On other possible solutions for the GRANCHIO (CRAB)-waist transformation

A. Bogomyagkov and E. Levichev
BINP

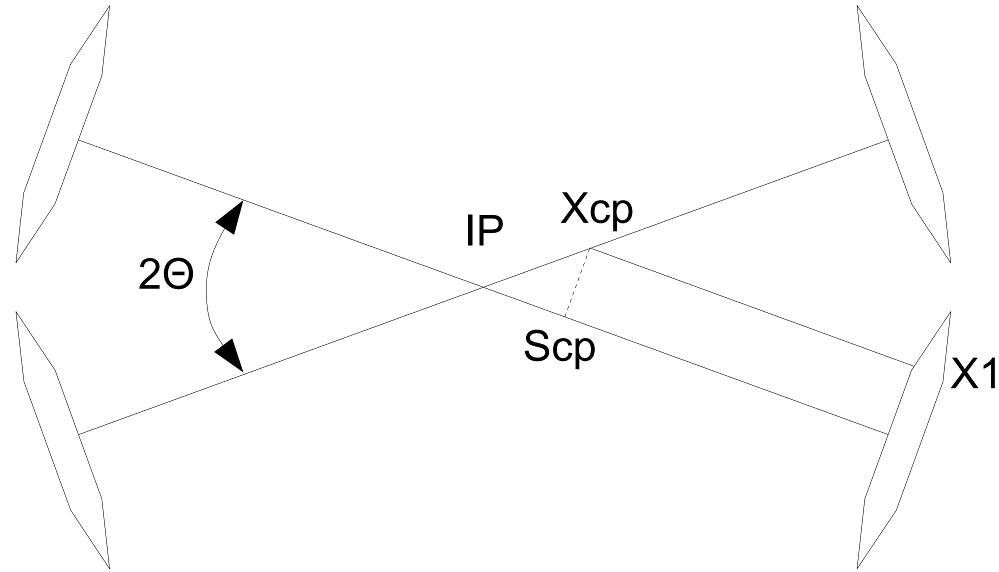
Goals

1. To study higher order aberrations of conventional CRAB sextupole scheme

2. To find another solutions with different phase advances, with different number of sextupoles to obtain CRAB waist condition

Conventional CRAB scheme

$$\mu_x = \pi \qquad \mu_y = \frac{\pi}{2}$$



1. $$s_{cp} = \frac{-x_{cp}}{2\theta}$$

2. $$\tan\mu_y = \frac{-\beta_{y0}}{s_{cp} - K_2 L\, x_1 \beta_{y0} \beta_{y1}} \qquad s_{cp} = K_2 L\, x_1 \beta_{y0} \beta_{y1}$$

3. $$K_2 L = \frac{1}{2\theta \beta_{y0} \beta_{y1}} \sqrt{\frac{\beta_{x0}}{\beta_{x1}}} + \frac{(x_1^2 - y_1^2)}{2} K_4 L$$

$$K_4 L = \frac{-(x_1^2 - y_1^2)}{2} \frac{1}{(2\theta)^3 \beta_{y0}^2 \beta_{y1}^{\ 2}} \sqrt{\frac{\beta_{x0}}{\beta_{x1}}}$$

CRAB sextupoles with arbitrary phase

1. $s_{cp} = \dfrac{-x_{cp}}{2\theta}$

2. $\tan\mu_{ycp} = \tan\mu_y + \dfrac{s}{\beta_{y0}} + \dfrac{s\tan^2(\mu_y)}{\beta_{y0}} - K_2 L x_1 \beta_{y1} \tan^2(\mu_y)$

$$s_{cp} = K_2 L x_1 \beta_{y0} \beta_{y1} \frac{1-\cos(2\mu_y)}{2}$$

3.

