EoS and symmetry energy with modern nucleon-nucleon potentials

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Outline

- The nuclear many-body problem
- The EoS of nuclear and neutron matter
- The symmetry energy
- Thermal effects on EoS and symmetry energy
- Constraints from HIC
- Constraints from astrophysical observables
- Summary and prospects

The non-relativistic nuclear many-body problem

• Non-relativistic pointlike protons and neutrons interacting through the Hamiltonian

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j}^{A} v_{ij}$$

• v_{ij} strongly constrained by the most recent data (few thousands, up to 350 MeV) on NN phase shifts, energy scattering parameters, and deuteron binding energies.

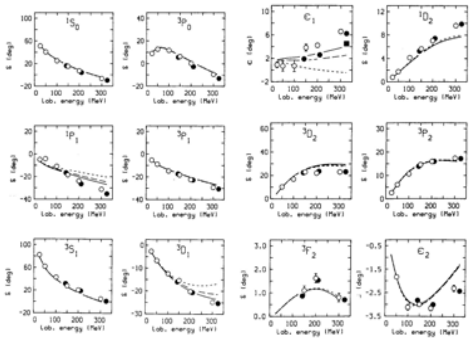
As an example, Argonne v_{18} potential :

$$v_{ij} = \sum_{p=1,18} v_p(r_{ij}) O^p_{ij}$$

$$O^p_{ij} = [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i \cdot \sigma_j), (\mathbf{L}\cdot\mathbf{S})^2] \otimes [1, \tau_i \cdot \tau_j],$$
$$[1, \sigma_i \cdot \sigma_j, S_{ij}] \otimes T_{ij}\ , \text{ and } (\tau_{zi} + \tau_{zj})$$

Wiringa, Stoks, Schiavilla, PRC**51**, (1995) 38

Nucleon-Nucleon phase shifts



Due to the short range repulsive core of the NN interaction, standard perturbation theory is not applicable.

Non-relativistic approach:

- **Brueckner-Bethe-Goldstone expansion**
- **Variational method**

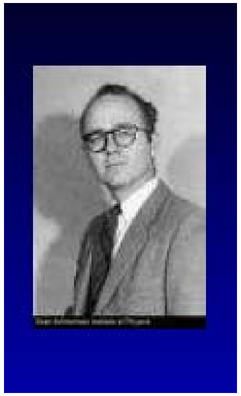
Relativistic approach :

- **DBHF method**

Ab initio approaches, realistic NN potentials, no free parameters

More in M. Baldo & C. Maieron, J. Phys. G **34,** (2007) R243

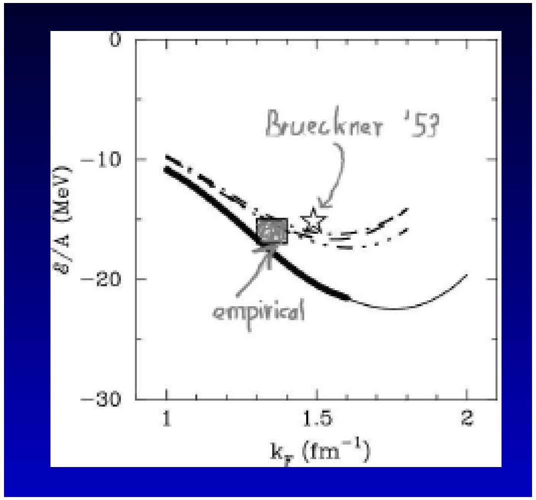
The Brueckner-Bethe-Goldstone theory of Nuclear Matter



$$H = \sum_{i=1}^{A} T_i + \sum_{i<j}^{A} v_{ij} = H_0 + H_1$$

- Introducing the auxiliary single-particle potential U

$$H = (H_0 + U) + (H_1 - U) = H'_0 + H'_1$$



- The diagrams in the expansion are grouped according to the order of correlations they describe (two-body, three-body ……)

At first order :
(2-body correlations)

$$\frac{E}{A}(\rho) = \frac{3}{5}\frac{k_F^{\,2}}{2m} + \frac{1}{2\rho}\sum_{k,k'\le k_F} \langle kk' | G[\rho;\omega] | kk' \rangle_a$$

Bethe-Goldstone equation :

$$G[\rho;\omega] = V_{NN} + \sum_{k_a k_b} V_{NN} \frac{|k_a k_b\rangle Q \langle k_a k_b|}{\omega - e(k_a,\rho) - e(k_b,\rho)} G[\rho;\omega]$$

V_{NN} : bare nucleon – nucleon potential

Q : Pauli operator

$e(k,\rho) = \frac{k^2}{2m} + U(k;\rho)$, s.p. energy

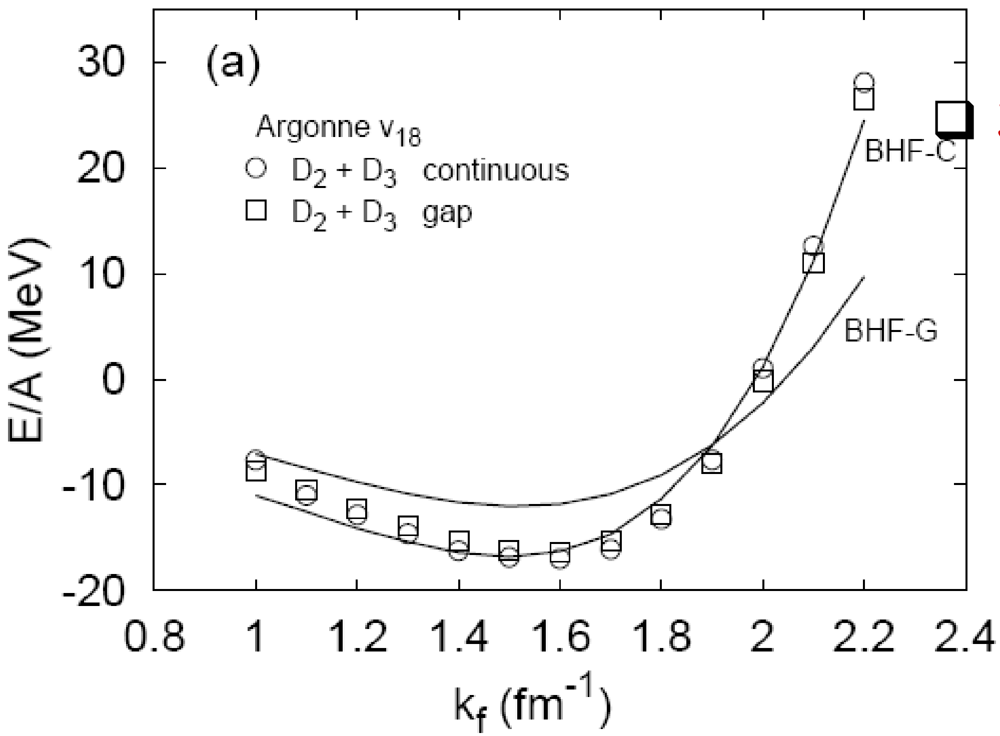
$U(k,\rho) = \mathrm{Re} \sum_{k' \le k_F} \langle kk' | G[\rho;\omega] | kk' \rangle_a$, s.p.potential

Equation of state :

$$P(\rho) = \rho^2 \frac{d(E/A)}{d\rho}$$

Convergence of the perturbative expansion

Phys. Rev. C65, 017303 (2001).



❑ 3-body correlations : Bethe-Fadeev

FIGURE 1. Comparison of BHF two hole-line (lines) and three hole-line (markers) results for symmetric nuclear matter, using continuous and gap choice for the single-particle potentials.

- EoS independent on the choice of the single-particle potential
- 3-body correlations give a small contribution

The variational method in its practical form

Pandharipande & Wiringa, 1979; Lagaris & Pandharipande, 1981

Method used to calculate the upper bound to the ground state energy:

$$E = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \geq E_0$$

Parameters in ψ are varied to minimize E.

Ψ is constructed from a symmetrized product of two-body correlation functions acting on an unperturbed ground state Φ:

$$\psi(r_1, r_2, \ldots) = \prod_{i<j} f(r_{ij}) \phi(r_1, r_2, \ldots.)$$

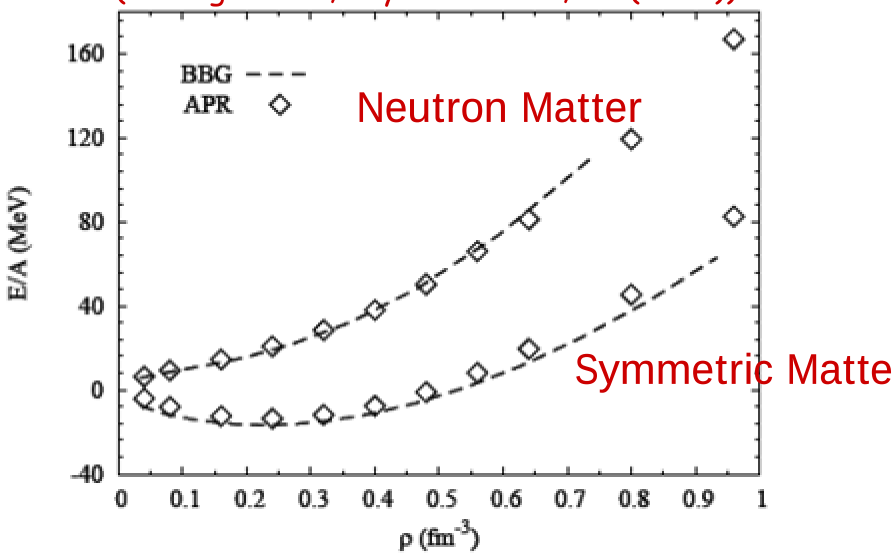
The correlation function f represents the correlations induced by the two-body potential. It is expanded in the same spin-isospin, spin-orbit and tensor operators appearing in the NN interaction.

