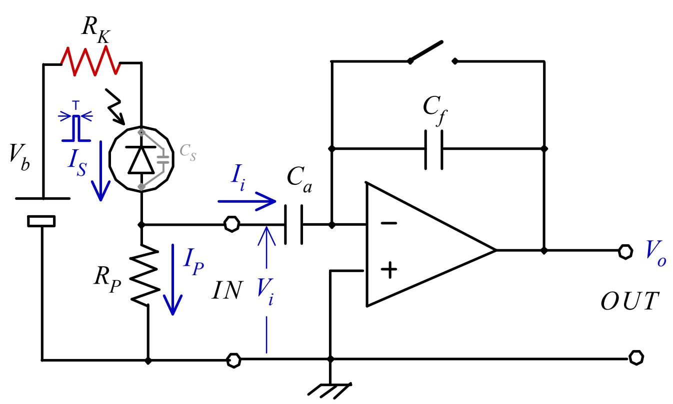
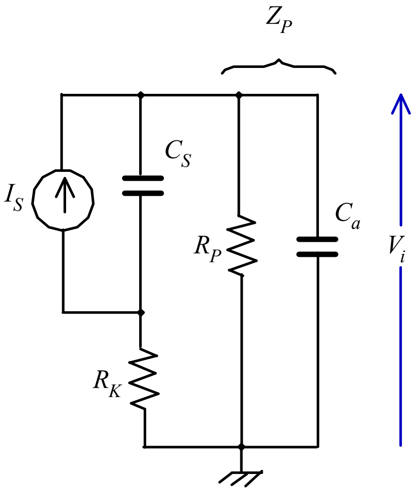
Valutazione globale di risposta dell'amplificatore di carica all'impulso rettangolare di corrente, in considerazione della presenza di "R_K "

Premessa:
Questo studio sfrutterà lavori già compiuti (valutazione velocità di risposta amplificatore di carica, implicazione R_K) per realizzare un'analisi complessiva (ammesso sia possibile perché molto complessa). Tale studio dovrà dividersi tra una risposta a gradino complessiva per la fase di "nascita" dell'impulso di corrente di ingresso, e una fase di rilascio (fine dell'impulso) che sfrutta i residui di accumulo energetico conservati (rappresentate dalle variabili di stato delle equazioni integro-differenziali) ma che possono essere rappresentate con generatori distribuiti intorno ad ogni componente implicato nell'accumulo di un residuo.

Sistema:



Sezione di ingresso (V_i).



$$Z_P=\frac{1}{\frac{1}{R_P}+S\cdot C_a}\qquad Z_P=\frac{R_P}{S\cdot C_a\cdot R_P+1}$$

$$Z_T=\frac{(Z_P+R_K)\cdot\frac{1}{S\cdot C_S}}{Z_P+R_K+\frac{1}{S\cdot C_S}}\qquad Z_T=\frac{\frac{Z_P+R_K}{\cancel{S\cdot C_S}}}{\frac{S\cdot C_S\cdot Z_P+S\cdot C_S\cdot R_K+1}{\cancel{S\cdot C_S}}}\qquad Z_T=\frac{Z_P+R_K}{S\cdot C_S\cdot Z_P+S\cdot C_S\cdot R_K+1}$$

$$V_i=\frac{I_S\cdot Z_T}{R_K+Z_P}\cdot Z_P\qquad V_i=I_S\cdot\frac{\frac{1}{C_S\cdot C_a\cdot R_K}}{S^2+S\cdot\frac{R_P\cdot(C_a+C_S)+C_S\cdot R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}+\frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}}$$

$$\begin{cases}\dfrac{R_P\cdot(C_a+C_S)+C_S\cdot R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}=\omega_o\\[2ex]\dfrac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}=\omega_n^2\end{cases}$$

$$V_i=I_S\cdot\frac{\omega_n^2\cdot R_P}{S^2+S\cdot\omega_o+\omega_n^2}\qquad £^{-1}\{V_i\}=I_S\cdot\frac{\omega_n^2\cdot R_P}{\sqrt{\Delta}}\cdot\mathrm{sh}(\sqrt{\Delta}\cdot t)\cdot e^{-\frac{\omega_o}{2}\cdot t}\qquad \Delta=\frac{\omega_o^2}{4}-\omega_n^2$$

$$\Delta=\frac{\omega_o^2-4\cdot\omega_n^2}{4}\qquad £^{-1}\{V_i\}=I_S\cdot\frac{2\cdot\omega_n^2\cdot R_P}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\frac{1}{2}\cdot\left(e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}\right)\cdot e^{-\frac{\omega_o}{2}\cdot t}$$

$$£^{-1}\{V_i\}=I_S\cdot R_P\cdot\frac{\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}\right)\cdot e^{-\frac{\omega_o}{2}\cdot t}\qquad £^{-1}\{V_i\}=I_S\cdot R_P\cdot\frac{\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(e^{-\frac{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-e^{-\frac{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}\right)$$

Risposta al gradino

$$V_{iG}=\int £^{-1}\{V_i\}\,\mathrm{d}(t)+K\qquad V_{iG}=I_S\cdot R_P\cdot\frac{\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\int\left(e^{-\frac{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-e^{-\frac{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}\right)\mathrm{d}t+K$$

$$V_{iG}=I_S\cdot R_P\cdot\frac{\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(\frac{2}{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot e^{-\frac{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-\frac{2}{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot e^{-\frac{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}\right)+K$$

COSTANTI DI TEMPO

$$\tau_1=\frac{2}{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}\qquad \tau_2=\frac{2}{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}$$

$$\tau_1=\frac{2\cdot C_S\cdot C_a\cdot R_P\cdot R_K}{R_P\cdot(C_a+C_S)+C_S\cdot R_K+\sqrt{R_P^2\cdot(C_a+C_S)^2+C_S\cdot R_K\cdot[C_S\cdot R_K+2\cdot R_P\cdot(C_S-C_a)]}}$$

$$\tau_2=\frac{2\cdot C_S\cdot C_a\cdot R_P\cdot R_K}{R_P\cdot(C_a+C_S)+C_S\cdot R_K-\sqrt{R_P^2\cdot(C_a+C_S)^2+C_S\cdot R_K\cdot[C_S\cdot R_K+2\cdot R_P\cdot(C_S-C_a)]}}$$

Valutazione costante K

$$\lim_{t\to\infty}(V_{iG}) = I_S\cdot R_P \qquad \lim_{t\to\infty}\left[I_S\cdot R_P\cdot\frac{\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(\frac{2}{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot e^{-\frac{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-\frac{2}{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot e^{-\frac{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}\right)+K\right]=I_S\cdot R_P$$

$$I_S\cdot R_P\cdot\frac{\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(\frac{2}{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot 0-\frac{2}{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot 0\right)+K=I_S\cdot R_P \qquad I_S\cdot R_P\cdot\frac{\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot 0+K=I_S\cdot R_P \qquad K=I_S\cdot R_P$$

$$V_{iG}=I_S\cdot R_P\cdot\left[\frac{2\cdot\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(\frac{e^{-\frac{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}-\frac{e^{-\frac{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+1\right]$$

$$\sqrt{\omega_o^2-4\cdot\omega_n^2}=\frac{\sqrt{R_P^2\cdot(C_a+C_S)^2+C_S\cdot R_K\cdot(C_S\cdot R_K+2\cdot R_P\cdot(C_S-C_a))}}{C_S\cdot C_a\cdot R_P\cdot R_K}$$

