Dynamic aperture studies with MADX

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XIV SuperB general meeting
Frascati
28th September 2010

Hamiltonian

Hamiltonian of the system:

$$H = -(1+hx)\left[\sqrt{1+\frac{2\delta}{\beta_0}+\delta^2-(P_x-K_x)^2-(P_y-K_y)^2}+K_s\right]+\frac{\delta}{\beta_0}+1$$

with : $K_{x,y,s} = \frac{qA_{x,y,s}}{p_0}$, $P_{x,y} = \frac{p_{x,y}}{p_0}$, p_0 the total momentum

The Taylor expansion of the Hamiltonian of an ideal straight magnet is:

$$H = \frac{P_x^2+P_y^2}{2} + K_1\frac{x^2-y^2}{2} \qquad \text{Linear terms}$$

$$+K_2\frac{x^3-3xy^2}{6} \qquad \text{Sextupoles}$$

$$+\left(P_x^2+P_y^2\right)^2/8 \qquad \text{Kinematic term}$$

$$+K_3\frac{x^4-6x^2y^2+y^4}{24} \qquad \text{Octupoles}$$

$$+\text{Chromatic terms}+\mathcal{O}(5)$$

Derivatives of the tune

Action-Angle variables:
The position and momentum of the particle can be written under the form:

$$x = \sqrt{2\beta_x J_x}\sin(2\pi\mu_x + \phi_x)$$

$$P_x = \sqrt{\frac{2J_x}{\beta_x}}\left(\cos(2\pi\mu_x + \phi_x) - \alpha_x \sin(2\pi\mu_x + \phi_x)\right)$$

$$\mu_x = \int_0^s \frac{dt}{\beta_x(t)}$$

At first order, the tune shift due to a perturbation ΔH of the Hamiltonian is given by:

$$\Delta\nu_x = \frac{1}{2\pi}\oint\left\langle\frac{\partial\Delta H}{\partial J_x}\right\rangle_\phi ds,\ \Delta\nu_y = \frac{1}{2\pi}\oint\left\langle\frac{\partial\Delta H}{\partial J_y}\right\rangle_\phi ds$$

Kinematic term

The perturbation of the Hamiltonian by this effect is:

$$\Delta H = \frac{\left(P_x^2 + P_y^2\right)^2}{8}$$

The first-order contribution of the kinematic terms to the tune is then:

$$C_{xx} = \frac{\partial \nu_x}{\partial J_x} = \frac{3}{16\pi} \oint \gamma_x^2 \mathbf{d}s \qquad C_{xy} = \frac{\partial \nu_x}{\partial J_y} = \frac{1}{8\pi} \oint \gamma_x \gamma_y \mathbf{d}s$$

$$C_{yy} = \frac{\partial \nu_y}{\partial J_y} = \frac{3}{16\pi} \oint \gamma_y^2 \mathbf{d}s$$

The main contribution is where the gamma function is the largest: at the IP.

Thick sextupole vs thin sextupole

The strongest sextupoles are in the interaction region to cancel the local chromaticity and the betatron function are high too. These sextupoles are then the main contribution to the non linearities. To kill the second order terms the phase advance between them is π.
The kick in position due to this kind of structure is then:

$$x_1 = -x_0 - P_{x,0}L - \frac{(K_2 L)^2}{12}(x_0^3 + x_0 y_0^2)L^2 + \mathcal{O}(5)$$

The second order is effectively killed.
BUT: Third order terms are not canceled for thick sextupoles.
The effect is quadratic and not proportional to the strength of the sextupole.

