GRAN SASSO SCIENCE INSTITUTE
CENTER FOR ADVANCED STUDIES
INFN

Sensitivity Study of CYGNO

G. Dho, E. Baracchini

Summary

- Introduction
- Fake Experiments
  - Theoretical Distributions
  - Procedure
- Discrimination Technique
- Results

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Goal of The Work

- The goal is to find the curve in the cross section–mass plot, representing the 3 σ sensitivity of the experiment

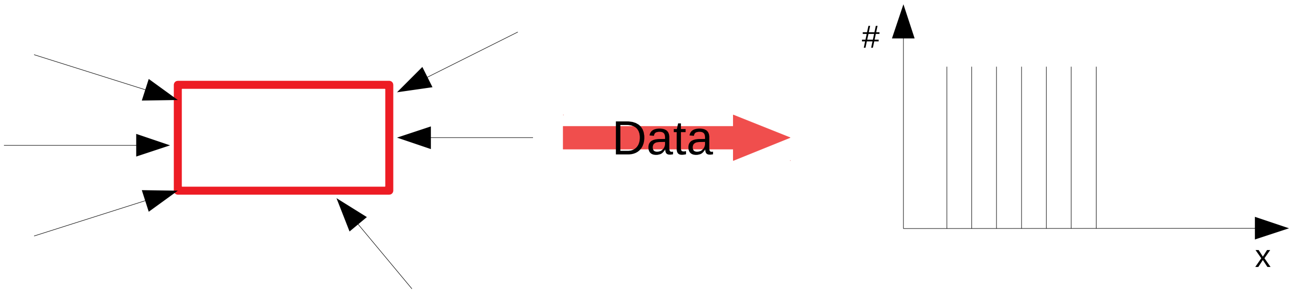
It answers the question: **How many events of WIMP–induced recoils do we need to determine there is a signal with a significance of 3 σ, when expecting μ_b background recoils?**

- The method used exploits a MC technique in a frequentist approach

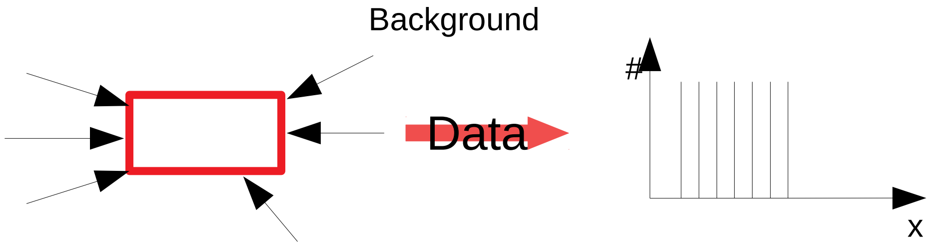
Based on repetition of a MC procedure to extract statistical information

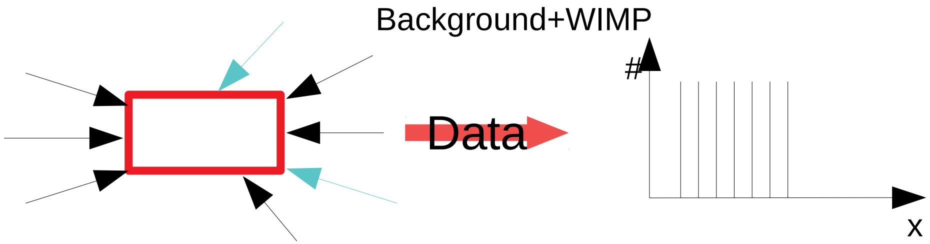
Basic Working Operation

- The MC procedure consists in the repetition of fake experiments



- Experiments with pure background or background +WIMP





- The extended profile likelihood ratio method will be used to find the 3 σ significance

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Background

- The background expected for the two possible kind of information from the detector is:

Energy

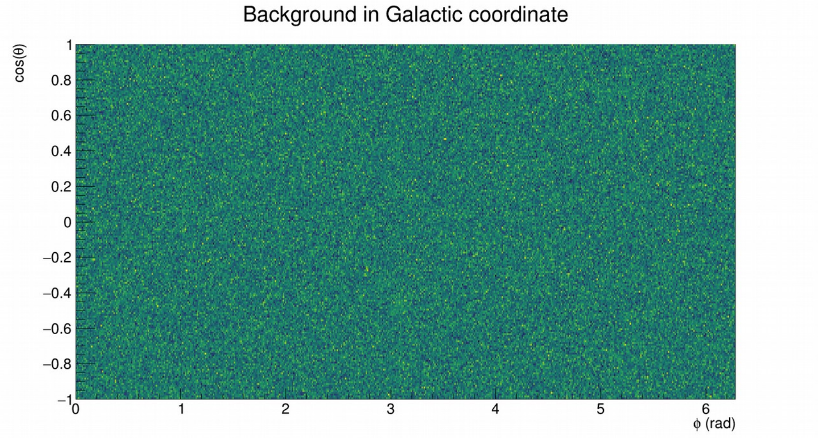
– Unknown shape (similar to an exponential)

– Less discriminative power than angle

2D angle

– Unknown shape

– At first order, in Galactic coordinates most of the background should dilute and look isotropic

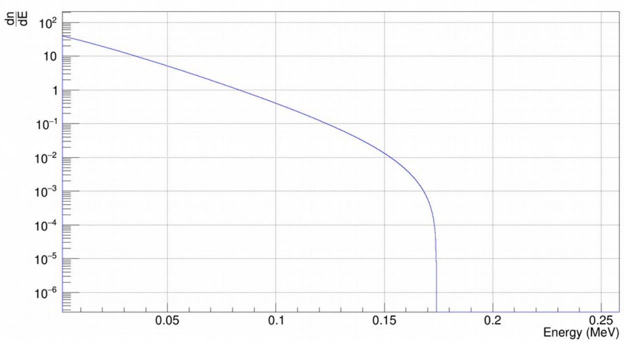


Being easier to model and more powerful in discrimination, the angular information will be used

