IBS Studies for SuperB

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Outline

- IBS different approaches:
  - **Tracking simulation code** most realistic one
  - ***Bane* model** consolidated method for growth times estimates
  - ***Chao* model** novel differential equation system

- Different methods described and compared, they are in agreement

- Results of IBS studies:
  - Bane model for LER SuperB
  - Tracking code, both Bane and Chao models for DAFNE crab-waist lattice

- Plans for future work on IBS

Three methods for IBS

- ***Bane* model**

  Consolidated procedure that allows fast growth times estimates

  High energy approximation for Gaussian beams

- ***Chao* model**

  Novel analytical model able to predict ε_x vs time

  Coupled differential equations valid for Gaussian beams

- **Tracking Simulation code**

  6-D Monte Carlo, it allows the most realistic studies to be done on IBS

  It aims at exploring final equilibrium non-Gaussian tails,

  non-nominal behavior e.g. when vertical emittance gets very small,

  ε_x, ε_y and ε_z evolution in time

Three methods are in good agreement

Bane model* for IBS growth rates calculations

* Ref. [K. Bane, EPAC02, p.1442] , [K. Kubo, S.K. Mtingwa and A. Wolski, PRSTAB **8**, 081001 (2005)]

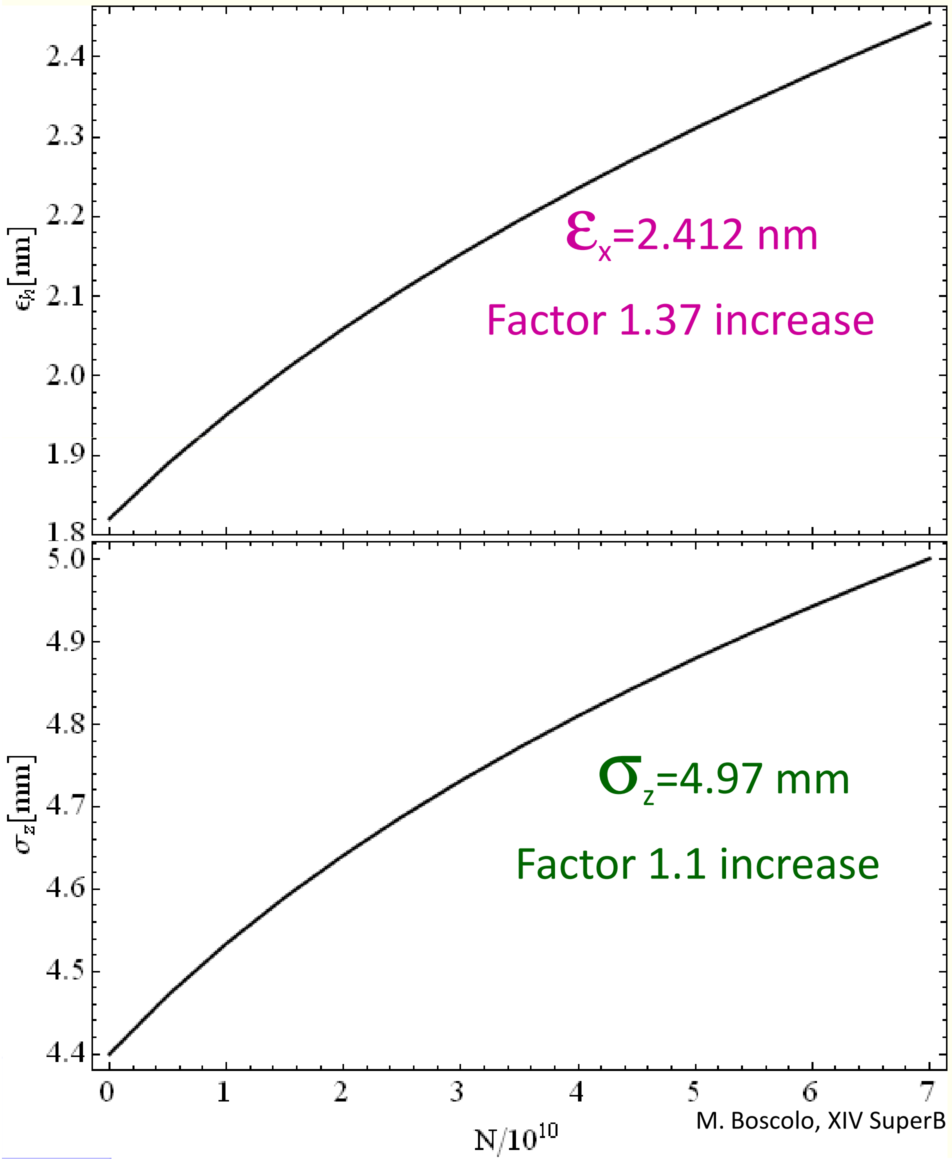
1. Evaluate equilibrium emittances ε_i and radiation damping times τ_i at low bunch charge
2. Evaluate the IBS growth rates $1/T_i(\varepsilon_i)$ for the given emittances, averaged around the lattice, using the **K. Bane high energy approximation**
3. Calculate the "new equilibrium" emittance from: $$\varepsilon_i' = \frac{1}{1-\tau_i/T_i}\varepsilon_i$$

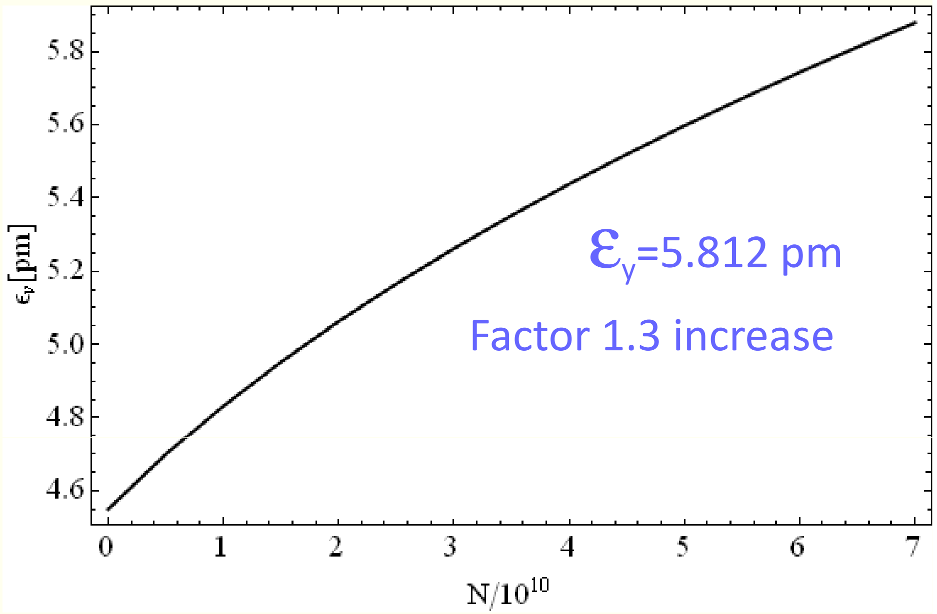
   - For the vertical emittance we use* : $$\varepsilon_y' = (1-r_\varepsilon)\frac{1}{1-\tau_y/T_y}\varepsilon_y + r_\varepsilon\frac{1}{1-\tau_x/T_x}\varepsilon_y$$

     where r_ε varies from 0 (ε_y generated from dispersion) to 1 (ε_y generated from betatron coupling)
4. Iterate from step 2

Equilibrium transverse emittances and rms bunch length vs bunch charge with *standard* procedure

SuperB LER V12 lattice @N=6.5e10





Vertical emittance generated 50% by coupling and 50% vertical dispersion (r_ε=0.5)

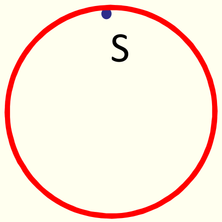
Open questions for further investigations

- This approach doesn't tell us how emittance increases vs time, nor the final beam distribution after IBS.
  In addition, Bane's approximation works for Gaussian beams →

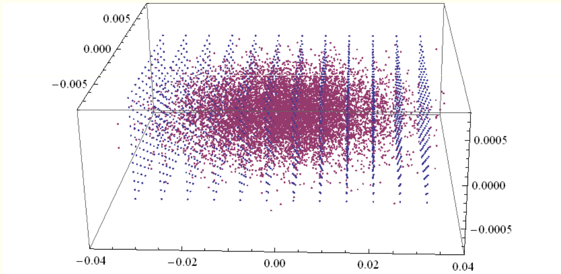
- Monte Carlo approach opens new possibilities for studying IBS –non Gaussian- beam tails distribution
- Tracking code that reads a MAD lattice gives large possibilities for investigations IBS effect on vertical emittance sources (vertical dispersion, misalignments, …)
- What happens when vertical emittance is very small as in SuperB?

