost consistent with a “constant” (within errors) (**uncorrelated ansatz usually assumed!**)...Some inconsistencies in production
- different ranges in X_i accessed in different experiments.

Within our CQM framework, we can calculate σ_{eff} without any approximations!



PRODUZIONE DI DUE MESONI W DELLO STESSO SEGNO AD LHC

32



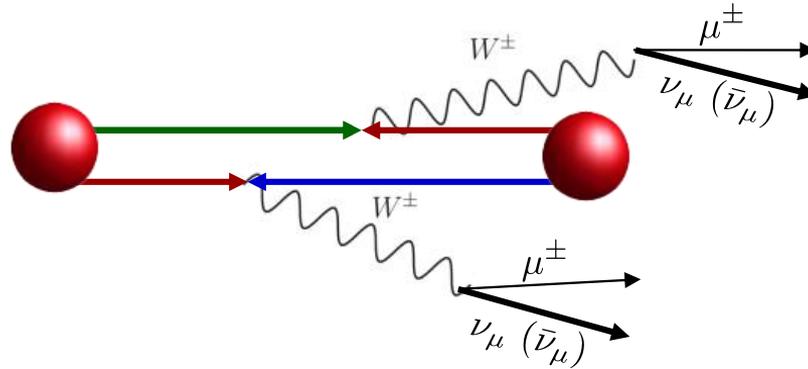
In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.



"Same-sign W boson pairs production is a promising channel to look for signature of double Parton interactions at the LHC."

PRODUZIONE DI DUE MESONI W DELLO STESSO SEGNO AD LHC

32

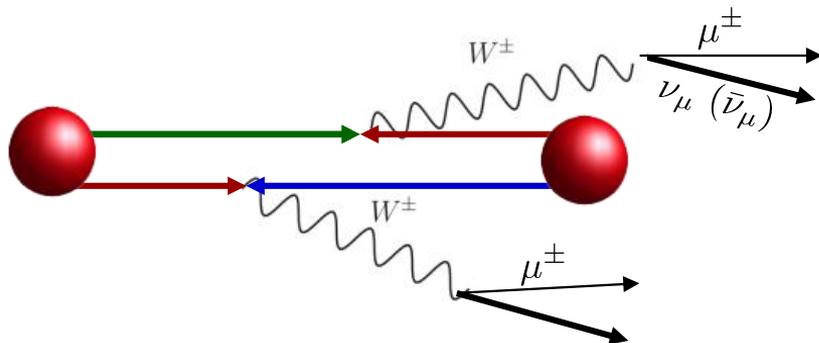


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Can double parton correlations be observed for the first time in the next LHC run ?

PRODUZIONE DI DUE MESONI W DELLO STESSO SEGNO AD LHC



Kinematical cuts

$$\begin{aligned}
 & pp, \sqrt{s} = 13 \text{ TeV} \\
 & p_{T,\mu}^{\text{leading}} > 20 \text{ GeV}, \quad p_{T,\mu}^{\text{subleading}} > 10 \text{ GeV} \\
 & |p_{T,\mu}^{\text{leading}}| + |p_{T,\mu}^{\text{subleading}}| > 45 \text{ GeV} \\
 & |\eta_{\mu}| < 2.4 \\
 & 20 \text{ GeV} < M_{\text{inv}} < 75 \text{ GeV} \text{ or } M_{\text{inv}} > 105 \text{ GeV}
 \end{aligned}$$

DPS cross section:

$$\frac{d^4 \sigma_{pp \rightarrow \mu^{\pm} \mu^{\pm} X}}{d\eta_1 dp_{T,1} d\eta_2 dp_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2 \vec{b}_{\perp} F_{ij}(x_1, x_2, \vec{b}_{\perp}, M_W) F_{kl}(x_3, x_4, \vec{b}_{\perp}, M_W) \frac{d^2 \sigma_{ik}^{pp \rightarrow \mu^{\pm} X}}{d\eta_1 dp_{T,1}} \frac{d^2 \sigma_{jl}^{pp \rightarrow \mu^{\pm} X}}{d\eta_2 dp_{T,2}} \mathcal{I}(\eta_i, p_{T,i})$$

$M_W \longrightarrow$ Momentum scale

In order to estimate the role of double parton correlations we have used as input of dPDFs:

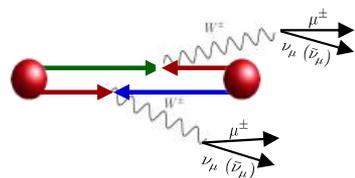
1) Longitudinal and transverse correlations arise from the relativistic CQM model describing three valence quarks

2) These correlations propagate to sea quarks and gluons through pQCD evolution

PRODUZIONE DI DUE MESONI W DELLO STESSO SEGNO AD LHC

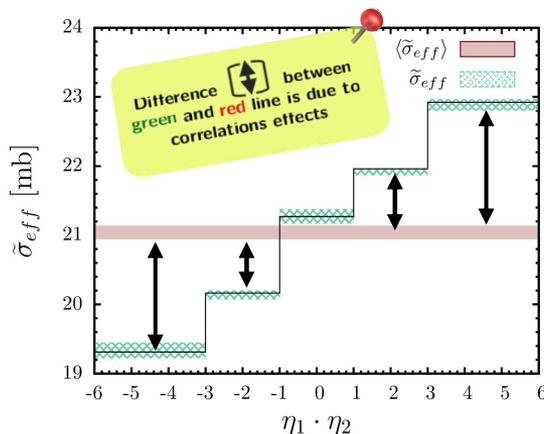
34

In order to understand whether correlations can be accessed in experimental observations, using dPDF evaluated within the QM model, the effective cross section has been calculated for this process and compared with its mean value:



$$\eta_1 \cdot \eta_2 \simeq \frac{1}{4} \ln \frac{x_1}{x_3} \ln \frac{x_2}{x_4}$$

$$\langle \tilde{\sigma}_{eff} \rangle = 21.04^{+0.07}_{-0.07} (\delta Q_0) {}^{+0.06}_{-0.07} (\delta \mu_F) \text{ mb} .$$



$$\tilde{\sigma}_{eff} = \frac{m}{2} \frac{\sigma_A^{pp'} \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$

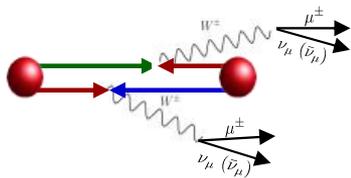
PRODUZIONE DI DUE MESONI W DELLO STESSO SEGNO AD LHC

34

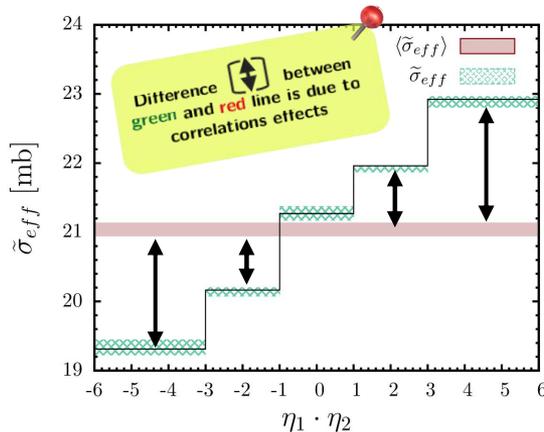
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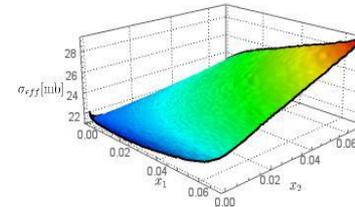
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x- dependence of effective x-section consistent with analyses:
 M.Rinaldi et al PLB 752,40 (2016)
 M. Traini, M. R. et al, PLB 768, 270 (2017)
 M.R. and F. A. Ceccopieri JHEP 1909 (2019) 097



Gluons ⊗ Gluons

“Assuming that the results of the first and the last bins can be distinguished if they differ by 1 sigma, we estimated that

$$\mathcal{L} = 1000 \text{ fb}^{-1}$$

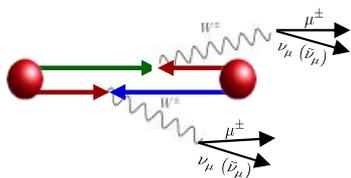
is necessary to observe correlations”

PRODUZIONE DI DUE MESONI W DELLO STESSO SEGNO AD LHC

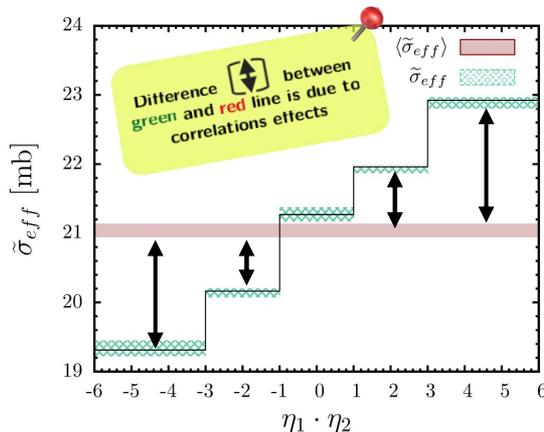
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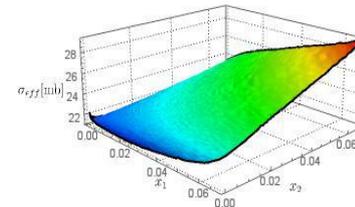


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Gluons ⊗ Gluons

To observe correlations,

$\mathcal{L} = 1000 \text{ fb}^{-1}$ is needed!