The main differences between BBG and Variational method:

a) In BBG the kinetic energy contribution is kept at its unperturbed value at all orders of the expansion, while all correlations are embodied in the interaction energy part. In the variational, both kinetic and interaction parts are directly modified by the correlation factors.

b) No single particle potential is introduced in variational. In BBG the s.p. potential is introduced in the expansion and improves the rate of convergence.

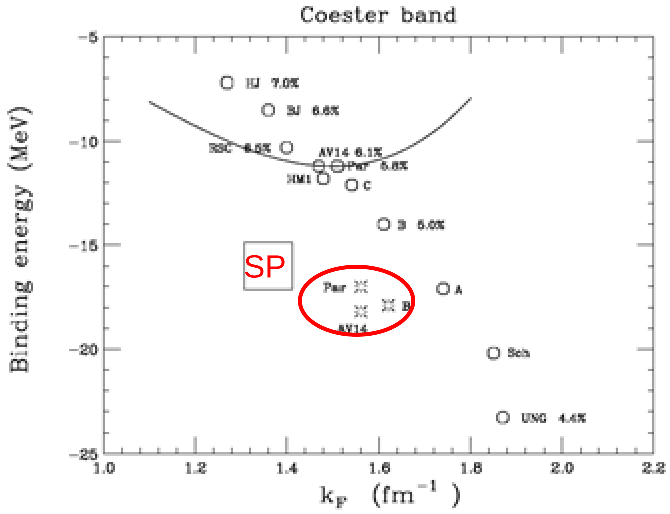
NN interaction: *Argonne v18*
(Wiringa et al., Phys. Rev. C**51**, 38 (1995))



At two-body level, both methods give very close results.

Missing the saturation point ....

Coester et al., Phys. Rev. **C1**, 769 (1970)



- When three hole-line diagrams are included and modern NN interactions are used the Coester band reduces to a Coester island".
- The saturation "point" is still missed.

Including three-body forces

- **They must allow to reproduce "reasonably well" also the data on three nucleon systems**

- **They must be consistent with the two-body force adopted. Partially explored !**

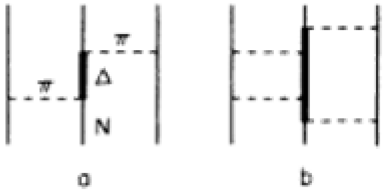
Three-nucleon forces (TBF)

(no complete theory available yet !)

M. Baldo et al., A&A 328, 274 (1997)

W. Zuo et al., Nucl. Phys. A706, 418 (2002)

Urbana IX phenomenological model :

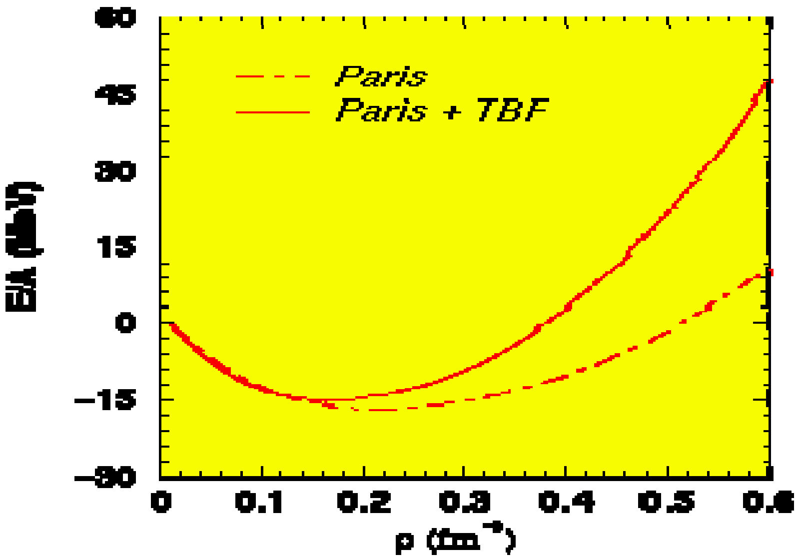


Carlson et al., NP A401,(1983) 59

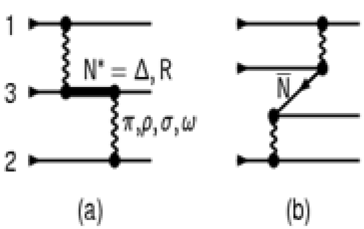
(a) : 2π exchange (attractive)
(b) : Roper R resonance (repulsive)

Fit saturation point !

In BHF, the TBF's are averaged over the position of the third nucleon, hence are reduced to an effective two-body force which is added to the bare NN interaction



Microscopic model :



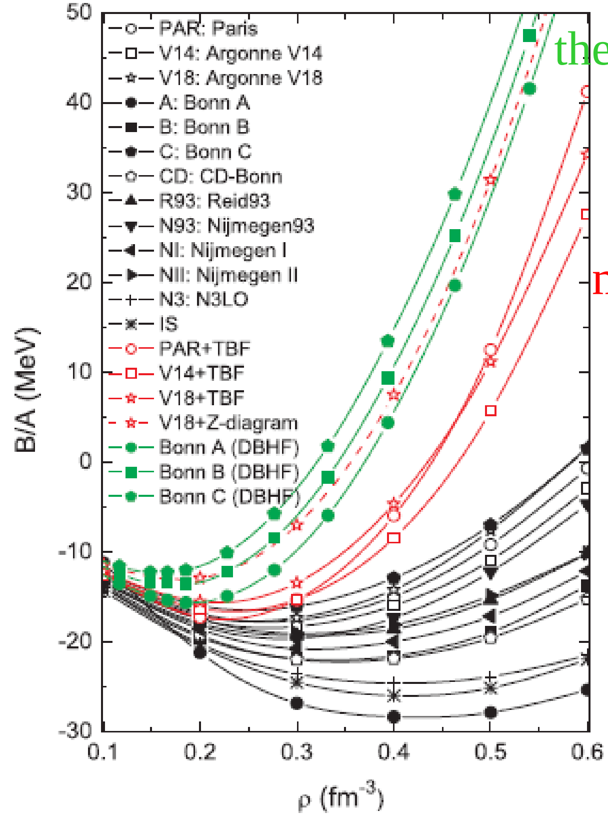
P. Grange' et al, PR C40, (1989) 1040

Exchange of π, ρ, σ, ω via Δ(1232), R(1440), N$\bar{N}$
Parameters compatibile with two-nucleon potential, where possible.

The most recent compilation of NN potentials

Z.H. Li et al., PRC 74, 047304 (2006)

- Paris (Lacombe, 1980)
- Argonne v_{14} (Wiringa,1984) and v_{18} (Wiringa,1995)
- Bonn A, B, C (Machleidt, 1989, 1990)
- Reid 93, Nijmegen I, and II (Stoks, 1994)
- CD-Bonn (Machleidt, 2001)
- N3LO (Entem, 2003)
- IS (Doleschall, 2003)