Macroparticle Monte Carlo Tracking simulation

- The lattice is read from a MAD (X or 8) file containing the Twiss functions.
- A particular ring location is selected as an IBS Interaction Point (S).



- 6D macroparticles coordinates are extracted randomly from a **Gaussian** distribution generated at the chosen location S.
- The **IBS routine** *(Binary Collision Algorithm)* is called once per turn at S, recalculated at each turn using different random number seeds:



  - Beam macroparticles are grouped in cells
  - Macroparticles inside a cell are coupled
  - Momentum of particles is changed due to scattering
- **Radiation damping** and **quantum excitation** are evaluated at each turn at S
- Macroparticles are tracked through a 1-turn 6D R matrix starting from S for as many turns as needed
- Invariants of particles and corresponding growth rates are recalculated at S each turn

Binary Collision algorithm for the IBS*

* Ref. [P. Yu, Y. Wang and W. Huang , PRST-AB , **12**, 061301 (2009)]
[N. Alekseev, A. Bolshakov, E. Mustafin and P. Zenkevich , in *Space Dominated Beam Physics for Heavy Ion Fusion*, ed. Y.K. Batygin AIP, New York, **480** (1999) p.31-41]

For two particles colliding with each other, the changes in momentum for particle 1 can be expressed as:

$$\Delta P_{1x} = \frac{P}{2}\left[\zeta\sqrt{1+\frac{\xi^2}{4\alpha^2}}\sin\phi - \frac{\xi\theta}{2\alpha}\cos\phi\right]\sin\varphi + \theta(\cos\varphi - 1)$$

$$\Delta P_{1y} = \frac{P}{2}\left[\theta\sqrt{1+\frac{\xi^2}{4\alpha^2}}\sin\phi - \frac{\xi\zeta}{2\alpha}\cos\phi\right]\sin\varphi + \zeta(\cos\varphi - 1)$$

$$\Delta P_{1s} = \frac{P}{2}[2\alpha\gamma\sin\varphi\cos\phi + \gamma\xi(\cos\varphi - 1)],$$

$$\xi = \frac{P_1 - P_2}{\gamma P},\ \theta = x_1' - x_2',\ \zeta = y_1' - y_2',\ \alpha = \frac{\sqrt{\theta^2+\zeta^2}}{2}$$

with the equivalent polar angle φ_{eff} and the azimuthal angle ϕ distributing uniformly in [0; 2π], the invariant changes caused by the equivalent random process are the same as that of the IBS in the time interval Δt_s

$$\varphi_{\text{eff}} = \frac{2r_0}{\gamma}\sqrt{\frac{\pi c\rho\Delta t_s}{\bar{\beta}^3}L_c}$$

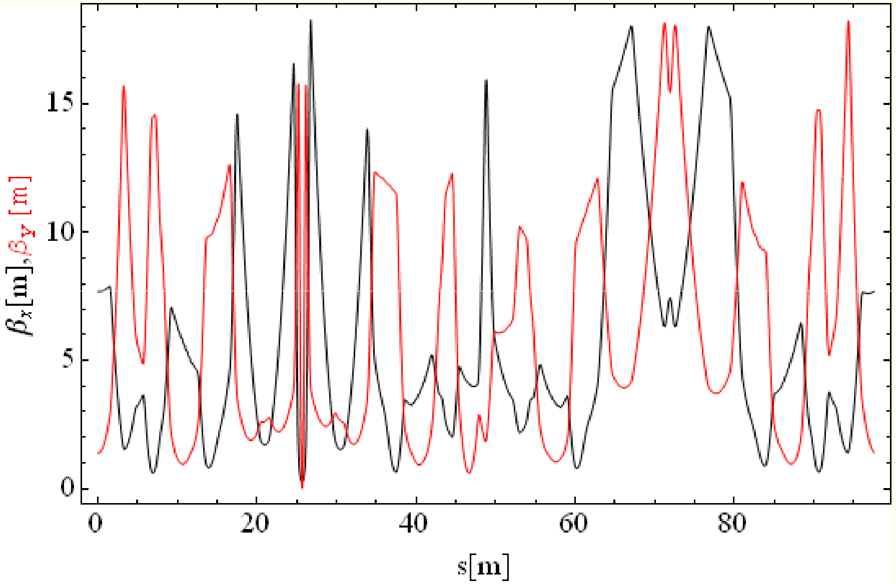
$$\frac{dJ_{1x}}{dt} = \frac{\pi r_0^2}{4\gamma^2\bar{\beta}^3}c\rho L_c\left[-4x_1'\theta + \xi^2 + \zeta^2 + 4\frac{x_{\beta 1}D_x}{\beta_x^2}\gamma\xi + \frac{D_x^2\gamma^2}{\beta_x^2}(\theta^2+\zeta^2)\right]$$

$$\frac{dJ_{1y}}{dt} = \frac{\pi r_0^2}{4\gamma^2\bar{\beta}^3}c\rho L_c(-4y_1'\zeta + \xi^2 + \theta^2)$$

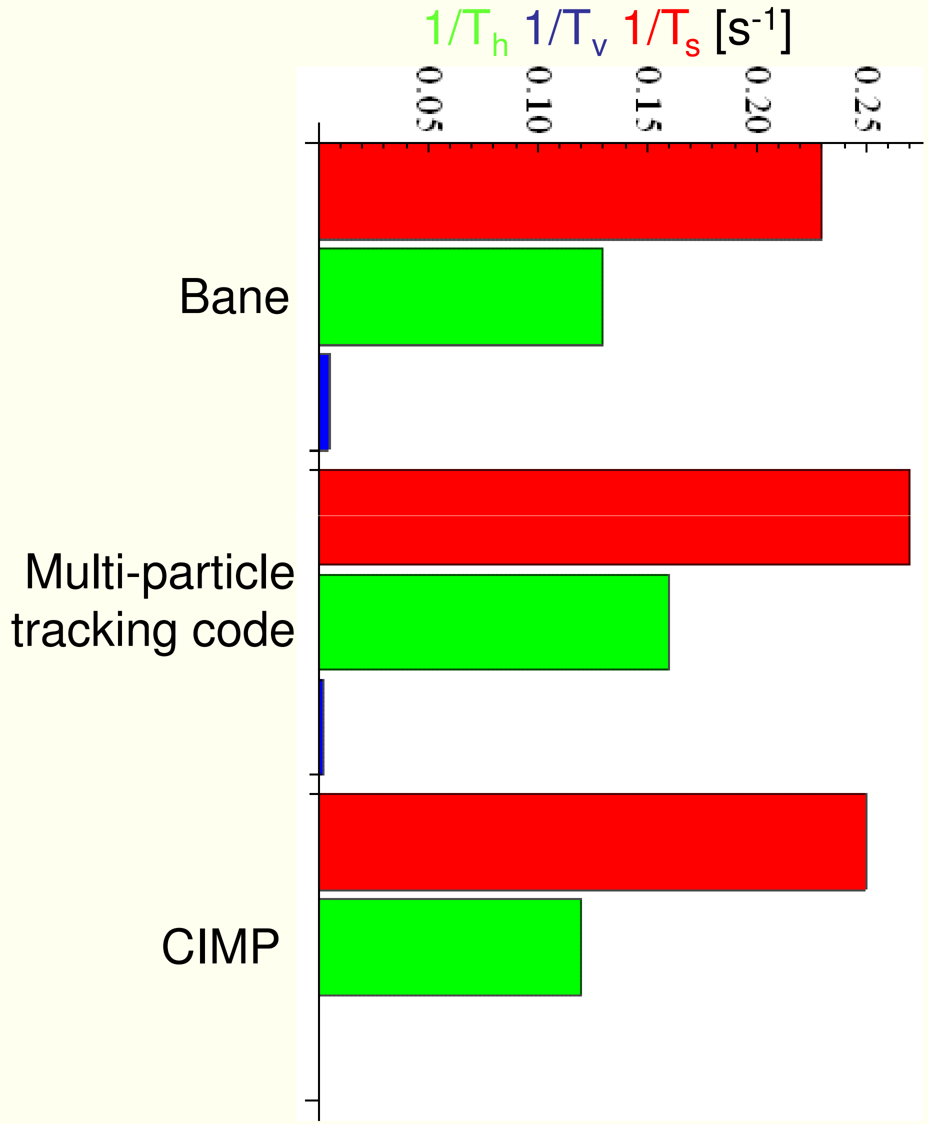
$$\frac{dJ_{1s}}{dt} = \frac{\pi r_0^2}{4\gamma^2\bar{\beta}^3}c\rho L_c\left(-4\frac{\delta_1}{\gamma}\xi + \zeta^2 + \theta^2\right),$$