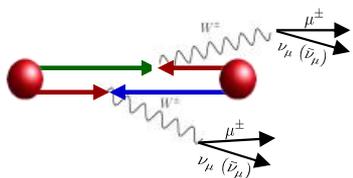
REACHABLE IN THE PLANNED LHC RUN

PRODUZIONE DI DUE MESONI W DELLO STESSO SEGNO AD LHC

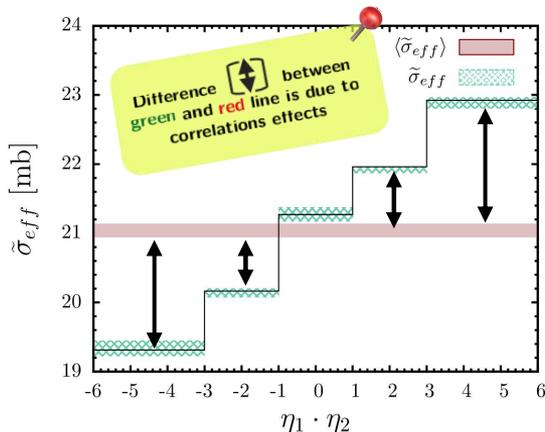
In order to understand whether correlations can be accessed in experimental observations, using dPDF evaluated within the QM model, the effective cross section has been calculated for this process and compared with its mean value:

$$\langle \tilde{\sigma}_{eff} \rangle = 21.04^{+0.07}_{-0.07} (\delta Q_0) {}^{+0.06}_{-0.07} (\delta \mu_F) \text{ mb} .$$

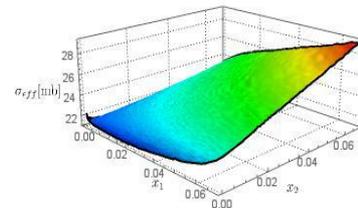
$$\tilde{\sigma}_{eff} = \frac{m}{2} \frac{\sigma_A^{pp'} \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$



$$\eta_1 \cdot \eta_2 \simeq \frac{1}{4} \ln \frac{x_1}{x_3} \ln \frac{x_2}{x_4}$$



x- dependence of effective x-section consistent with analyses:
 M.Rinaldi et al PLB 752,40 (2016)
 M. Traini, M. R. et al, PLB 768, 270 (2017)
 M.R. and F. A. Ceccopieri JHEP 1909 (2019) 097



Gluons ⊗ Gluons

IN THIS CHANNEL, WE ESTABLISHED THE POSSIBILITY TO OBSERVE, FOR THE FIRST TIME, TWO-PARTON CORRELATIONS IN THE NEXT LHC RUN!

Considering the factorization ansatz, for which some estimates of σ_{eff} are available, one has:

$$\longrightarrow \sigma_{eff} = \left[\int \frac{d\vec{k}_{\perp}}{(2\pi)^2} \tilde{T}(\vec{k}_{\perp}) \tilde{T}(-\vec{k}_{\perp}) \right]^{-1}$$

Effective form factor (Eff)

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Eff can be formally defined as **FIRST MOMENT** of dPDF (like for GPDs) through the proton wave function:

$$\tilde{T}(k_\perp) = \frac{1}{2} \int dx_1 dx_2 F(x_1, x_2, k_\perp) = \int d\vec{k}_1 d\vec{k}_2 \Psi(\vec{k}_1 + \vec{k}_\perp, \vec{k}_2) \Psi^\dagger(\vec{k}_1, \vec{k}_2 + \vec{k}_\perp)$$

From the above quantity the mean distance in the transverse plane (not necessary close to the proton radius) between two partons can be defined: $\langle b^2 \rangle \sim -2 \frac{d}{dk_\perp} \tilde{T}(k_\perp) \Big|_{k_\perp=0}$

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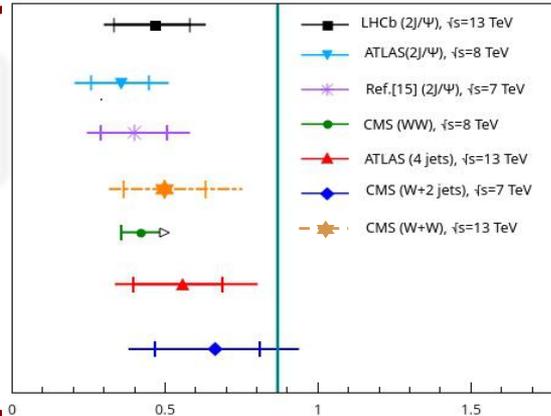
Eff is unknown but using general model independent properties and comparing Eff with standard proton ff, we found:



$$\frac{\sigma_{eff}}{3\pi} \leq \langle b^2 \rangle \leq \frac{\sigma_{eff}}{\pi}$$

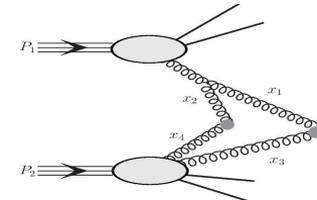
DPS processes:

The vertical line stands for the transverse proton radius



We also:
M. R. and F. A. Ceccopieri, JHEP 1909 (2019) 097

Extended the approach including splitting term



Extended the approach to the most general unfactorized case

Considering the factorization ansatz, for which some estimates of σ_{eff} are available, one has: $\rightarrow \sigma_{eff} = \left[\int \frac{d\vec{k}_\perp}{(2\pi)^2} \tilde{T}(\vec{k}_\perp) \tilde{T}(-\vec{k}_\perp) \right]^{-1}$ → Effective form factor (Eff)

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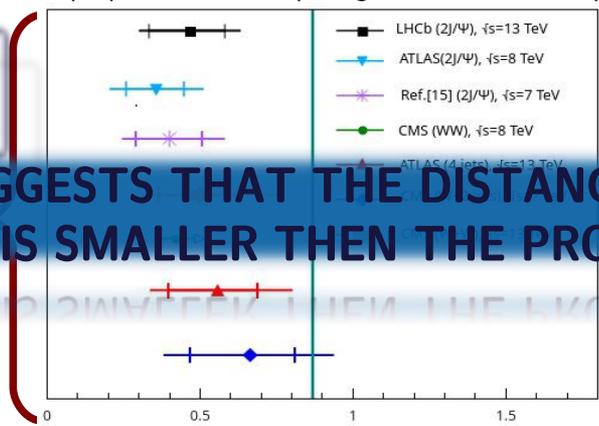


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DATA SUGGESTS THAT THE DISTANCE OF THESE PARTONS IS SMALLER THEN THE PROTON RADIUS

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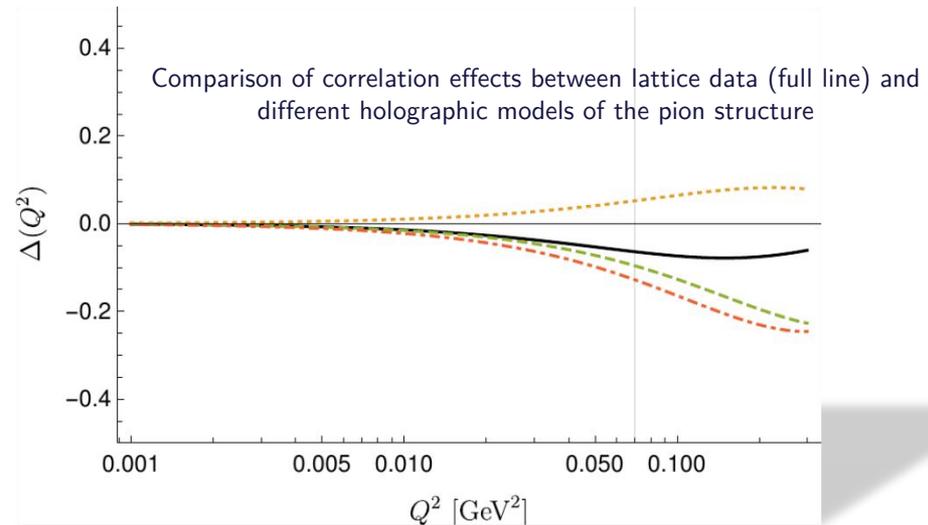
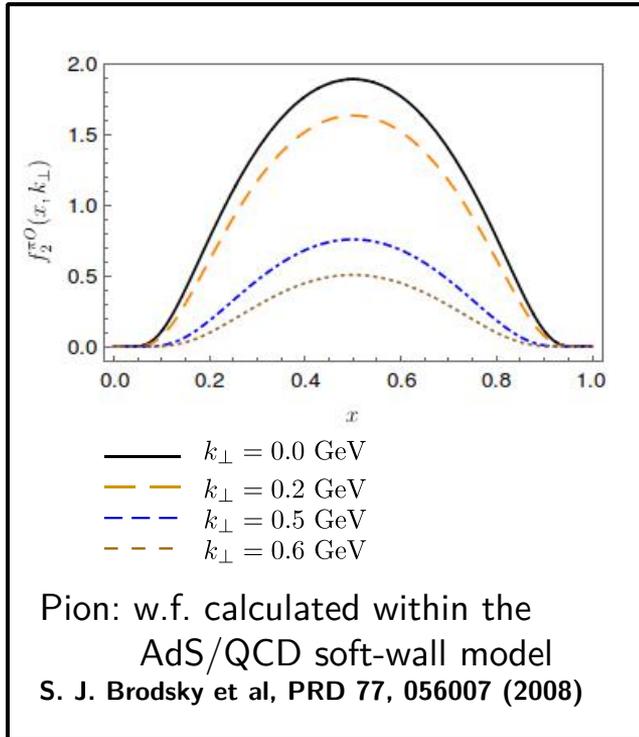