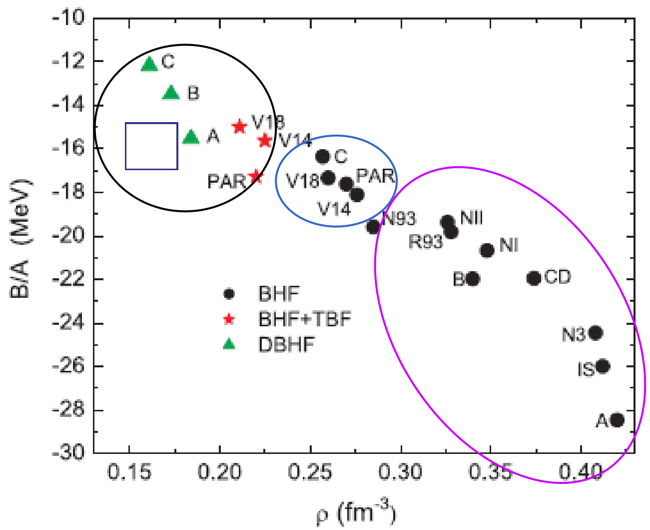


DBHF : the most repulsive one

2BF + micro TBF: more repulsion added

Only 2BF : too strong binding

The "new" Coester band

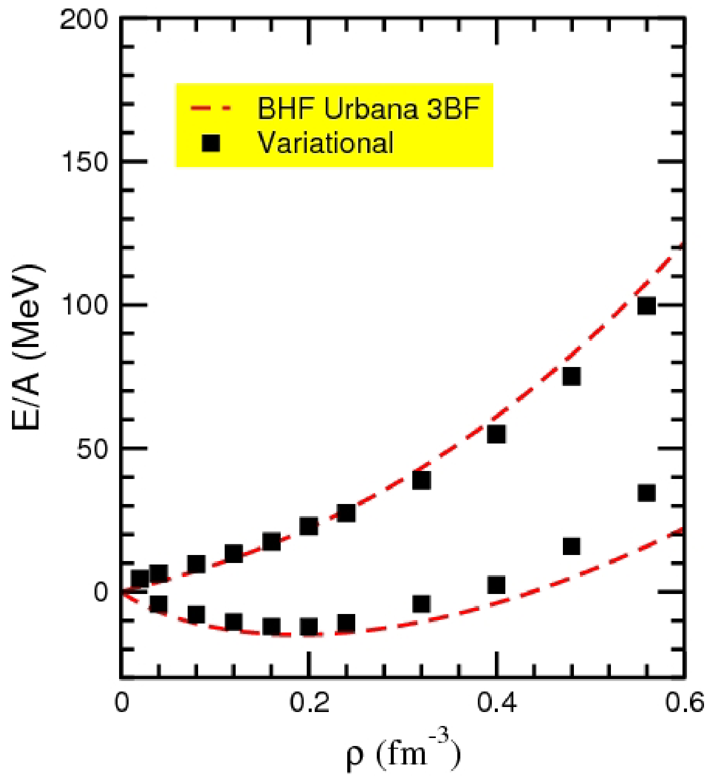


DBHF, BHF + TBF, minimum very close to SP

BHF with Paris, Argonne, larger binding & SP around 0.27 fm^{-3}

BHF with CD-Bonn, Reid, N3LO, IS, strong overbinding and SP more than twice saturation density.

BHF vs. Variational with Av18 plus Urbana IX TBF



CAVEAT : TBF are not exactly the same !

Urbana IX TBF contain two parameters, A and U,
i.e. the strengths of the attractive and the repulsive part.

In BHF, A and U are fitted on the SP of nuclear matter.
In Variational, the fit is on the triton binding energy, and on the saturation density of NM , hence TBF are different.

Variational : A=-0.0293, U=0.0048
BHF: A= -0.033, U= 0.00038

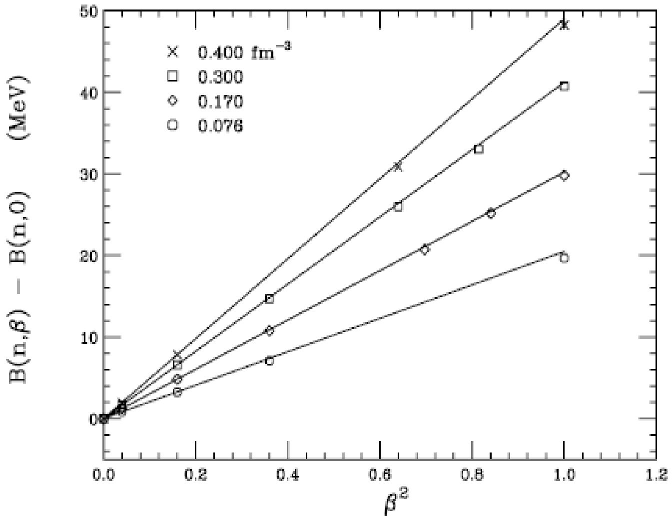
- Good agreement between Variational and BHF up to 0.4 fm-3 in SNM, better in PNM
- Main uncertainty is TBF at high density (above 0.4 fm-3).

Symmetry energy

$$At \quad \rho = \rho_0, \quad E_{sym} \cong 28 - 32\ MeV$$

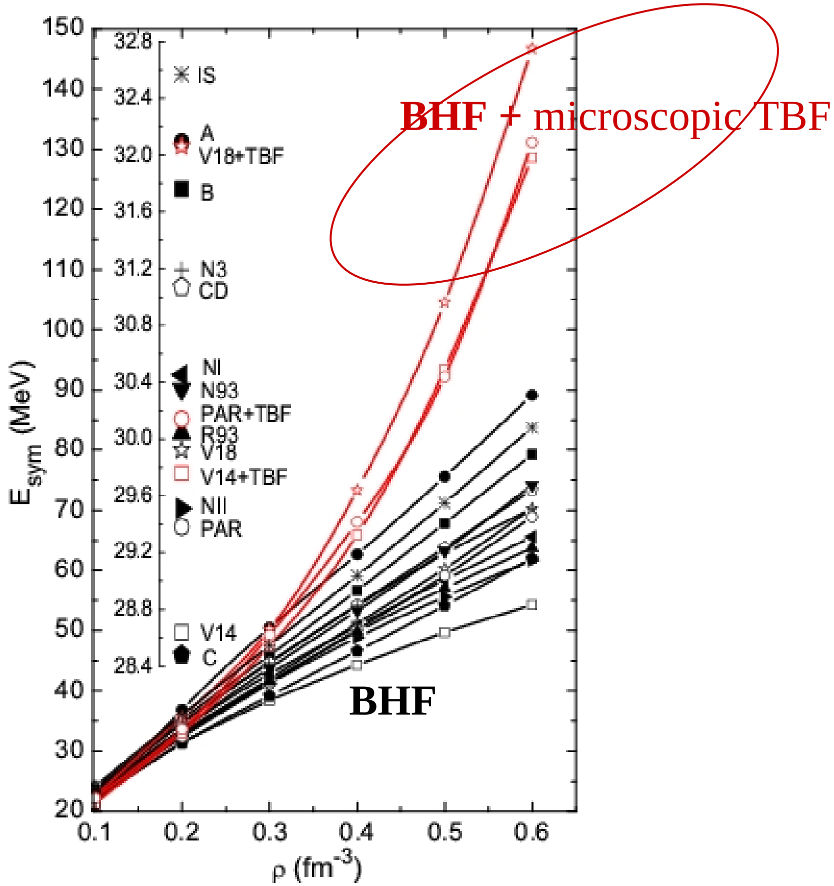
Asymmetry parameter $\beta = \frac{\rho_n - \rho_p}{\rho} = 1\text{-}2x_p$

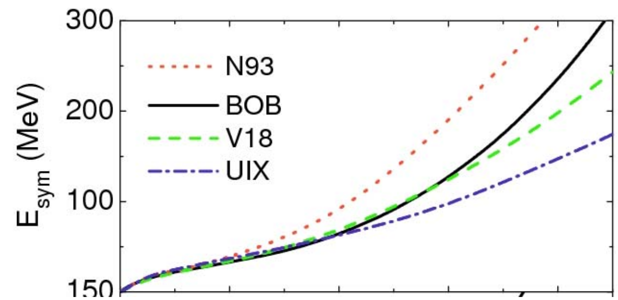
Parabolic approximation
(checked microscopically both in
BBG and Variational)