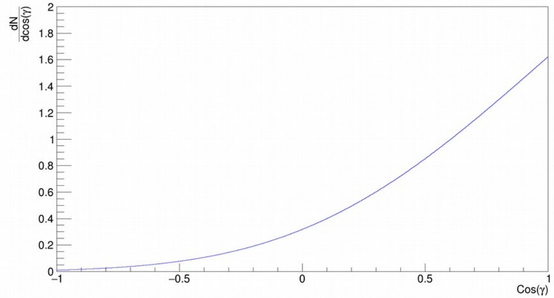
SIGNAL

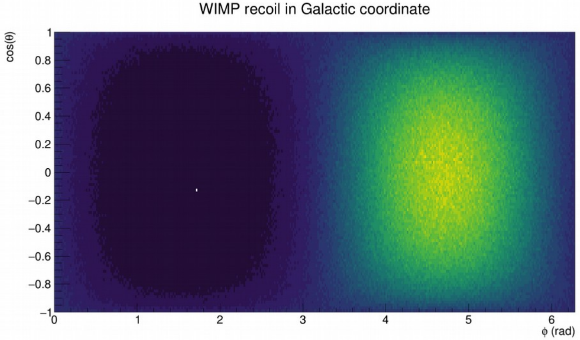
- From WIMP theory, it is possible to calculate:

Energy spectrum



Angular spectrum





Probability of hitting an element

Important during simulation to extract the element hit by the DM particle

Energy Spectrum Calculation

- There are different simple ways to obtain the spectra when neglecting the Galactic escape velocity. When considering it, it becomes harder.

- Following Lewin-Smith paper (https://doi.org/10.1016/S0927-6505(96)00047-3)

$$dR = \rho_p \times \rho_t \times f(v)\, d^3 v$$

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of projectile (DM)

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Velocity distribution of projectile

- Which becomes

$$\frac{dR}{dE_R} = \sigma_0 \times C \times g(El, m_\chi) \times \frac{f(v)}{v} d^3 v$$

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WIMP-proton cross-section

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Constant factors

Function depending on the element of target and of WIMP mass

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Velocity distribution integral

Velocity Integral

- To perform the velocity integral, both Sun velocity (v_E) and escape velocity (v_{esc}) must be considered

$$\int_{v_{min}(E_R)}^{\infty} \frac{v}{k_1} e^{-(|\vec{v}+\vec{v}_E|/v_0)^2} \Theta\left(v_{esc}-|\vec{v}+\vec{v}_E|\right) dv\, d\cos\theta\, d\varphi$$

$$|\vec{v}+\vec{v}_E| = \sqrt{v^2+v_E^2+2\,v\,v_E\cos\theta}$$

- So the energy spectrum is:

$$\frac{dR_Z}{dE_R} = \frac{N_0}{A_M} N_Z \frac{2\rho_0}{m_\chi^2 r} \sigma_p A^2 F^2 \left(\frac{\mu_A}{\mu_p}\right)^2 \frac{k_0}{2v_e k_1} \times$$

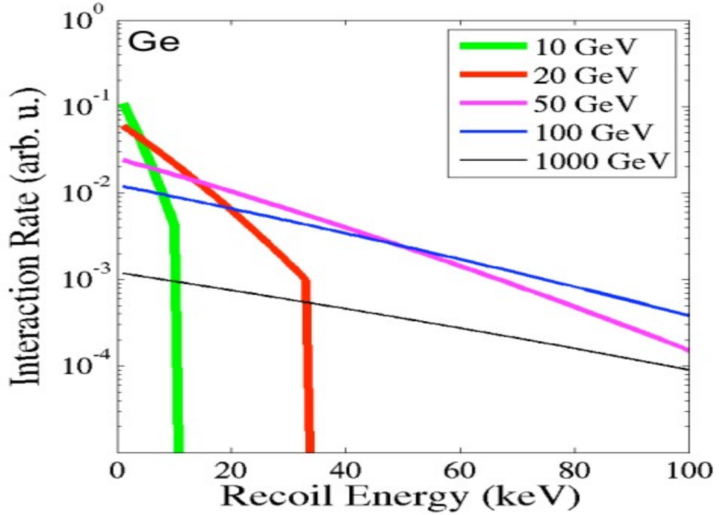
$$\begin{cases} \left[\mathrm{erf}\left(\frac{v_{min}(E_R)+v_e}{v_0}\right) - \mathrm{erf}\left(\frac{v_{min}(E_R)-v_e}{v_0}\right) - \frac{4v_e}{\sqrt{\pi}v_0} e^{-\frac{v_{esc}^2}{v_0^2}}\right] & v_{min} < v_{esc} - v_e \\ \left[\mathrm{erf}\left(\frac{v_{esc}}{v_0}\right) - \mathrm{erf}\left(\frac{v_{min}(E_R)-v_e}{v_0}\right) - \frac{2}{\sqrt{\pi}v_0}(v_{esc}+v_e-v_{min}) e^{-\frac{v_{esc}^2}{v_0^2}}\right] & v_{esc} - v_e < v_{min} < v_{esc} + v_e \\ 0 & v_{min} > v_{esc} + v_e \end{cases}$$

VESC EFFECT ON ENERGY SPECTRUM

- Cut on the maximum energy of recoil spectrum

$$E_{R_{max}} = 2 m_\chi^2 \frac{M_{Target}}{(M_{Target} + m_\chi)^2} (v_{esc} + v_E)^2$$

El/WIMP	0,5 GeV	1 GeV	10 GeV	100 GeV	
He	0,7	2,2	26,5	46,4	keV
F	0,17	0,67	30,9	171,1	keV



DOI: 10.1142/9789814327183_0014

arXiv:1101.5205v1

- Considering the low energy threshold this could revert to a limit in WIMP mass sensitivity

Thr	1 keV	2 keV	5 keV	15 keV	
He	0,62	0,93	1,72	4,52	GeV
C	0,99	1,46	2,51	5,18	GeV
F	1,23	1,79	3,01	5,95	GeV

Angular Spectrum

- To have the angular spectrum in the lab RF, the integration must be modified to include the angle between recoil and Sun direction. I adapted the calculation from *DOI:10.1103/PhysRevD.66.103513*

$$\frac{dR}{dE_R\, d\cos\gamma} = \left(e^{-\frac{(v_{min}-v_E\cos\gamma)^2}{v_0^2}} - e^{-\frac{v_{esc}^2}{v_0^2}}\right)\Theta\left(\cos\gamma - \frac{v_{min}-v_{esc}}{v_E}\right)$$

