

Isotopic effects in nuclear reactions and the symmetry energy at sub-saturation densities

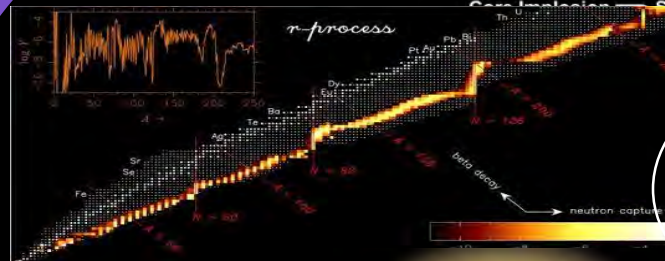
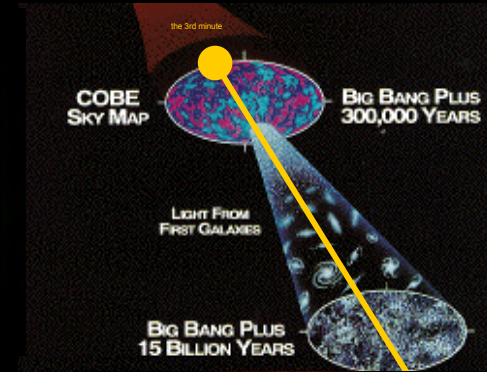
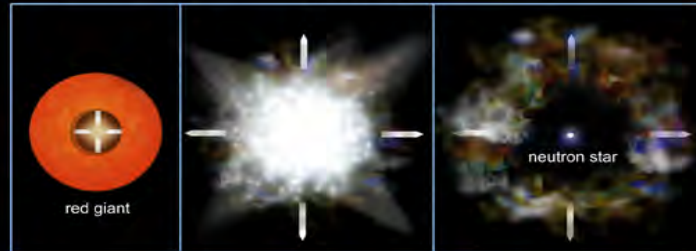
S. Yennello

Texas A&M University

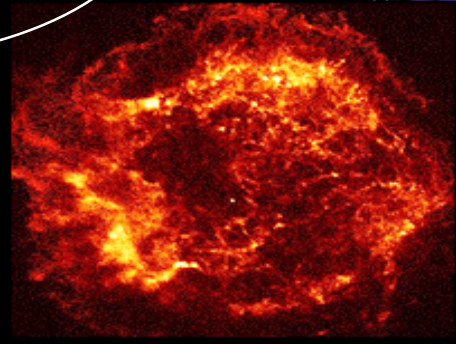
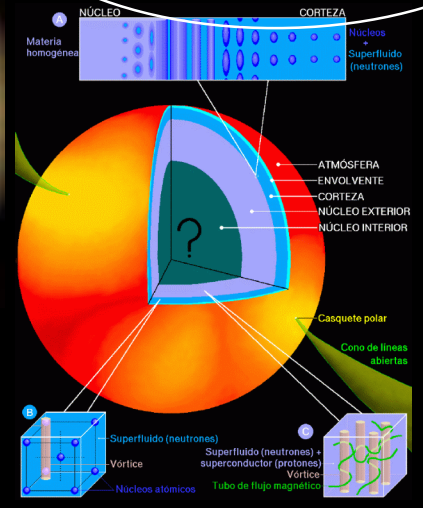
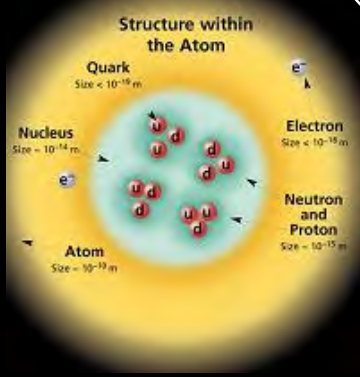


from Atomic Nuclei to Neutron Stars

Birth of a Neutron Star and Supernova Remnant
(not to scale)