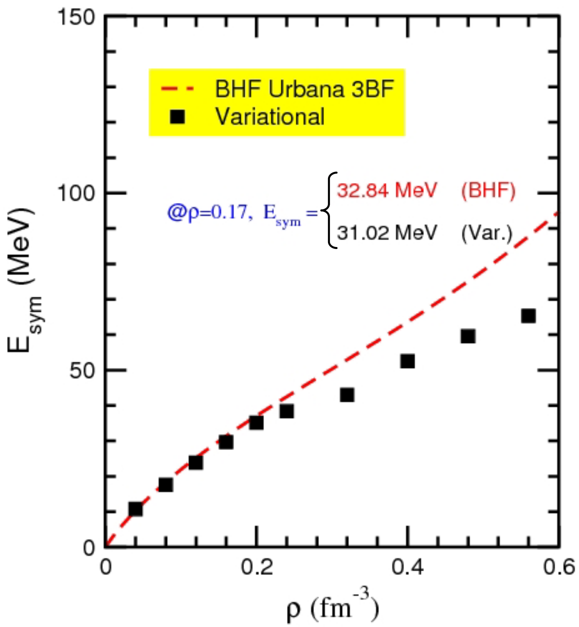


$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + E_{sym}(\rho)\beta^2$$

$$E_{sym}(\rho) = \frac{E}{A}(\rho,\beta=1) - \frac{E}{A}(\rho,\beta=0)$$

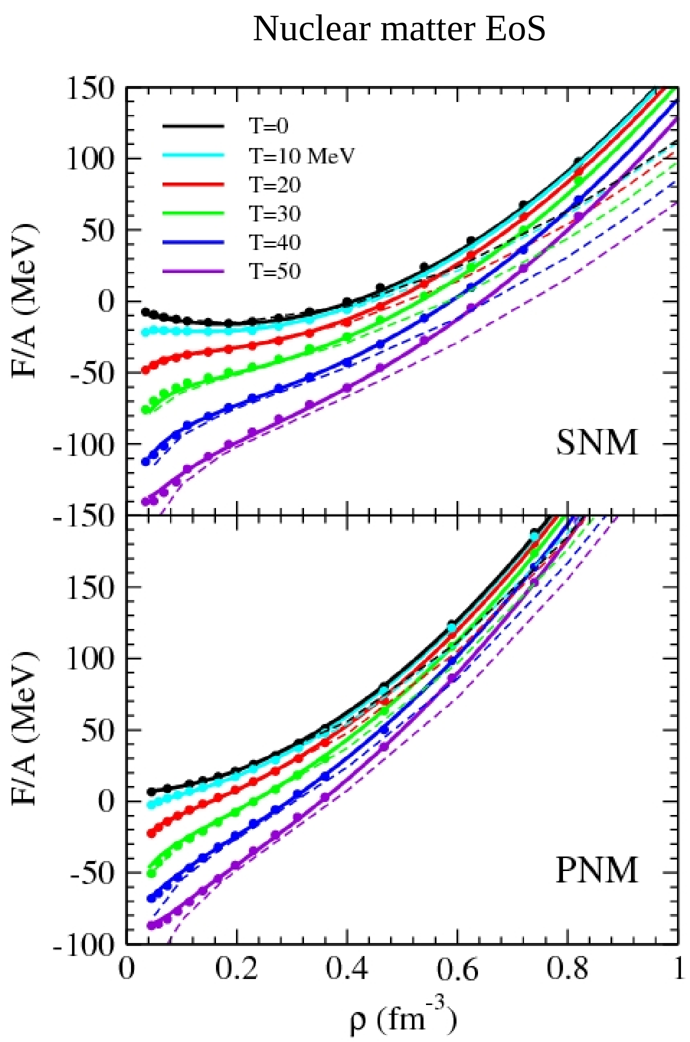




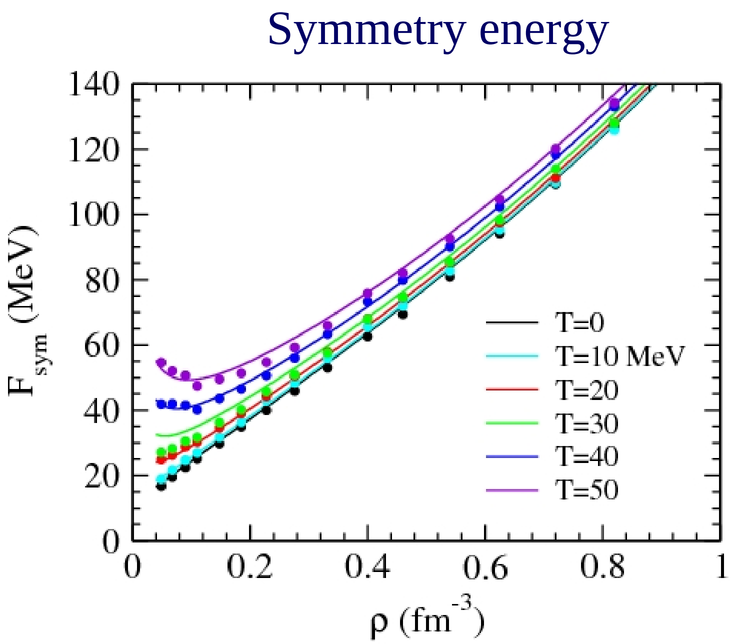


- Symmetry energy : monotonically increasing function of the density
- At saturation, 28.5 < Esym< 32.6 MeV
- Including TBF enhances the symmetry energy at high density

Thermal effects on the EoS and symmetry energy



- Extension of BHF calculations to finite T
- In SNM, typical Van der Waals behavior, LG phase transition with T_c=19 MeV and ρ_c~0.06 fm^{-3}
- Parabolic approximation still OK.



Possible tests of EOS from H.I. collisions and from observations on astrophysical compact objects

- Compressibility : H.I.   Flows in H.I.
  NS   Masses
- Symmetry energy : H.I.   Particle production
  Isotopic distributions
  NS   DU process and cooling
- EOS at finite temperature : H.I.   Multifragmentation
  Limiting temperature
  NS   Proto-neutron stars

Determination of the Equation of State of Dense Matter

Science **298**, 1592 (2002)

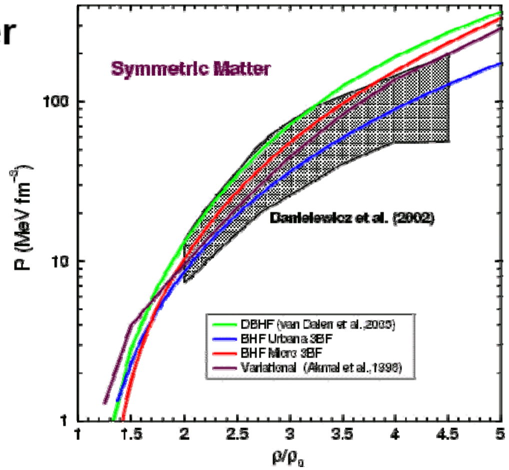
Pawel Danielewicz[1,2], Roy Lacey[3] & William G. Lynch[1*]

- Transverse flow measurements in Au + Au collisions at E/A=0.5 to 10 GeV
- Pressure determined from simulations based on the Boltzmann-Uehling-Uhlenbeck transport theory

Flow data exclude stiff equations of state at low density

Urbana IX TBF

Microsc. TBF



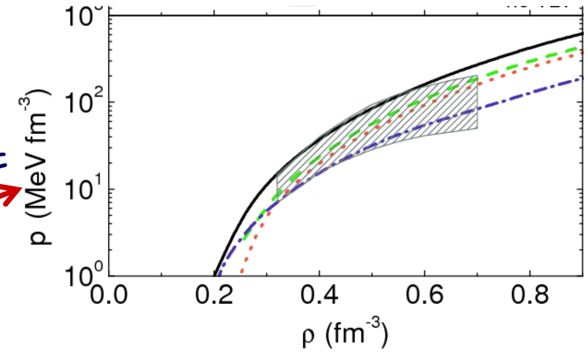


TABLE I. Parameters of the EOS fit, Eq. (7), for symmetric nuclear matter and pure neutron matter and saturation properties of nuclear matter using different interactions.

	Symmetric matter			Neutron matter			$[\rho, E/A]_0$ (fm^{-3}, MeV)	K (MeV)	E_{sym} (MeV)	E'_{sym} (MeV)
	α	β	γ	α	β	γ				
UIX	−452.5	556.0	1.24	78.0	232.9	2.24	[0.18, −15.3]	192	33.5	24.5
V18	−123.2	407.9	2.38	55.9	532.3	2.68	[0.20, −14.7]	226	30.6	33.8
BOB	−130.4	537.0	2.39	31.0	780.2	2.77	[0.17, −15.9]	244	29.4	24.8
N93	−152.5	343.3	1.94	72.3	693.6	2.67	[0.18, −15.4]	216	34.0	35.5

The value of the compressibility at saturation does not fix the EoS behaviour at high density