- And rearranging

$$\frac{dR}{d\cos\gamma} = \int_{E_{thr}}^{\frac{1}{2}m_\chi r(v_E\cos\gamma+v_{esc})^2} \left(e^{-\frac{(v_{min}-v_E\cos\gamma)^2}{v_0^2}} - e^{-\frac{v_{esc}^2}{v_0^2}}\right) dE_R$$

If low integral extreme is less than high integral extreme

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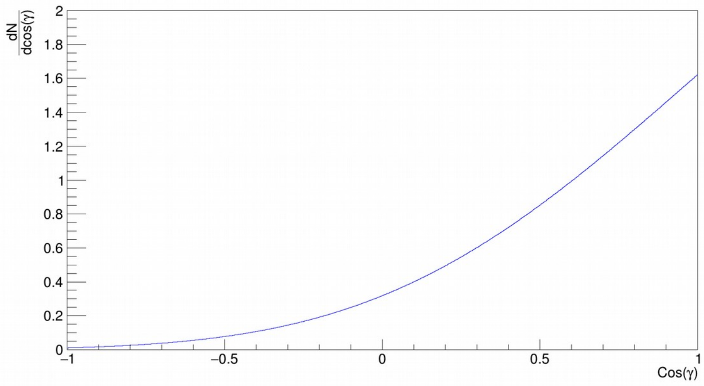
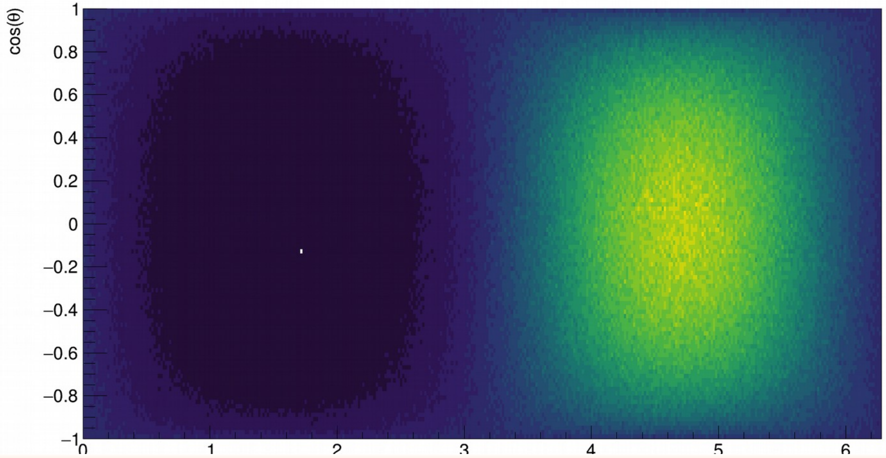
If low integral extreme is less than high integral extreme

According to the angle, not all the energies can contribute, with v_{esc} different than 0

- This allows us to calculate the specta of cosine in lab RF



- **Assuming the detector in the centre of the Sun,** the spectra can be transformed in the Galactic coordinates



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Fake Experiment Creation

- The experiment data will consist in a 2D angular histogram with the values obtained from the simulated events

- Background experiment

1. Expecting μ_b background events
2. Extract x events from Poisson(x,μ_b)
3. For each event the 2D coordinates are extracted from background distribution

- Procedure for background+WIMP experiment

1. Same for background
2. Extract y events from Poisson(y,μ_s,m_χ), with μ_s the expected WIMP events
3. For each event the 2D coordinates are extracted from signal distribution

Assumptions on the Simulation

- **Angular resolution σ_a** is taken as 30x30 deg^2 and is <u>independent</u> on the energy of the recoil
- The histogram containing the data in **binned** so that each bin is ± 1σ_a (so twice as resolution)
- The angular resolution effect is applied to the simulation with a **gaussian smear** centred in zero and with σ_a as standard deviation
- Perfect **head-tail** recognition is considered
- **Low energy threshold** determines the minimum energy detectable, thus the angular spectrum shape
- **High energy threshold** determines is up to the end of the spectrum (200 keV recoils are expected to be visible and contained)
- The probability of hitting a specific element is considered in the simulation, taking into account the **gas mixture**

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Likelihood ratio test

- We planned on using this test:

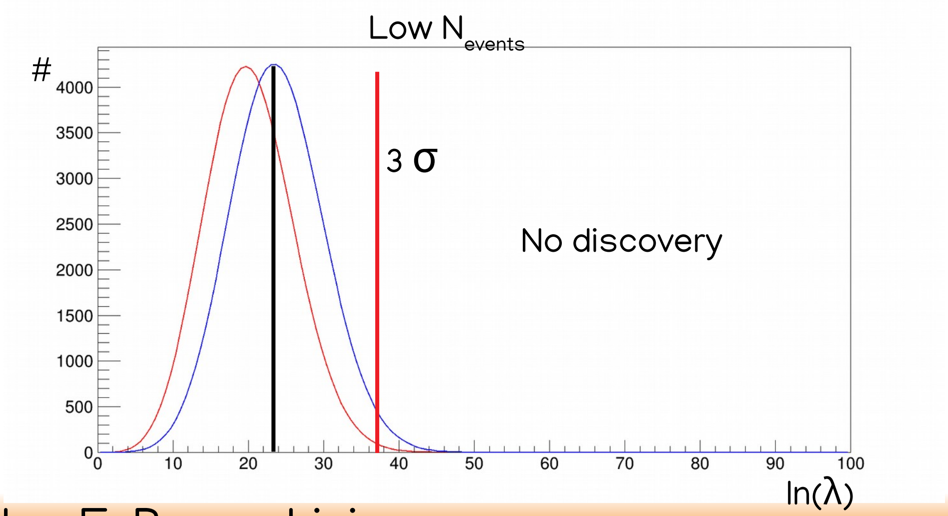
Partially already implemented

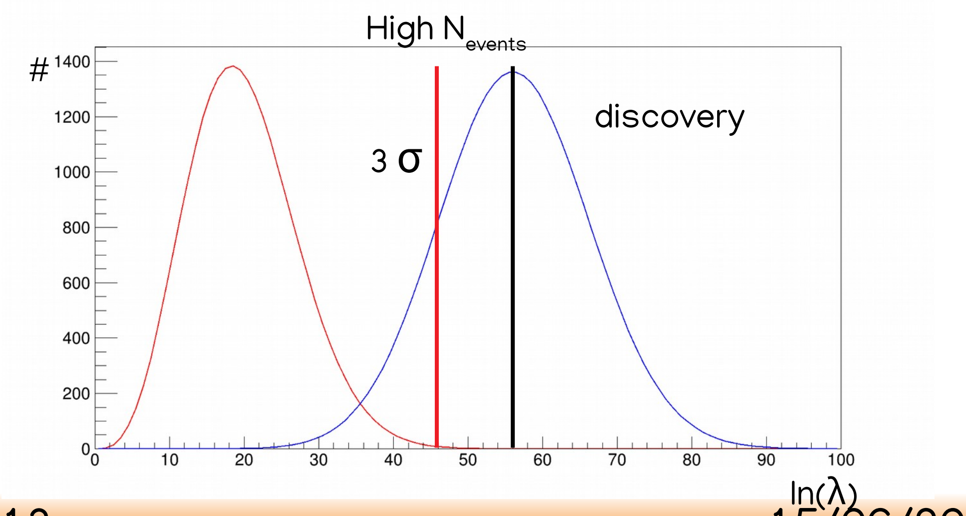
Well established and used technique

- Basic working:
  - You have two hypothesis H_0 (only background), H_1 (background +WIMP)
  - For each fake experiment you calculate the likelihood of data being from H_0 (L_{H0}) and the likelihood of data being from H_1 (L_{H1})
  - It is calculated the ratio of the two $L_r = L_{H1}/L_{H0}$
  - If the simulation was of only background, we call it $L_{r,b}$, otherwise $L_{r,s}$

Likelihood ratio test

- The distribution of $L_{r,b}$ (red) and $L_{r,s}$ (blue) are obtained from the repetition of fake experiments
- Given WIMP mass and μ_s under test, the most probable value of $L_{r,s}$, $\mathcal{L}_{r,s}$ (black), is found with a gaussian fit
- If the p-value of $\mathcal{L}_{r,s}$, estimated on the red distribution, is less than the one tail 3 σ, then μ_s is enough to have 3 σ significativity





- A very simplistic likelihood would be based on the poissonian expectation of the events

$$L_{H0}=\frac{\mu_b^{N_{evt}}}{N_{evt}!}e^{-\mu_b} \qquad L_{H1}=\frac{(\mu_b+\mu_s)^{N_{evt}}}{N_{evt}!}e^{-(\mu_b+\mu_b)}$$

- We can include the directional (angular) information, by multiplying by the probability of having obtained data distributed as the histogram of the fake experiment

$$L_{H0}=\mu_b^{N_{evt}}e^{-\mu_b}\times\prod_{i=1}^{N_{bin}}P_{i,b}^{n_i}\frac{1}{n_i!}$$

n_i: number of events occurred in bin i;

$P_{i,x}$: probability of ending up in bin i according to distribution x

$$L_{H1}=(\mu_b+\mu_s)^{N_{evt}}e^{-(\mu_b+\mu_s)}\times\prod_{i=1}^{N_{bin}}\left(\frac{\mu_b}{\mu_b+\mu_s}P_{i,b}^{n_i}+\frac{\mu_s}{\mu_b+\mu_s}P_{i,s}^{n_i}\right)\frac{1}{n_i!}$$

Likelihood Building

- At first approximation $P_{i,x}$ will represent the probability due to the theoretical distribution the data are extracted from

- To match the input from simulation, the likelihood must take into account:
  - Migration of events from one bin to another due to resolution
  - Probability of the element hit

$$P_{i,x}=\sum_{j=He}^{El}\left(0{,}683\,P'_{i,x,j}+0{,}158\left(P'_{i+1,x,j}+P'_{i-1,x,j}\right)\right)P_j$$

Probability of migration (gaussian approx)

Probability of the event to end up in bin i+1, according to distribution x, and having hit element j