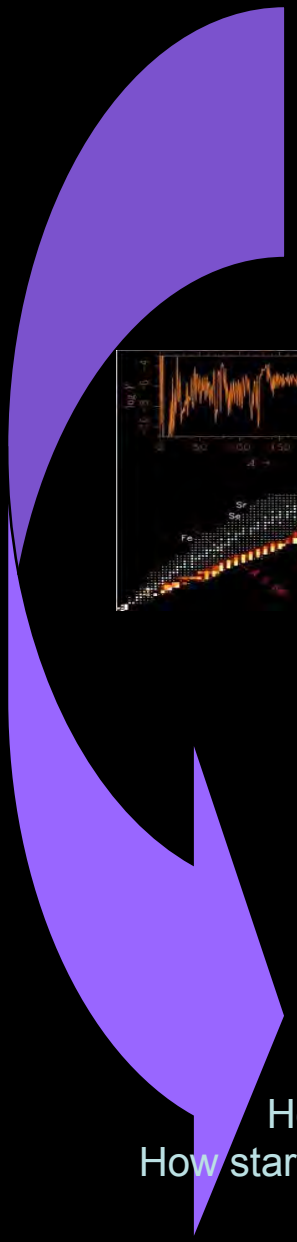


Nuclear Equation of State

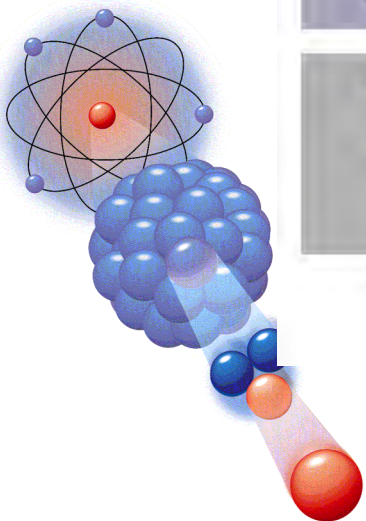
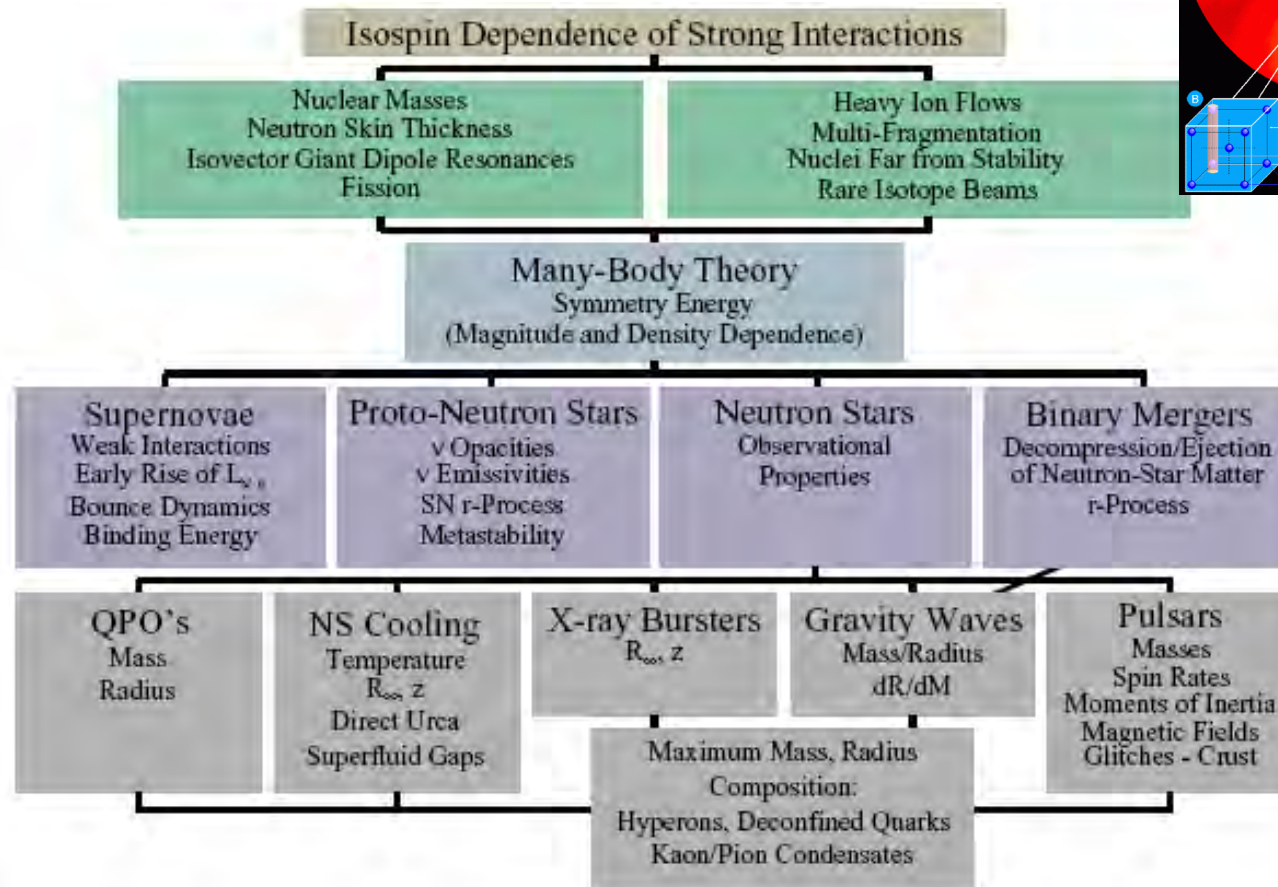
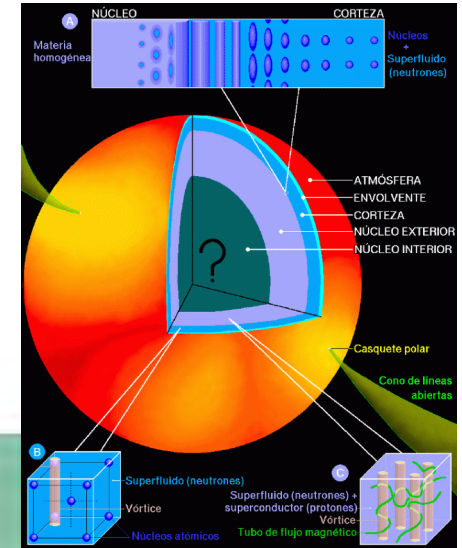