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dPDFs of the pion within holographic models:

M. R., S. Scopetta, M. Traini and V.Vento, EPJC 78, no. 9,782 (2018)

M. R., submitted to EPJC (Minor corrections requested)





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Relativistic effects in dPDFs calculations:

M. R. and F. A. Ceccopieri, PRD 95 (2017) 034040

M. R. and F. A. Ceccopieri, JHEP 09 (2019) 097

$$\underbrace{F_{ij}(x_1, x_2, k_\perp; Q^2)}_{\text{pheno}} = \underbrace{q_i(x_1; Q^2)q_j(x_2; Q^2)}_{\text{phenomenology from PDFs}} \underbrace{\theta(1 - x_1 - x_2)}_{\text{good support}} \underbrace{f(x_1, x_2, Q^2)}_{\text{sum rules}} \underbrace{R(x_1, x_2, k_\perp)}_{\text{Melosh effects correlations!}} \underbrace{F(k_\perp)}_{\text{}}$$

To be modeled:
CQMs, GPDs...



4

GLUEBALLS STUDIATE CON MODELI OLOGRAFICI

Lavori ed articoli prodotti:

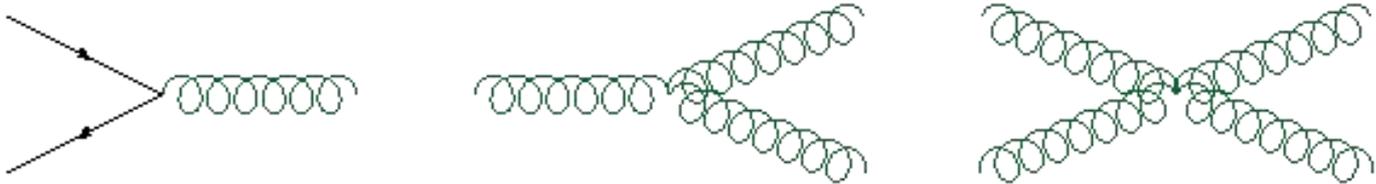
- M. R. and V. Vento, EPJA 54 (2018) 151
- M. R. and V. Vento, J. Phys. G 47 (2020) no.5, 055104
- M. R. and V. Vento, arXiv: 2002.11720, sottomesso a J. Phys. G

The QCD, the gauge theory describing strong interactions

$$\mathcal{L} = -\frac{1}{4} \text{Tr} G_{\mu\nu} G^{\mu\nu} + \sum \bar{\Psi} (i\gamma \cdot D - m) \Psi$$

gluon field strength tensor:

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$$

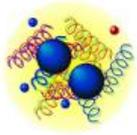


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Mesons

$$3 \otimes \bar{3}$$



Baryons

$$3 \otimes 3 \otimes 3$$

Exotic states

$$3 \otimes \bar{3} \otimes 8$$

$$8 \otimes 8$$

$$8 \otimes \dots \otimes 8$$

HYBRIDS

GLUEBALLS

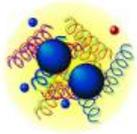
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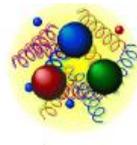
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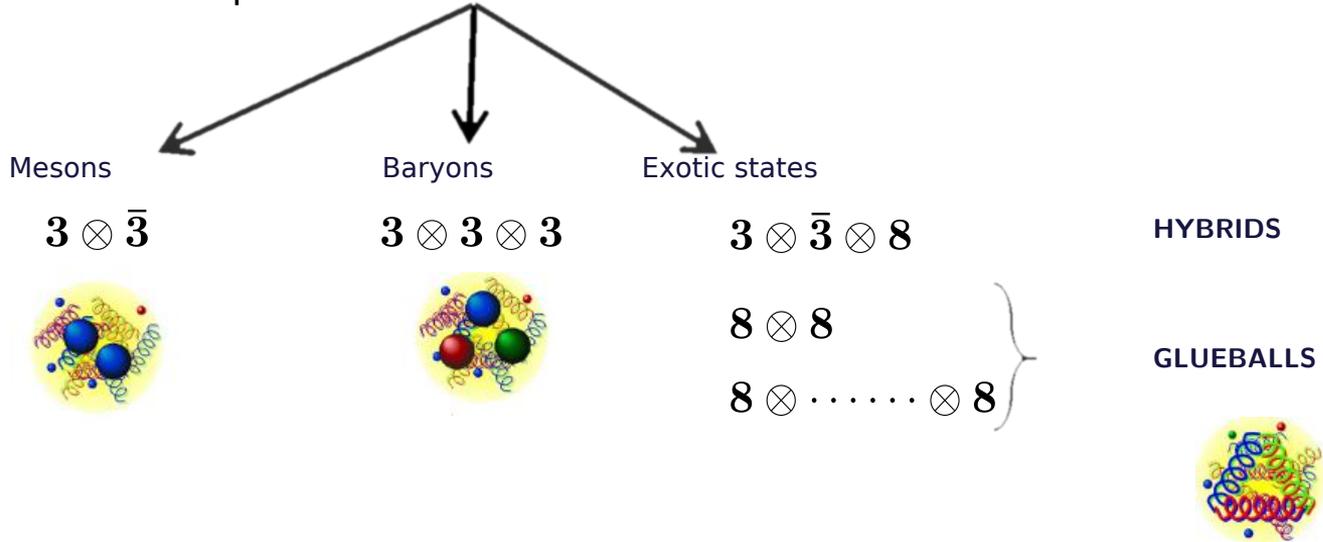
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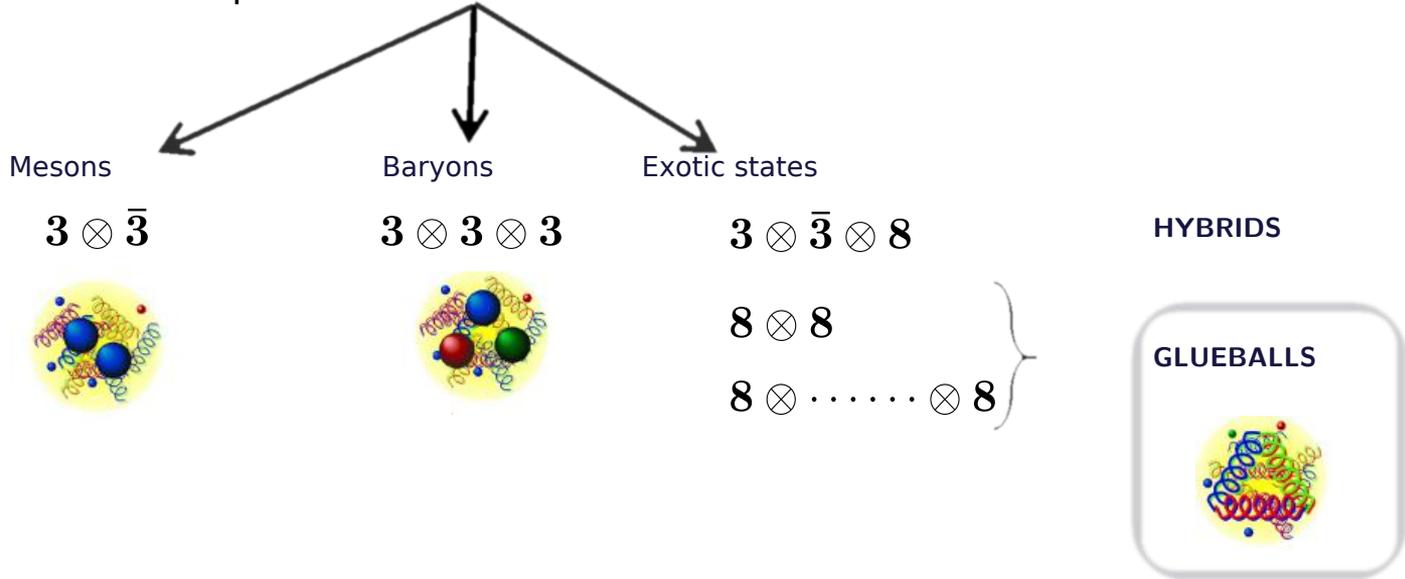
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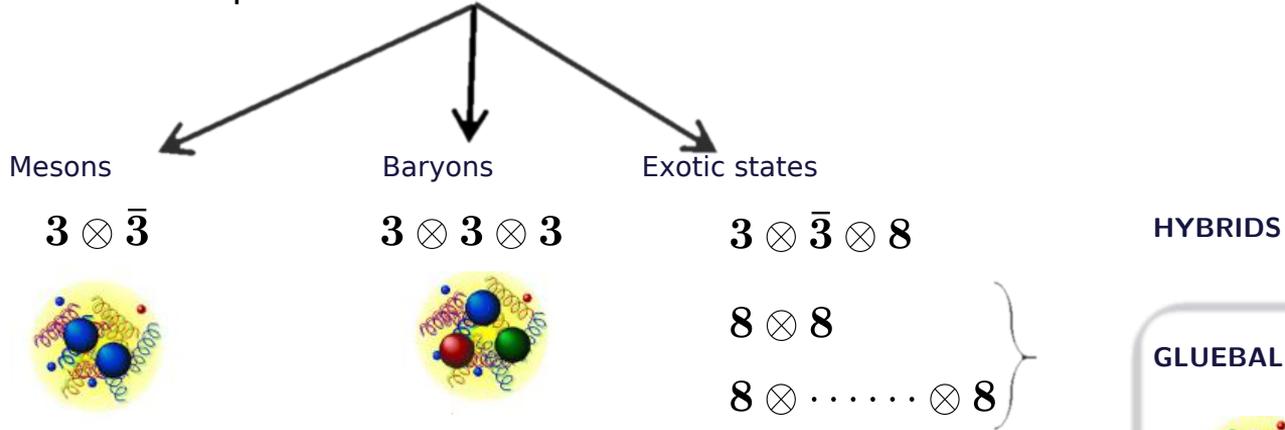


Why Glueballs?