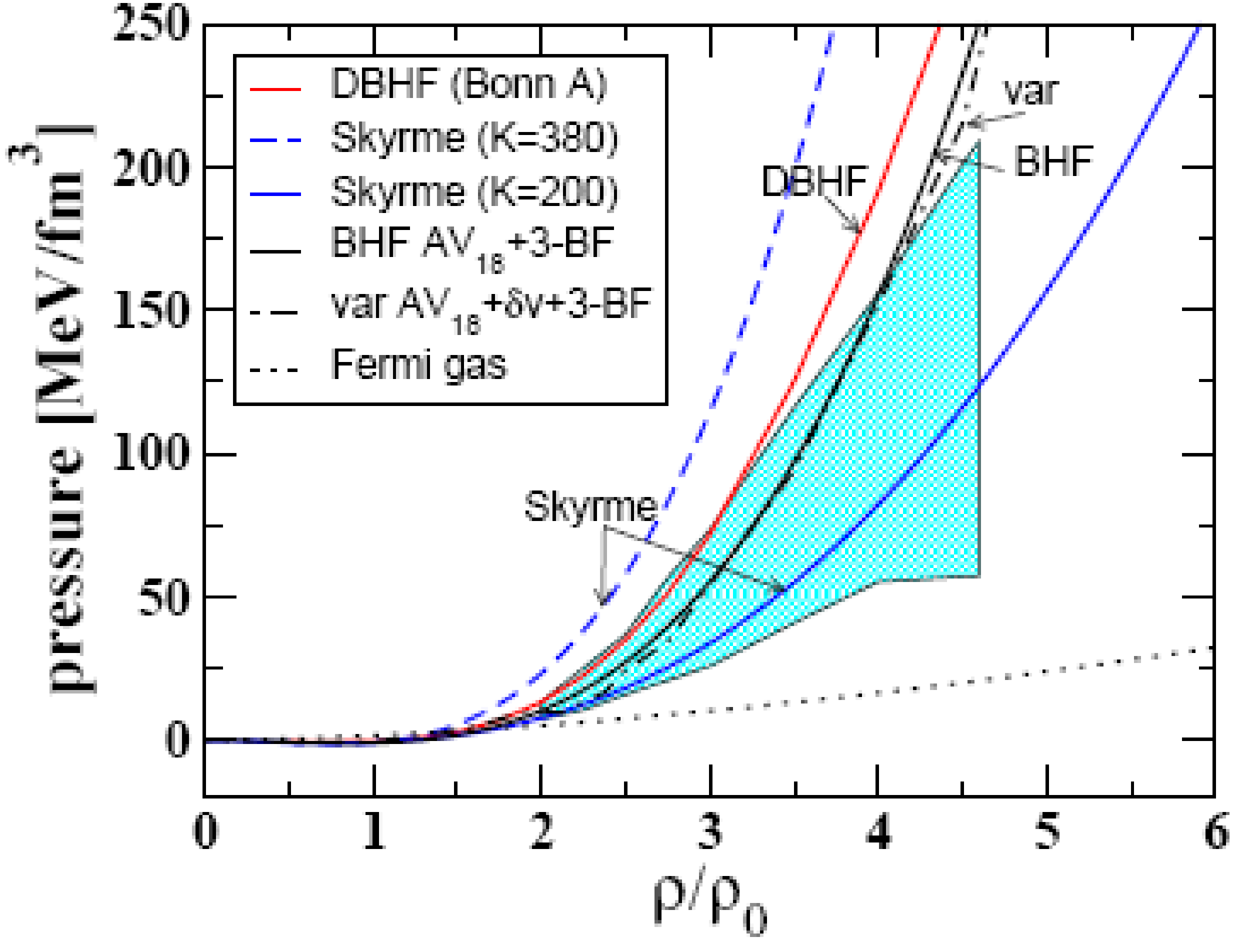
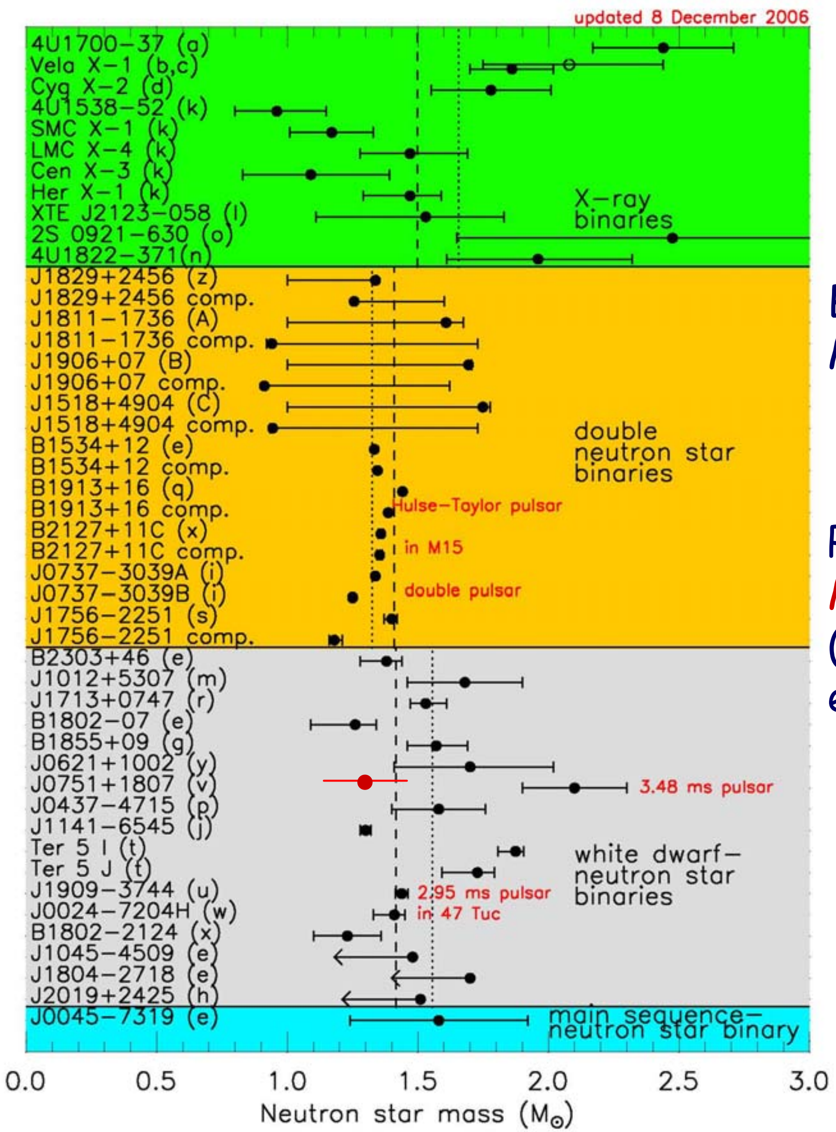


Figure from C. Fuchs et al., nucl-th/0511070



Binary radio pulsars:

$M_{BRP} = 1.35 \pm 0.04\ M_0$

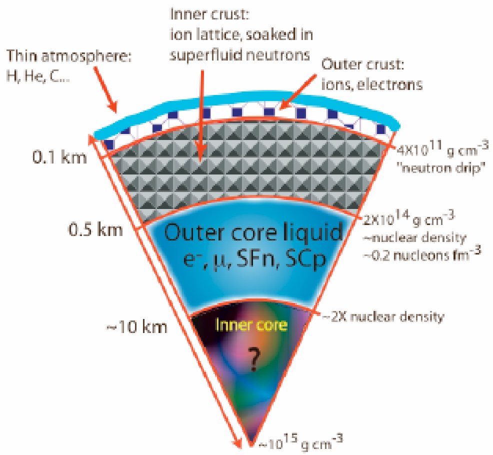


Figure 1. Current theoretical understanding of the interior composition of neutron stars. Uncertainty increases with depth.

PSR J1903+0327

$M=1.67 \pm 0.01\ M_0$

(P.Freire et al., arXiv:0907.3219)

excludes “soft” EoS

J. Lattimer and M. Prakash, PRL 94, 111101 (2005)

- «Recipe» for neutron star structure calculation:

Brueckner calculation: $\epsilon(\rho, x_e, x_p, x_\Lambda, x_\Sigma, \ldots);\ x_i = \frac{\rho_i}{\rho}$

Chemical potentials: $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$

Beta-equilibrium: $\mu_i = b_i \mu_n - q_i \mu_e$

Charge neutrality: $\sum_i x_i q_i = 0$

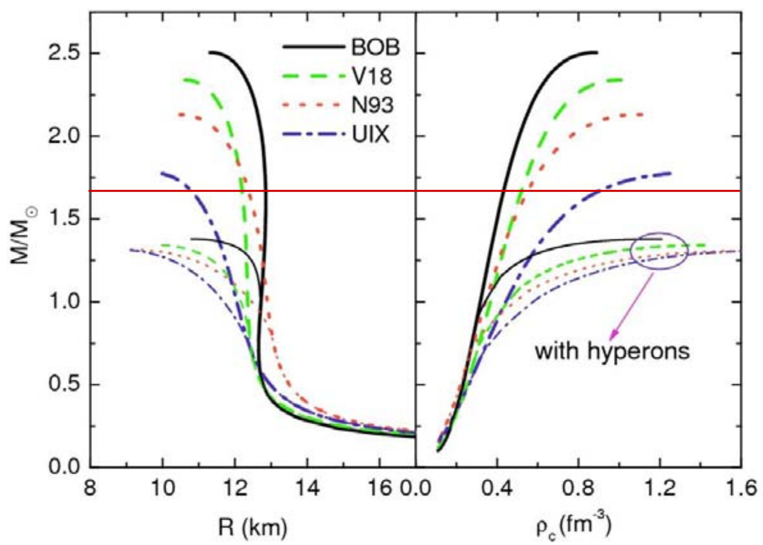
$$\mu_e = \mu_\mu = \mu_n - \mu_p$$
$$\mu_\Sigma = 2\mu_n - \mu_p$$
$$\mu_{\Sigma^0} = \mu_\Lambda = \mu_n$$
$$\mu_{\Sigma^+} = \mu_p$$

Composition: $x_i(\rho)$

Equation of state: $p(\rho) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$

TOV equations:
$$\frac{dp}{dr} = -\frac{Gm}{r^2}\frac{(\epsilon + p)(1 + 4\pi r^3 p/m)}{1 - 2Gm/r}$$
$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

Structure of the star: $\rho(r)$, $M(R)$ etc.



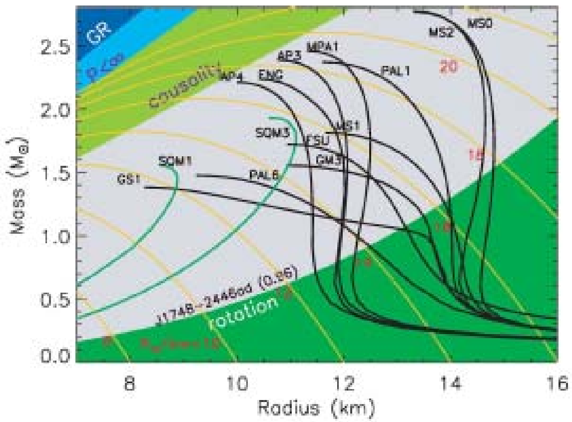


Figure 2: The mass-radius relationship for neutron stars reflects the equation of state for cold superdense matter. Mass-radius trajectories for typical EOSs are shown as black curves. Green curves (SQM1, SQM3) are self-bound quark stars. Orange lines are contours of radiation radius, $R_\infty = R/\sqrt{1 - 2GM/Rc^2}$. The dark blue region is excluded by the GR constraint $R > 2GM/c^2$, the light blue region is excluded by the finite pressure constraint $R > (9/4)GM/c^2$, and the light green region is excluded by causality, $R > 2.9GM/c^2$. The green region in the right-hand corner shows the region $R > R_{max}$ excluded by the 716 Hz pulsar J1748-2446ad (Hessels et al. 2006). From Lattimer and Prakash (2007).