What kind of matter exists inside a neutron star ?

How elements are formed ?
How stars explode into supernova ?



Nuclear Equation of State



Nuclear Equation of State (EoS)

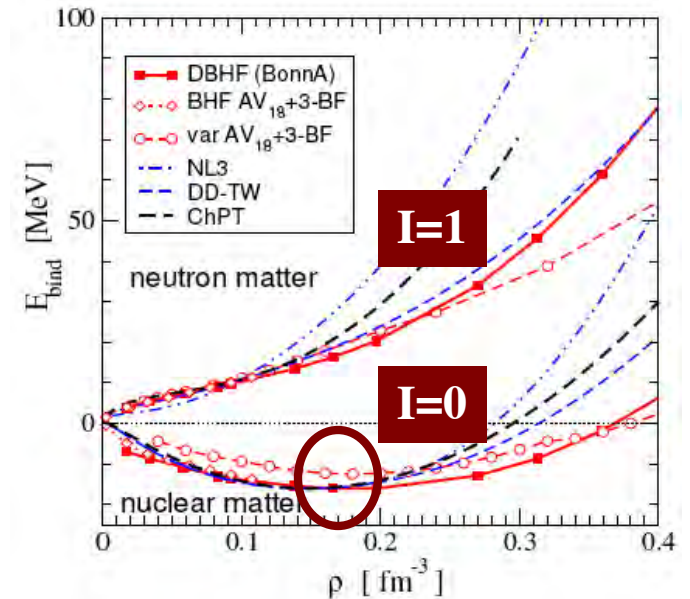
$$E(\rho, I) = E(\rho) + E_{sym}(\rho) I^2$$

with $I = \frac{\rho_n - \rho_p}{\rho_{Total}} \approx \frac{N - Z}{A}$

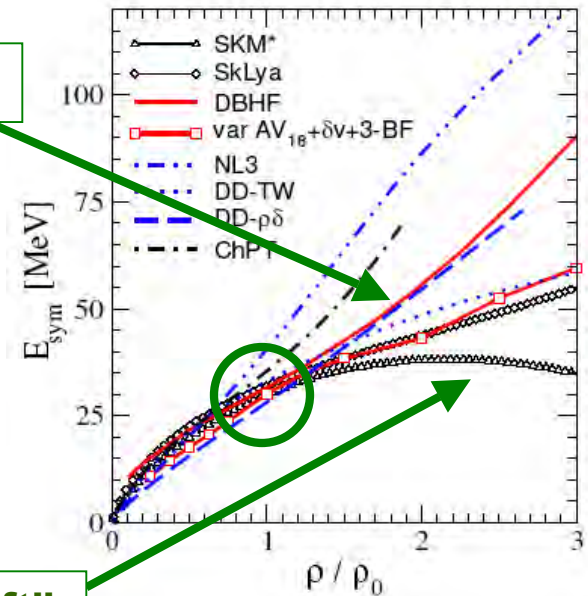
Binding Energy of Symmetric Nuclear Matter