First Application: DAΦNE

DAΦNE Crab Waist (Siddharta model)



β_x	α_x	η_x	η_x	β_y	α_y	η_y	η_y
4.96	0.33	2.15	0.11	1.37	-0.31	0	0

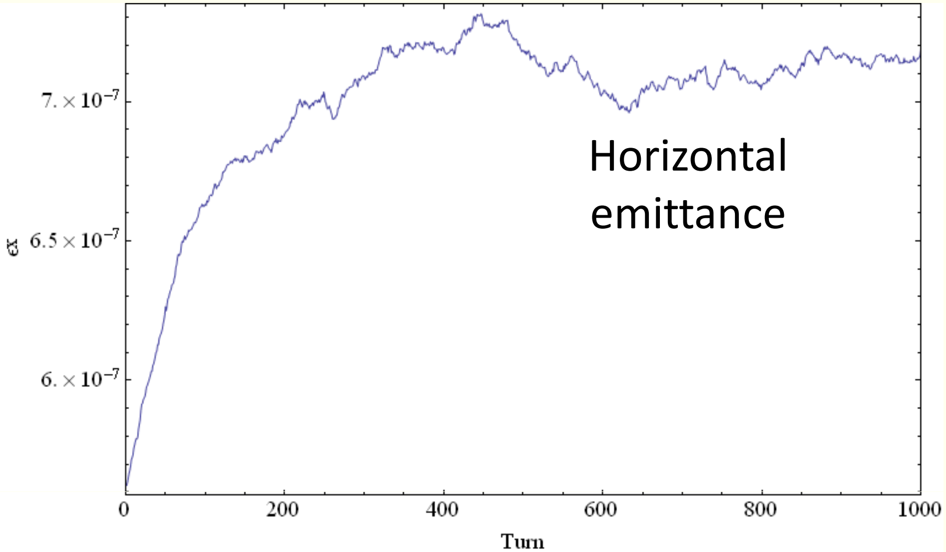


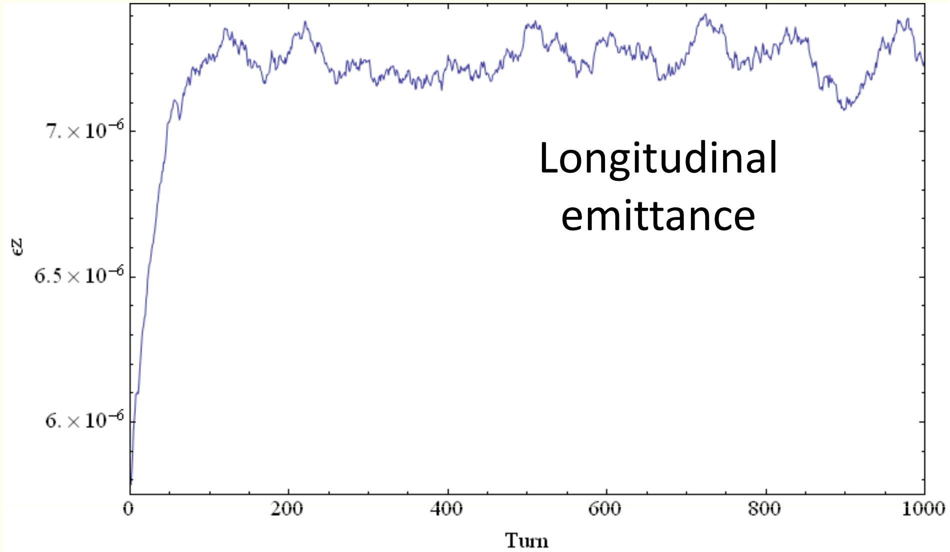
\# of macroparticles: 10^4

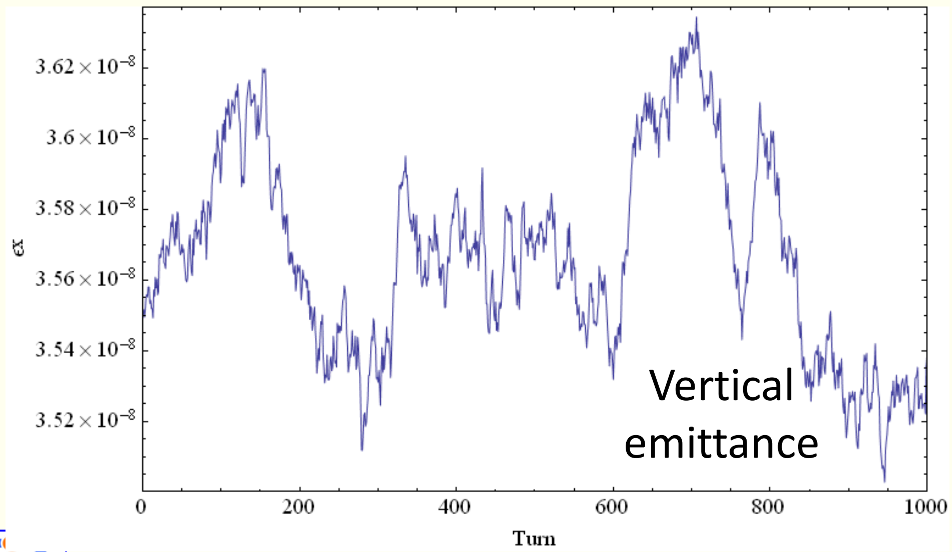
Grid size: $5\sigma_x x 5\sigma_y x 5\sigma_z$

Cell size: $\sigma_x/2 x \sigma_y/2 x \sigma_z/2$

Tracking Simulation result for DAΦNE CW lattice







N_{bunch}=**10000***2.1*10^{10}

MacroParticleNumber=40000

NTurn=1000 (≈10 damping times)