Cooling of NS : Direct Urca process

- NS are born with an internal temperature T~10^{11}-10^{12} K
- During the initial 10^5-10^6 yrs, cooling via neutrino emission

$$n \rightarrow p + e^- + \bar{\nu} \quad , \quad p + e^- \rightarrow n + \nu$$

This is the main mechanism of NS cooling which dominates all other processes, whenever it is possible. Imposing momentum and energy conservation, it requires in nucleonic matter a proton fraction ***x_p>11 %*** *(Lattimer et al., PRL **66**, (1991) 2701)*

1) Chemical equilibrium

$$\mu_n = \mu_p + \mu_e$$
$$\mu_e = \mu_\mu$$

2) Charge neutrality

$$\rho_p = \rho_e + \rho_\mu$$

3) Baryon number cons.

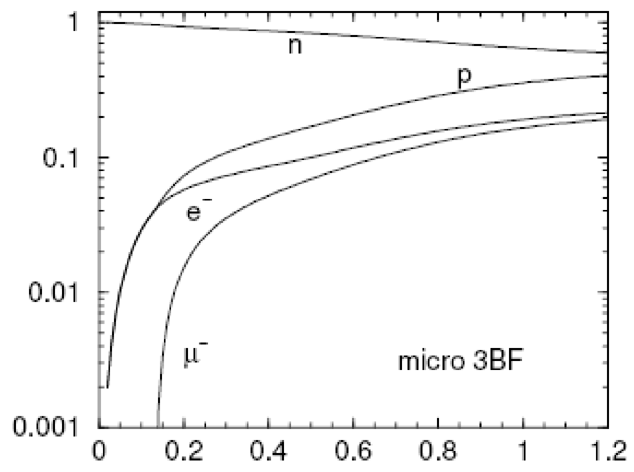
$$\rho = \rho_p + \rho_n$$

Asymmetry parameter $\beta = \frac{\rho_n - \rho_p}{\rho} = 1 - 2x_p$

$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + E_{sym}(\rho)\beta^2$$

$$\mu_n(\rho, \beta) = \frac{\partial E}{\partial N_n}, \mu_p(\rho, \beta) = \frac{\partial E}{\partial N_p}$$

$$[\mu_n - \mu_p](\rho, \beta) = 4\beta E_{sym}(\rho)$$



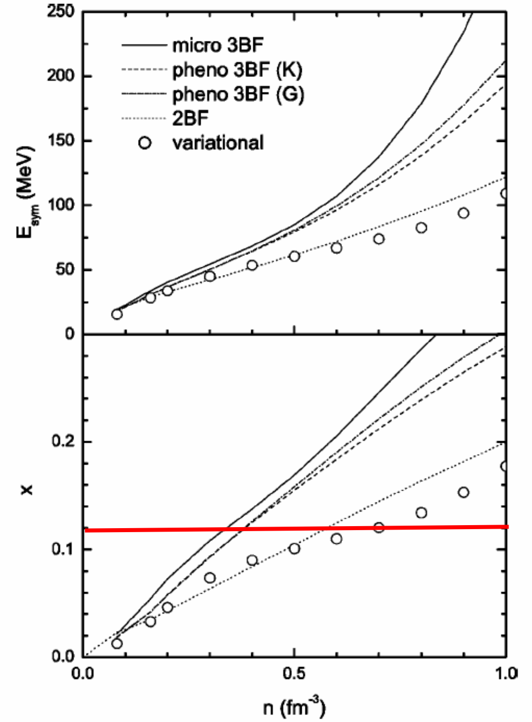
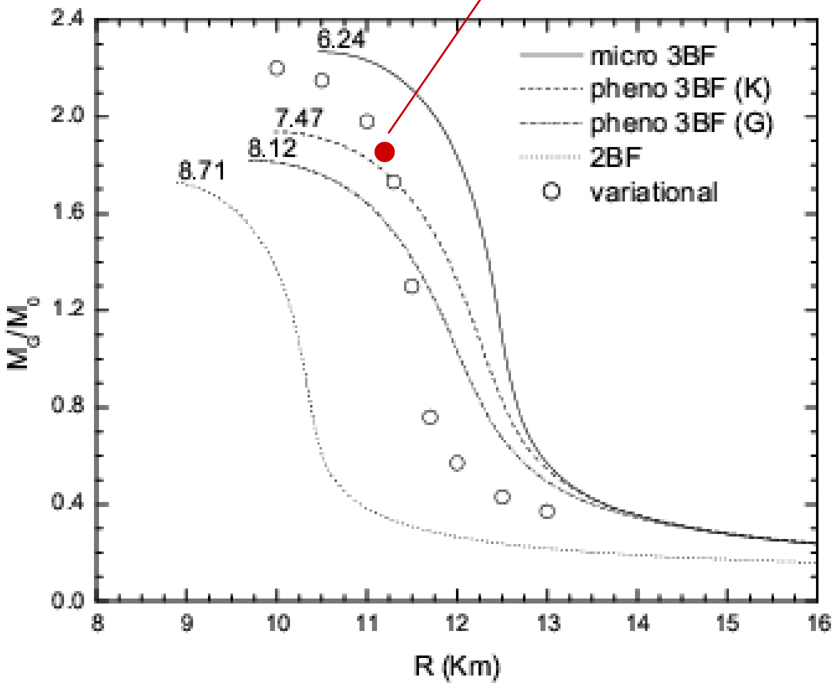


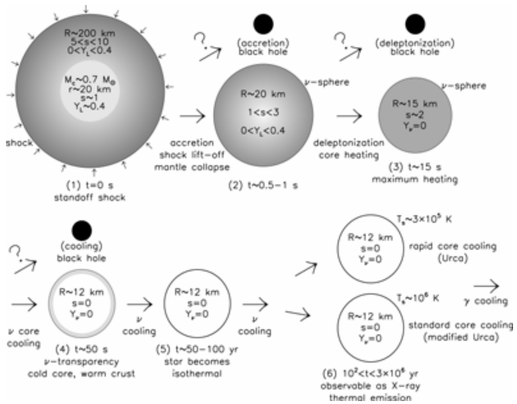
FIG. 4. Symmetry energy (upper panel) and proton fraction (lower panel) of β-stable matter using different TBF.

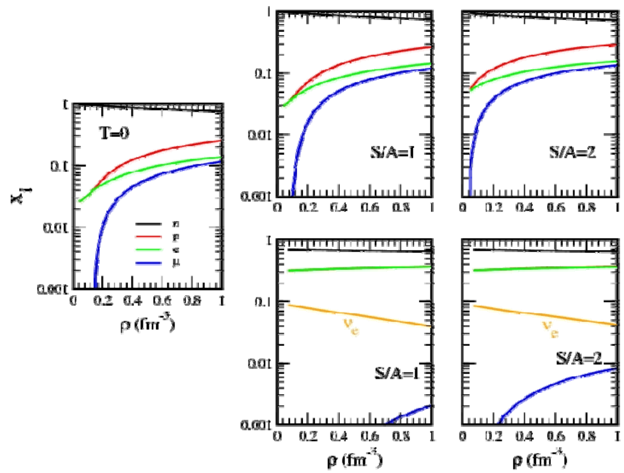
$M_{Urca} \sim 1.8$

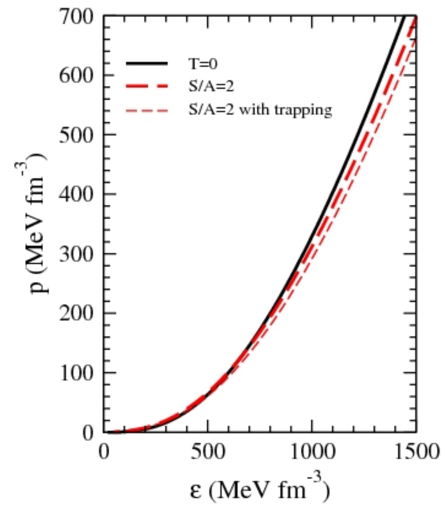


Variational has almost no direct URCA processes

EoS at finite T : Structure of Protoneutron stars







- T ~ 20-30 MeV, S ~1-2
  ν–trapping

$$n \rightarrow p + e^- + \bar{\nu}_e$$
$$e^- \rightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

- ν– free
  formation of a cold **NS**

- Increase of particles' fractions at low density
- More symmetric matter when neutrinos are trapped

Softer EoS in neutrino-trapped matter

		minimum mass			maximum mass		
		$M/M_\odot$	R (km)	ρ_c/ρ_0	$M/M_\odot$	R (km)	ρ_c/ρ_0
untrapped T=0	LS				2.03	9.86	10.55
	SKa				2.03	9.86	10.42
	Shen				2.03	9.93	10.42
trapped S/A=1	LS	0.58	40	1.02	1.95	10.2	11.34
	SKa	0.60	38	1.08	1.95	10.2	11.20
	Shen	0.58	44	1.02	1.95	10.3	11.20
trapped S/A=2	LS	0.70	44	0.90	1.95	10.7	10.85
	SKa	0.77	42	0.90	1.95	10.8	10.70
	Shen	0.75	52	0.77	1.95	10.8	10.80

Detection of neutrinos from a Galactic Supernova

Summary….plus a few hints

- High level of accuracy in the many-body technique.