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Glueball spectroscopy is a unique laboratory to test non perturbative QCD and CONFINEMENT

However :

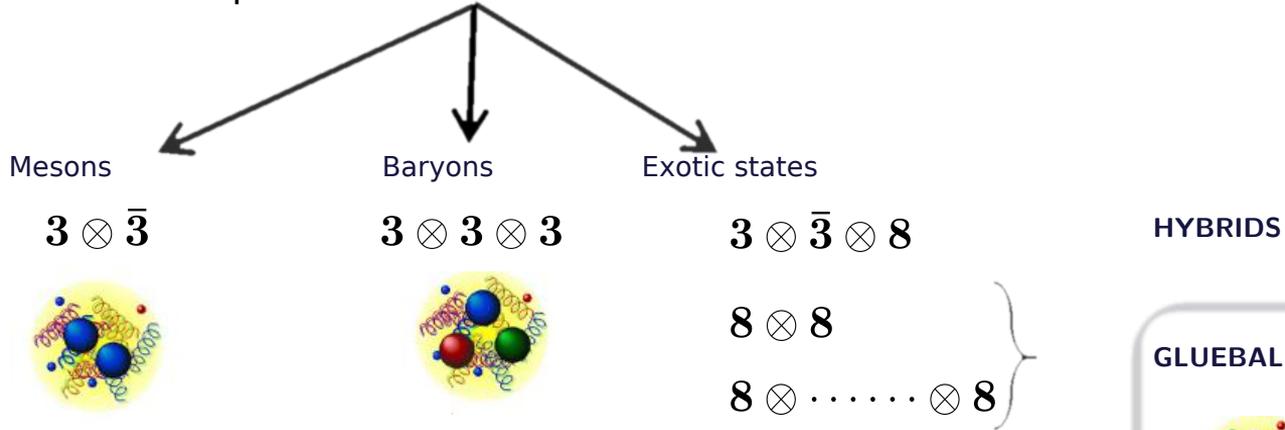
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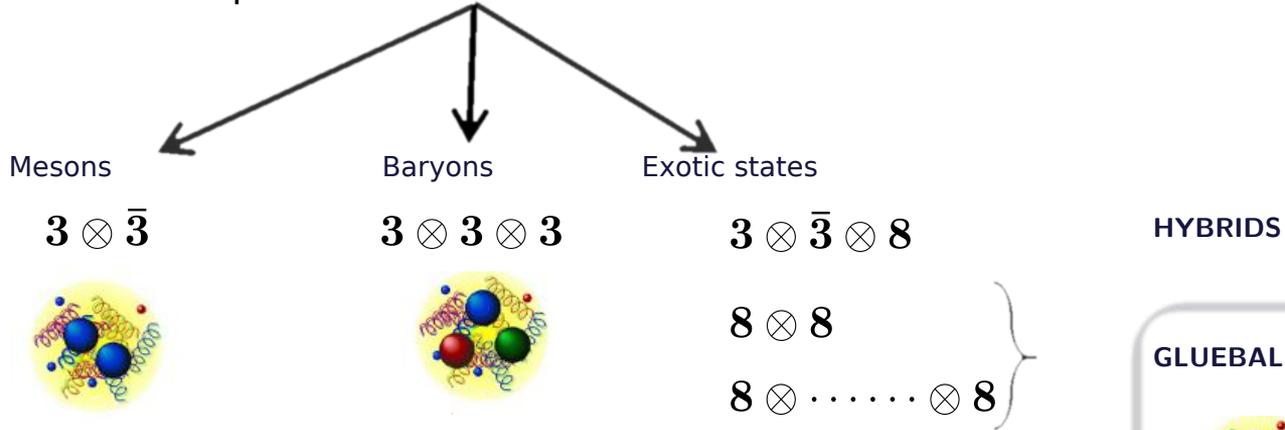
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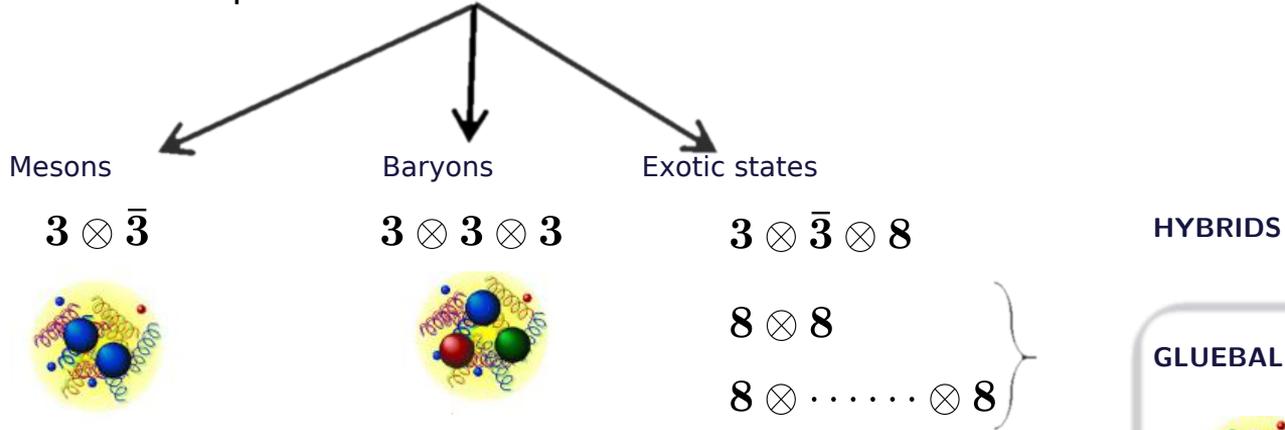
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MP: C.J. Morningstar et al, PRD 60, 034509 (1999)

YC: Y. Chen et al, PRD 73, 014516 (2006)

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	0^{++}	2^{++}	0^{++}	2^{++}	0^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222			
YC	1719 ± 94	2390 ± 124				
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Could model help in this scenario?
We used AdS/QCD models!

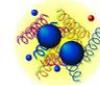
One of the main difficulties in the observation of glueballs is related to their mixing with mesons!

For example:

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Meson	$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(2020)$	$f_0(2100)$	$f_0(2200)$
PDG	475 ± 75	990 ± 20	1350 ± 150	1504 ± 6	1723 ± 6	1992 ± 16	2101 ± 7	2189 ± 13



Mixing?

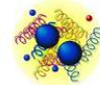
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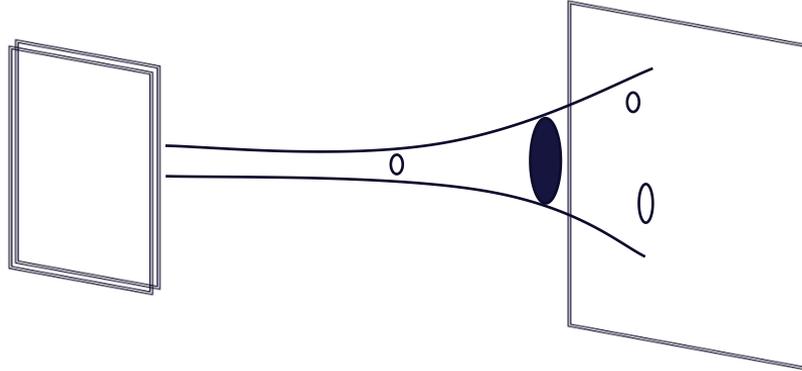
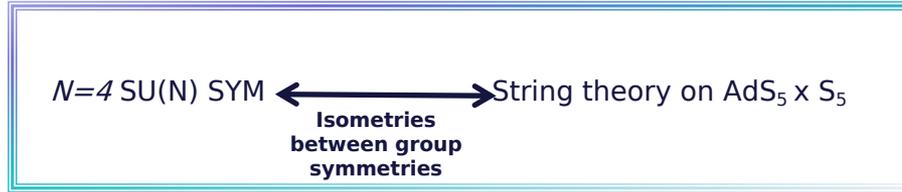
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Mixing?

We use AdS/QCD models to study the MIXING problems and “predict” the kinematic conditions where pure glueball states could be observed.