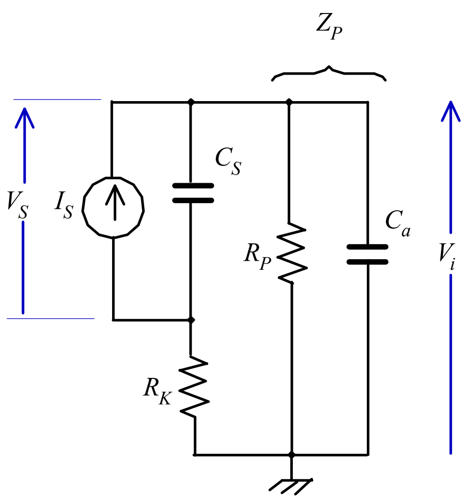
$$\frac{\omega_n^2}{\sqrt{\omega_o^2-\omega_n^2}}=\frac{1}{\sqrt{R_P^2\cdot(C_a+C_S)^2+C_S\cdot R_K\cdot(C_S\cdot R_K+2\cdot R_P\cdot(C_S-C_a))}}$$

$$\frac{\omega_n^2}{\sqrt{\omega_o^2-\omega_n^2}}=\frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}\cdot\frac{C_S\cdot C_a\cdot R_P\cdot R_K}{\sqrt{R_P^2\cdot(C_a+C_S)^2+C_S\cdot R_K\cdot(C_S\cdot R_K+2\cdot R_P\cdot(C_S-C_a))}}$$

Fase di rilascio ,circuito di ingresso (t ≥ T):

La rete di ingresso sarà alimentata dalle cariche accumulate sui condensatori al momento t=T; queste cariche possono essere rappresentate da generatori di tensione in serie ai condensatori con valore corrispondente alle tensioni dei condensatori calcolate con il transitorio a gradino al momento t=T. Per stabilire il valore sul condensatore di ingresso (C_S), dovrà essere calcolato un transitorio anche per la sua tensione (V_S).

Calcolo transitorio V_S



$$V_S=I_S+Z_T$$

$$Z_P=\frac{1}{S\cdot C_a+\frac{1}{R_P}} \qquad Z_P=\frac{R_P}{S\cdot C_a\cdot R_P+1} \qquad Z_T=\frac{1}{S\cdot C_S+\frac{1}{R_K+Z_P}}$$

$$R_K+Z_P=\frac{S\cdot C_a\cdot R_P\cdot R_K+R_P+R_K}{S\cdot C_a\cdot R_P+1} \qquad \frac{1}{R_K+Z_P}=\frac{S\cdot C_a\cdot R_P+1}{S\cdot C_a\cdot R_P\cdot R_K+R_P+R_K}$$

$$Z_T=\frac{1}{S\cdot C_S+\frac{S\cdot C_a\cdot R_P+1}{S\cdot C_a\cdot R_P\cdot R_K+R_P+R_K}}$$

$$Z_T=\frac{S\cdot C_a\cdot R_P\cdot R_K+R_P+R_K}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[C_S\cdot(R_P+R_K)+C_a\cdot R_P]+1} \qquad Z_T=\frac{\frac{S}{C_S}+\frac{R_P+R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}}{S^2+S\cdot\frac{C_S\cdot(R_P+R_K)+C_a\cdot R_P}{C_S\cdot C_a\cdot R_P\cdot R_K}+\frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}}$$

$$V_S = I_S \cdot \frac{\frac{S}{C_S} + \frac{R_P+R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}}{S^2 + S\cdot\frac{C_S\cdot(R_P+R_K)+C_a\cdot R_P}{C_S\cdot C_a\cdot R_P\cdot R_K} + \frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}}$$

$$V_S = I_S\cdot\frac{\frac{S}{C_S}+\omega_n^2\cdot(R_P+R_K)}{S^2+S\cdot\omega_o+\omega_n^2}$$

$$\Delta = \omega_n^2 - \frac{\omega_o^2}{4}\ (<0) \qquad |\Delta| = \frac{\omega_o^2-4\cdot\omega_n^2}{4}$$

$$\frac{V_{SG}}{I_S} = £^{-1}\left\{\frac{1}{S}\cdot\frac{\frac{S}{C_S}}{S^2+S\cdot\omega_o+\omega_n^2}\right\} + \int £^{-1}\left\{\frac{\omega_n^2\cdot(R_P+R_K)}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\mathrm{d}t + K$$

$$\frac{V_{SG}}{I_S} = \frac{1}{C_S}\cdot £^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\} + \omega_n^2\cdot(R_P+R_K)\cdot\int £^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\mathrm{d}t + K$$

$$£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\} = \frac{1}{\sqrt{|\Delta|}}\cdot\frac{e^{\sqrt{|\Delta|}\cdot t}-e^{-\sqrt{|\Delta|}\cdot t}}{2}\cdot e^{-\frac{\omega_o}{2}\cdot t}$$

$$£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\} = \frac{2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\frac{1}{2}\cdot\left(e^{\frac{1}{2}\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot t}-e^{-\frac{1}{2}\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot t}\right)\cdot e^{-\frac{\omega_o}{2}\cdot t}$$

$$£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\} = \frac{1}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left[e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}\right]$$

$$£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\} = \frac{e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}$$

$$\int £^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\mathrm{d}t = \frac{1}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left[\int e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}\mathrm{d}t-\int e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}\mathrm{d}t\right]$$

$$\int £^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\mathrm{d}t = \frac{1}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left[\frac{e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)}-\frac{e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)}\right]$$

$$\int £^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\mathrm{d}t = \frac{2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\frac{\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{\cancel{\omega_o^2}-\cancel{\omega_o^2}+4\cdot\omega_n^2}$$

$$\int £^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\mathrm{d}t = \frac{\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{2\cdot\omega_n^2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}$$

$$\frac{V_{SG}}{I_S} = \frac{1}{C_S}\cdot\frac{e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}+(R_P+R_K)\cdot\cancel{\omega_n^2}\cdot\frac{\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{2\cdot\cancel{\omega_n^2}\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+K$$

$$\frac{V_{SG}}{I_S} = \frac{2\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-2\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+C_S\cdot(R_P+R_K)\cdot\left[\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}\right]}{2\cdot C_S\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+K$$

$$\frac{V_{SG}}{I_S}=\frac{\left[2-C_S\cdot(R_P+R_K)\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\right]\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+\left[C_S\cdot(R_P+R_K)\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-2\right]\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{2\cdot C_S\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+K$$

Valutazione K:

$$\lim_{t\to 0}\left(\frac{V_{SG}}{I_S}\right)=0 \qquad \frac{2-C_S\cdot(R_P+R_K)\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)+C_S\cdot(R_P+R_K)\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-2}{2\cdot C_S\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+K=0$$

$$\frac{\cancel{C_S}\cdot(R_P+R_K)}{2\cdot\cancel{C_S}\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(\cancel{\omega_o}-\sqrt{\omega_o^2-4\cdot\omega_n^2}-\cancel{\omega_o}-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)+K=0 \qquad -\frac{\cancel{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot(R_P+R_K)}{\cancel{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}}+K=0 \qquad \boxed{K=R_P+R_K}$$

$$\frac{V_{SG}}{I_S}=\frac{\left[2-C_S\cdot(R_P+R_K)\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\right]\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+\left[C_S\cdot(R_P+R_K)\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-2\right]\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{2\cdot C_S\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+R_P+R_K$$

$$\boxed{V_{SG}=I_S\cdot\left\{\frac{\left[2-C_S\cdot(R_P+R_K)\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\right]\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+\left[C_S\cdot(R_P+R_K)\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-2\right]\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{2\cdot C_S\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+R_P+R_K\right\}}$$