σ_z=12.0*10^{-3}

δp=4.8*10^{-4}

Grid size: $6\sigma_x x 6\sigma_y x 6\sigma_z$

Cell size: $\sigma_x/2x\sigma_y/2x\sigma_z/2$

$\epsilon_x=(5.63*10^{-4})/\gamma$

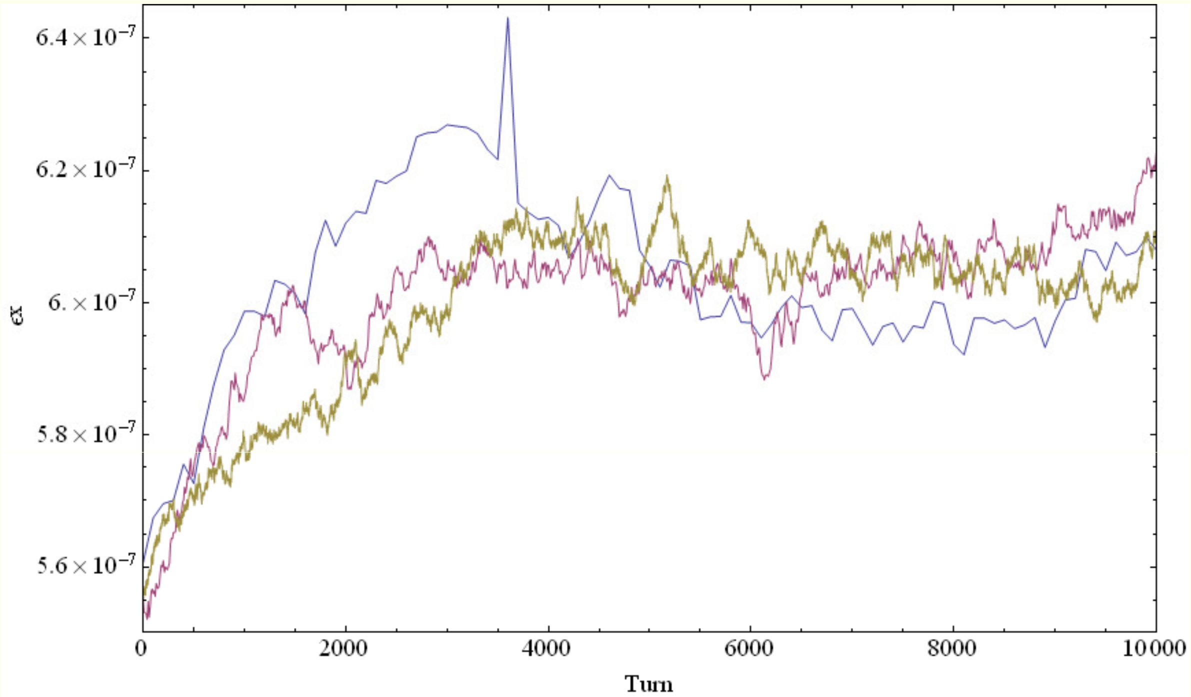
$\epsilon_y=(3.56*10^{-5})/\gamma$

$\tau_x = 1000^{-1} * 42.028822 * 10^{-3}$

$\tau_y = 1000^{-1} * 37.161307 * 10^{-3}$

$\tau_s = 1000^{-1} * 17.563599 * 10^{-3}$

Simulation results: ε_x vs t for different N_{bunch} and τ



Macroparticles =40000

Nturns ≈10 damping times

σ_z=12.0*10^{-3}

δp=4.8*10^{-4}

$\epsilon_x = (5.63*10^{-4})/\gamma$

$\epsilon_y = (3.56*10^{-5})/\gamma$

Grid size: $6\sigma_x$x$6\sigma_y$x$6\sigma_z$

Cell size: $\sigma_x/2$x$\sigma_y/2$x$\sigma_z/2$

Blue (100*dt):

N_{bunch}=**10^5***2.1*10^{10}

τ_x = 10^{-4} * 42.02 * 10^{-3}

τ_y = 10^{-4} * 37.16 * 10^{-3}

τ_s = 10^{-4} * 17.56 * 10^{-3}

Magenta (10*dt):

N_{bunch}=**10^4***2.1*10^{10}

τ_x = 10^{-3} * 42.02 * 10^{-3}

τ_y = 10^{-3} * 37.16 * 10^{-3}

τ_s = 10^{-3} * 17.56 * 10^{-3}

Gold (1*dt):

N_{bunch}=**10^3***2.1*10^{10}

τ_x = 10^{-2} * 42.02 * 10^{-3}

τ_y = 10^{-2} * 37.16 * 10^{-3}

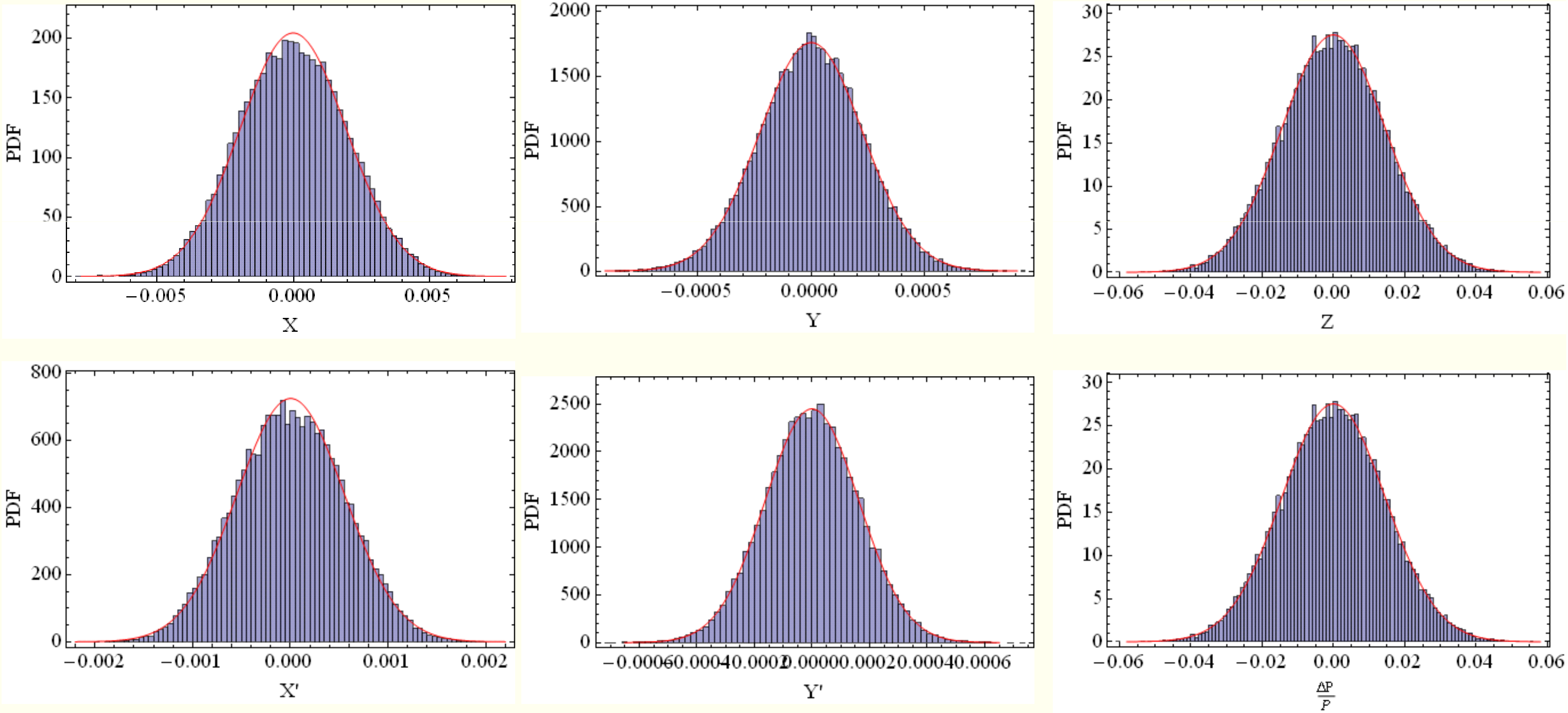
τ_s = 10^{-2} * 17.56 * 10^{-3}

Example: Beam distribution studies with Monte Carlo

N_{bunch}=10000*2.1*10^{10}
# Macroparticles =40000
NTurn=10000

τ_x(ms) = 1000^{-1} *42.029
τ_y(ms) = 1000^{-1} *37.161
τ_s (ms) = 1000^{-1} *17.563

Last tracking Turn



The Kolmogorov-Smirnov Normality Test gives a confidence level >99% in all cases

Simulation results compared to *Chao* model

- Monte Carlo simulation needed for realistic IBS studies

But, on the other hand,

- Emittance evolution estimates with Monte Carlo require very long CPU time (i.e. ≈ 20 hours for $4 \cdot 10^4$ macropart. and 7.5 damping times)

  → Translate to FORTRAN to speed up Mathematica Code

  → it can be useful to extrapolate Monte Carlo results using a scaling law from Chao model

- Performed a study of the scaling law accuracy on Monte Carlo simulations by varying relevant parameters in a wide range (within CPU time constraints)

Chao Model: differential equation system for ε_x and ε_z

Radial and longitudinal emittance growths can be predicted by a model that takes the form of a coupled differential equations:

$$\begin{cases} \dot{\varepsilon}_x = -\dfrac{1}{\tau_x / T_{rev}} \left(\varepsilon_x(t) - \varepsilon_{xeq} \right) + \dfrac{Na}{\varepsilon_x^{3/4}(t)\ \varepsilon_z(t)} \\ \dot{\varepsilon}_z = -\dfrac{1}{\tau_z / T_{rev}} \left(\varepsilon_z(t) - \varepsilon_{zeq} \right) + \dfrac{Nb}{\varepsilon_x^{3/4}(t)\ \varepsilon_z(t)} \end{cases}$$