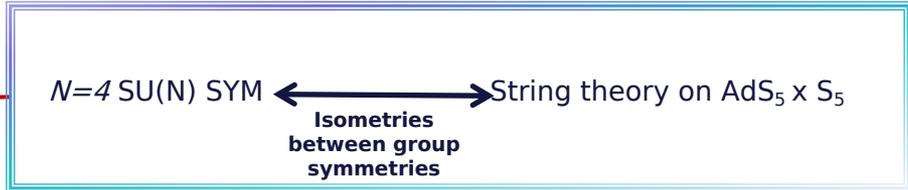
From Maldacena conjecture: AdS/CFT



$$g_{\text{YM}}^2 N \underset{N \rightarrow \infty}{=} \frac{R^4}{l^4}$$

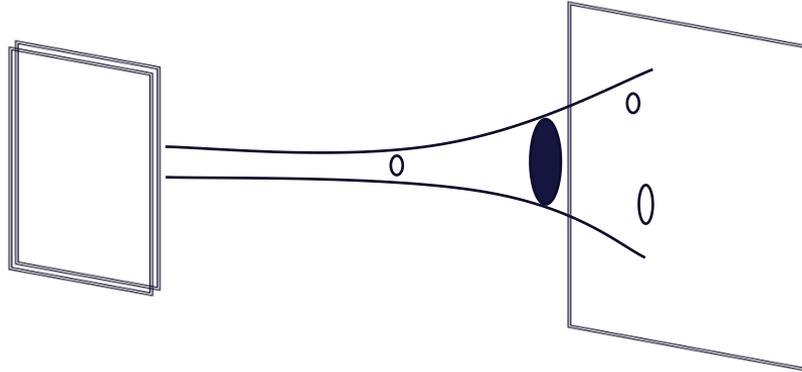
R = radius of the manifold
l = length

From Maldacena conjecture: AdS/CFT



This is not QCD!

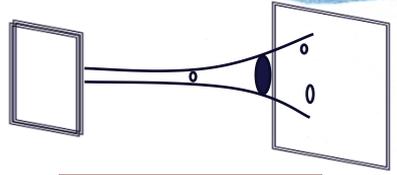
- No supersymmetry
- Confinement
- Conformal symmetry broken
- N is finite



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The dream is to understand QCD using its dual gravity theory!



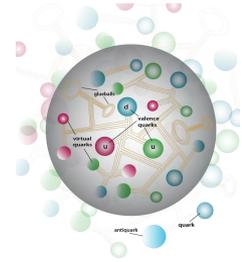
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Top-down Approach:
Find a gravitational theory dual to QCD

Bottom-up Approach:
Starts from QCD and attempts to construct a five dimensional holographic dual

Confinement

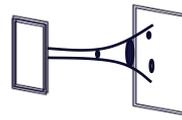
Hard-wall model
Compactification of the 5th dimension by hand. AdS geometry cut by two branes: UV and IR.



Confinement

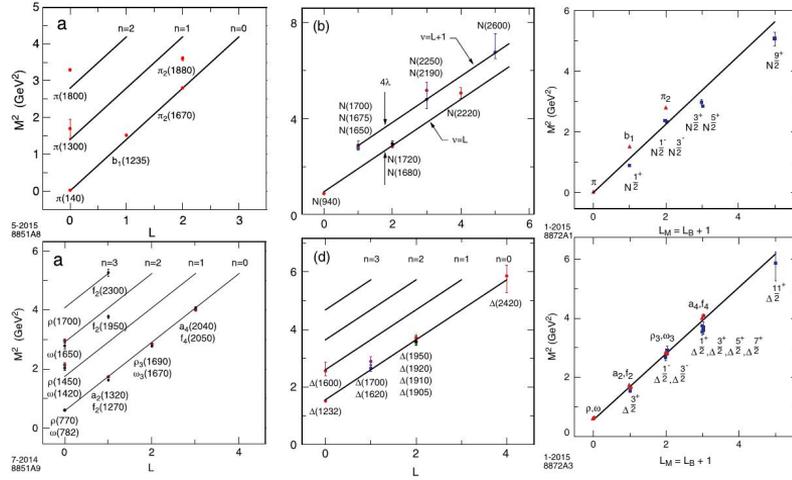
Soft-wall model
Soft cutoff of AdS space by introducing a dilaton field.
 $e^{-\varphi}$

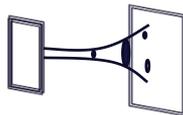
AdS/QCD: PREDIZIONI



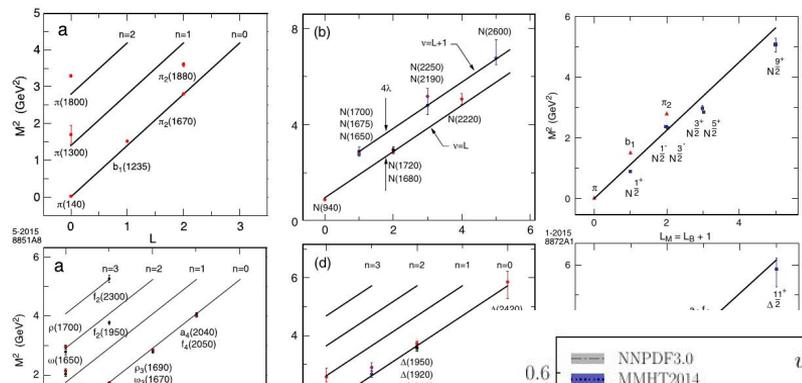
HADRON SPECTRUM:
 S.J. Brodsky et al, Phys. Rep. 584 (2015)
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 085016 (2015)

see Brodsky's talk on Thursday



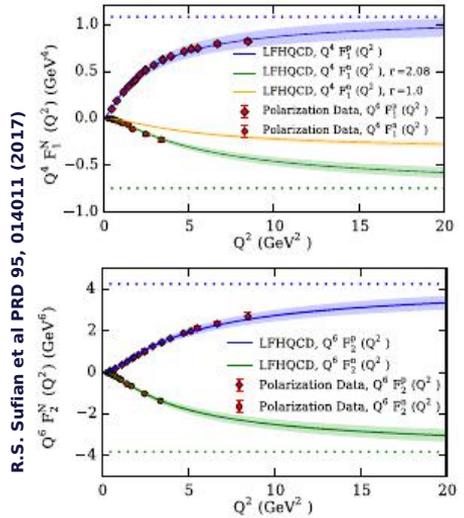


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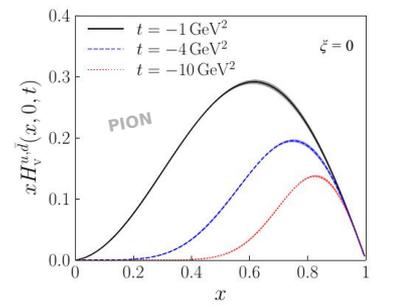
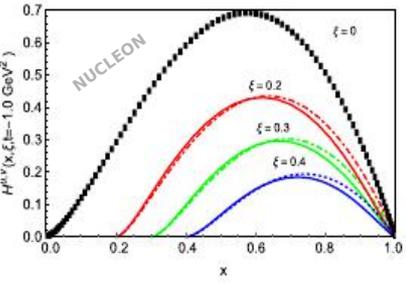
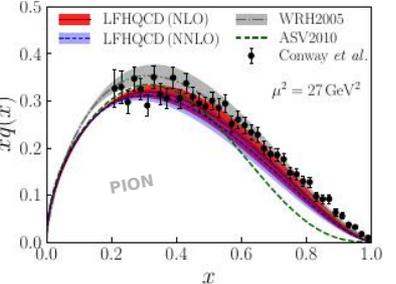
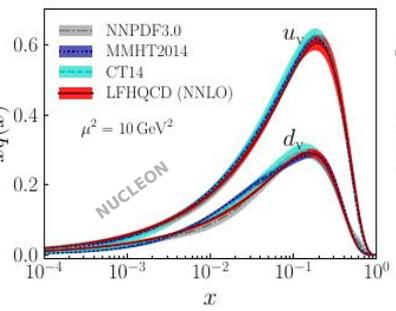
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FORM FACTORS, PDFs & GPDs

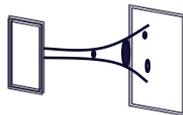


R.S. Sufian et al *PRD* **95**, 014011 (2017)

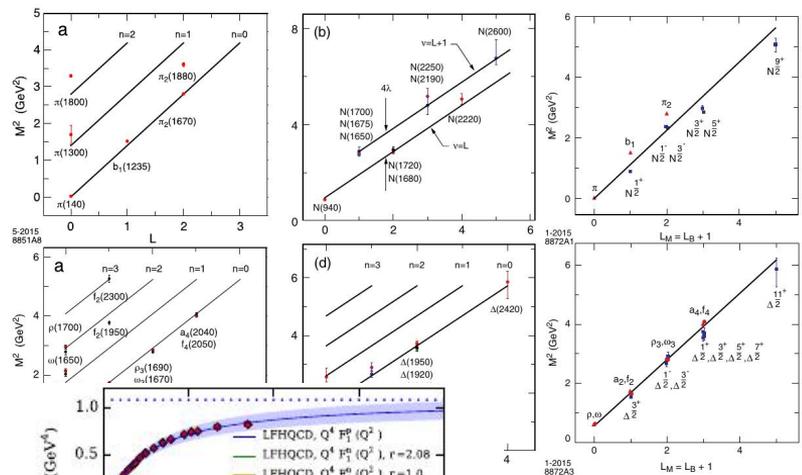
de Teramond et al, *PRL* **120**, 182001 (2018)