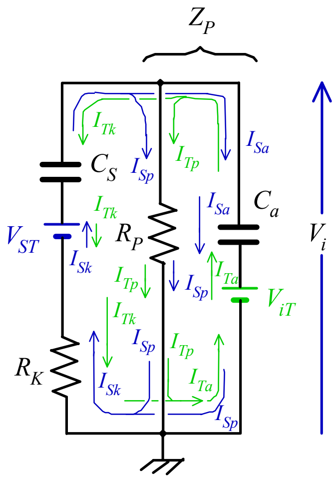
$$\begin{cases}\omega_o=\dfrac{C_S\cdot(R_P+R_K)+C_a\cdot R_P}{C_S\cdot C_a\cdot R_P\cdot R_K}\\[2ex] \omega_n^2=\dfrac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}\end{cases}$$

Ratifica potenziali di accumulo iniziali sulla fase di rilascio (accumulo durante "T"):

$$\boxed{V_{iT}=I_S\cdot R_P\cdot\left[\frac{2\cdot\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(\frac{e^{-\frac{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot T}}{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}-\frac{e^{-\frac{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot T}}{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+1\right]}$$

$$\boxed{V_{ST}=I_S\cdot\left\{\frac{\left[2-C_S\cdot(R_P+R_K)\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\right]\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot T}+\left[C_S\cdot(R_P+R_K)\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-2\right]\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot T}}{2\cdot C_S\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+R_P+R_K\right\}}$$

Dinamica fase di rilascio (sovrapposizione degli effetti)



$$\frac{V_{ST}}{R_K+\frac{1}{S\cdot C_S}+Z_P}\cdot Z_P\cdot S\cdot C_a = I_{Sa}$$

$$I_{Sa} = V_{ST}\cdot\frac{Z_P\cdot S\cdot C_a}{\frac{S\cdot C_S\cdot(Z_P+R_K)+1}{S\cdot C_S}}$$

$$I_{Sa} = V_{ST}\cdot Z_P\cdot S\cdot C_a\cdot\frac{S\cdot C_S}{S\cdot C_S\cdot(Z_P+R_K)+1}$$

$$I_{Sa} = V_{ST}\cdot\frac{Z_P\cdot S^2\cdot C_S\cdot C_a}{S\cdot C_S\cdot(Z_P+R_K)+1}$$

$$I_{Sa} = V_{ST}\cdot\frac{R_P}{S\cdot C_a\cdot R_P+1}\cdot\frac{S^2\cdot C_S\cdot C_a}{S\cdot C_S\cdot\left(\frac{R_P}{S\cdot C_a\cdot R_P+1}+R_K\right)+1}$$

$$I_{Sa} = V_{ST}\cdot\frac{R_P}{S\cdot C_a\cdot R_P+1}\cdot\frac{S^2\cdot C_S\cdot C_a}{S\cdot C_S\cdot\frac{R_P+S\cdot C_a\cdot R_P\cdot R_K+R_K}{S\cdot C_a\cdot R_P+1}+1}$$

$$I_{Sa} = V_{ST}\cdot\frac{R_P}{\cancel{S\cdot C_a\cdot R_P+1}}\cdot\frac{S^2\cdot C_S\cdot C_a}{\frac{S\cdot C_S\cdot(S\cdot C_a\cdot R_P\cdot R_K+R_P+R_K)+S\cdot C_a\cdot R_P+1}{\cancel{S\cdot C_a\cdot R_P+1}}}$$

$$I_{Sa} = V_{ST}\cdot\frac{S^2\cdot C_S\cdot C_a\cdot R_P}{S\cdot C_S\cdot(S\cdot C_a\cdot R_P\cdot R_K+R_P+R_K)+S\cdot C_a\cdot R_P+1}$$

$$\boxed{I_{Sa} = V_{ST}\cdot\frac{S^2\cdot C_S\cdot C_a\cdot R_P}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[C_S\cdot(R_P+R_K)+C_a\cdot R_P]+1}}$$

$$I_{Ta} = \frac{V_{iT}}{\frac{1}{S\cdot C_a}+\frac{R_P\cdot\left(\frac{1}{S\cdot C_S}+R_K\right)}{\frac{1}{S\cdot C_S}+R_P+R_K}}$$

$$I_{Ta} = \frac{V_{iT}}{\frac{1}{S\cdot C_a}+\frac{R_P\cdot(S\cdot C_S\cdot R_K+1)}{S\cdot C_S\cdot(R_P+R_K)+1}}$$

$$I_{Ta} = V_{iT}\cdot\frac{S^2\cdot C_S\cdot C_a\cdot(R_P+R_K)+S\cdot C_a}{S\cdot C_S\cdot(R_P+R_K)+1+S\cdot C_a\cdot R_P\cdot(S\cdot C_S\cdot R_K+1)}$$

$$\boxed{I_{Ta} = V_{iT}\cdot\frac{S\cdot C_a\cdot[S\cdot C_S\cdot(R_P+R_K)+1]}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[C_S\cdot(R_P+R_K)+C_a\cdot R_P]+1}}$$

$$\boxed{V_i = V_{iT}+\frac{I_{Sa}-I_{Ta}}{S\cdot C_a}}$$

$$V_i = V_{iT}+\frac{1}{S\cdot C_a}\cdot\left\{V_{ST}\cdot\frac{S^2\cdot C_S\cdot C_a\cdot R_P}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[C_S\cdot(R_P+R_K)+C_a\cdot R_P]+1}-V_{iT}\cdot\frac{S\cdot C_a\cdot[S\cdot C_S\cdot(R_P+R_K)+1]}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[C_S\cdot(R_P+R_K)+C_a\cdot R_P]+1}\right\}$$

$$V_i = V_{iT}+\frac{1}{S\cdot C_a}\cdot\frac{V_{ST}\cdot S^2\cdot C_S\cdot C_a\cdot R_P-V_{iT}\cdot S\cdot C_a\cdot[S\cdot C_S\cdot(R_P+R_K)+1]}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[C_S\cdot(R_P+R_K)+C_a\cdot R_P]+1}$$

$$V_i = V_{iT}+\frac{\cancel{S\cdot C_a}}{\cancel{S\cdot C_a}}\cdot\frac{V_{ST}\cdot S\cdot C_S\cdot R_P-V_{iT}\cdot[S\cdot C_S\cdot(R_P+R_K)+1]}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[C_S\cdot(R_P+R_K)+C_a\cdot R_P]+1}$$

$$V_i = V_{iT}+\frac{S\cdot C_S\cdot[V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)]-V_{iT}}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[C_S\cdot(R_P+R_K)+C_a\cdot R_P]+1}$$

$$\boxed{V_i = V_{iT}+\frac{S\cdot\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{C_a\cdot R_P\cdot R_K}-\frac{V_{iT}}{C_S\cdot C_a\cdot R_P\cdot R_K}}{S^2+S\cdot\frac{C_S\cdot(R_P+R_K)+C_a\cdot R_P}{C_S\cdot C_a\cdot R_P\cdot R_K}+\frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}}}$$

...ricordando

$$\frac{R_P\cdot(C_a+C_S)+C_S\cdot R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}=\omega_o \qquad \frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}=\omega_n^2$$

ma: $\dfrac{C_S\cdot(R_P+R_K)+C_a\cdot R_P}{C_S\cdot C_a\cdot R_P\cdot R_K}=\dfrac{R_P\cdot(C_a+C_S)+C_S\cdot R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}$

...allora:

$$V_{id}=V_{iT}+\frac{S\cdot\left(\frac{V_{ST}}{C_a\cdot R_K}-V_{iT}\cdot\frac{R_P+R_K}{C_a\cdot R_P\cdot R_K}\right)-V_{iT}\cdot\omega_n^2}{S^2+S\cdot\omega_o+\omega_n^2}$$

$$\frac{V_{ST}}{C_a\cdot R_K}-V_{iT}\cdot\frac{R_P+R_K}{C_a\cdot R_P\cdot R_K}=\gamma$$

NOTA: V_i è stata cambiata con V_{id} per distinguerla come appartenente alla fase "discendente".

$$V_{id}=V_{iT}+\frac{S\cdot\gamma-V_{iT}\cdot\omega_n^2}{S^2+S\cdot\omega_o+\omega_n^2}$$

...separazione in frazioni parziali per agevolare l'analisi nel dominio t

$$V_{id}=V_{iT}+\frac{S\cdot\gamma}{S^2+S\cdot\omega_o+\omega_n^2}-\frac{V_{iT}\cdot\omega_n^2}{S^2+S\cdot\omega_o+\omega_n^2}$$

NOTA:

I denominatori sono gli stessi per lo sviluppo di V_{ST} e la forma complessiva presenta l'analoga forma precedente.