Symmetry Energy Term for Asymmetric Matter

$$E_{sym}(\rho) = E(\rho, 1) - E(\rho, 0)$$



“Stiff”



“Soft”

C. Fuchs and H.H. Wolter, Eur. Phys. J. A **30**, 5 (2006).

V. Baran *et al.*, Phys. Rep. **410**, 335 (2005).



Observables sensitive to the asymmetry term in the EOS ?

Moderate density ($\rho < 1.5 \rho_0$) :

Neutron-skin thicknesses

Pygmy resonances

Fragment isotope distribution, isotopic & isobaric yield ratios

Isospin distillation/fractionation, relative n & p densities

Isospin transport / diffusion / migration

Nuclear stopping & N/Z equilibration

Pre-equilibrium emission

Particle - particle correlation

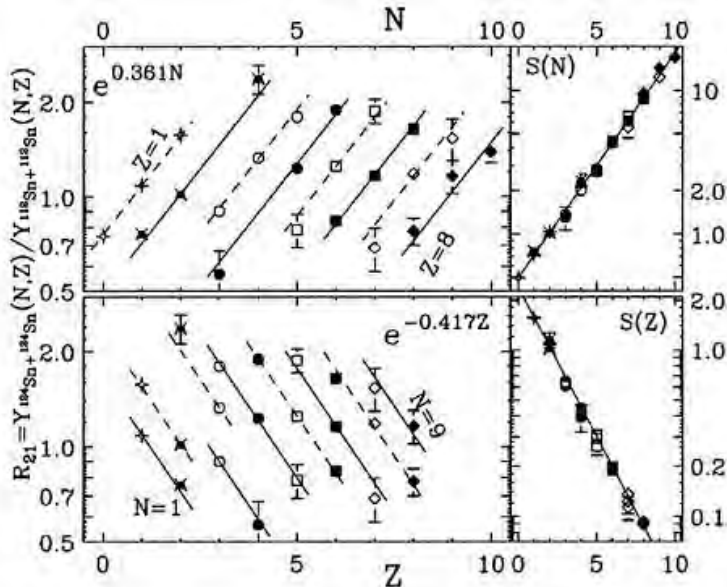
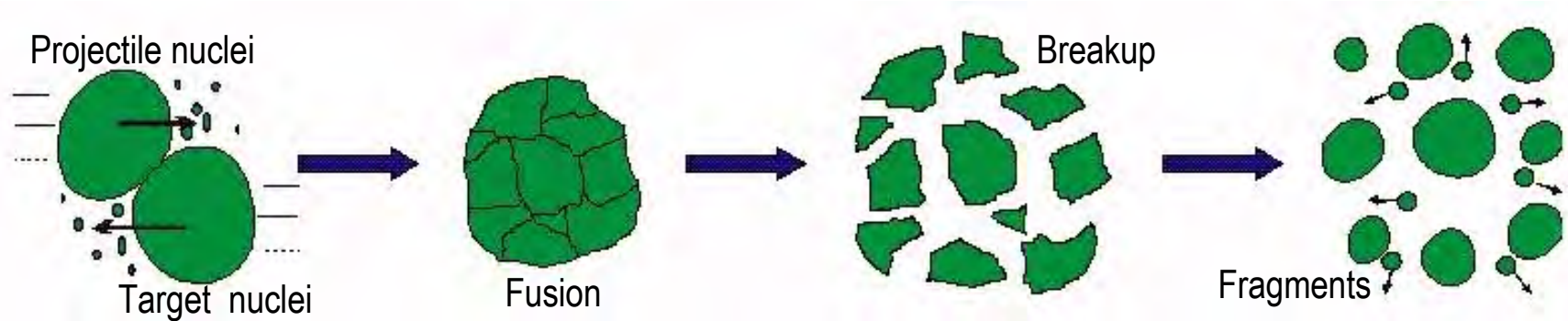
Light cluster production