  Use of realistic NN potentials, no free parameters.

Useful Parametrizations of the EoS for your simulations !

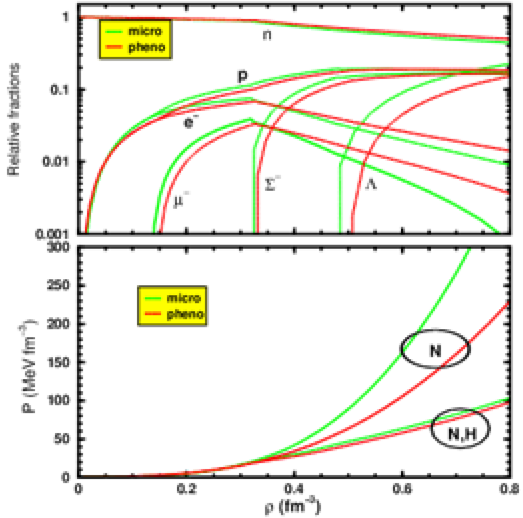
$$\frac{E}{A}(\rho) = \alpha\rho + \beta\rho^{\gamma}$$

		Symmetric matter			Neutron matter		
		α	β	γ	α	β	γ
Av_{18} + Urbana IX	UIX	−452.5	556.0	1.24	78.0	232.9	2.24
Av_{18} + micro	V18	−123.2	407.9	2.38	55.9	532.3	2.68
Bonn B + micro	BOB	−130.4	537.0	2.39	31.0	780.2	2.77
Nijmegen 93 + micro	N93	−152.5	343.3	1.94	72.3	693.6	2.67

….Similar polynomial fits available at finite T

- <u>Main problem : TBF's at density ρ > 0.4 fm-3</u>

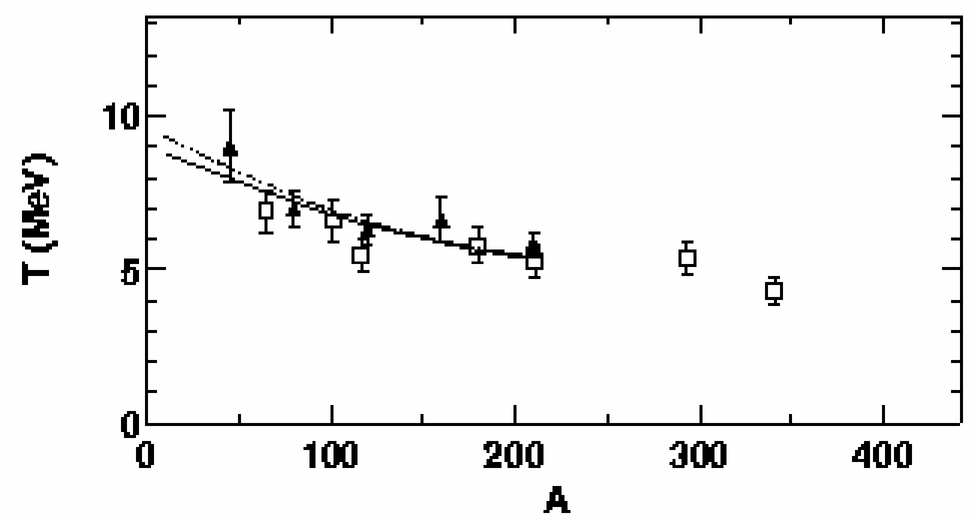
Appearance of other particle species : hyperons, kaons, quark matter ….
Many problems : nucleon-hyperon interaction,
hyperon-hyperon interaction,
kaon-nucleon interaction,
QM Equation of state ….



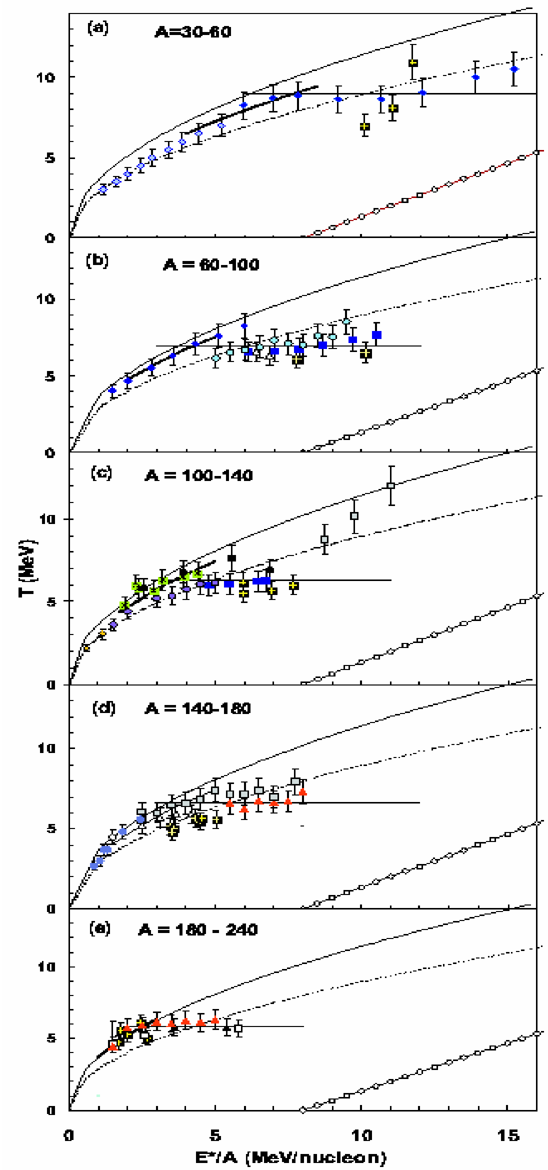
- Symmetry energy : monotonically increasing function of density. NO ASY-SOFT behaviour!

Limiting temperature and caloric curves

Limiting Temperature: the maximal temperature that a nucleus can sustain before reaching mechanical instability. It represents the temperature at which the **nuclear caloric curve** shows a plateau.



Natowitz et al. PRL **89** 212701 (2002)



Natowitz et al. PRC **65** 034618 (2002)

➢ Nuclear matter ⟺ Finite nuclei

(S.Levit, P.Bonche, Nucl.Phys. **A437**, 426 (1985))

Liquid drop Corr.'s ⇨ $\begin{cases} P+\delta P \\ \mu+\delta\mu \end{cases}$

$$\delta P = \left(\frac{Z^2e^2}{5A}\rho - 2\alpha(T)\right)\frac{1}{R}$$

$$\delta\mu = \frac{6Z^2e^2}{5AR}$$

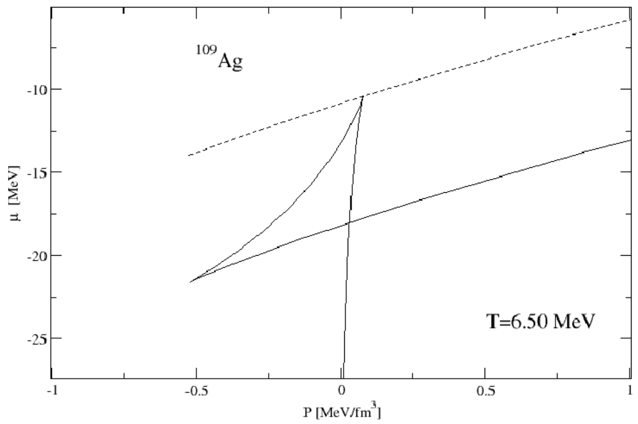
$$\alpha(T) = \alpha_0\left(1+\frac{3T}{2T_C}\right)\left(1-\frac{T}{T_C}\right)^{3/2}$$