M. Rinaldi, *PLB* **771** (2017)

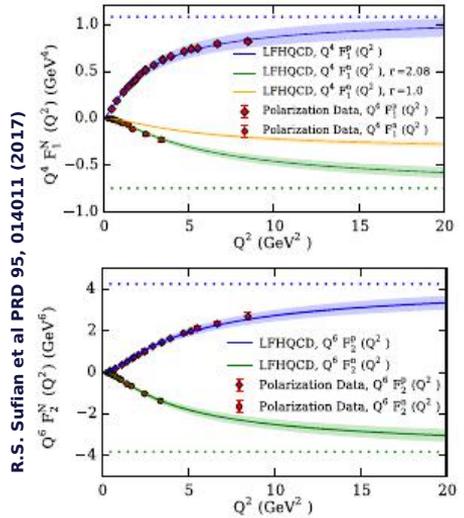


HADRON SPECTRUM:
 S.J. Brodsky et al, *Phys. Rep.* **584** (2015)
 H.G. Dosh et al *PRD* **91**, 045040 (2015),
 085016 (2015)

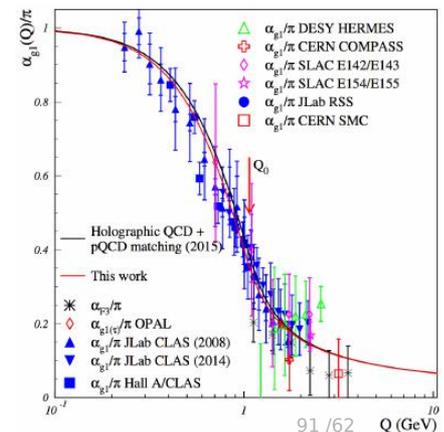


see Brodsky's talk on Thursday

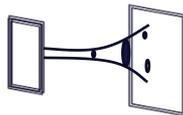
FORM FACTORS, PDFs & GPDs



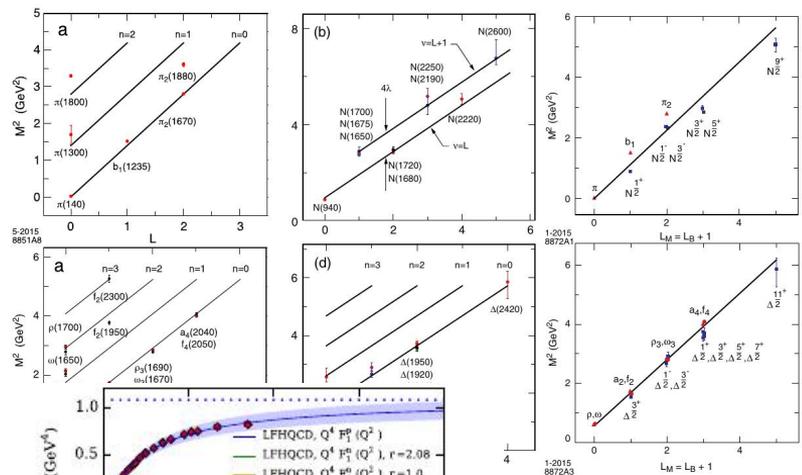
MATCHING THE RUNNING COUPLING



de Teramond et al, *PRL* **120**, 182001 (2018)

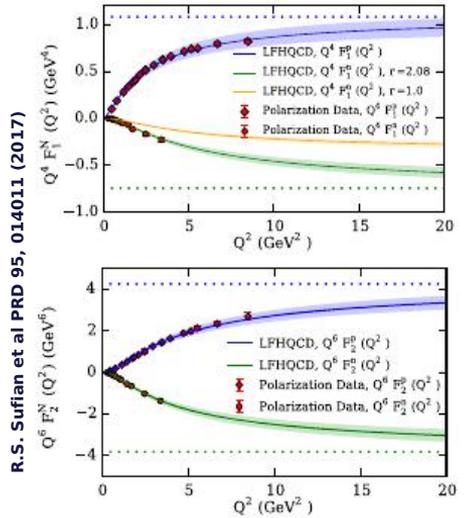


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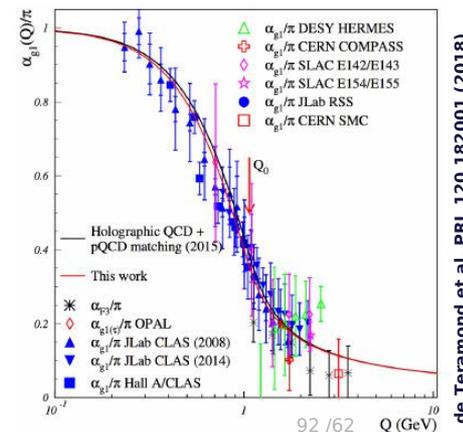
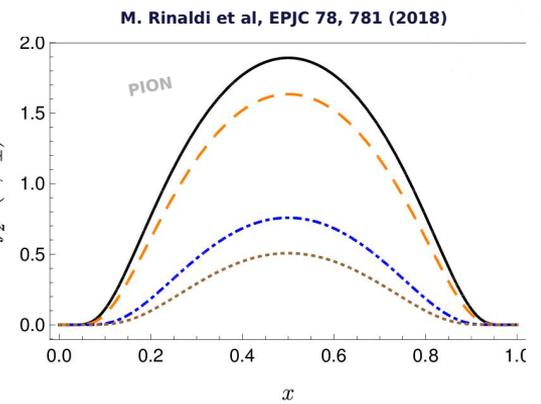
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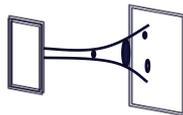


R.S. Sufian et al PRD 95, 014011 (2017)

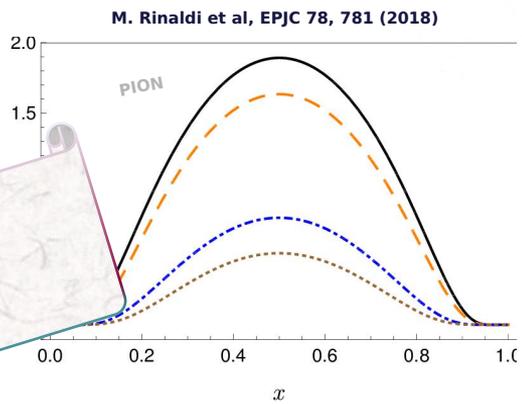
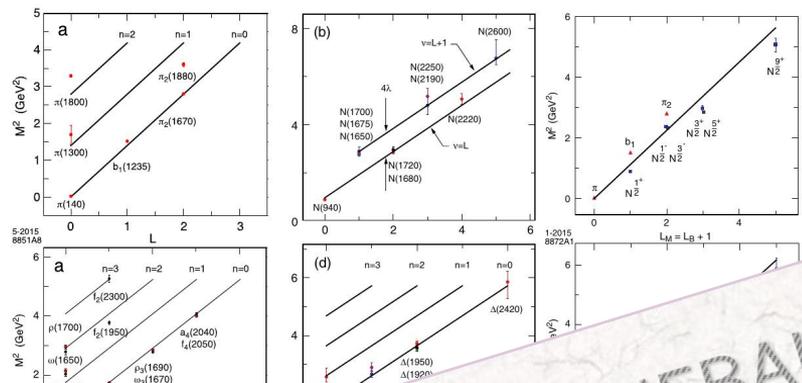
MATCHING THE RUNNING COUPLING



de Teramond et al, PRL 120,182001 (2018)



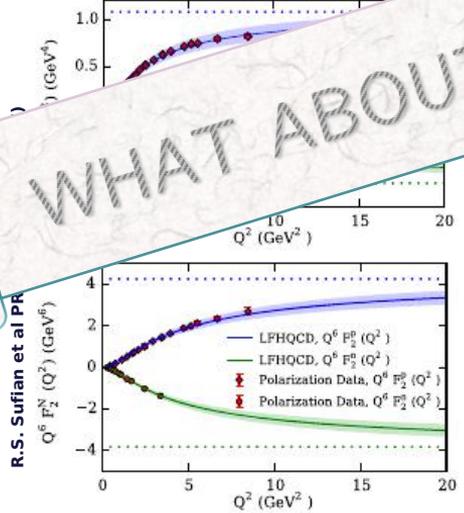
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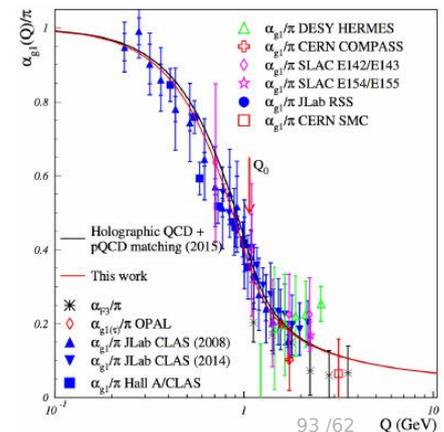
see Brodsky's talk on Thursday

WHAT ABOUT GLUEBALLS?