Crab sextupoles

Parameters		Units	HER
Length	L	m	35
Strength	K_2	m^{-3}	16.67
Horizontal beta at the crab	β_x	m	14.6
Vertical beta at the crab	β_y	m	200
Horizontal beta at the IP	β_x^*	cm	2.6
Vertical beta at the IP	β_y^*	cm	0.0274
Full crossing angle	θ	mrad	66

The crab sextupole strength is given by [1]:

$$K_2 L = \frac{1}{\theta} \frac{1}{\beta_y^* \beta_y} \sqrt{\frac{\beta_x^*}{\beta_x}}$$

[1] "SUPPRESSION OF BEAM-BEAM RESONANCES IN CRAB WAIST COLLISIONS", P. Raimondi *et al.*, EPAC08, Genoa

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A factor 2 is missing. To take into account in the future.

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Fringe field in the quadrupoles

The perturbated Hamiltonian is:

$$\Delta H = -\frac{K_1'}{12}(xp_x(x^2-3y^2) - yp_y(y^2-3x^2)) - \frac{K_1''}{48}(x^4-y^4) + \mathscr{O}(5)$$

The derivative of the tune with the amplitude is then:

$$C_{xx} = \frac{1}{8\pi}\sum_{QP} K_1(\alpha_{x,o}\beta_{x,o} - \alpha_{x,i}\beta_{x,i})$$

$$C_{xy} = \frac{1}{8\pi}\sum_{QP} K_1(\alpha_{x,o}\beta_{y,o} - \alpha_{x,i}\beta_{y,i} - \alpha_{y,o}\beta_{x,o} + \alpha_{y,i}\beta_{x,i})$$

$$C_{yy} = \frac{-1}{8\pi}\sum_{QP} K_1(\alpha_{y,o}\beta_{y,o} - \alpha_{y,i}\beta_{y,i})$$

The main contribution is where the beta function is the largest with K_1: at the quadrupole QD0 near the IP.

Octupoles

The perturbation of the Hamiltonian is:

$$\Delta H = K_3 \frac{x^4 - 6x^2y^2 + y^4}{24}$$

After calculation, we have:

$$C_{xx} = \frac{1}{16\pi} \oint K_3 \beta_x^2 ds \qquad C_{xy} = \frac{-1}{8\pi} \oint K_3 \beta_x \beta_y ds$$

$$C_{yy} = \frac{1}{16\pi} \oint K_3 \beta_y^2 ds$$

Remark: The kick in x due to a thin octupole is proportional to $x^3 - 3xy^2$ whereas the kick due to the fringe field is $x^3 + 3xy^2$.

$\Rightarrow$ It is not possible to compensate the fringe field with octupoles.

MAD8 or MADX?

The aim was to cross check the results from the simulations of Piminov and Levichev.
The first idea was to use the very powerful package PTC to make the study of the dynamic aperture. The big advantages are:

- Possibility to track at a very high order.
- The values of the derivatives of the tune with amplitude and momentum can be calculated at the order we wish.
- The fringe field can be simulated in the quadrupoles.

BUT: The fringe field in the quadrupoles is taken into account neither by MADX alone nor by MAD8.
⇒ The source code of Mad8 was modified. The tracking is then with Lie4 method.

Summary of the contributions HER

Accelerаticum		C_{xx}	C_{xy}	C_{yy}
IP Sextupoles	cm^{-1}	-25	-51	-8300
Crab Sextupoles	cm^{-1}	-0.6	-5.5	-114
Arc Sextupoles	cm^{-1}	270	-330	-71
Sub Total	cm^{-1}	253	-402	-6510
Octupoles	cm^{-1}	-124	142	365
QP Fringe Field	cm^{-1}	240	1510	5830
Kinematic term	cm^{-1}	0.6	35	4440
Total	cm^{-1}	375	1320	3390

Analytical formula:	C_{xx}	C_{xy}	C_{yy}
Octupoles	-116	124	392
QP Fringe field	253	1614	6369
Kinematic term	100	149	6969

Average agreement Acceleraticum with analytical formulae (kinematic and fringe field term).