Probability of hitting element j

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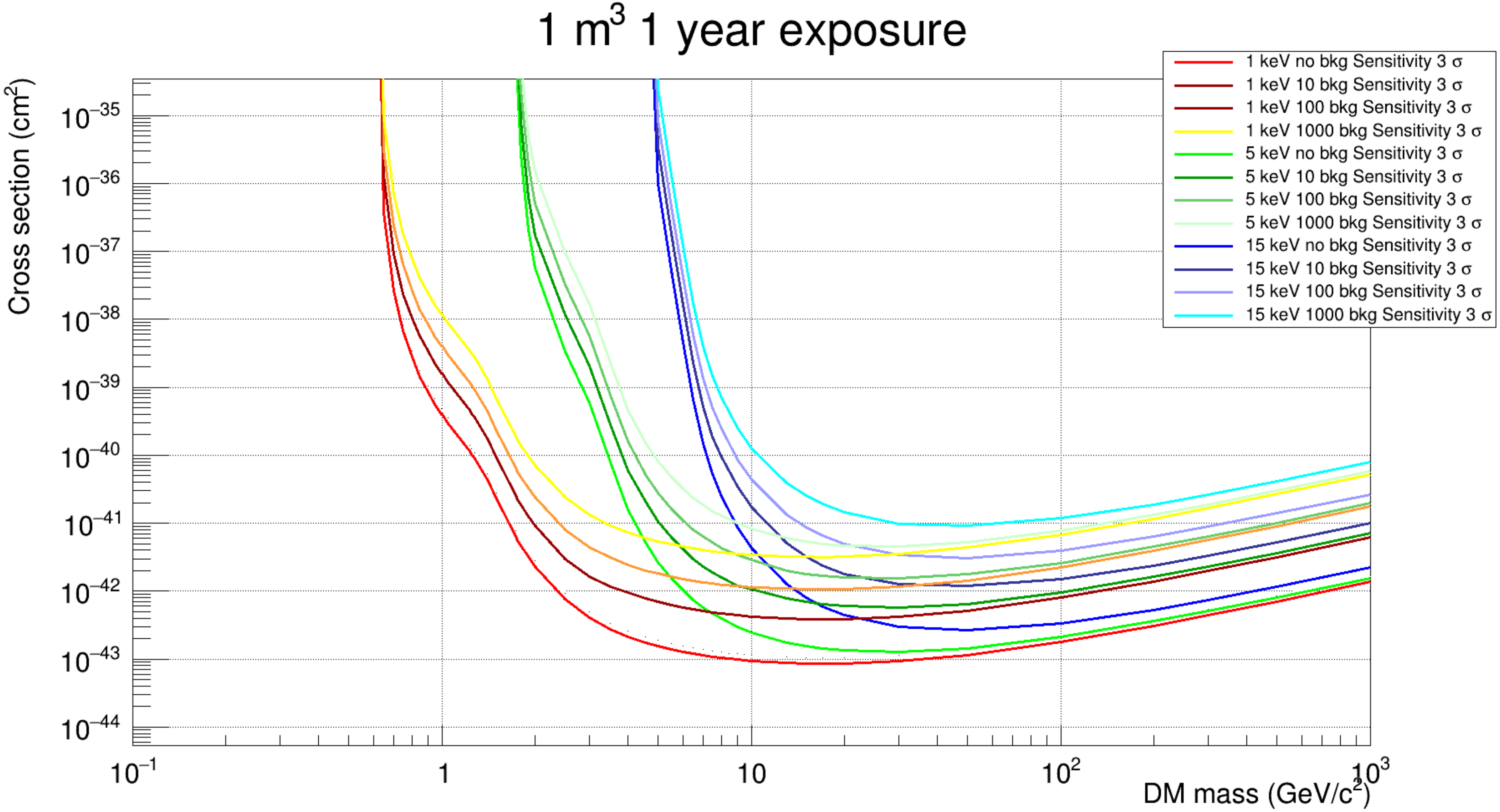
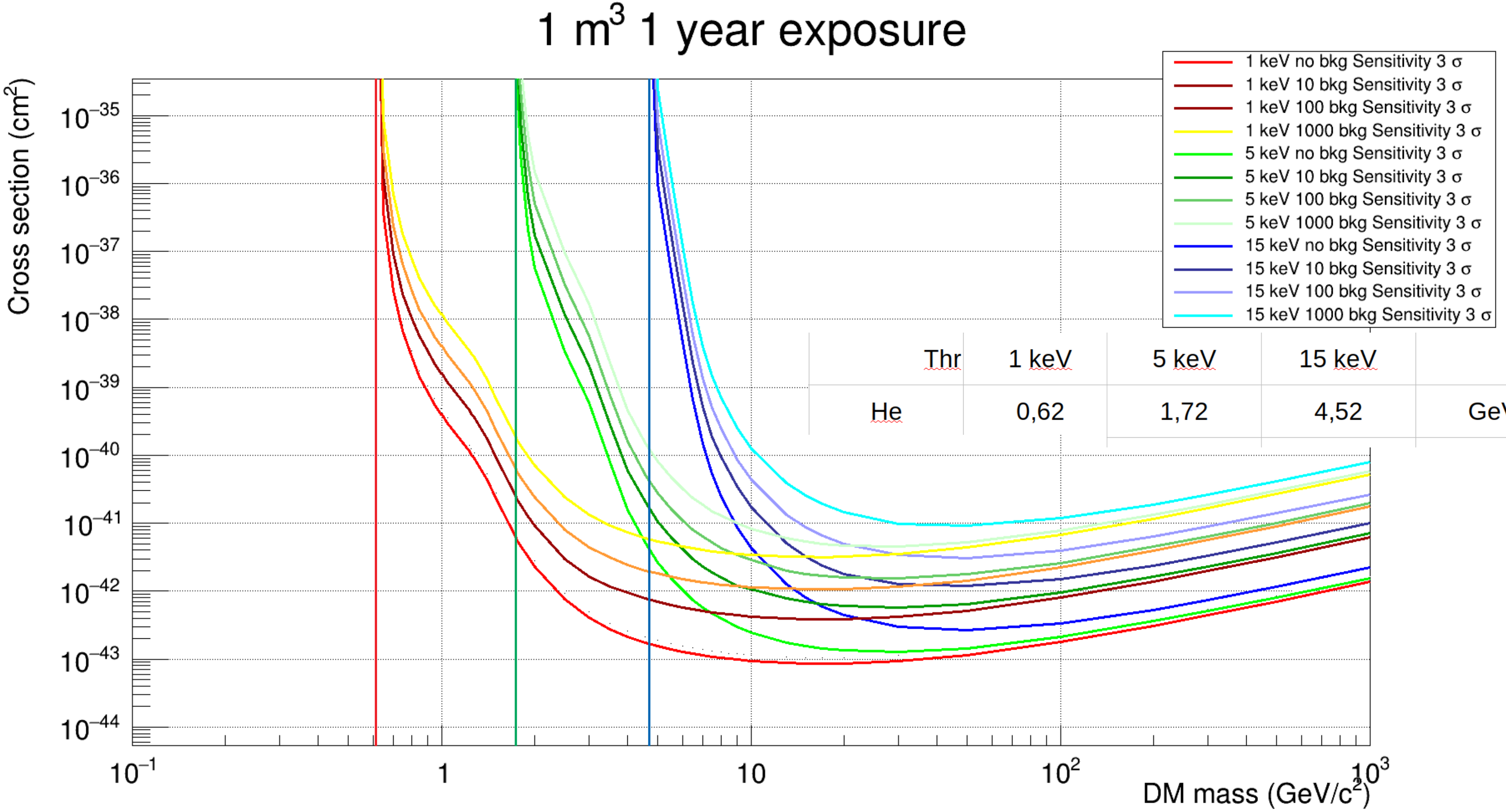
Sensitivity Plot