$V_{id}=V_{iT}+\alpha-\beta$ dove: $\alpha=\dfrac{S\cdot\gamma}{S^2+S\cdot\omega_o+\omega_n^2}$ e $\beta=\dfrac{V_{iT}\cdot\omega_n^2}{S^2+S\cdot\omega_o+\omega_n^2}$

Quindi: $V_{idG}=V_{iT}+\pounds^{-1}\left(\frac{1}{S}\cdot\alpha\right)+\int\pounds^{-1}(\beta)\,\mathrm{d}t+K_d$

cioè

$$V_{idG}=V_{iT}+\gamma\cdot\frac{e^{\sqrt{\left(\frac{\omega_o}{2}\right)^2-\omega_n^2}\cdot t}-e^{-\sqrt{\left(\frac{\omega_o}{2}\right)^2-\omega_n^2}\cdot t}}{2\cdot\sqrt{\left(\frac{\omega_o}{2}\right)^2-\omega_n^2}}\cdot e^{-\frac{\omega_o}{2}\cdot t}-V_{iT}\cdot\omega_n^2\cdot\int\frac{e^{\sqrt{\left(\frac{\omega_o}{2}\right)^2-\omega_n^2}\cdot t}-e^{-\sqrt{\left(\frac{\omega_o}{2}\right)^2-\omega_n^2}\cdot t}}{2\cdot\sqrt{\left(\frac{\omega_o}{2}\right)^2-\omega_n^2}}\cdot e^{-\frac{\omega_o}{2}\cdot t}\,\mathrm{d}t+K_d$$

$$V_{idG}=V_{iT}+\gamma\cdot\frac{e^{\frac{1}{2}\cdot\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}-\omega_o\right)\cdot t}-e^{-\frac{1}{2}\cdot\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}+\omega_o\right)\cdot t}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}-$$
$$-V_{iT}\cdot\cancel{\omega_n^2}\cdot\frac{\cancel{2}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\frac{\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}+\omega_o\right)\cdot e^{\frac{1}{2}\cdot\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}-\omega_o\right)\cdot t}+\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}-\omega_o\right)\cdot e^{-\frac{1}{2}\cdot\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}+\omega_o\right)\cdot t}}{\cancel{\omega_o^2}-\cancel{4}^2\cdot\cancel{\omega_n^2}-\cancel{\omega_o^2}}+K_d$$

$$V_{idG}=V_{iT}+\frac{\left[V_{iT}\cdot\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}+\omega_o\right)+2\cdot\gamma\right]\cdot e^{-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+\left[V_{iT}\cdot\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}-\omega_o\right)-2\cdot\gamma\right]\cdot e^{-\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+K_d$$

$$V_{idG}=V_{iT}+\frac{V_{iT}\cdot\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}+\omega_o\right)+2\cdot\left(\frac{V_{ST}}{C_a\cdot R_K}-V_{iT}\cdot\frac{R_P+R_K}{C_a\cdot R_P\cdot R_K}\right)}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot e^{-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+$$
$$+\frac{V_{iT}\cdot\left(\sqrt{\omega_o^2-4\cdot\omega_n^2}-\omega_o\right)-2\cdot\left(\frac{V_{ST}}{C_a\cdot R_K}-V_{iT}\cdot\frac{R_P+R_K}{C_a\cdot R_P\cdot R_K}\right)}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot e^{-\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+K_d$$

$$V_{idG}=V_{iT}+\left[V_{iT}\cdot\left(\frac{1}{2}+\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot C_a\cdot R_P\cdot R_K}\right]\cdot e^{-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+$$
$$+\left[V_{iT}\cdot\left(\frac{1}{2}-\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)-\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot C_a\cdot R_P\cdot R_K}\right]\cdot e^{-\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+K_d$$

Valutazione costante K_d

La valutazione può essere eseguita all'istante t_0 (dopo T) per corrispondere V_{idG} al valore dei contributi V_{iT} e V_{ST} senza i condensatori (cioè V_{idG} = V_{iT})

$$\lim_{t\to 0}(V_{idG})=V_{iT}$$

$$\lim_{t\to 0}\left(\begin{array}{l}V_{iT}+\left[V_{iT}\cdot\left(\frac{1}{2}+\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot C_a\cdot R_P\cdot R_K}\right]\cdot e^{-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+\\ +\left[V_{iT}\cdot\left(\frac{1}{2}-\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)-\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot C_a\cdot R_P\cdot R_K}\right]\cdot e^{-\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+K_d\end{array}\right)=V_{iT}$$

$$V_{iT}+V_{iT}\cdot\left(\frac{1}{2}+\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot C_a\cdot R_P\cdot R_K}+V_{iT}\cdot\left(\frac{1}{2}-\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)-\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot C_a\cdot R_P\cdot R_K}+K_d=V_{iT}$$

$$V_{iT}\cdot\left(1+\frac{1}{2}+\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+\frac{1}{2}-\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+K_d=V_{iT}$$

$$2\cdot V_{iT}-V_{iT}=-K_d$$

$$\boxed{K_d=-V_{iT}}$$

$$V_{idG}=\left[V_{iT}\cdot\left(\frac{1}{2}+\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot C_a\cdot R_P\cdot R_K}\right]\cdot e^{-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+$$
$$+\left[V_{iT}\cdot\left(\frac{1}{2}-\frac{\omega_o}{2\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)-\frac{V_{ST}\cdot R_P-V_{iT}\cdot(R_P+R_K)}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot C_a\cdot R_P\cdot R_K}\right]\cdot e^{-\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}$$