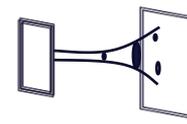
FORM FACTORS, PDFs & GPDs



MATCHING THE RUNNING COUPLING



de Teramond et al, PRL 120,182001 (2018)

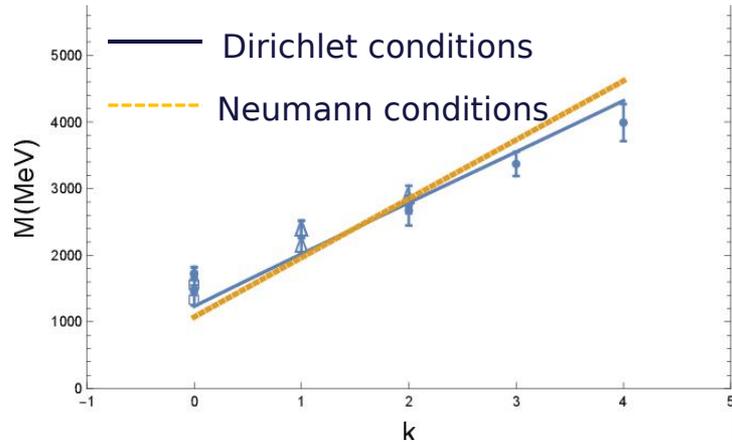


In this case we have the following $AdS_5 \times S_5$ metric : $ds^2 = g_{MN}dx^M dx^N + R^2 d\Omega_5 = \frac{R^2}{z^2} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu) + R^2 d\Omega_5$

In the **hard-wall (HW)** model confinement is implemented by imposing the following IR cutoff: $0 \leq z \leq z_{\max} = \frac{1}{\Lambda_{QCD}}$

0^{++} \otimes 2^{++} GLUEBALL SPECTRA

M.Rinaldi and V. Vento EPJA 54 (2018)



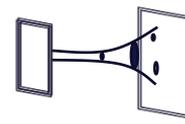
Good agreement!

However the **HW** model does not reproduce the meson spectrum.

$$M_n^2 \sim n^2$$

In order to have a unified view we need another model, i.e.: the **Soft-wall** model!





karch et al, PRD 74, 015005 (2006)

In the original model we have: $g_{MN}dx^M dx^N = \frac{R^2}{z^2} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$

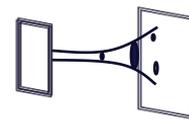
but a soft **cutoff** to space-time is obtained by adding a **dilaton** field in the action:

$$\mathcal{I} = \int d^5x \sqrt{-g} e^{-\varphi(x)} \mathcal{L}$$

Successful in describing the Regge behavior of the spectrum:

$$M_{n,j}^2 \sim n + j, \quad j \geq 0$$

WHAT ABOUT GLUEBALLS?



In this case we have the following $AdS_5 \times S_5$ metric: $ds^2 = g_{MN}dx^M dx^N + R^2 d\Omega_5 = \frac{R^2}{z^2} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu) + R^2 d\Omega_5$

We consider the profile function: $\varphi(z) = \kappa z^2$

SCALAR FIELD EQUATION:

Equation of motion of the scalar glueball can be obtained:

$$I = \int d^5x \sqrt{g} e^{-\varphi(z)} \left[g^{MN} \partial_M \mathcal{G} \partial_N \mathcal{G} + M_5^2 \mathcal{G}^2 \right] \Delta = \text{conformal dimension}$$

Dilaton field $\Delta = 2 + \sqrt{4 + M_5^2 R^2}$

- 1) scalar glueball state 0^{++} is represented by: $\mathcal{O}_{\Delta=4} = \text{Tr}(F^{\mu\nu} F_{\mu\nu})$
- 2) For example for even spin J : $\mathcal{O}_{\Delta=4+J} = \text{FD}_{\{\mu_1 \dots \mu_J\}} F$

The equation of motion for the scalar is:

$$-\Psi''(z) + \left[z^2 + \frac{15}{4z^2} + 2 \right] \Psi(z) = M^2 \Psi(z)$$

where:

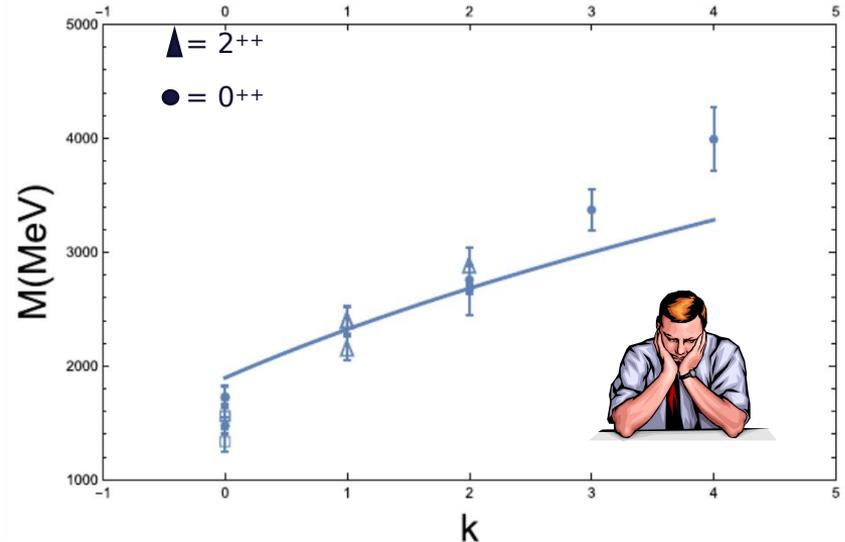
$$\mathcal{G}(x, z) = e^{iP_\mu x^\mu} \left(\frac{z}{R} \right)^{3/2} e^{\kappa^2 z^2 / 2} \Psi(z), \quad P^2 = -M^2$$

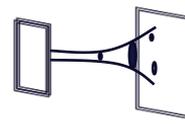
Eduardo Folco Capossoli et al, PLB 753 (2019) 419-423

SCALAR GLUEBALL SPECTRUM:

$$M_J^2 = 4k + 4 + 2\sqrt{4 + J(J+4)} = 4k + 8$$

$\begin{cases} k = 0, 1, \dots & \text{scalar} \\ k = 1, 2, \dots & \text{tensor} \end{cases}$





In **M.Rinaldi and V. Vento EPJA 54 (2018)** we propose to use a soft-wall graviton (GSW) model.
In this case a dilatonic cutoff is incorporated in the metric:

$$\tilde{g}_{MN} dx^M dx^N = e^{-\alpha\varphi(z)} \frac{R^2}{z^2} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

However, a dilatonic contribution in the action can still be kept:

$$\tilde{\mathcal{I}} = \int d^5x \sqrt{-\tilde{g}} e^{-\beta\varphi(x)} \mathcal{L}$$

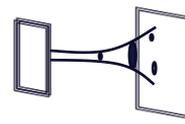
In order to preserve the good description of the hadronic spectrum we require:

$$\int d^5x \sqrt{-\tilde{g}} e^{-\beta\varphi(x)} \mathcal{L} \sim \int d^5x \sqrt{-g} e^{-\varphi(x)} \mathcal{L}$$

kinetic term

$$\frac{3\alpha}{2} + \beta = 1$$

WHAT ABOUT GLUEBALLS?

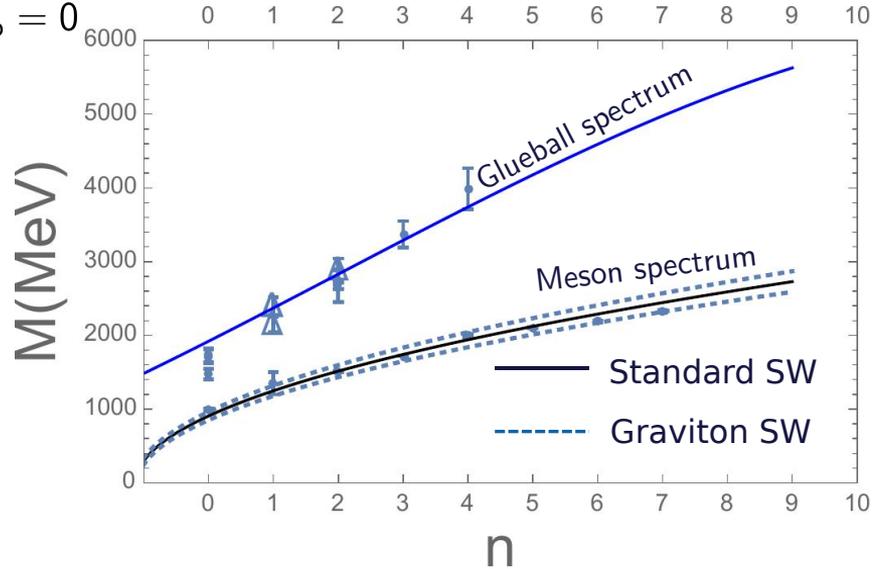


In this case we have the following AdS₅ metric : $\tilde{g}_{MN}dx^M dx^N = \underbrace{e^{-\alpha\varphi(z)}}_{\alpha k^2} \frac{R^2}{z^2} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$

In **M.Rinaldi and V. Vento EPJA 54 (2018)** we consider αk^2 as the only **one parameter!**

GRAVITON EoM and SPECTRUM

$$-\frac{1}{2}\tilde{h}_{ab;c}^c - \frac{1}{2}\tilde{h}_{c;ab}^c + \frac{1}{2}\tilde{h}_{ac;b}^c + \frac{1}{2}\tilde{h}_{bc;a}^c + 4\tilde{h}_{ab} = 0$$



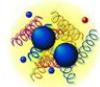
Also in this case we have a good description of data, but now (w.r.t. the HW model): **we have a complete description of the meson and glueball spectra**

Glueball and meson states could mix!