Flow

Neck emission

Fusion vs Deep Inelastic reactions

Studying density dependence of symmetry energy : Multifragmentation

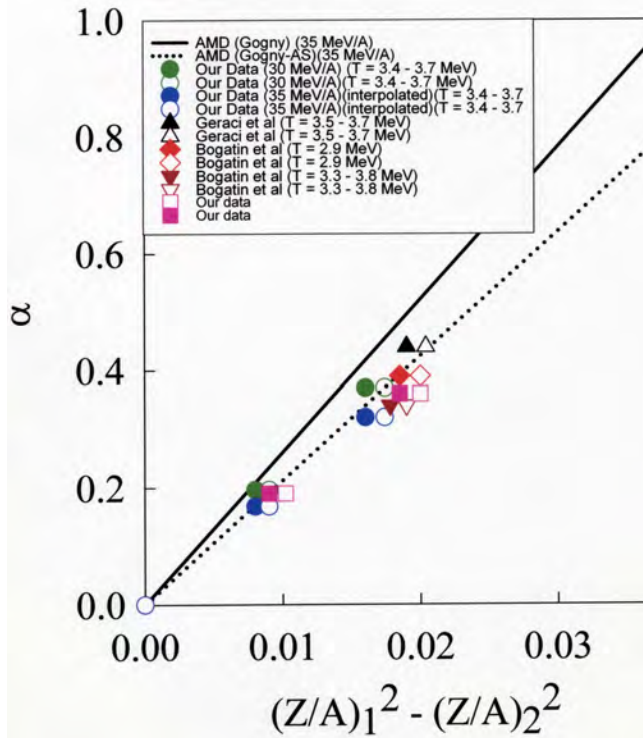


$$\alpha = \frac{4C_{sym}}{T} \left(\frac{Z_1^2}{A_1^2} - \frac{Z_2^2}{A_2^2} \right)$$