Sintesi rete d'ingresso

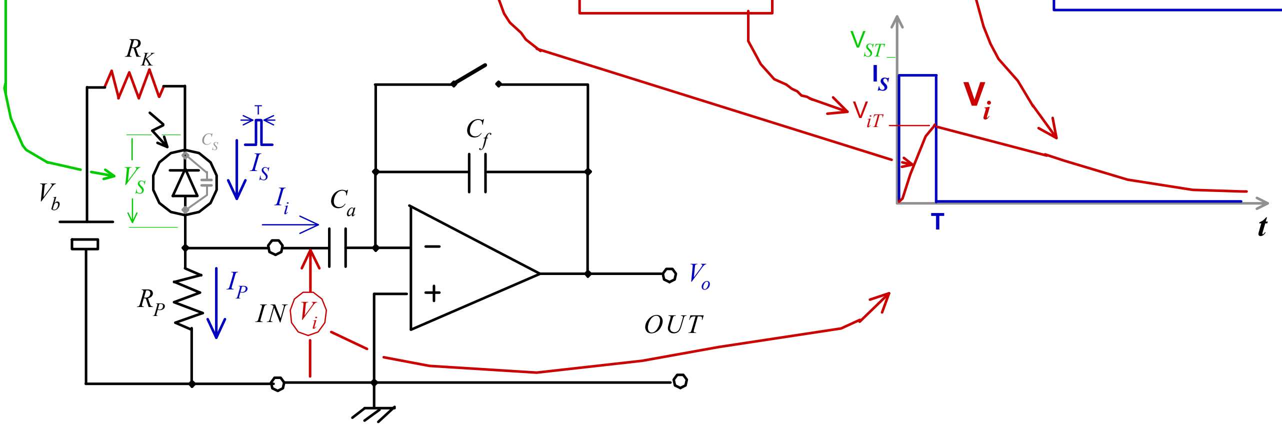
$$V_{ST}=I_S\cdot\left\{\frac{\left[2-C_S\cdot(R_P+R_K)\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\right]\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot T}+\left[C_S\cdot(R_P+R_K)\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-2\right]\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot T}}{2\cdot C_S\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+R_P+R_K\right\}$$

$$V_{iG}=I_S\cdot R_P\cdot\left[\frac{2\cdot\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(\frac{e^{-\frac{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}-\frac{e^{-\frac{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+1\right]$$

$$\omega_o=\frac{R_P\cdot(C_a+C_S)+C_S\cdot R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}$$

$$\omega_n^2=\frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}$$

$$V_{iT}=V_{iG}\Big|_{t=T}$$



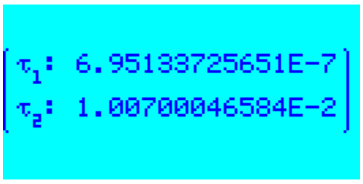
Valutazioni numeriche (assegnazione valori di esempio)

I_S =300nA (15fC accumulati per 50ns) **T=50ns**

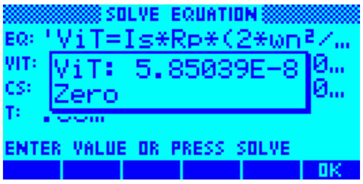
$$\begin{cases} R_K = 10\,K\Omega \\ R_P = 1\,M\Omega \\ C_S = 70\,pF \\ C_a = 10\,nF \end{cases}$$

ω_0 = 1.438671 Mrad/s

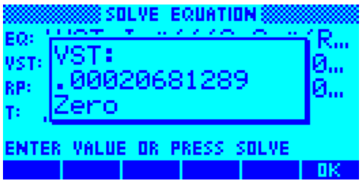
ω_n^2 = 142.857143 Mrad^2/s^2



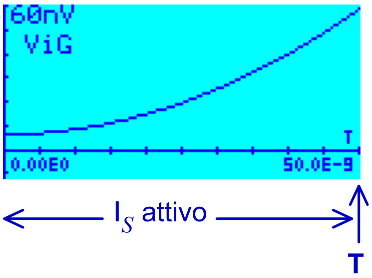
Calcolo V_{iT}



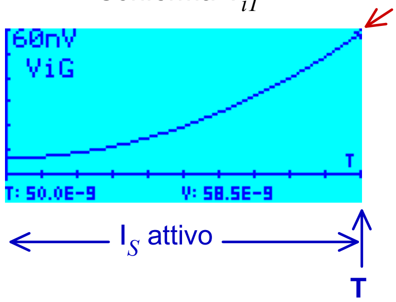
Calcolo V_{ST}



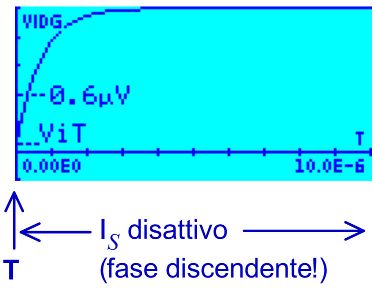
Plot V_{iG}



Conferma V_{iT}

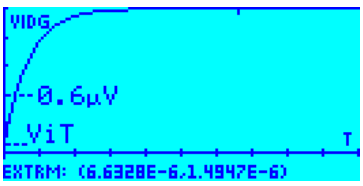


Plot V_{idG}

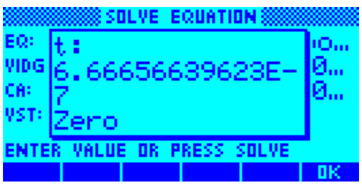


Notare che questa fase, in cui il generatore I_S è spento, il potenziale continua a crescere (non è un errore)... Poi, il potenziale decrescerà, ma il fenomeno determina un massimo (e un segnale ritardato) che è ben superiore al valore di fine impulso (V_{iT}).

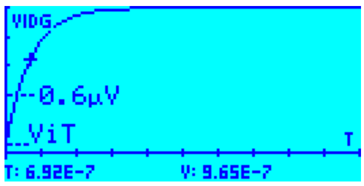
Localizzazione massimo



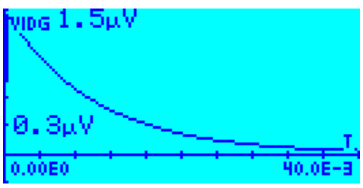
Costante di tempo τ_R calcolata per via grafica



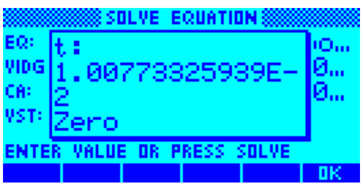
Costante di tempo



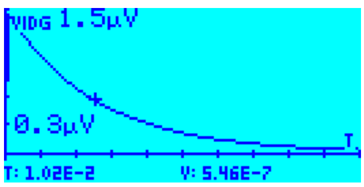
Rilevazione fase di discesa:



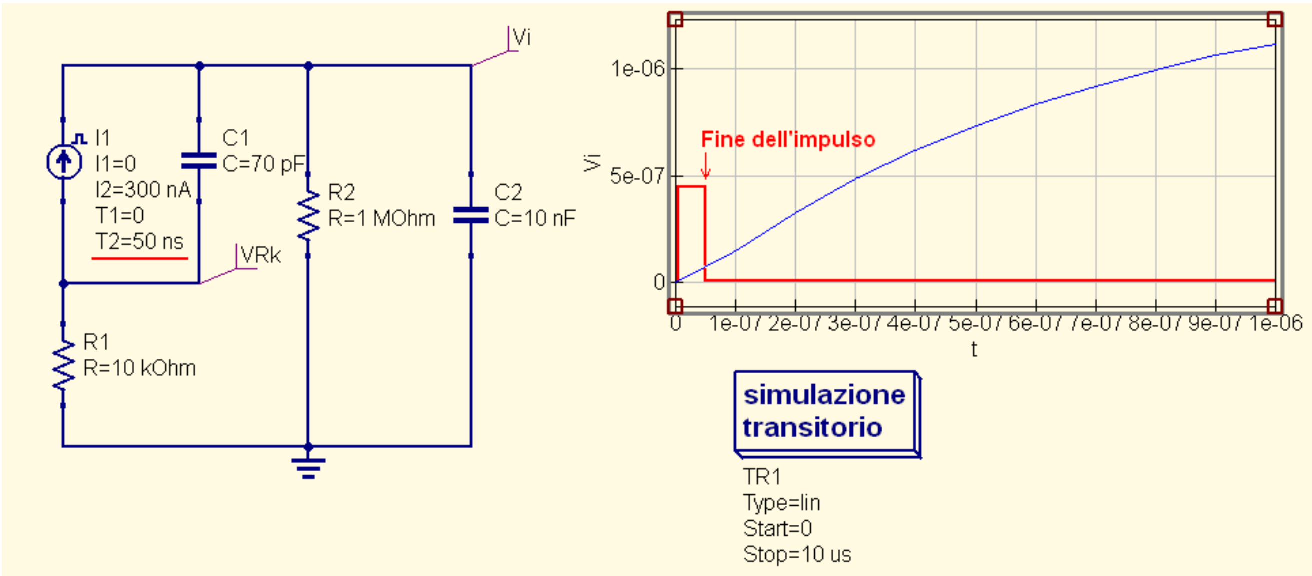
Costante di tempo τ_F calcolata per via grafica