J^{PC}	0^{++}	2^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222
YC	1719 ± 94	2390 ± 124	
LTW	1475 ± 72	2150 ± 104	2755 ± 124

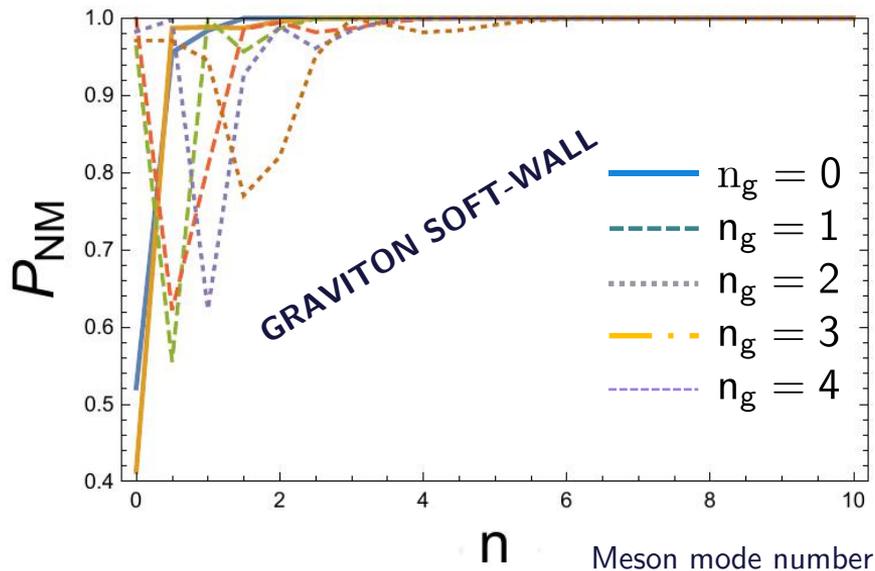


Meson	$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(2020)$	$f_0(2100)$	$f_0(2200)$
PDG	475 ± 75	990 ± 20	1350 ± 150	1504 ± 6	1723 ± 6	1992 ± 16	2101 ± 7	2189 ± 13



We define the probability for NO MIXING as: $P_{mg} \equiv 1 - |\langle \psi^g | \psi^m \rangle|^2$

M. Rinaldi and V. Vento arXiv:1803.05738



Meson wave function

Gluon wave function

Within the Soft-Wall AdS/QCD models (standard and with graviton) pure glueballs in the scalar sector exist in the mass range above 2 GeV!



5

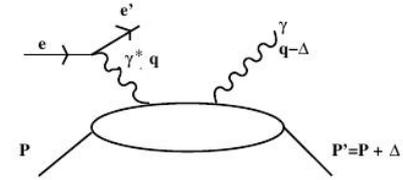
**CONCLUSIONI:
AGGIORNAMENTI
E
PROSPETTIVE**



In seguito ai lavori di successo di Sergio Scopetta e Sara Fucini, riguardo il calcolo e il confronto con i dati delle asimmetrie (combinazioni e rapporti di sezioni d'urto) del processo DVCS su ^4He , ho iniziato a provare interesse per questo argomento:

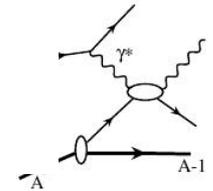
■ Coherent case S. Fucini, S. Scopetta and M. Viviani, PRC98 (2018), no.1, 093001

- * I.A. calculation of the GPD H within a non-diagonal spectral function based on the AV18 + UrbanaIX interaction, realistic only in the ground part; Nucleonic model: GK
- * Forward limit and nuclear FFs recovered, momentum SR slightly violated
- * Numbers for CFFs, X-sections, BSA



■ Incoherent case S. Fucini, S. Scopetta and M. Viviani, PRD101 (2020) no.7, 071501

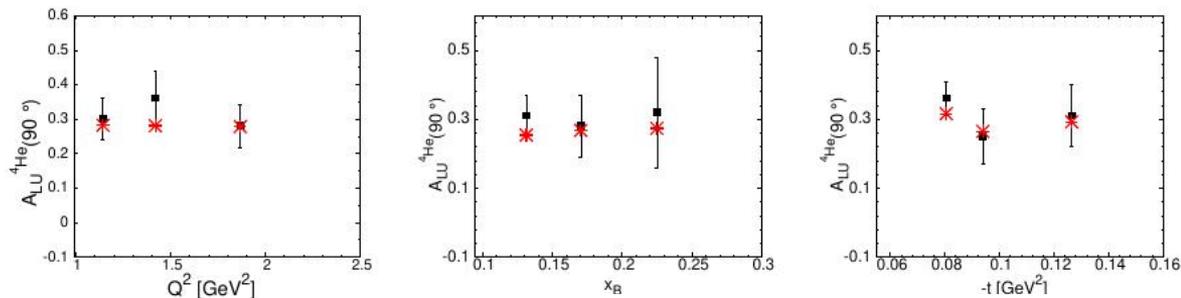
- * I.A. calculation of cross sections within a diagonal spectral function based on the AV18 + UrbanaIX interaction, realistic only in the ground part; cross section developed for a bound proton; Nucleon model: GK, MMS; numbers for X-sections, BSA



In seguito ai lavori di successo di Sergio Scopetta e Sara Fucini, riguardo il calcolo e il confronto con i dati delle asimmetrie (combinazioni e rapporti di sezioni d'urto) del processo DVCS su ^4He , ho iniziato a provare interesse per questo argomento:

COERENTE

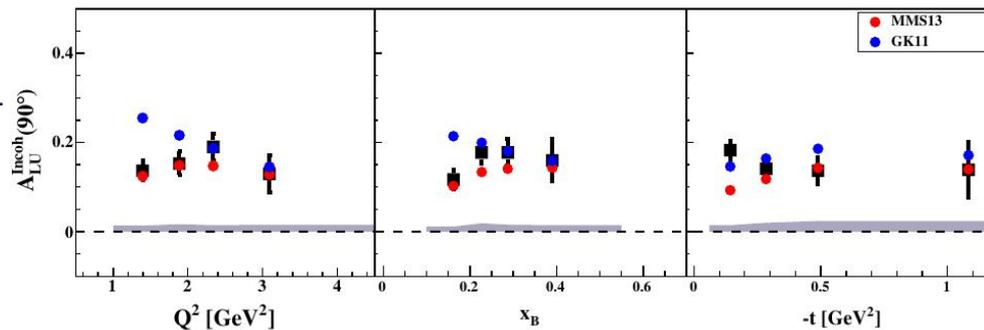
S. Fucini, S. Scopetta and M. Viviani, PRC98 (2018), no.1, 093001



$$A_{LU}(\phi) = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

INCOERENTE

S. Fucini, S. Scopetta and M. Viviani, PRD101 (2020) no.7, 071501



Dati i positivissimi risultati, è in corso una collaborazione con i colleghi sperimentali di Orsay per costruzione un **generatore Monte Carlo di eventi!** Questo passo è fondamentale perché:

- Mandatory for the EIC setup
- To realize the feasibility of relevant measurements (e.g., at ξ and $-t$ high enough, look for non-nucleonic degrees of freedom at parton level (Berger, Cano, Diehl and Pire PRL 2002))
- Very useful in general (JLab @ 12 GeV!)
- Ultimate Goal:
a *new* event generator, based on the FOAM library, *flexible* (different light nuclei (d , ^3He , ^4He ... ^7Li ?); different setups (fixed target, collider)), *open* (different available models to be implemented), *accessible* to interested colleagues for their studies
- Medium term (weeks? Months?):
 - introduce shadowing in the description (a discussion with M. Strikman has started);
 - introduce other nuclei (^3He cross section evaluation in progress from our GPDs (with M. Rinaldi));
 - incoherent channels (S. Fucini, S.S., M. Viviani, Phys. Rev. C101 (2020) no.7, 071501)
 - other people's models for the cross sections

Calcolo sezioni d'urto ed asimmetrie per il processo DVCS su ^3He

Implementazione del generatore Monte Carlo per la simulazione di dati per il processo DVCS su ^3He .

Introduzione degli effetti dei gluoni nel calcolo delle GPDs nucleari in collaborazione con Sergio Scopetta, Michele Viviani e Mark Strikman

Studio dell'effetto EMC attraverso la funzione spettrale di ^3He includendo formalmente effetti relativistici tramite l'approccio Light-Front, in collaborazione con Sergio Scopetta, Emanuele Pace e Giovanni Salmè.

Confronto con i dati di lattice per il nucleone: C. Zimmermann, arXiv:1911.05051

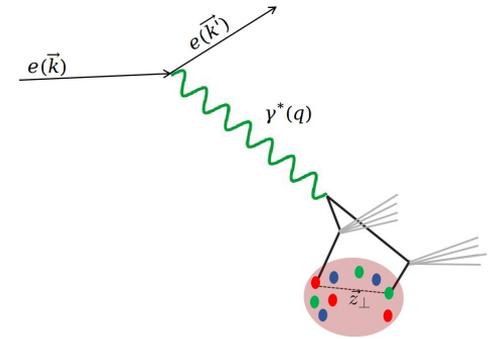
Studio del DPS ad EIC via processi iniziati da Fotoni.

Questo progetto è finanziato dai “Fondi Ricerca di Base” dell’Università degli Studi di Perugia:

“Photon initiated double parton scattering: illuminating the proton parton structure”

PI: Matteo Rinaldi

Collaborators: Sergio Scopetta, Livio Fanò, Simone Pacetti ed Alessandro Rossi



GRAZIE PER L'ATTENZIONE!!