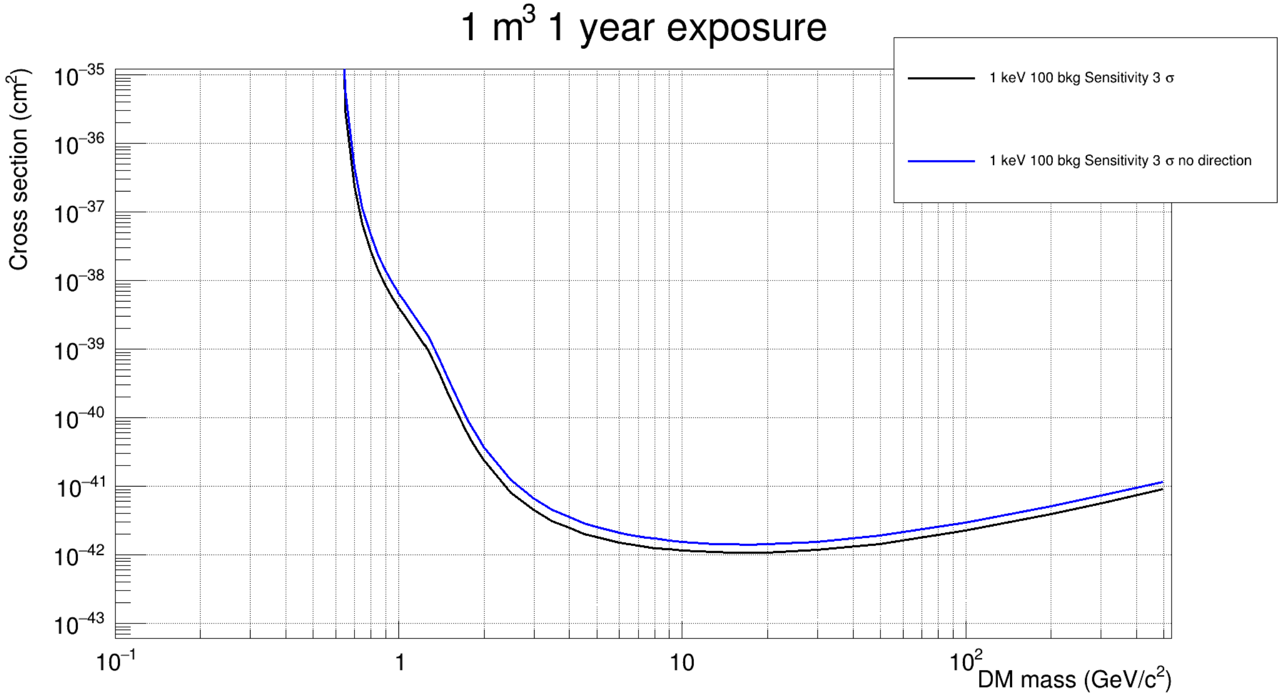
Expected Kinemati Limits



	Thr	1 keV	5 keV	15 keV	
He		0,62	1,72	4,52	GeV

Directionality Effect on Limits

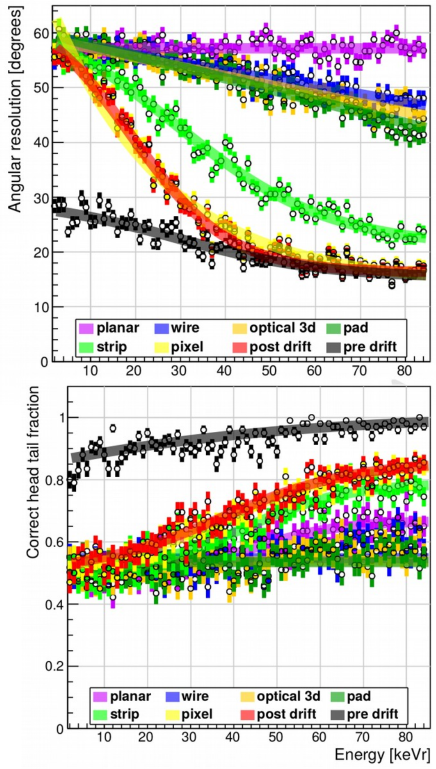
- If the likelihood with the simple Poissonian distribution is taken into account, the sensitivity worsens



- It may look a small effect on log-scale, but the directionality reduces the amount of events needed for discrimination (in this example) from 33 down to 16–22.

Possible Improvements

- It is understood that not all variables are considered and could be added:
  - Dependence of **angular resolution** on energy
  - Dependence of **head-tail recogntion** on energy
  - More precise expectation of the **background shape**
  - More precise effect of the **background rejection**
  - **Combination** of energetic and angular information



CAVEAT: these plots are not definitive and use different gas-mixture. Here just to exemplify what we need

- All of these need input from other works (some are already available, others do not)

Next

- More important and more often shown are the 90% confidence level limits.

- To obtain them, the Bayesian approach should help smoothing some calculation for the likelihood part, while exploiting the simulation structure already implemented.

- All the possible improvements will be added to the new approach.

- This work will be used to cross-check at the beginning the consistency of the two methods.

- Prof. Andrea Messina will help us in this task.

Conclusion

- A study on the 3 σ sensitivity of the CYGNO was performed.

- This study exploits the correct calculation of recoil spectra.

- It exploits the extended profile likelihood ratio in a frequentist approach.

- It has a solid simulation structure that can be used to generate data samples.

- Though not all variables are included, it has interesting results and further complications will be included in the future work on the CL with a Bayesian approach.

Formulas

$$\frac{dR_Z}{dE_R} = \frac{N_0}{A_M} N_Z \frac{2\rho_0}{m_X^2 r} \sigma_p A^2 F^2 \left(\frac{\mu_A}{\mu_p}\right)^2 \frac{k_0}{2 v_e k_1} \times$$

$$\begin{cases} \left[\operatorname{erf}\left(\frac{v_{min}(E_R)+v_e}{v_0}\right) - \operatorname{erf}\left(\frac{v_{min}(E_R)-v_e}{v_0}\right) - \frac{4v_e}{\sqrt{\pi}v_0} e^{-\frac{v_{esc}^2}{v_0^2}}\right] & v_{min} < v_{esc} - v_e \\ \left[\operatorname{erf}\left(\frac{v_{esc}}{v_0}\right) - \operatorname{erf}\left(\frac{v_{min}(E_R)-v_e}{v_0}\right) - \frac{2}{\sqrt{\pi}v_0}(v_{esc} + v_e - v_{min}) e^{-\frac{v_{esc}^2}{v_0^2}}\right] & v_{esc} - v_e < v_{min} < v_{esc} + v_e \\ 0 & v_{min} > v_{esc} + v_e \end{cases}$$