Summary of the contributions HER

MADX-PTC		C_{xx}	C_{xy}	C_{yy}
IP Sextupoles	cm^{-1}	-21	-43	-7301
Crab Sextupoles	cm^{-1}	-0.5	-4.9	-88
Arc Sextupoles	cm^{-1}	277	-360	-70
Sub Total	cm^{-1}	256	-408	-7459
Octupoles	cm^{-1}	-116	124	392
QP Fringe Field	cm^{-1}	253	1614	6369
Kinematic term	cm^{-1}	100	149	6967
Total	cm^{-1}	493	1479	6270

Analytical formula:	C_{xx}	C_{xy}	C_{yy}
Octupoles	-116	124	392
QP Fringe field	253	1614	6369
Kinematic term	100	149	6969

Very good agreement MADX-PTC with analytical formulae.

Summary of the contributions LER

Acceleraticum		C_{xx}	C_{xy}	C_{yy}
IP Sextupoles	cm^{-1}	-25	-51	-8300
Crab Sextupoles	cm^{-1}	-0.6	-5.5	-114
Arc Sextupoles	cm^{-1}	273	-450	-93
Sub Total	cm^{-1}	247	-510	-8520
Octupoles	cm^{-1}	-120	112	384
QP Fringe Field	cm^{-1}	240	1440	5750
Kinematic term	cm^{-1}	0.6	35	5090
Total	cm^{-1}	370	1205	3380

Analytical formula:	C_{xx}	C_{xy}	C_{yy}
Octupoles	-119	107	405
QP Fringe field	254	1616	6372
Kinematic term	100	149	6969

Average agreement Acceleraticum with analytical formulae (kinematic and fringe field term).

Summary of the contributions LER

MADX-PTC		C_{xx}	C_{xy}	C_{yy}
IP Sextupoles	cm^{-1}	-21	-43	-7301
Crab Sextupoles	cm^{-1}	-0.5	-4.9	-88
Arc Sextupoles	cm^{-1}	274	-433	-92
Sub Total	cm^{-1}	256	-481	-7482
Octupoles	cm^{-1}	-118	107	405
QP Fringe Field	cm^{-1}	254	1616	6372
Kinematic term	cm^{-1}	100	149	6967
Total	cm^{-1}	488	1390	6262

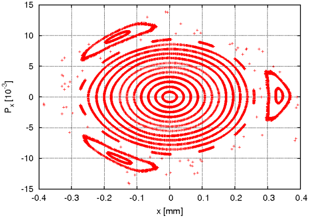
Analytical formula:	C_{xx}	C_{xy}	C_{yy}
Octupoles	-119	107	405
QP Fringe field	254	1616	6372
Kinematic term	100	149	6969

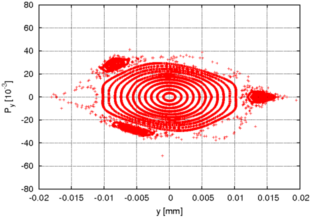
Very good agreement MADX-PTC with analytical formulae.

Comparison MADX-PTC/MAD8

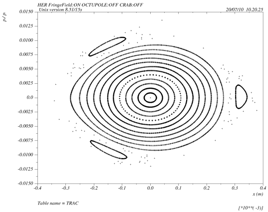
With Fringe Field/ No crab sextupole HER

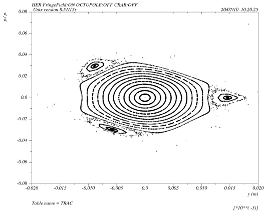
MADX-PTC 6th order





MAD8 with Lie4



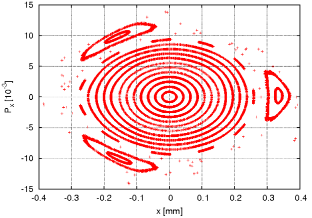


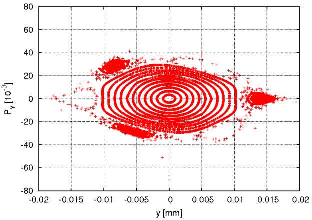
Good agreement between both codes.

