QD0 Italian style

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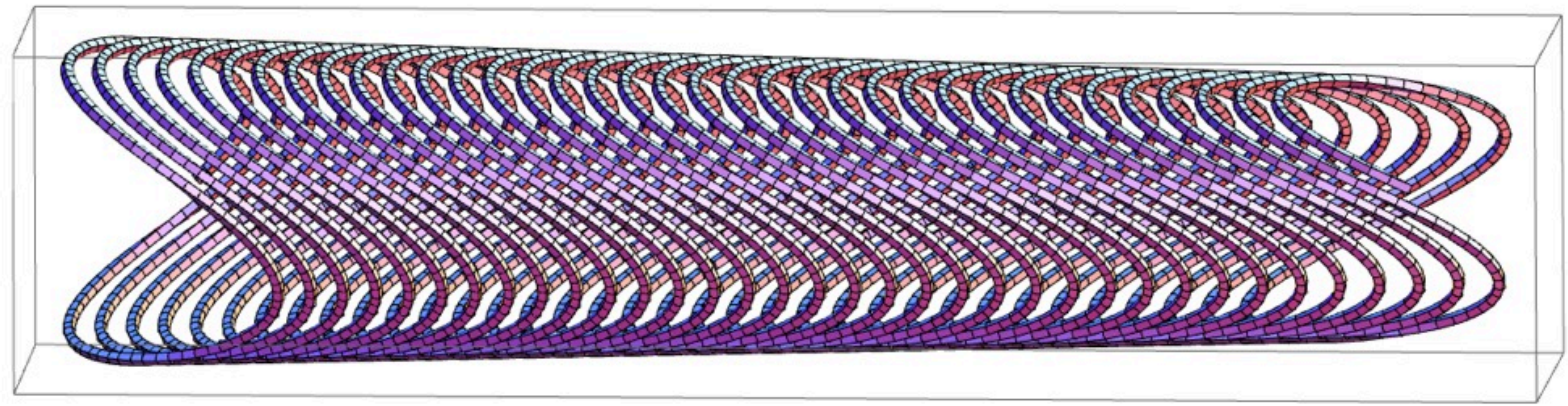
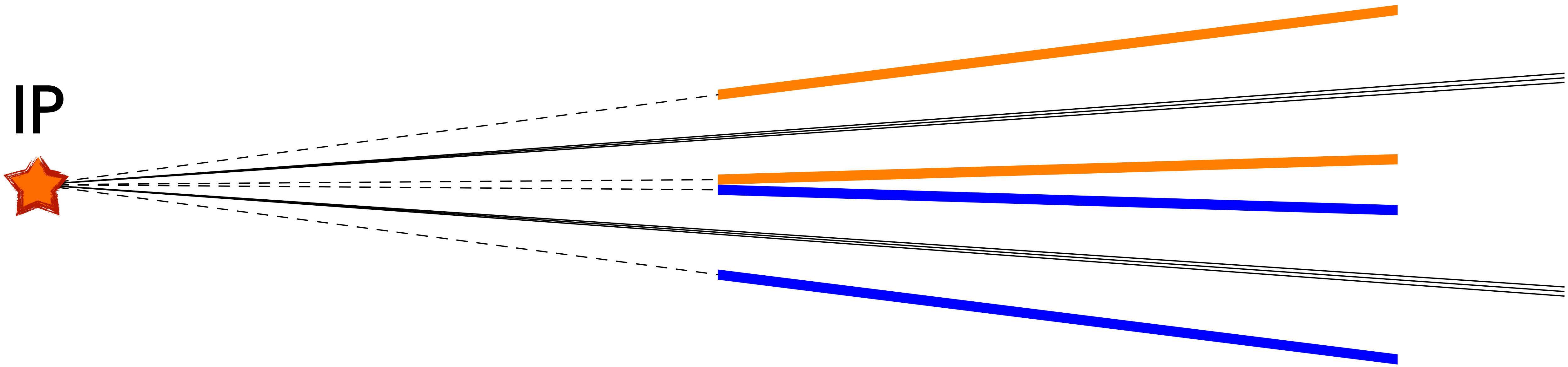
SC wire landed in Genova yesterday!

Conical QD0



- Long standing, pressing request from Pantaleo: "Why not a conical shaped QD0 in Italian sauce?"

Pros & Cons



- The magnet can follow the BSC in a closer way with respect to a cylindrical geometry
- The wires are closer to the beam line, an higher gradient is achievable / less current is needed
- The magnetic axis naturally follows the beam line
- The gradient is not constant along the length
- The field is not a pure quadrupole
- The algebra is harder

Current Source

- 2D Approximation:
  infinite long double cone
- Current density vector tangent to the cone and directed along the cone axis
- Current density concentrated on the cone surface

$$\vec{j} = \delta(\sqrt{x^2 + y^2} - z \tan\alpha) \left[\hat{z}\, \frac{j(\varphi)}{z}\right]$$

$$\nabla \cdot \vec{j} = 0$$

General characteristic of the field

- The source is an homogeneous function:

$$\vec{j}(\beta x, \beta y, \beta z) = \beta \vec{j}(x, y, z)$$

- The magnetic field reflects this scaling

$$\vec{B}(\beta x, \beta y, \beta z) = \beta \vec{B}(x, y, z)$$

- Hence, if the field looks like a good quadrupole at a given z, it will look like a good quadrupole wherever

Poisson equation in "2D"

- In the vacuum chamber we can use the scalar potential to express the B field

$$\vec{B} = \nabla \tilde{\varphi}$$

- The 2D problem in "conical geometry" is:

$$\tilde{\varphi}(x, y, z) = \phi(x\frac{z_0}{z}, y\frac{z_0}{z})$$

$$\left(x^2+1\right)\varphi^{(2,0)} + \left(y^2+1\right)\varphi^{(0,2)} + 2x\varphi^{(1,0)} + 2xy\varphi^{(1,1)} + 2y\varphi^{(0,1)}$$

Does it looks like a good quadrupole?