N_0 *Avogadro number in kg^{-1}*

A_M *Molar mass of molecule of gas*

N_Z *Number of atoms of an element in gas molecule*

ρ_0 *Dark matter mass density next to Solar system*

m_X *Dark matter particle mass*

m_A *Target particle mass*

r *Recoil factor* $r = \frac{4 m_A m_X}{(m_A + m_X)^2}$

A *Mass number of the element*

μ_A *Reduced mass of element*

μ_p *Reduced mass of proton*

v_e *Lab velocity*

v_0 *Galactic thermal dispersion velocity*

v_{min} *Minimum velocity to have recoil energy of E_R*

$$v_{min} = \frac{\sqrt{2 m_A E_R}}{2 \mu_A}$$

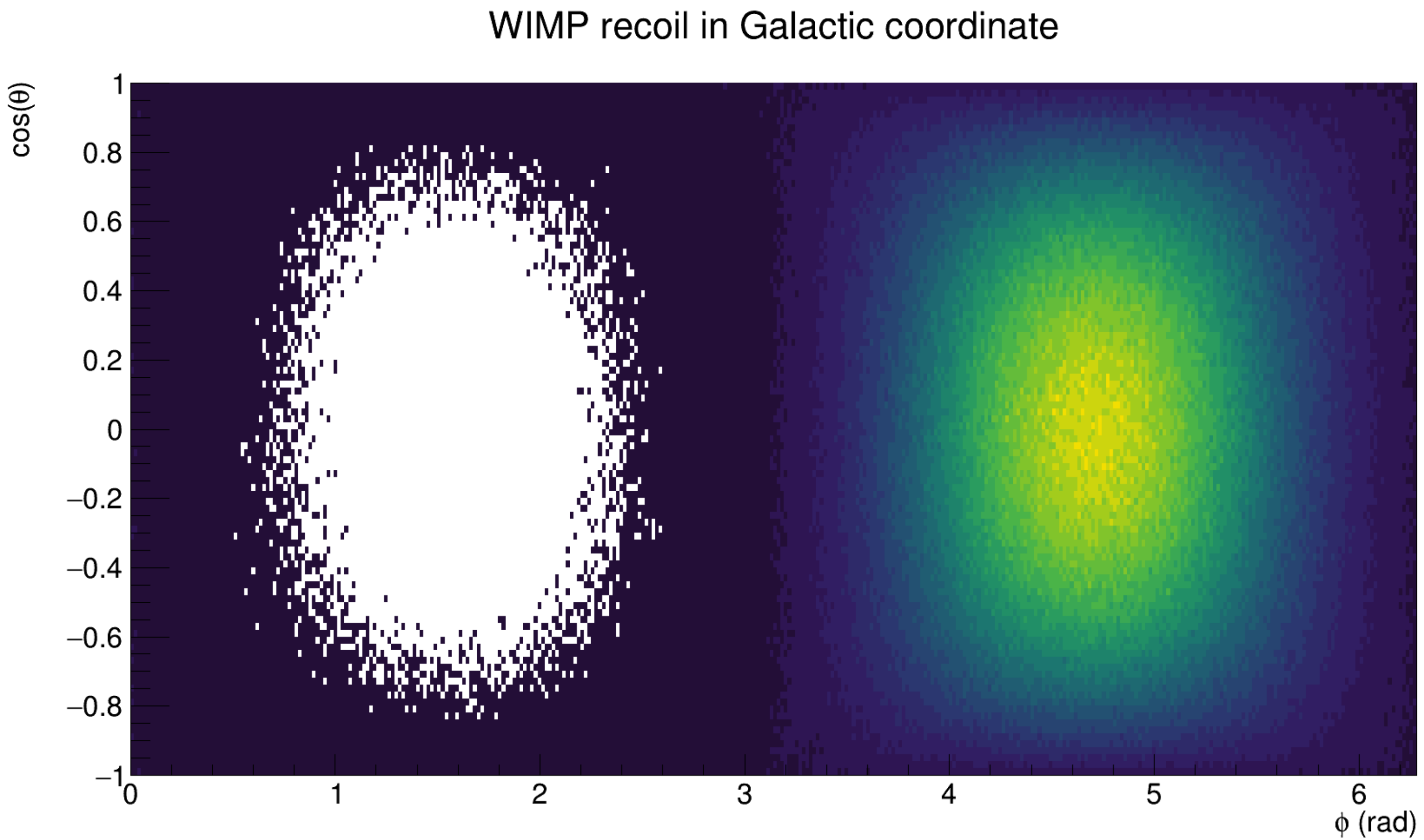
v_{esc} *Galactic escape velocity*

F *Form factor*

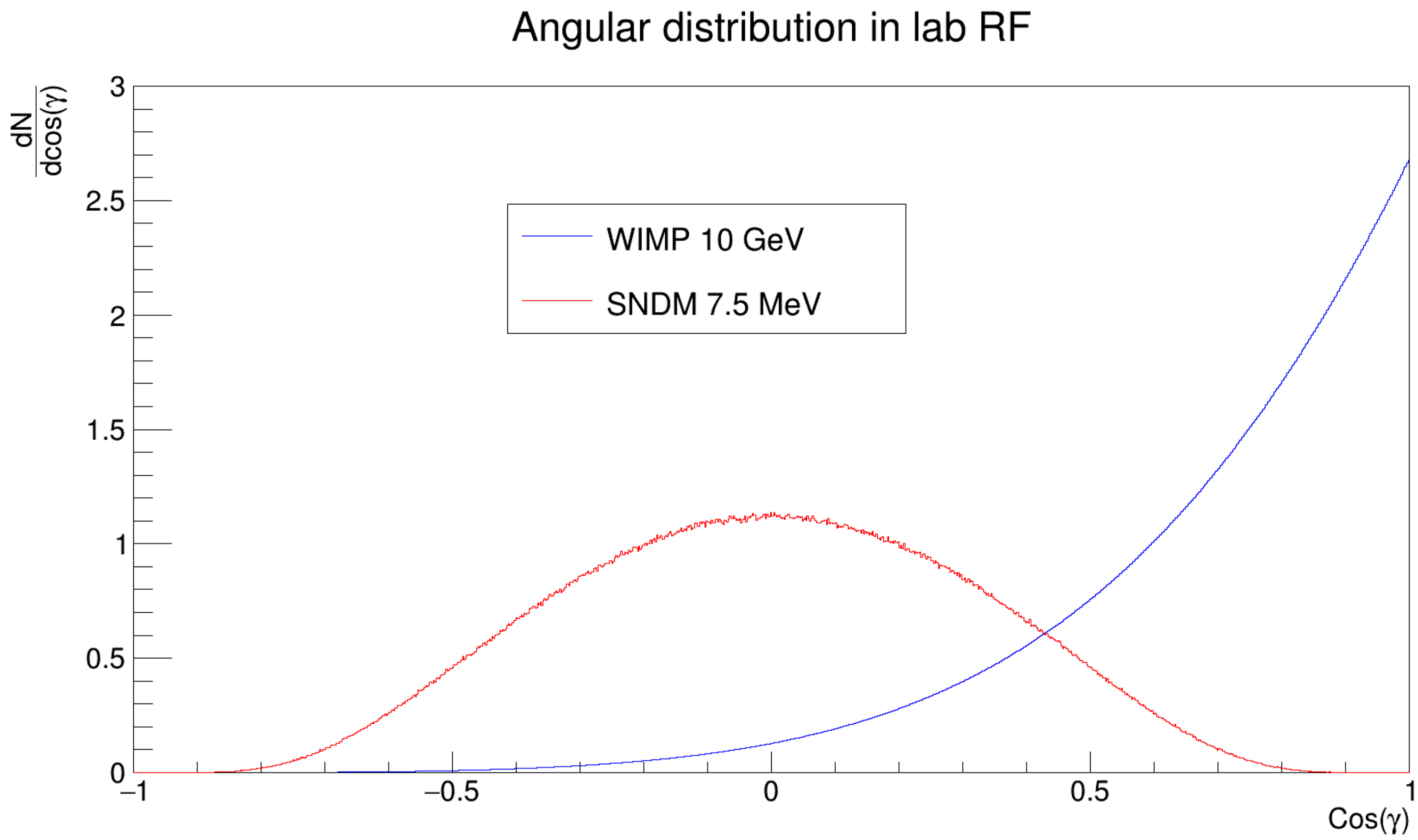
k_0 *Normalization of regular velocity distribution*