Comparison MADX-PTC/Acceleraticum

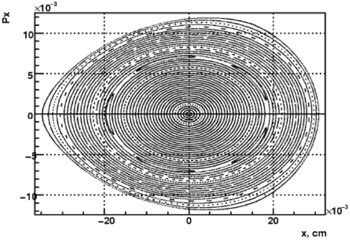
With Fringe Field/ No crab sextupole HER

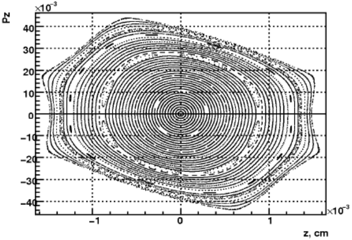
MADX-PTC 6th order





Acceleraticum





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Comparison MADX-PTC/Acceleraticum

With Fringe Field/ With crab sextupole HER

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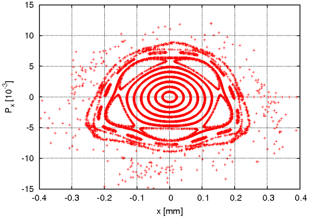
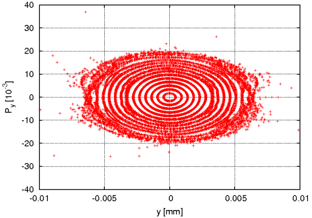
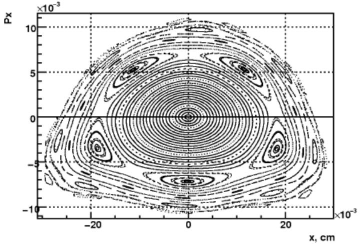
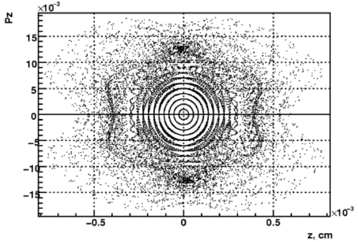




Acceleraticum



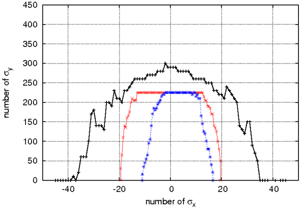


Good agreement between both codes.