$$K_2 L = -\frac{1}{2\theta\beta_{y0}\beta_{y1}}\sqrt{\frac{\beta_{x0}}{\beta_{x1}}}\frac{\cos\mu_x}{\sin^2\mu_y}$$

$$-\frac{\alpha_{x1}}{2\theta\beta_{y0}\beta_{y1}}\sqrt{\frac{\beta_{x0}}{\beta_{x1}}}\frac{\sin\mu_x}{\sin^2\mu_y} - \frac{p_{x1}}{x_1}\frac{\sqrt{\beta_{x0}\beta_{x1}}}{2\theta\beta_{y0}\beta_{y1}}\frac{\sin\mu_x}{\sin^2\mu_y}$$

Numerical examples

1. the second from IP edge of QD1

   $\mu_x = 0.243 \cdot 2\pi$, $\mu_y = 0.2501 \cdot 2\pi$,

   $\beta_{x1} = 113.74\,m$, $\beta_{y1} = 175.4\,m$, $\alpha_{x1} = 14.9$,

   $$K2L = -16.6311 - \frac{126.553\, p_{x1}}{x_1}$$

2. the middle point of CRAB SEXTUPOLE

   $\mu_x = 3.5 \cdot 2\pi$, $\mu_y = 3.25 \cdot 2\pi$,

   $\beta_{x1} = 5.016\,m$, $\beta_{y1} = 39.79\,m$, $\alpha_{x1} = 0$,

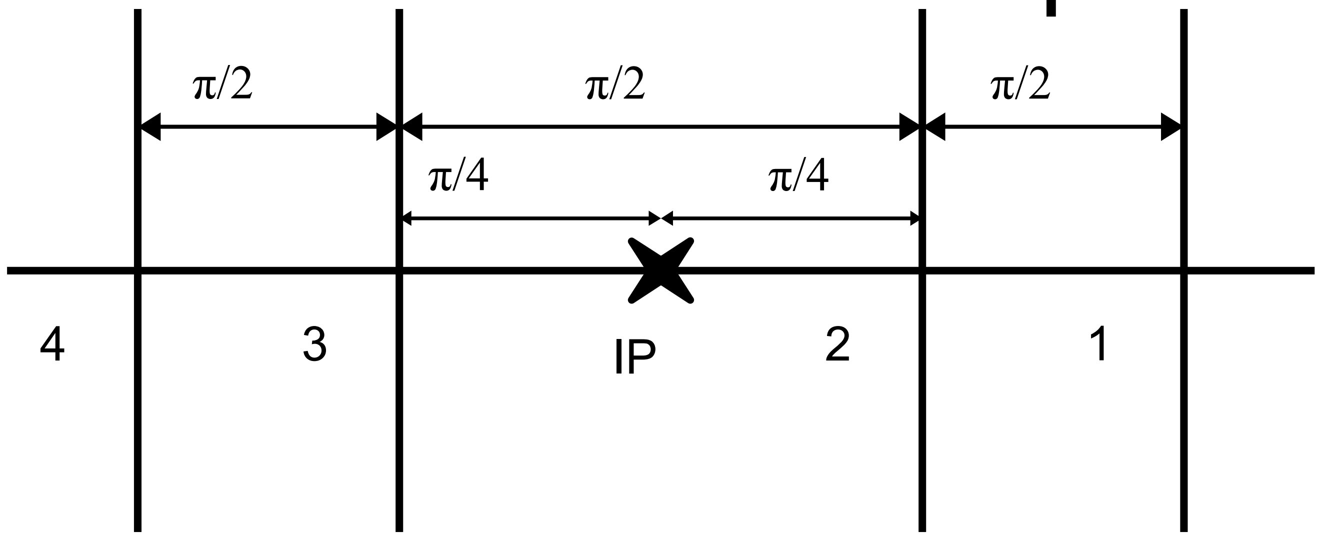
   $K2L = 23.38$

General condition of second order terms cancellation

1. Sextupoles with equal strengths and the same optical functions will have no second order aberrations if vector summation is equal zero in ψ and 3ψ diagrams.