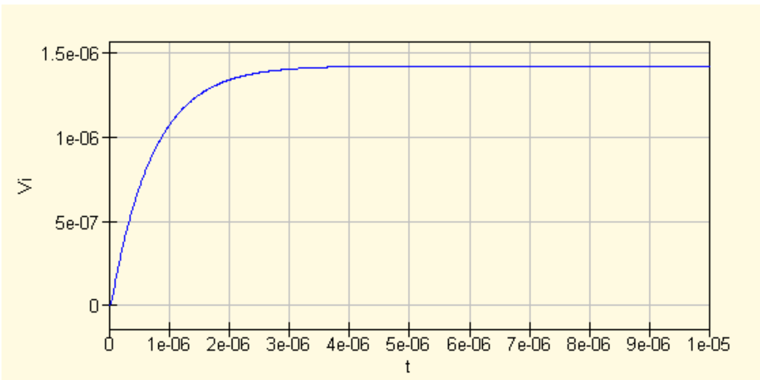


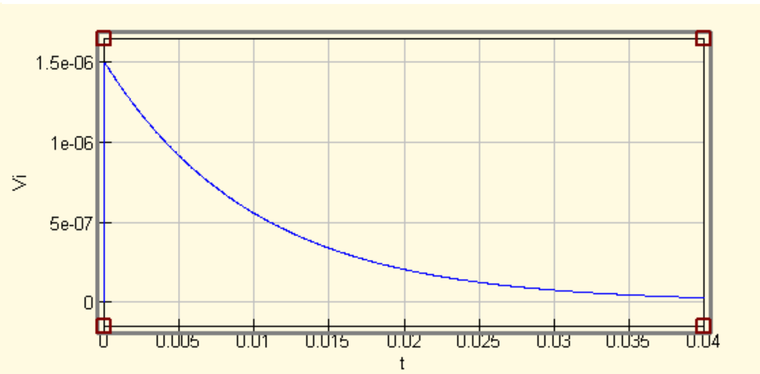
Costante di tempo



Conferme con simulatore







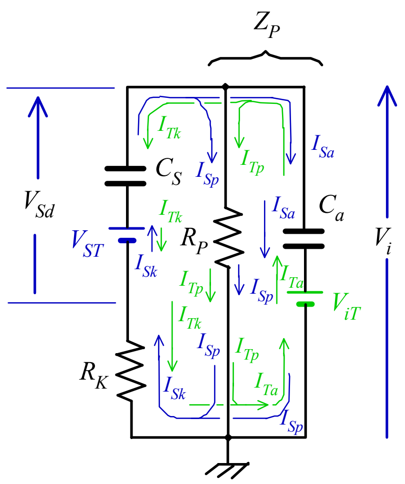
Verifica sulla quantità di carica accumulata per la massima tensione

Carica generata totale: $Q_T = I_S \cdot T \rightarrow 1.5 \cdot 10^{-14}$ C

Massima V_i :

```
Vim: 1.49473444396E-6
tm: 6.63277778827E-6
```

Per la verifica, serve il calcolo della distribuzione delle cariche al tempo t_M , su entrambi i condensatori. Per la fase di "rilascio" non è stato necessario il calcolo di V_S (su C_S) ma gli effetti della corrente per la valutazione di V_{idG} . Pertanto deve essere sviluppata l'equazione di V_S per la fase di rilascio (V_{SdG}).



$$V_{Sd}=V_{ST}+\frac{I_{Tk}-I_{Sk}}{S\cdot C_S}$$

$$I_{Tk}=\frac{V_{iT}}{\frac{1}{S\cdot C_a}+\frac{R_P\cdot\left(\frac{1}{S\cdot C_S}+R_K\right)}{\frac{1}{S\cdot C_S}+R_P+R_K}}\cdot\frac{R_P\cdot\cancel{\left(\frac{1}{S\cdot C_S}+R_K\right)}}{\frac{1}{S\cdot C_S}+R_P+R_K}\cdot\frac{1}{\cancel{\frac{1}{S\cdot C_S}+R_K}}$$

$$I_{Tk}=V_{iT}\cdot\frac{S\cdot C_a\cdot\cancel{\left(\frac{1}{S\cdot C_S}+R_P+R_K\right)}}{\frac{1}{S\cdot C_S}+R_P+R_K+S\cdot C_a\cdot R_P\cdot\left(\frac{1}{S\cdot C_S}+R_K\right)}\cdot\frac{R_P}{\cancel{\frac{1}{S\cdot C_S}+R_P+R_K}}$$

$$I_{Tk}=V_{iT}\cdot\frac{S\cdot C_a\cdot R_P}{\frac{1+S\cdot C_S\cdot(R_P+R_K)+S\cdot C_a\cdot R_P+S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K}{S\cdot C_S}}$$

$$I_{Tk}=V_{iT}\cdot\frac{S^2\cdot C_S\cdot C_a\cdot R_P}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}$$

$$I_{Sk}=\frac{V_{ST}}{Z_P+R_K+\frac{1}{S\cdot C_S}}\qquad Z_P=\frac{R_P}{S\cdot C_a\cdot R_P+1}\qquad I_{Sk}=\frac{V_{ST}}{\frac{R_P}{S\cdot C_a\cdot R_P+1}+\frac{1}{S\cdot C_S}+R_K}\qquad I_{Sk}=V_{ST}\cdot\frac{S\cdot C_S\cdot(S\cdot C_a\cdot R_P+1)}{S\cdot C_S\cdot R_P+S\cdot C_a\cdot R_P+1+S\cdot C_S\cdot R_K\cdot(S\cdot C_a\cdot R_P+1)}$$

$$I_{Sk}=V_{ST}\cdot\frac{S\cdot C_S\cdot(S\cdot C_a\cdot R_P+1)}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}$$

$$V_{Sd}=V_{ST}+V_{iT}\cdot\frac{S\cdot C_a\cdot R_P}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}-V_{ST}\cdot\frac{S\cdot C_a\cdot R_P+1}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}$$

$$V_{Sd}=V_{ST}\cdot\frac{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+\cancel{1}-S\cdot C_a\cdot R_P-\cancel{1}}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}+V_{iT}\cdot\frac{S\cdot C_a\cdot R_P}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}$$

$$V_{Sd}=V_{ST}\cdot\frac{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+\cancel{C_a}-\cancel{C_a})+C_S\cdot R_K]}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}+V_{iT}\cdot\frac{S\cdot C_a\cdot R_P}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}$$

$$V_{Sd}=V_{ST}\cdot\frac{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot C_S\cdot(R_P+R_K)}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}+V_{iT}\cdot\frac{S\cdot C_a\cdot R_P}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}$$

$$V_{Sd}=V_{ST}\cdot\frac{S\cdot C_S\cdot(S\cdot C_a\cdot R_P\cdot R_K+R_P+R_K)}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}+V_{iT}\cdot\frac{S\cdot C_a\cdot R_P}{S^2\cdot C_S\cdot C_a\cdot R_P\cdot R_K+S\cdot[R_P\cdot(C_S+C_a)+C_S\cdot R_K]+1}$$