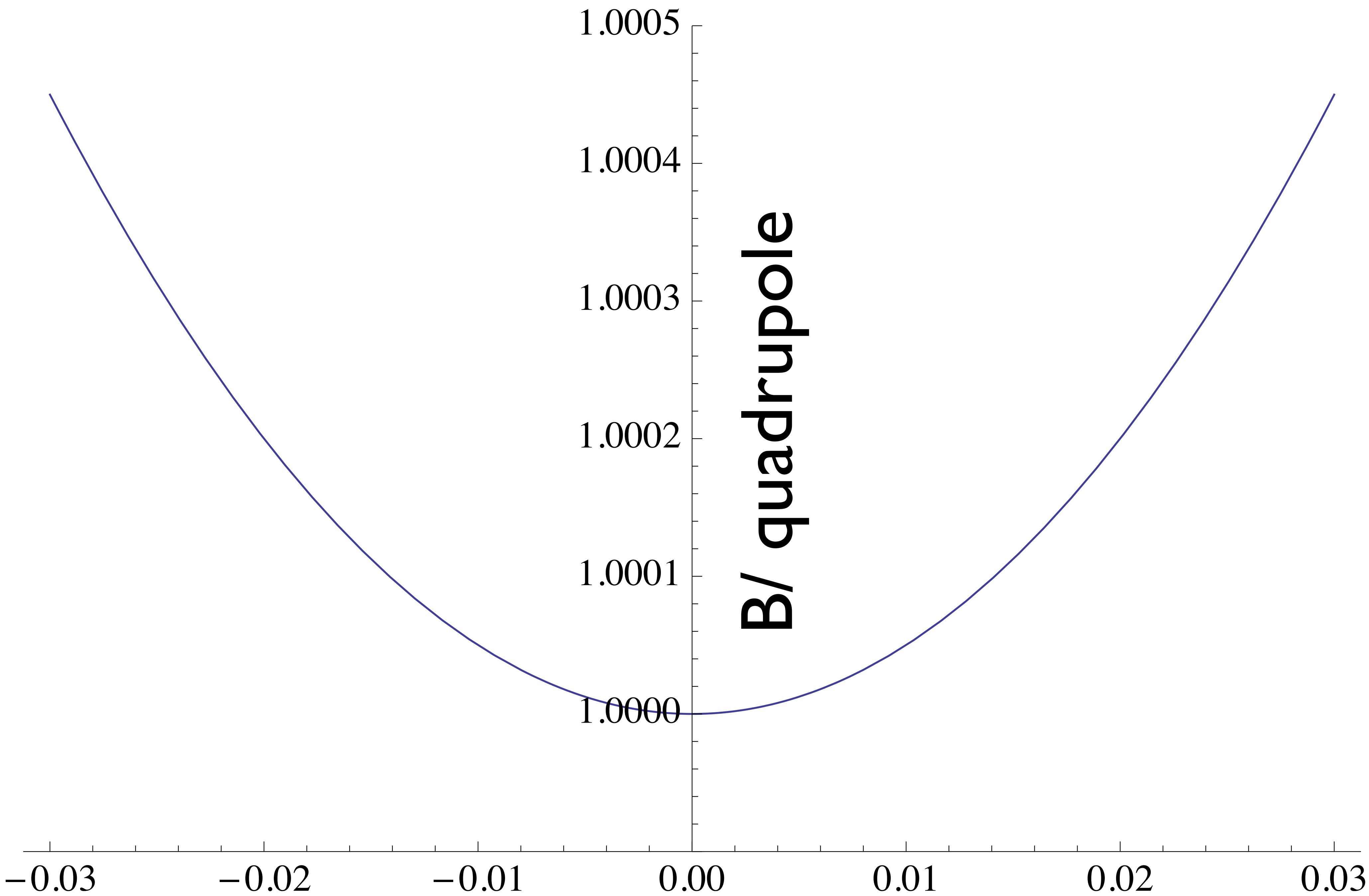
- Mathematica solves algebraically this “2D” Poisson equation. The quadrupole equivalent potential is:

$$\left\{\frac{C[1]+4\,C[2]}{(\mathrm{Cos}[\varphi]-i\,\mathrm{Sin}[\varphi])^2\,\rho^2}+\frac{C[1]+4\,C[2]}{2\,(\mathrm{Cos}[\varphi]-i\,\mathrm{Sin}[\varphi])^2}-\frac{C[1]\,\rho^2}{8\,(\mathrm{Cos}[\varphi]-i\,\mathrm{Sin}[\varphi])^2}+\frac{C[1]\,\rho^4}{16\,(\mathrm{Cos}[\varphi]-i\,\mathrm{Sin}[\varphi])^2}-\frac{5\,C[1]\,\rho^6}{128\,(\mathrm{Cos}[\varphi]-i\,\mathrm{Sin}[\varphi])^2}+O[\rho]^7\right\}$$

- The “octupole” coefficient is half of the quadrupole,

but rho (which is adimensional) cannot exceed the half opening angle…

hence the field is a quadrupole within 5 in 10^{-4}



Field characteristics II

- The gradient scales with the inverse of z^2
- A gradient of 1 T/cm should be feasible at the larger aperture, ~2 T/cm should be possible at the smaller aperture

Compensation

- I am still working on the compensation scheme, I am confident that it can be done
- Is the octupolar component small enough?