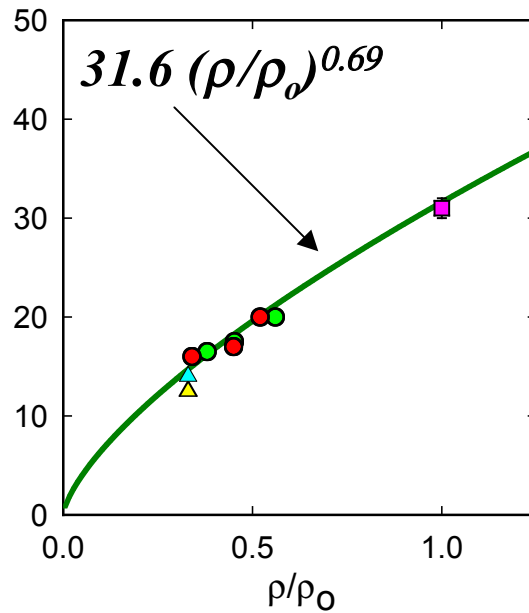
α – Scaling parameter

C_{sym} – Symmetry energy

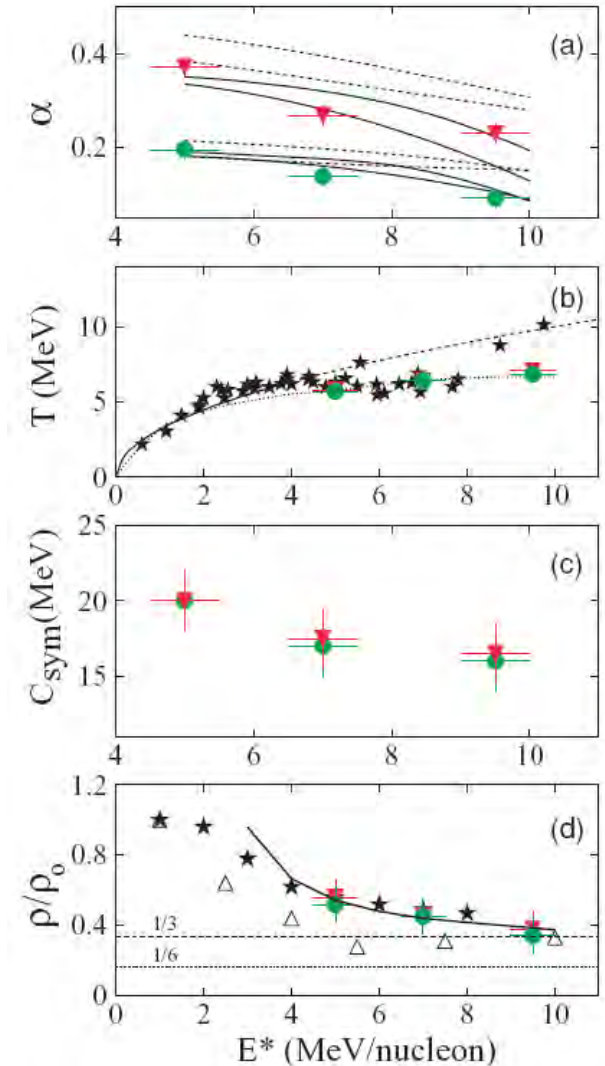
Dynamical (AMD) Isoscaling



D. V. Shetty et al, PRC 70 (2004) 011601(R)



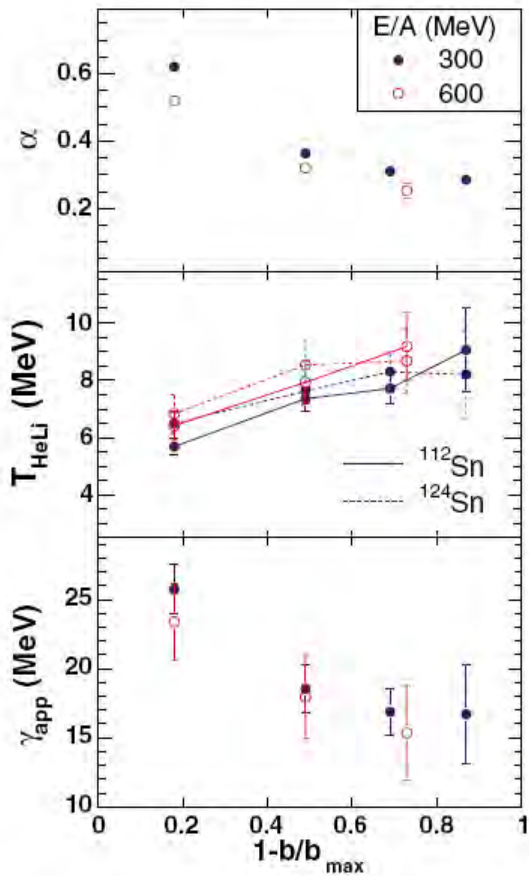
Statistical (SMM)



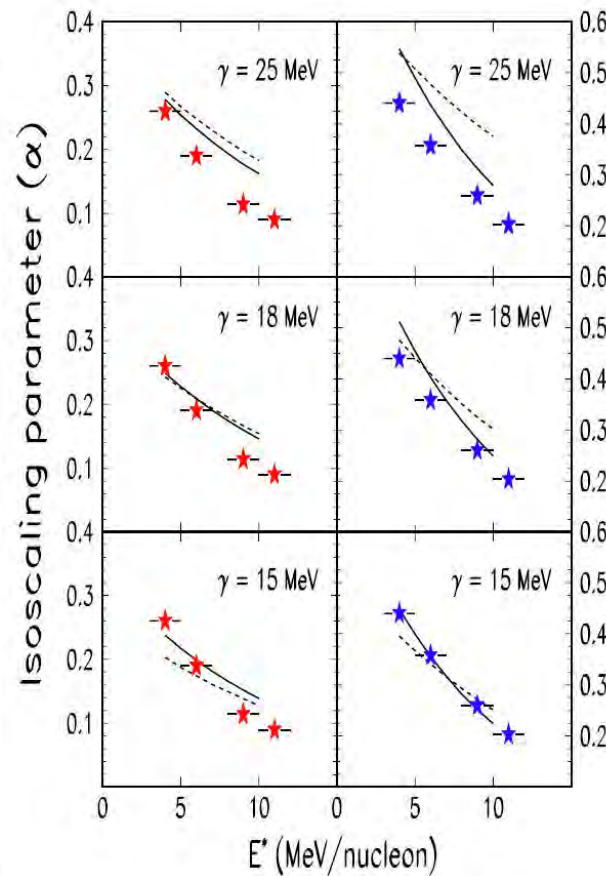
D.V. Shetty et al., PRC 76 (2007) 024606



Decrease in Symmetry energy (Expt. Observation)