k_0 *Normalization of cut velocity distribution*

BACKUP

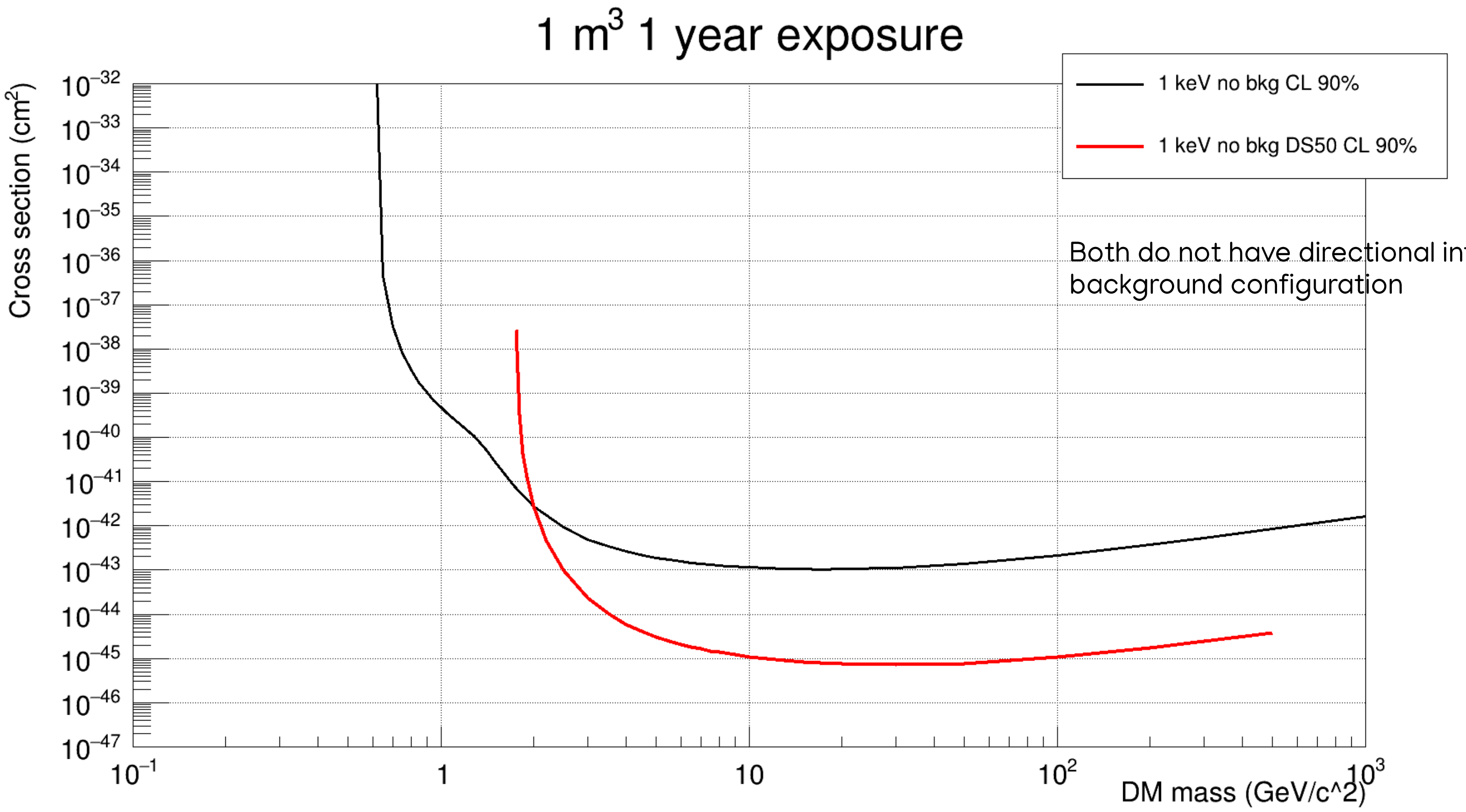

WIMP recoil in Galactic coordinate
cos(θ)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
1
2
3
4
5
6
ϕ (rad)

F 10 GeV



Angular distribution in lab RF
$\frac{dN}{d\cos(\gamma)}$
3
2.5
2
1.5
1
0.5
0
−1
−0.5
0
0.5
1
Cos(γ)
WIMP 10 GeV
SNDM 7.5 MeV

F 10 GeV



Both do not have directional information. Zero background configuration