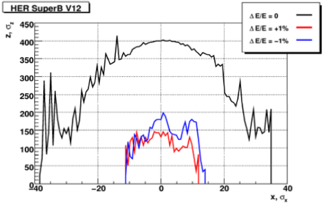
Dynamic aperture

No crab sextupole HER

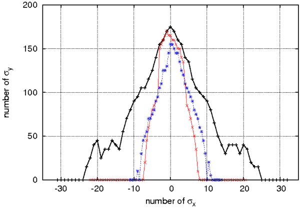
MADX-PTC 6th order

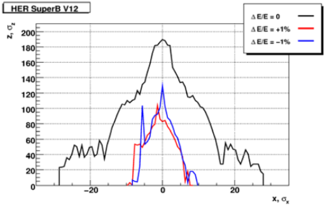


Acceleraticum



With crab sextupole HER





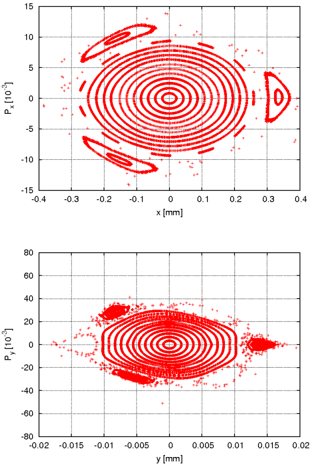
Enlargement dynamic aperture

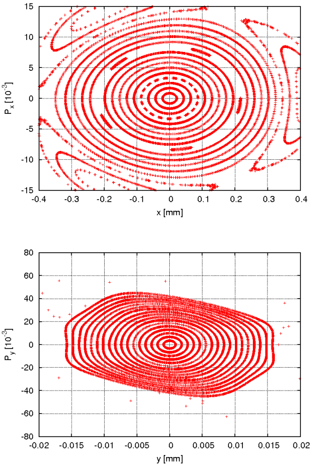
The dynamic aperture is a bit different between MADX-PTC and Acceleraticum.
The reason why we have this difference is a different treatment of the kinematic term but a new patch in Acceleraticum should reduce the difference (the kinematic term will be taken into account in the quadrupoles too).
A way to enlarge the dynamic aperture is to change the strength of the octupoles (one near the IP and the other near the crab sextupole).
The optimization was made “by hand”. A tracking was made for different parameter sets for the octupole strengthes. I have then chosen the parameter set which minimizes the non linearities.

Tracking with octupoles

No crab sextupoles HER

Reference octupoles / New octupoles



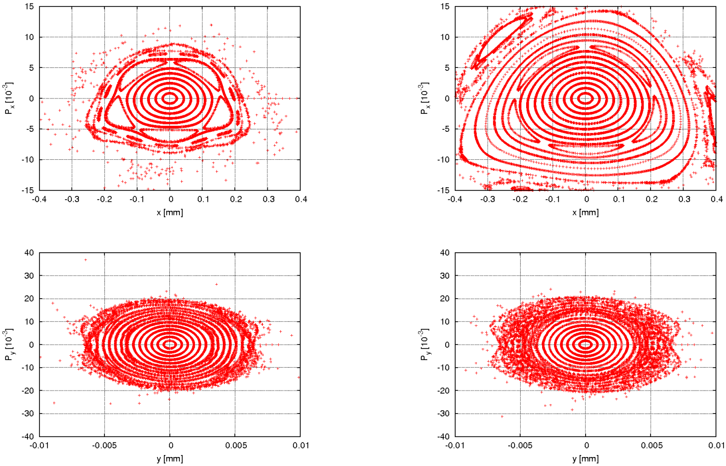


Very weak octupoles (-5 m^{-3} and -180 m^{-3})

Tracking with octupoles

With crab sextupoles HER

Reference octupoles | **New octupoles**



Very weak octupoles (-5 m^{-3} and -240 m^{-3})