N number of particles per bunch

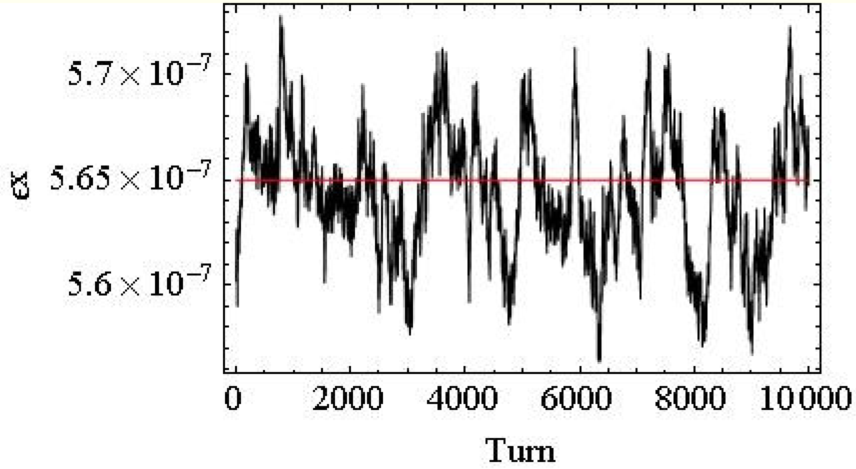
a and **b** coefficients characterizing IBS obtained once by fitting the tracking simulation data for a chosen benchmark case ($N_{bunch} = 10^4 * 2.1*10^{10}$)

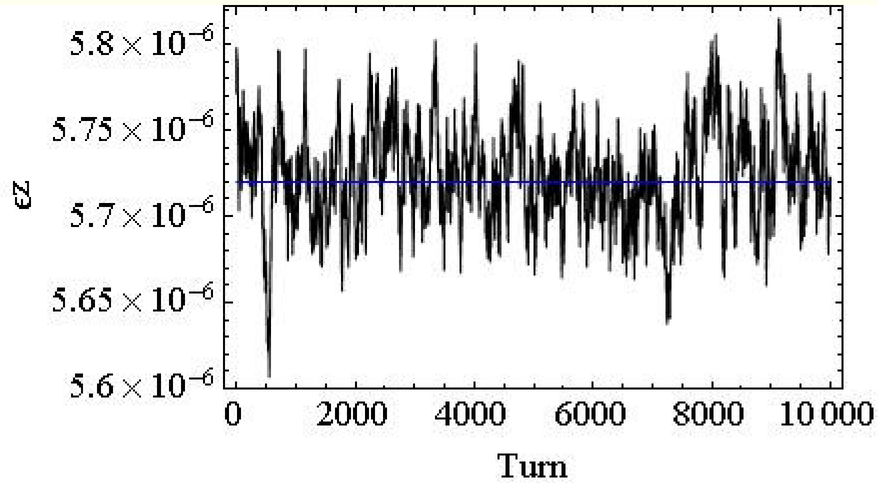
$$\begin{cases} \varepsilon_x(t=0) = \varepsilon_{x0} \\ \varepsilon_z(t=0) = \varepsilon_{z0} \end{cases}$$

Obtained by fitting the zero bunch intensity case (IBS =0)

$$\begin{cases} \varepsilon_x(t \to \infty) = \varepsilon_{xeq} \\ \varepsilon_z(t \to \infty) = \varepsilon_{zeq} \end{cases}$$

IBS = 0 (N_{bunch}=0)





MC Simulation parameters

MacroParticleNumber=40000

NTurn=10000 (≈77damping times)

σ_z=12.0*10^{-3}

δp=4.8*10^{-4}

$\epsilon_x=(5.63*10^{-4})/\gamma$

$\epsilon_y=(3.56*10^{-5})/\gamma$

τ_x = 1000^{-1} * 42.028822 * 10^{-3}

τ_y = 1000^{-1} * 37.161307 * 10^{-3}

τ_s = 1000^{-1} * 17.563599 * 10^{-3}

Cpu=20.10 hrs

$\varepsilon_{x0} = \varepsilon_{xeq}$ = **5.65** 10^{-7}

$\varepsilon_{z0} = \varepsilon_{zeq}$ = **5.72** 10^{-6}

Benchmark case

MC Simulation parameters

N_{bunch}=10000*2.1*10^{10}

MacroParticleNumber=40000

NTurn=1000 (≈10 damping times)