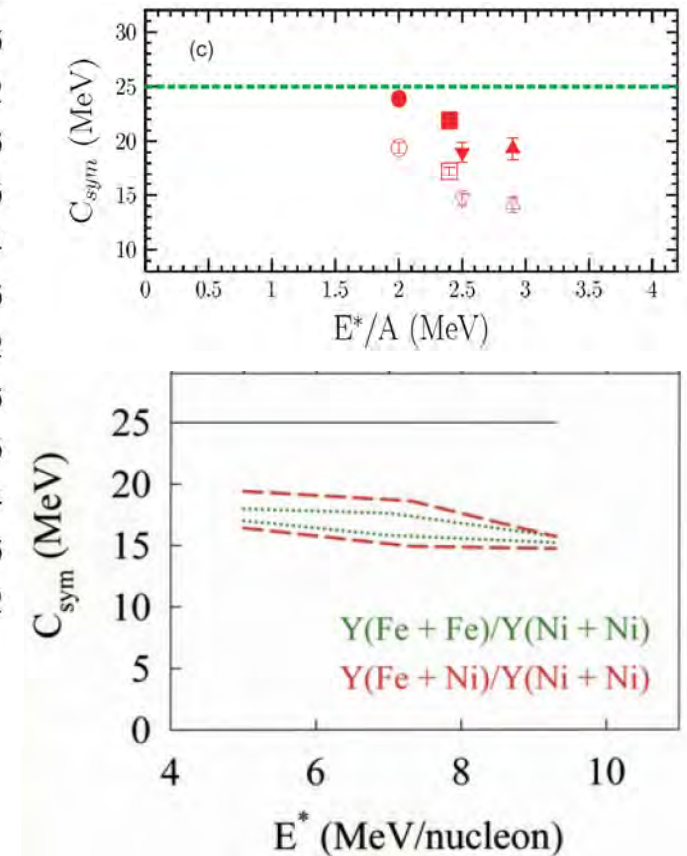
A. Le Fevre et al., PRL 94 (2005) 162701