Dynamic aperture with octupolar correction

No crab sextupole HER

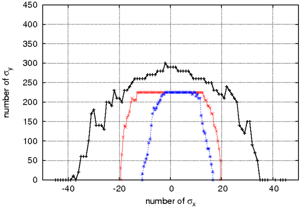
Reference octupoles

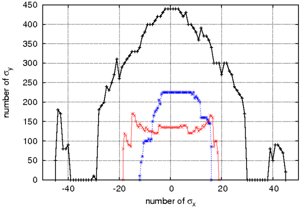
New octupoles

$\delta = 0\%$

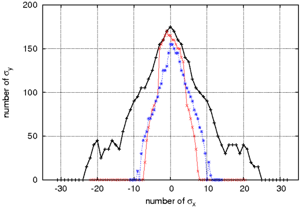
$\delta = +1\%$

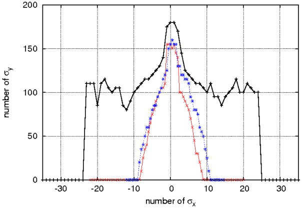
$\delta = -1\%$





With crab sextupole HER

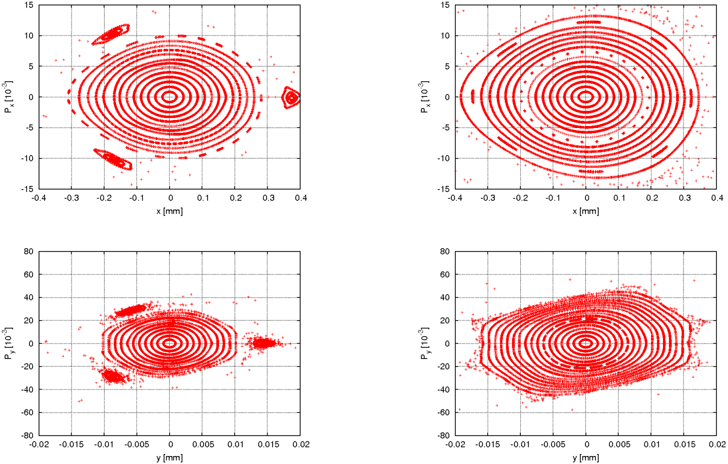




Tracking with octupoles

No crab sextupoles LER

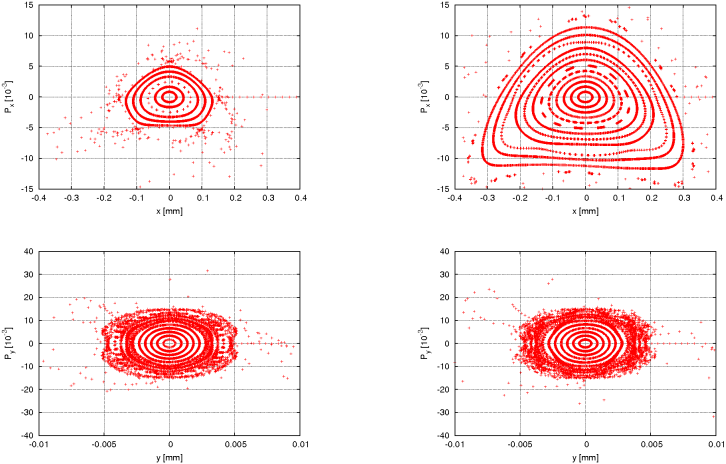
Reference octupoles | New octupoles



Very weak octupoles (-5 m^{-3} and -180 m^{-3})

With crab sextupoles LER

Reference octupoles | New octupoles



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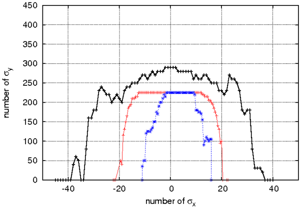
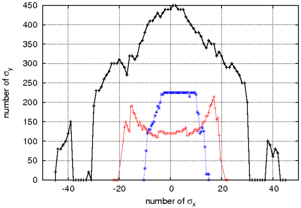
No crab sextupole LER

Reference octupoles | New octupoles



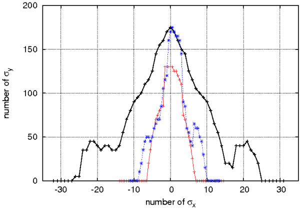


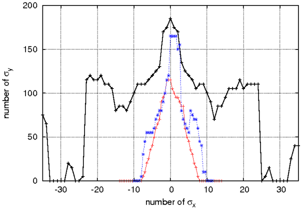
$\delta = 0\%$

$\delta = +1\%$

$\delta = -1\%$

With crab sextupole LER





Correction with decapoles

The dynamic aperture shrinks a lot with the momentum.
Optimizing the dynamic aperture on momentum with octupoles might shorten it off momentum.
An idea is to add decapoles in dispersive regions.
That enables to add an octupolar term proportional to the dispersion and δ at the decapole location.
For the moment, there is no significant gain.
Further studies (location and strength of the decapoles, …) are needed.

Conclusion

- Some simulations with MADX-PTC taking into account the fringe field were performed.
- A preliminary cross-checking of Piminov and Levichev's results was made by taking into account the fringe field effect in the quadrupoles.
- The order of magnitude for the dynamic aperture found with MADX-PTC is quite coherent with their results despite a few differences (kinematic term contribution).
- It is possible to enlarge significantly the on momentum dynamic aperture by adding weak octupoles in the structure.
- More studies must be pursued to evaluate if decapoles would be useful for the off momentum dynamic aperture.