σ_z=12.0*10^{-3}

δp=4.8*10^{-4}

$\epsilon_x=(5.63*10^{-4})/\gamma$

$\epsilon_y=(3.56*10^{-5})/\gamma$

Grid size: $6\sigma_x x 6\sigma_y x 6\sigma_z$

Cell size: $\sigma_x/2 x \sigma_y/2$

lost macroparticles =0

$\tau_x = 1000^{-1} * 42.028822 * 10^{-3}$

$\tau_y = 1000^{-1} * 37.161307 * 10^{-3}$

$\tau_s = 1000^{-1} * 17.563599 * 10^{-3}$

Scaling law parameters First Model

τ_{dx} = 129; τ_{dz} = 54

ε_{x0} = 5.65 10^{-7}

ε_{z0} = 5.7 10^{-6}

ε_{xeq} = 5.7*10^{-7}

ε_{zeq} = 5.75 * 10^{-6}

⬇

Scaling law parameters Modified Model

τ_{dx} = 129; τ_{dz} = 54

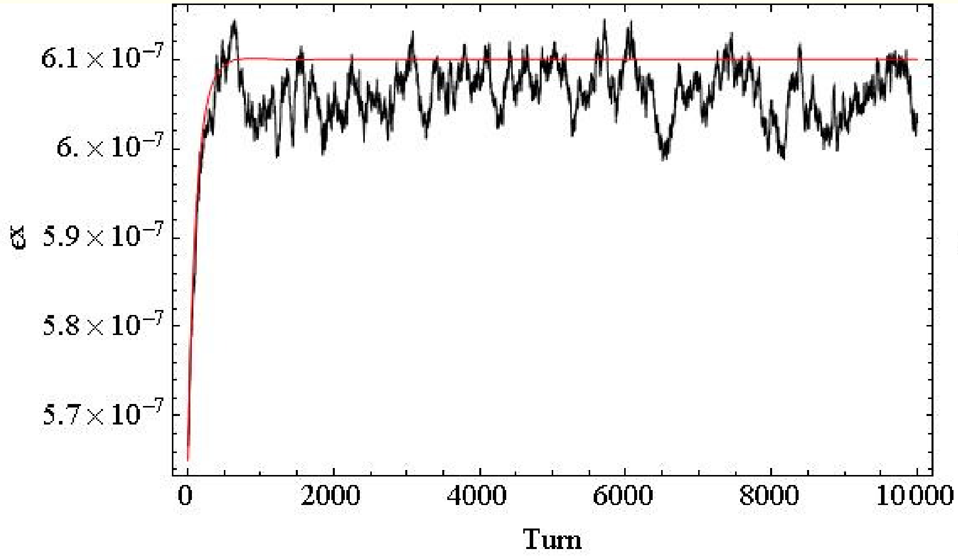
ε_{x0} = **5.65** 10^{-7}

ε_{z0} = **5.72** 10^{-6}

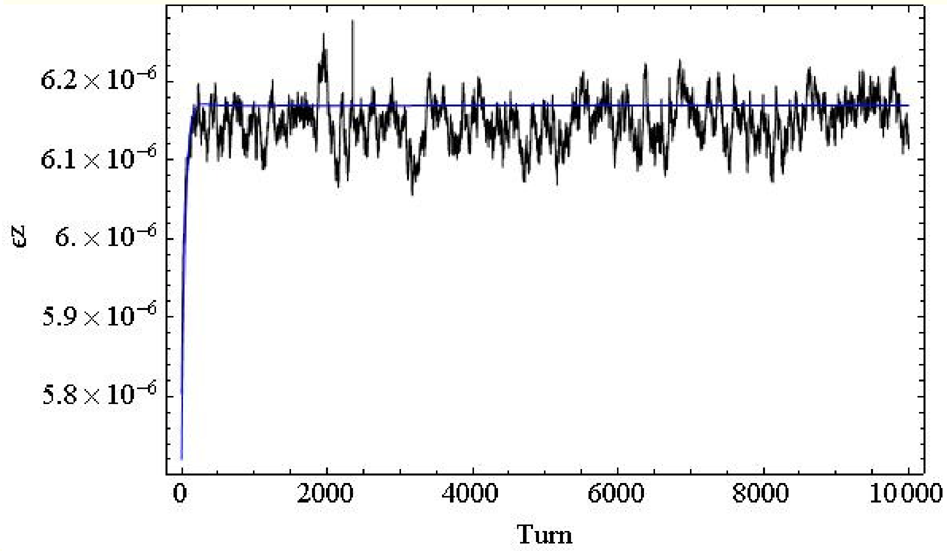
ε_{xeq} = **5.65***10^{-7}

ε_{zeq} = **5.72** * 10^{-6}

Benchmark case $I=10^4 * I_{nom}$



Horizontal
emittance

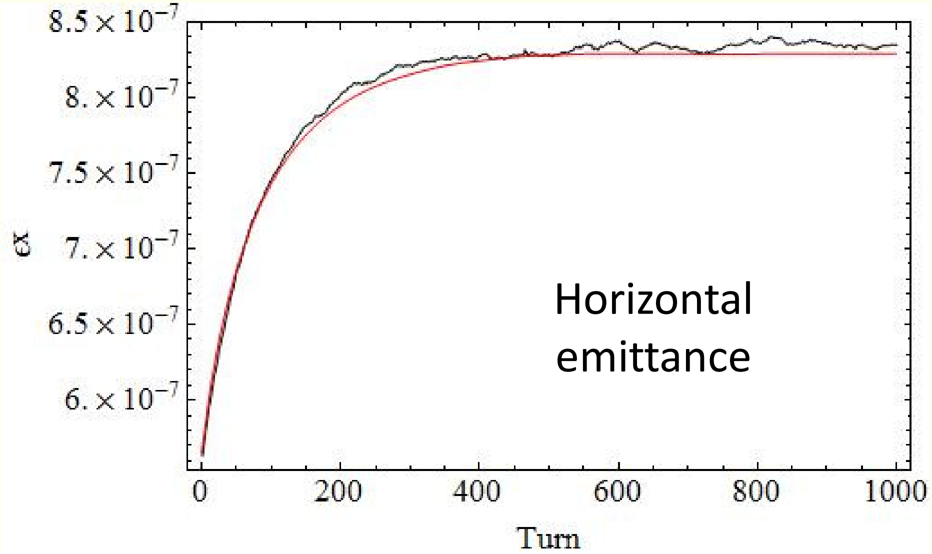


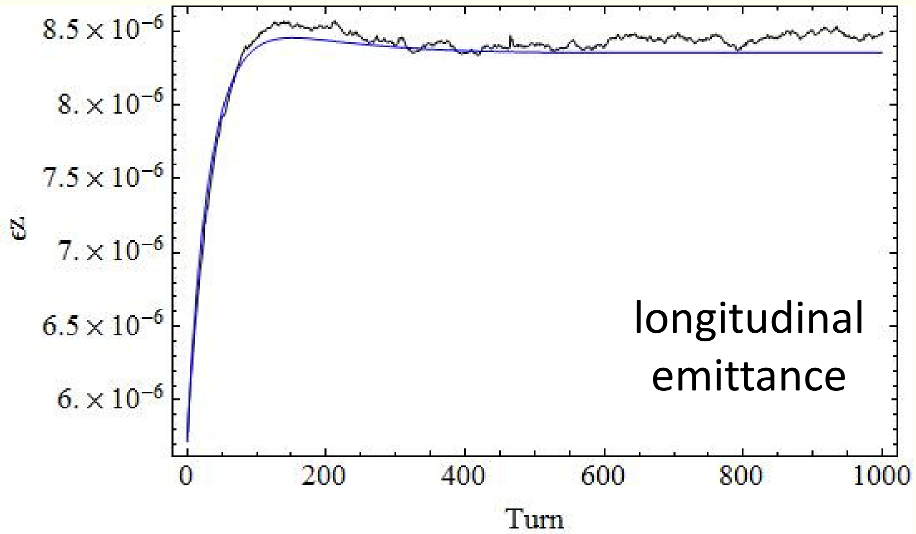
longitudinal
emittance

Na (BENCHMARK) = $4.7*10^{-20}$
Nb (BENCHMARK) = $1.12*10^{-18}$

Monte Carlo vs rescaled Chao model for $I=10^5 * I_{nom}$

Na = Na (BENCHMARK) *10
Nb = Nb (BENCHMARK)*10





MacroParticleNumber=40000
NTurn=1000 (≈7.7damping times)
$\sigma_z=12.0*10^{-3}$
$\delta p=4.8*10^{-4}$
$\epsilon_x=(5.63*10^{-4})/\gamma$
$\epsilon_y=(3.56*10^{-5})/\gamma$
$\tau_x = 1000^{-1} * 42.028822 * 10^{-3}$
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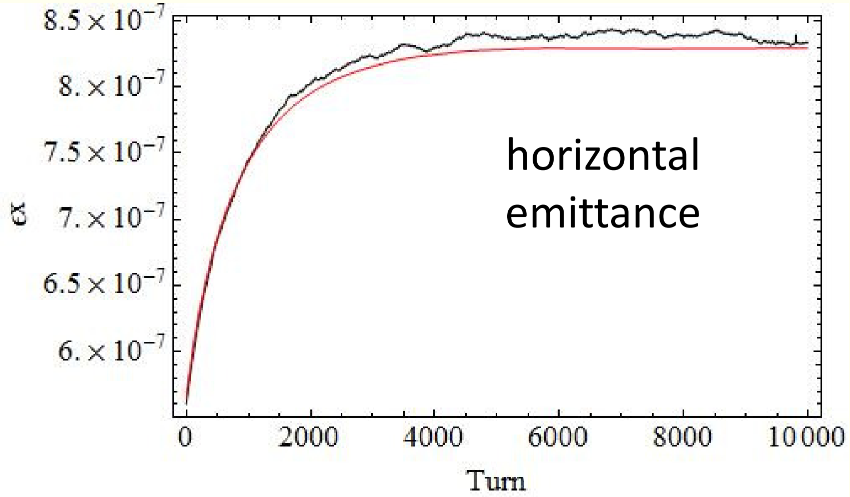
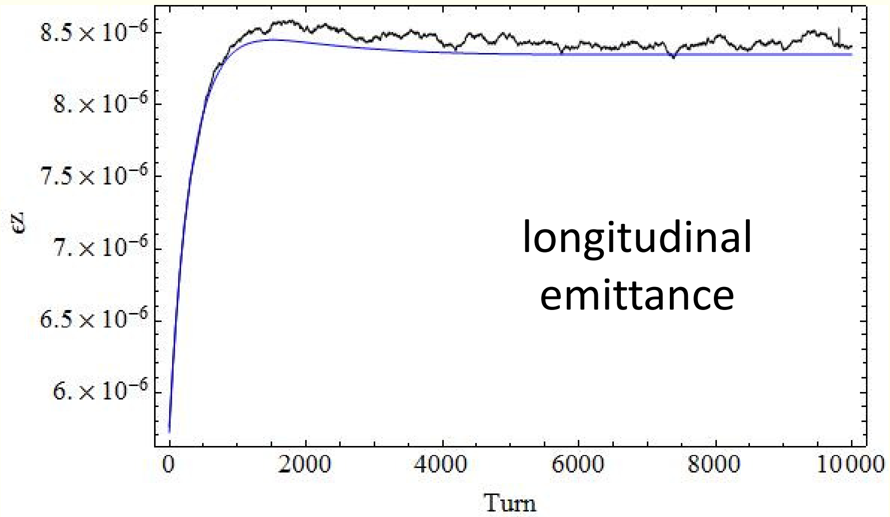
Monte Carlo vs scaling law prediction





Na (BENCHMARK) = $4.7*10^{-20}$
Nb (BENCHMARK)=$1.12*10^{-18}$

MacroParticleNumber=40000

NTurn=10000 (≈7.7damping times)