2. Examples
   1. Two sextupoles at π phase advance
   2. Four sextupoles at $\pi/4$ phase advance
   3. Six sextupoles at $\pi/3$ phase advance

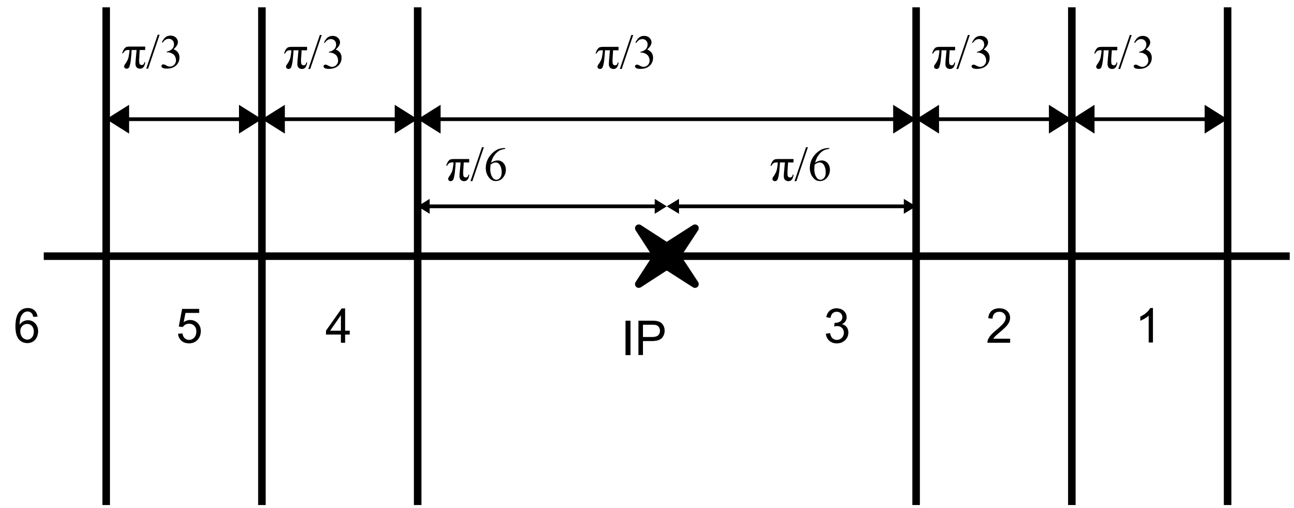
CRAB with four sextupoles



1. $$K_2 L = \frac{-1}{\sqrt{2}\,\theta\,\beta_{y0}\beta_{y1}}\sqrt{\frac{\beta_{x0}}{\beta_{x1}}} + \frac{x_1}{p_{x1}}\,\frac{\sqrt{2}}{\theta\,\beta_{y0}\beta_{y1}}\sqrt{\frac{\beta_{x0}}{\beta_{x1}^3}}$$

2. $$\left\langle \frac{x_1}{p_{x1}} \right\rangle = \frac{-\beta_{x1}}{(1+\alpha_{x1}^2)\sqrt{\alpha_{x1}^2}}$$

CRAB with six sextupoles



$$K_2 L = \frac{-2}{5\sqrt{3}\,\theta\,\beta_{y0}\beta_{y1}}\sqrt{\frac{\beta_{x0}}{\beta_{x1}}} + \frac{x_1}{p_{x1}}\frac{2}{5\,\theta\,\beta_{y0}\beta_{y1}}\sqrt{\frac{\beta_{x0}}{\beta_{x1}^3}}\left(1-\frac{\alpha_{x1}}{\sqrt{3}}\right)$$

Conclusion

1. Several CRAB waist schemes have been studied
   a) conventional with additional decapole
   b) two sextupoles placed at Final Focus quadrupoles
   c) 4 and 6 sextupoles

2. It is necessary to study more what condition gives suppression of resonances in CRAB waist

3. Proposed solutions need to be checked with beam-beam simulation