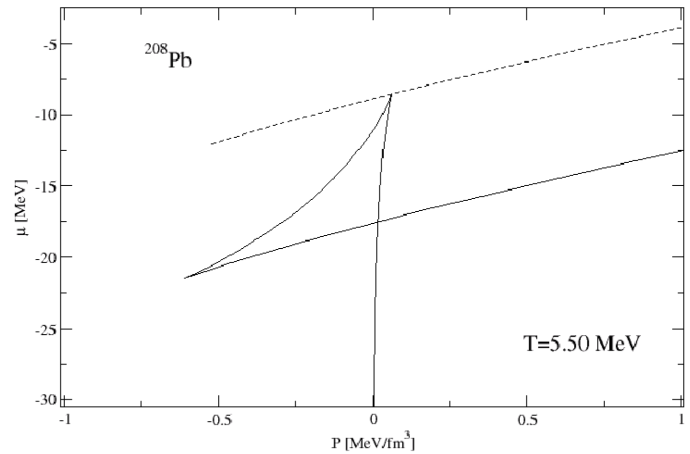
$$\alpha_0 = 1.14\ MeV\ fm^{-2}$$

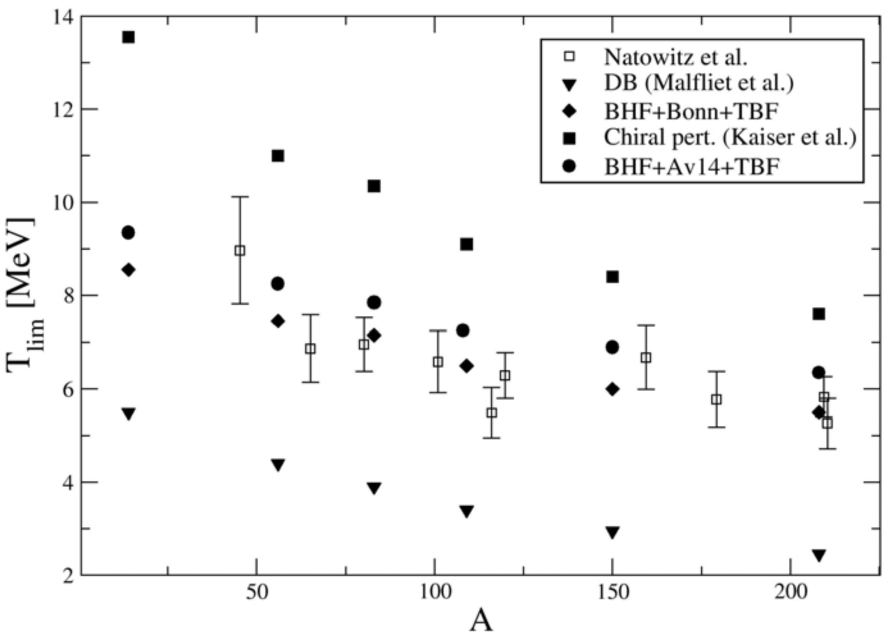
➢ Coexistence equations:

$$p(T,\rho_L) + p_{Coul}(\rho_L) + p_{surf}(T,p_L) = p(T,\rho_V)$$

$$\mu(T,p_L) + \mu_{Coul}(\rho_L) = \mu(T,\rho_V)$$





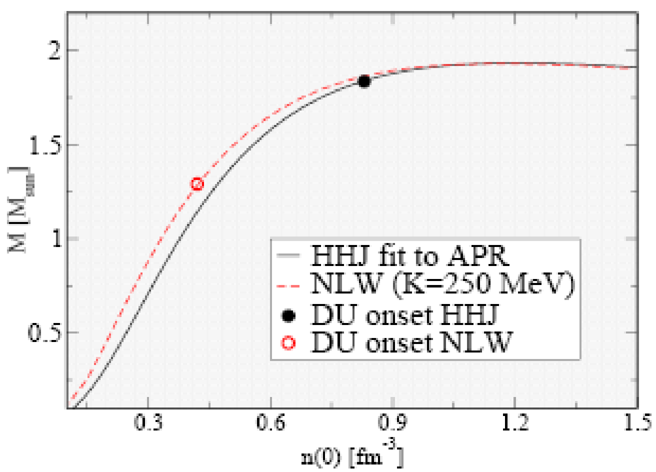


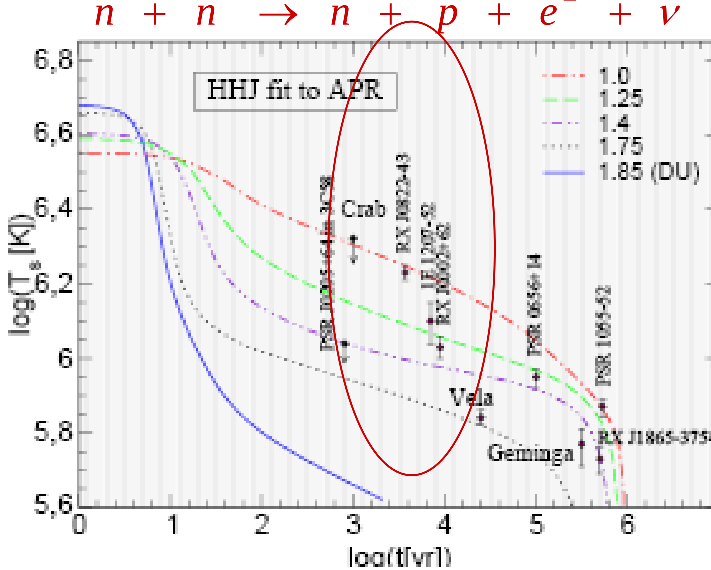
- Smaller values of T_c results in a smaller value of T_{lim}.
- Sensitivity of T_{lim} to the EoS.
- Some dependence on the N-N interaction.
- These results support the interpretation of T_{lim} as the temperature for the onset of multifragmentation regime.
- Phenomenology appears to favor non-relativistic BHF results.

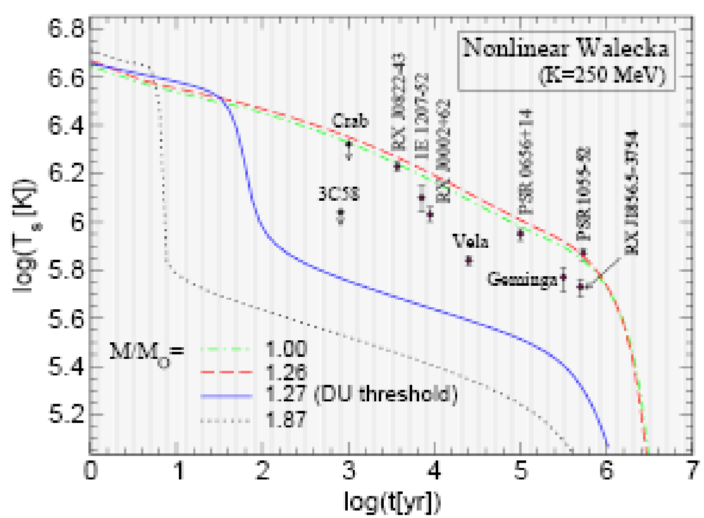
M.Baldo, L.S. Ferreira, O.E. Nicotra, **PRC 69** 034321 (2004) & **NPA 749** 118c (2005)

Indirect Urca process (much less efficient)

$$n + n \rightarrow n + p + e^{-} + \bar{\nu}$$







NS cooling – different masses

→ DU cools neutron stars too rapidly
Superfluidity ? Medium effects ?

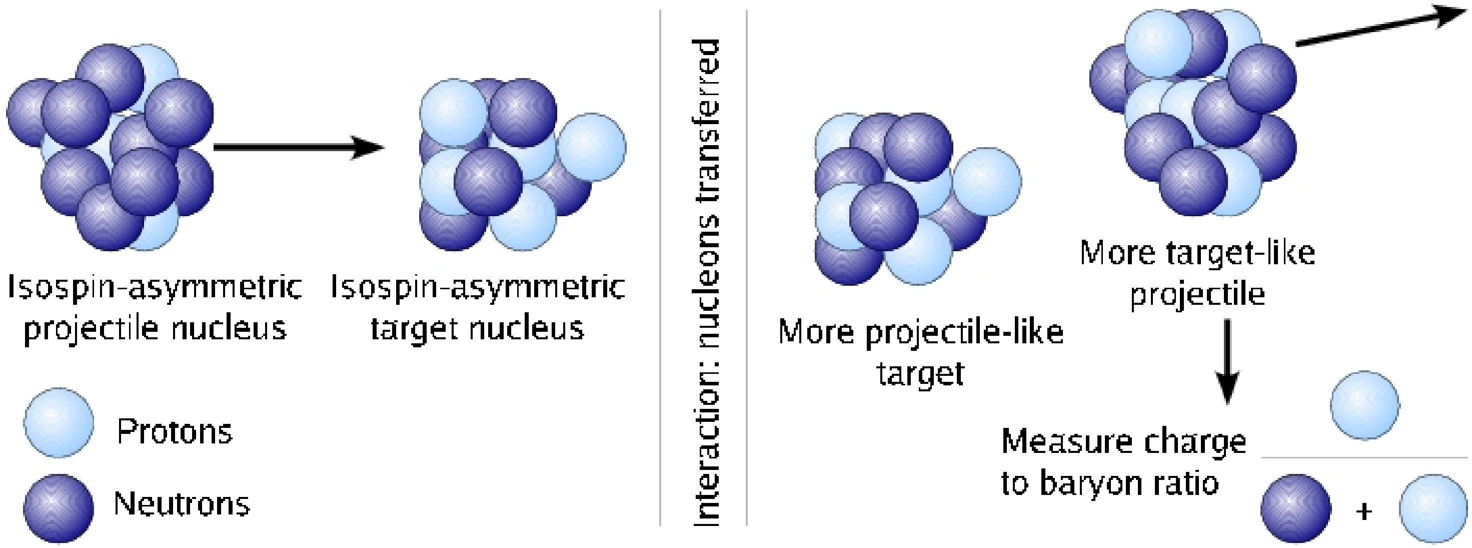
D. Blaschke, H. Grigorian, D. Voskresensky,
Astronomy & Astrophysics **424**, 979 (2004)

Symmetry energy in HIC

- Isospin Fractionation in multifragmentation processes

  Isotopic yields, isoscaling (Xu et al, PRL (200), Tsang et al., PRL (2001), Ono et al. PRC (2003))

- Isospin diffusion : the symmetry energy drives the exchange of neutrons and protons between nuclei in a HIC.



Complicated experimental task :
weak signals, several competing effects cancel out among each other