$\sigma_z=12.0*10^{-3}$

$\delta p=4.8*10^{-4}$

$\epsilon_x=(5.63*10^{-4})/\gamma$

$\epsilon_y=(3.56*10^{-5})/\gamma$

$\tau_x = 100^{-1} * 42.028822 * 10^{-3}$

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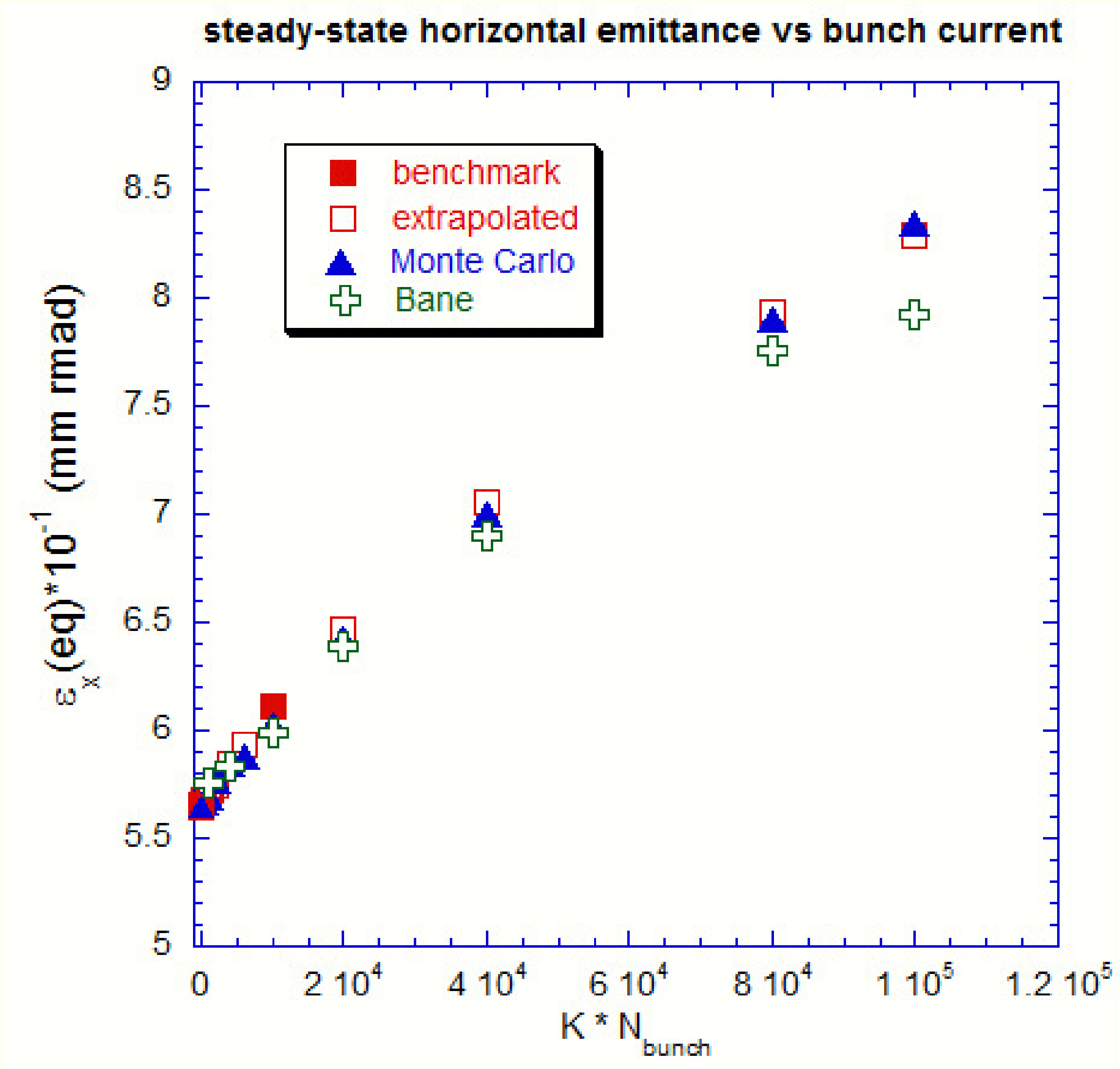
Summary plots: ε_x vs bunch current



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NTurn=1000 (≈10 damping times)

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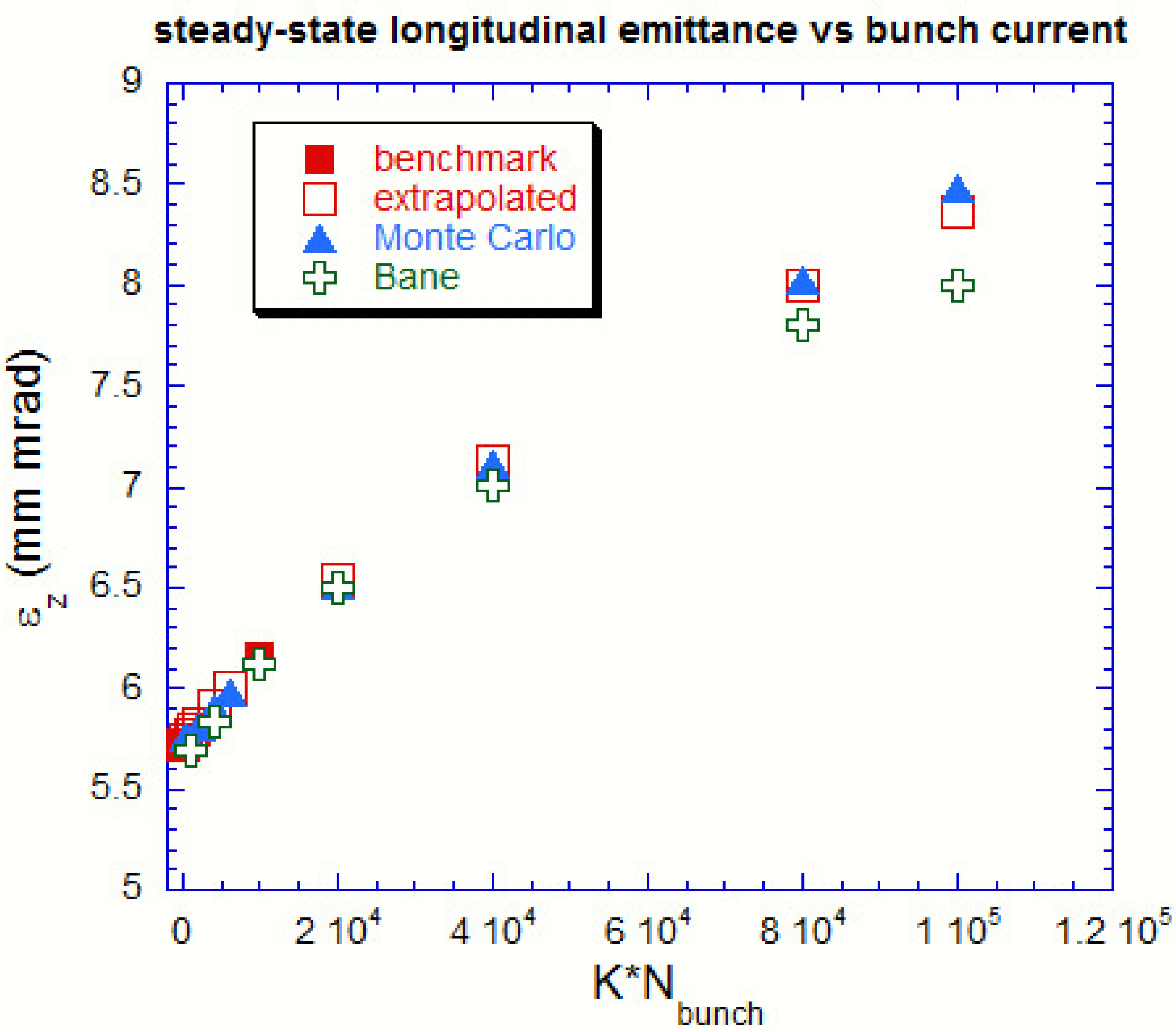
Summary plots: ε_z vs bunch current