Normalizzazione:

$$V_{Sd}=V_{ST}\cdot\frac{S\cdot\frac{S\cdot C_a\cdot R_P\cdot R_K+R_P+R_K}{C_a\cdot R_P\cdot R_K}}{S^2+S\cdot\frac{R_P\cdot(C_S+C_a)+C_S\cdot R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}+\frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}}+V_{iT}\cdot\frac{S\cdot\frac{1}{C_S\cdot R_K}}{S^2+S\cdot\frac{R_P\cdot(C_S+C_a)+C_S\cdot R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}+\frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}}$$

...solita storia:

$$\omega_o=\frac{R_P\cdot(C_a+C_S)+C_S\cdot R_K}{C_S\cdot C_a\cdot R_P\cdot R_K}$$

$$\omega_n^2=\frac{1}{C_S\cdot C_a\cdot R_P\cdot R_K}$$

$$V_{Sd}=V_{ST}\cdot\frac{S\cdot\left(S+\frac{R_P+R_K}{C_a\cdot R_P\cdot R_K}\right)}{S^2+S\cdot\omega_o+\omega_n^2}+V_{iT}\cdot\frac{S\cdot\frac{1}{C_S\cdot R_K}}{S^2+S\cdot\omega_o+\omega_n^2}$$

$$\boxed{V_{Sd}=V_{ST}\cdot\frac{S\cdot[S+\omega_n^2\cdot C_S\cdot(R_P+R_K)]}{S^2+S\cdot\omega_o+\omega_n^2}+V_{iT}\cdot\frac{S\cdot\frac{1}{C_S\cdot R_K}}{S^2+S\cdot\omega_o+\omega_n^2}}$$

Risposta al gradino

$$V_{SdG}=V_{ST}\cdot£^{-1}\left\{\frac{S}{S^2+S\cdot\omega_o+\omega_n^2}+\frac{\omega_n^2\cdot C_S\cdot(R_P+R_K)}{S^2+S\cdot\omega_o+\omega_n^2}\right\}+V_{iT}\cdot\frac{1}{C_S\cdot R_K}\cdot£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}$$

$$V_{SdG}=V_{ST}\cdot\left[\frac{d\left(£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\right)}{dt}+\omega_n^2\cdot C_S\cdot(R_P+R_K)\cdot£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\right]+V_{iT}\cdot\frac{1}{C_S\cdot R_K}\cdot£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}$$

...ricordando:

$$\boxed{£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}=\frac{e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}}$$

$$\frac{d\left(£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\right)}{dt}=\frac{\frac{d\left(e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}\right)}{dt}-\frac{d\left(e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}\right)}{dt}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}$$

$$\frac{d\left(£^{-1}\left\{\frac{1}{S^2+S\cdot\omega_o+\omega_n^2}\right\}\right)}{dt}=\frac{-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}$$

$$V_{SdG}=V_{ST}\cdot\frac{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}+\omega_n^2\cdot C_S\cdot(R_P+R_K)\cdot\left[e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}\right]}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}+$$

$$+V_{iT}\cdot\frac{e^{-\frac{1}{2}\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}-e^{-\frac{1}{2}\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot t}}{C_S\cdot R_K\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}+K_{ST}$$

$$V_{SdG}=e^{-\frac{1}{2}\cdot\omega_o\cdot t}\cdot\left\{V_{ST}\cdot\frac{-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}+\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\cdot e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}+\omega_n^2\cdot C_S\cdot(R_P+R_K)\cdot\left[e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}\right]}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}+V_{iT}\cdot\frac{e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{C_S\cdot R_K\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right\}+K_{ST}$$

$$\boxed{V_{SdG}=\left\{V_{ST}\cdot\frac{\left[\omega_n^2\cdot C_S\cdot(R_P+R_K)-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\right]\cdot e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}+\left[\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-\omega_n^2\cdot C_S\cdot(R_P+R_K)\right]\cdot e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}+V_{iT}\cdot\frac{e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{C_S\cdot R_K\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right\}\cdot e^{-\frac{1}{2}\cdot\omega_o\cdot t}+K_{ST}}$$

Valutazione costante K_{ST}

$$\lim_{t\to 0}(V_{SdG})=V_{ST}$$

$$\lim_{t\to 0}(V_{SdG})=V_{ST}\cdot\frac{\cancel{\omega_n^2\cdot C_S\cdot(R_P+R_K)}-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)+\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-\cancel{\omega_n^2\cdot C_S\cdot(R_P+R_K)}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}+\cancel{V_{iT}\cdot\frac{1-1}{C_S\cdot R_K\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}}+K_{ST}$$

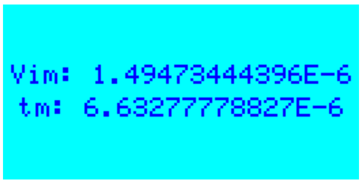
$$V_{ST}\cdot\frac{\cancel{\omega_o\cdot\left(\frac{1}{2}-\frac{1}{2}\right)}+\left(\frac{1}{2}+\frac{1}{2}\right)\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}+K_{ST}=V_{ST}\qquad V_{ST}\cdot\frac{\cancel{\sqrt{\omega_o^2-4\cdot\omega_n^2}}}{\cancel{\sqrt{\omega_o^2-4\cdot\omega_n^2}}}+K_{ST}=V_{ST}\qquad K_{ST}=V_{ST}-V_{ST}\qquad\boxed{K_{ST}=0}$$

$$V_{SdG}=\left\{V_{ST}\cdot\frac{\left[\omega_n^2\cdot C_S\cdot(R_P+R_K)-\frac{1}{2}\cdot\left(\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)\right]\cdot e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}+\left[\frac{1}{2}\cdot\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)-\omega_n^2\cdot C_S\cdot(R_P+R_K)\right]\cdot e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}+V_{iT}\cdot\frac{e^{\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}-e^{-\frac{\sqrt{\omega_o^2-4\cdot\omega_n^2}}{2}\cdot t}}{C_S\cdot R_K\cdot\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right\}\cdot e^{-\frac{1}{2}\cdot\omega_o\cdot t}$$

...tornando alla valutazione della distribuzione delle cariche:

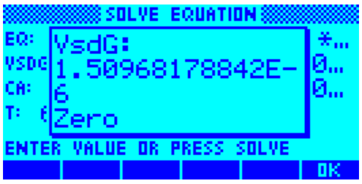
$Q_T = Q_{SM} + Q_{iM}$ dove: Q_T è la carica totale dall'impulso di ingresso; Q_{SM} è la carica accumulata su C_S al tempo t_M ; Q_{iM} la carica accumulata su C_a a t_M

Massima V_i :



$Q_T = I_S \cdot T \rightarrow$ **$1.5 \cdot 10^{-14}$ C**

Calcolo Q_{SM} $\quad Q_{SM} = V_{SdG}(t_M)\cdot C_S$ $\quad$ Calcolo $V_{SdG}(t_M)$:

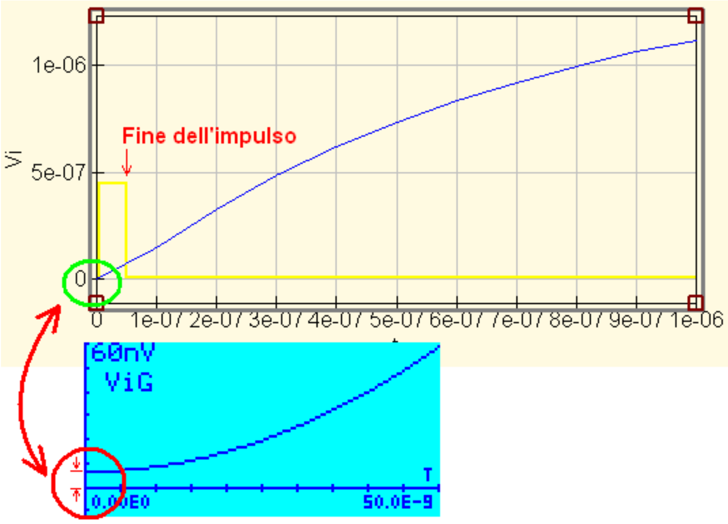


$Q_{SM} = 1.51 \cdot 10^{-6} \cdot 7 \cdot 10^{-11} \rightarrow$ **$1.0568 \cdot 10^{-16}$ C**

Calcolo Q_{iM} $\quad Q_{iM} = V_{iM} \cdot C_a \rightarrow$ **$1.4947 \cdot 10^{-14}$ C** $\quad Q_{SM} + Q_{iM} =$ **15.053 fC** $\quad$ ($\neq Q_T$ anche se vicino, +0.35%)

...imperfezioni numeriche:

L'approssimazione del calcolatore alla 11-esima cifra decimale e considerato i grandi intervalli di variabilità dei valori, oltre alle notevole complessità delle equazioni, determina degli errori di approssimazione che si propagano nei calcoli. Evidente un fenomeno di approssimazione che coinvolge già la fase di crescita e che sarà analizzato per dimostrare l'esclusione di errori umani.



Incongruenza del calcolatore; estrazione dell'equazione implicata (V_{iG}):

$$ViG=Is\cdot Rp\cdot\left(\frac{2\cdot wn^2}{\sqrt{wo^2-4\cdot wn^2}}\cdot\left(\frac{EXP\left(-\left(\frac{\left(wo+\sqrt{wo^2-4\cdot wn^2}\right)}{2}\right)\cdot t\right)}{wo+\sqrt{wo^2-4\cdot wn^2}}-\frac{EXP\left(-\left(\frac{\left(wo-\sqrt{wo^2-4\cdot wn^2}\right)}{2}\right)\cdot t\right)}{wo-\sqrt{wo^2-4\cdot wn^2}}\right)+1\right)$$

Imponendo t=0:

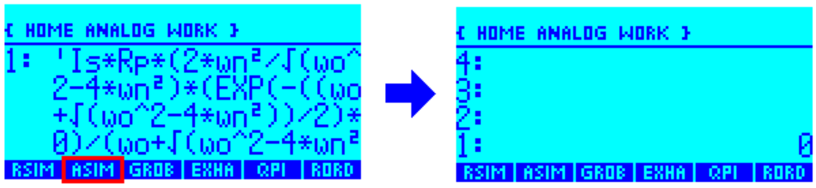
$$V_{iG} = I_S \cdot R_P \cdot \left(\frac{2\cdot\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\left(\frac{1}{\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}}-\frac{1}{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}}\right)+1\right)$$

$$V_{iG} = I_S \cdot R_P \cdot \left(\frac{2\cdot\omega_n^2}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\frac{\omega_o-\sqrt{\omega_o^2-4\cdot\omega_n^2}-\left(\omega_o+\sqrt{\omega_o^2-4\cdot\omega_n^2}\right)}{\omega_o^2-\left(\omega_o^2-4\cdot\omega_n^2\right)}+1\right) \qquad V_{iG} = I_S \cdot R_P \cdot \left(\frac{2\cdot\cancel{\omega_n^2}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}}\cdot\frac{\cancel{\omega_o}-\sqrt{\omega_o^2-4\cdot\omega_n^2}-\cancel{\omega_o}-\sqrt{\omega_o^2-4\cdot\omega_n^2}}{\cancel{\omega_o^2}-\cancel{\omega_o^2}+4\cdot\cancel{\omega_n^2}}+1\right)$$

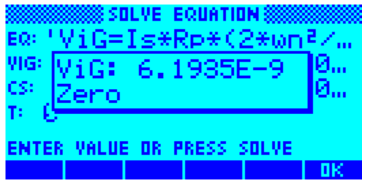
$$V_{iG} = I_S \cdot R_P \cdot \left(\frac{-2\cdot\sqrt{\omega_o^2 - 4\cdot\omega_n^2}}{\sqrt{\omega_o^2-4\cdot\omega_n^2}\cdot 2} + 1 \right) \qquad V_{iG} = I_S\cdot R_P\cdot(-1+1) \to V_{iG}=0$$

Il calcolatore provvede anche alla semplificazione simbolica con una funzione specifica (ASIM), dimostrando la validità dell'uguaglianza.

$$Is\cdot Rp\cdot\left(\frac{2\cdot\omega n^2}{\sqrt{\omega o^2-4\cdot\omega n^2}}\cdot\left(\frac{EXP\left(-\left(\frac{\omega o+\sqrt{\omega o^2-4\cdot\omega n^2}}{2}\right)\cdot\theta\right)}{\omega o+\sqrt{\omega o^2-4\cdot\omega n^2}}-\frac{EXP\left(-\left(\frac{\omega o-\sqrt{\omega o^2-4\cdot\omega n^2}}{2}\right)\cdot\theta\right)}{\omega o-\sqrt{\omega o^2-4\cdot\omega n^2}}\right)+1\right)$$

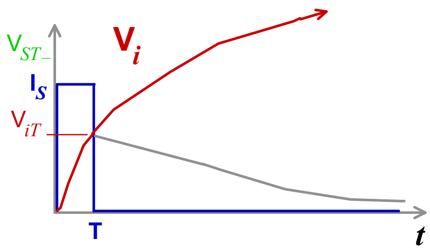


...ma con la valutazione numerica per t=0:



CONCLUSIONI

La presenza di R_K svincola l'accoppiamento dei due condensatori in gioco nei confronti del generatore di corrente (Fotodiodo), determinando inizialmente una rapida carica del condensatore "più leggero" (C_S), lasciando zavorrato a massa il condensatore di accoppiamento, per la maggiore inerzia alla carica. Questo comporta anche un forte dislivello di tensioni tra i due condensatori ai primi istanti. Quando l'impulso di corrente termina, anzichè osservare una decrescita di tutti i potenziali accumulati, si stabilisce una compensazione dello squilibrio dei potenziali sui condensatori, tramite R_K , determinando un trasferimento di carica tra C_S e C_a che fa insorgere il fenomeno di "lievitazione" ritardata della tensione su C_a per molto tempo dopo l'esaurimento della corrente I_S .



Ovviamente, avvenuto il trasferimento e la riequilibratura delle cariche si avvia l'estinzione graduale delle stesse, tramite le resistenze del circuito.
Tutto cambia se R_K svanisce (in corto) accoppiando i condensatori in una singola capacità globale che al momento dell'esaurimento della corrente, corrisponderebbe al contemporaneo avvio della riduzione dei potenziali.
Recuperando una visione generale del sistema, essendo questa la struttura di front-end dell'amplificatore di carica, il modello ideale corrisponde a una rampa di tensione crescente finché il generatore di corrente è attivo; poi, il mantenimento del potenziale (conservazione della carica) fino a una imposizione di "reset" per predisporre una nuova acquisizione.
La presenza della resistenza di polarizzazione R_P determina invece quella dispersione che consuma la conservazione della carica (motivo per cui deve avere valore elevato).
Esperimenti di laboratorio hanno mostrato un accenno del fenomeno anche in assenza di R_K che dovrà essere ulteriormente indagato. Certamente, il modello di studio del problema sarà di valido aiuto come riferimento orientativo per continuare le indagini.

Seba 24/05/2020