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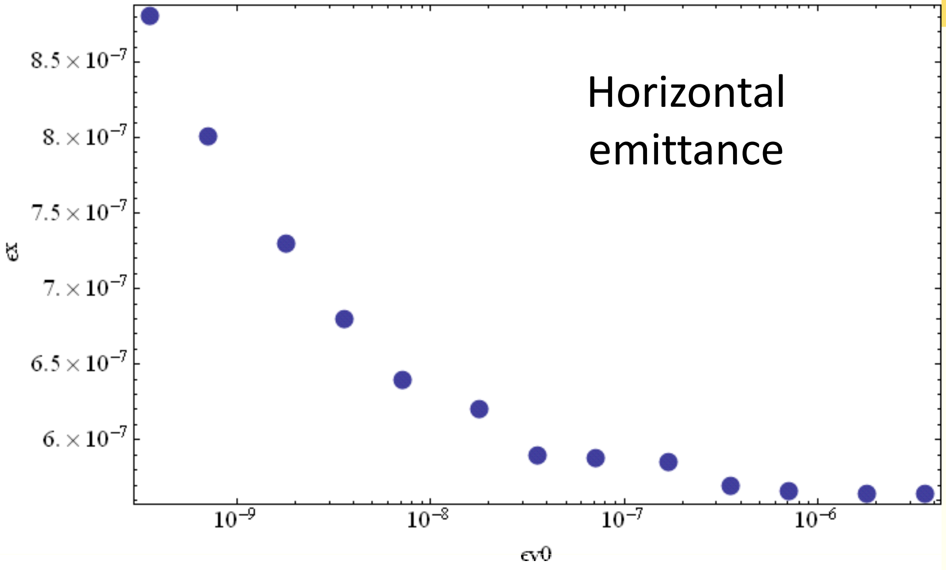
$\epsilon_x=(5.63*10^{-4})/\gamma$
$\epsilon_y=(3.56*10^{-5})/\gamma$

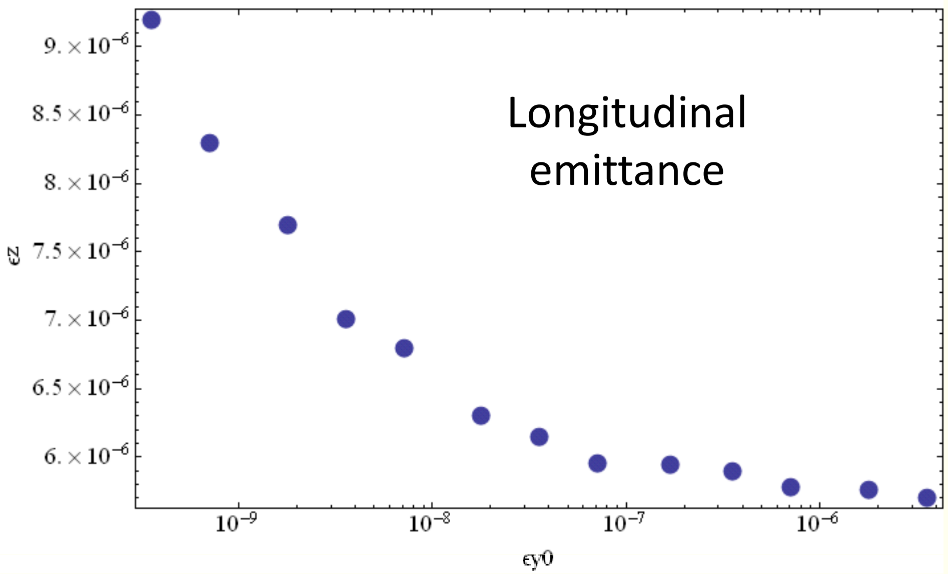
τ_x = 1000^{-1} * 42.028822 * 10^{-3}
τ_y = 1000^{-1} * 37.161307 * 10^{-3}
τ_s = 1000^{-1} * 17.563599 * 10^{-3}

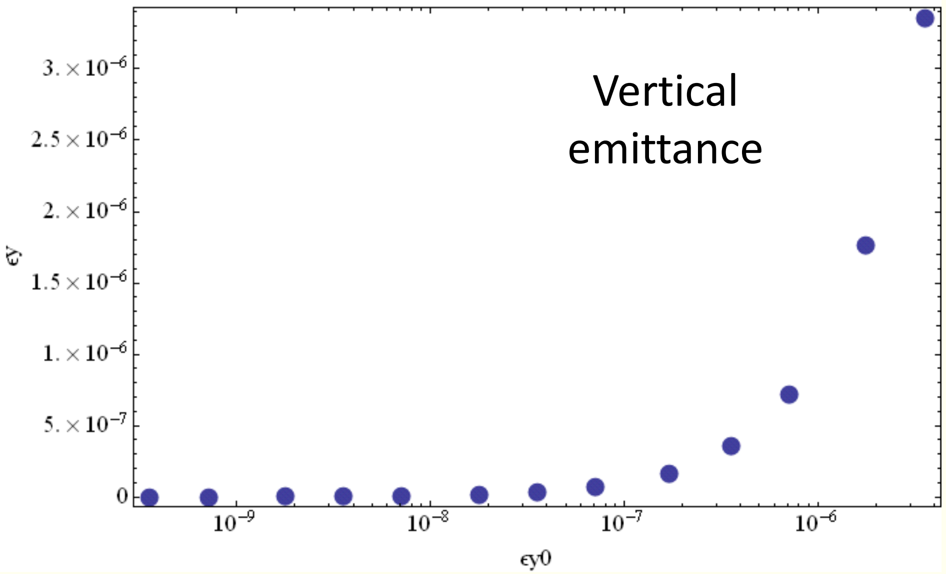
Prospects for IBS Monte Carlo

- Now written in Mathematica, translation to FORTRAN needed for long runs
- Studies with Vertical emittance in Monte Carlo tracking, with SuperB lattice
- Non-Gaussian tail distribution due to IBS
- First results for DAFNE (now completely uncoupled lattice (i.e. D_y=0 and κ=0), with the prospect comparison with real data, next simulations will be on SuperB
- Another physics issue is the behavior when ε_x and ε_y are reduced. They may be reduced together or reduced separately. Theo already studied the case when they are reduced separately. So we still want to see the behavior when they are reduced together.

IBS effect vs initial vertical emittance







N_{bunch}=**10000***2.1*10^{10}

MacroParticleNumber=40000

NTurn=1000 (≈10 damping times)

σ_z=12.0*10^{-3}

δp=4.8*10^{-4}

$\epsilon_x = (5.63*10^{-4})/\gamma$

Grid size: $6\sigma_x x 6\sigma_y x 6\sigma_z$

Cell size: $\sigma_x/2 x \sigma_y/2 x \sigma_z/2$

$\tau_x = 1000^{-1} * 42.028822 * 10^{-3}$

$\tau_y = 1000^{-1} * 37.161307 * 10^{-3}$

$\tau_s = 1000^{-1} * 17.563599 * 10^{-3}$

Conclusions

- The effect of IBS on the transverse emittances is about 30% in the LER and less then 5% in HER that is still reasonable if applied to lattice natural emittances values.
- Interesting aspects of the IBS such as its impact on damping process and on generation of non Gaussian tails may be investigated with a multiparticle algorithm.
- A code implementing the Zenkevich-Bolshakov algorithm to investigate IBS effects is being developed
    - Benchmarking with conventional IBS theories gave good results.
- Will continue paying attention to nonconventional effects as the vertical emittance continues to become smaller as in SuperB.
- Produce the FORTRAN version of the code, maybe a parallel implementation (CMAD?)
- Start studying SuperB full lattice (including coupling and errors?)
- Study the effect of IBS on bunch distribution

Back-up

Differential equation system initially considered

model presented in Elba

$$\begin{cases} \dot{\varepsilon}_x = -\dfrac{1}{\tau_x / T_{rev}}\left(\varepsilon_x(t) - \varepsilon_{xeq}\right) + \dfrac{Na}{\varepsilon_x(t)\varepsilon_z(t)} \\ \dot{\varepsilon}_z = -\dfrac{1}{\tau_z / T_{rev}}\left(\varepsilon_z(t) - \varepsilon_{zeq}\right) + \dfrac{Nb}{\varepsilon_x(t)\varepsilon_z(t)} \end{cases}$$

Now Slightly Modified model (wrt Elba) according to K. Bane's suggestions

First result from rescaled model

Na = Na(benchmark)*10

Nb = Nb(benchmark)*10

N_{bunch}=**100000***2.1*10^{10}

MacroParticleNumber=40000

NTurn=1000 (≈10 damping times)

$\sigma_z = 12.0*10^{-3}$

$\delta p = 4.8*10^{-4}$

$\epsilon_x = (5.63*10^{-4})/\gamma$

$\epsilon_y = (3.56*10^{-5})/\gamma$

$\tau_x = 1000^{-1} * 42.028822 * 10^{-3}$

$\tau_y = 1000^{-1} * 37.161307 * 10^{-3}$

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