J. Iglio et al., PRC 74 (2006) 024605

G.A. Souliotis et al., PRC 73 (2006) 024606

G.A. Souliotis et al., PRC 75 (2007) 011601

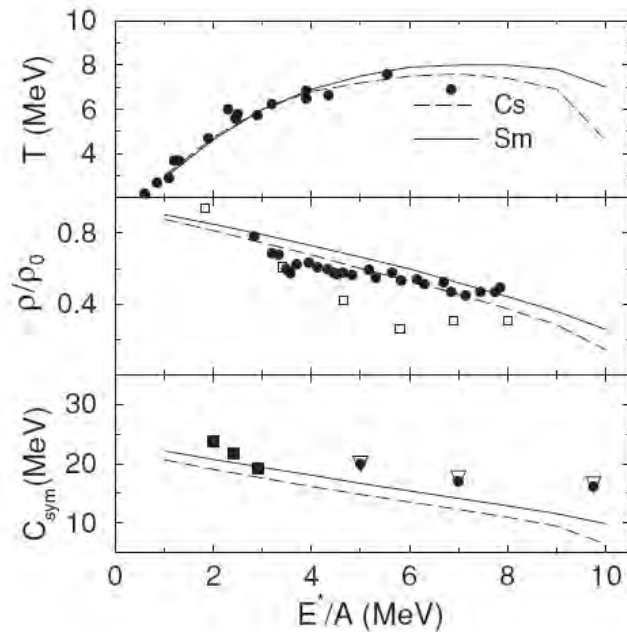


D.V. Shetty et al., PRC 74 (2005) 024602

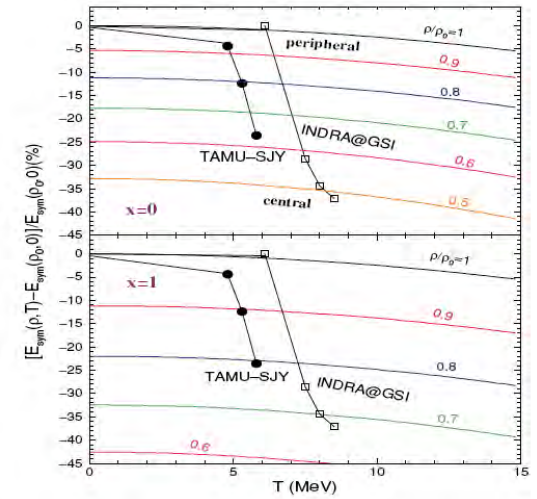
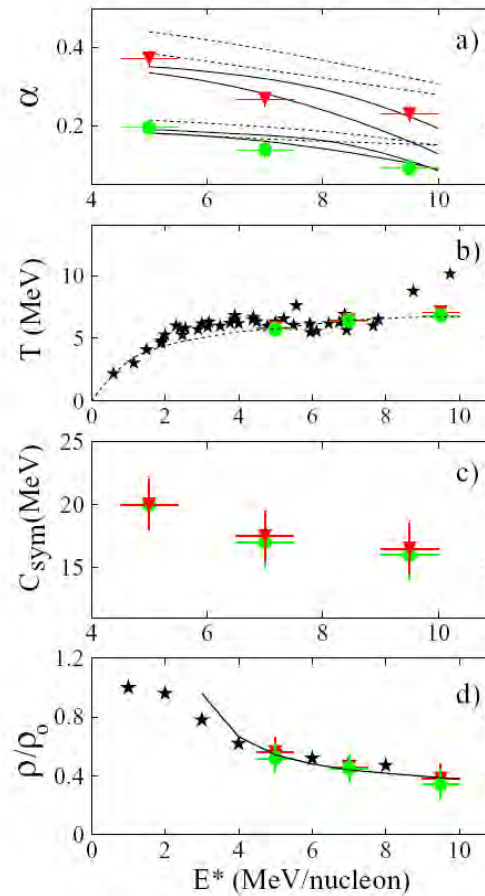


Decrease due to thermal expansion

- Finite T Thomas-Fermi Seyler Blanchard interaction



S.K. Samaddar et al., PRC 76 (2007) 041602



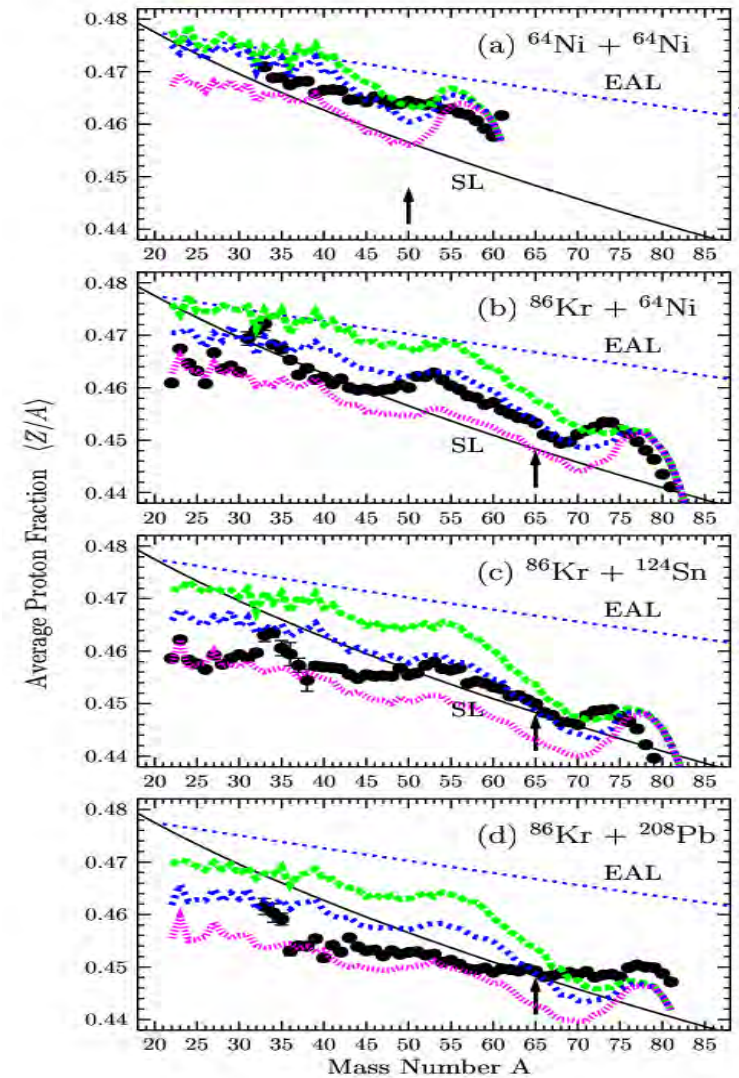
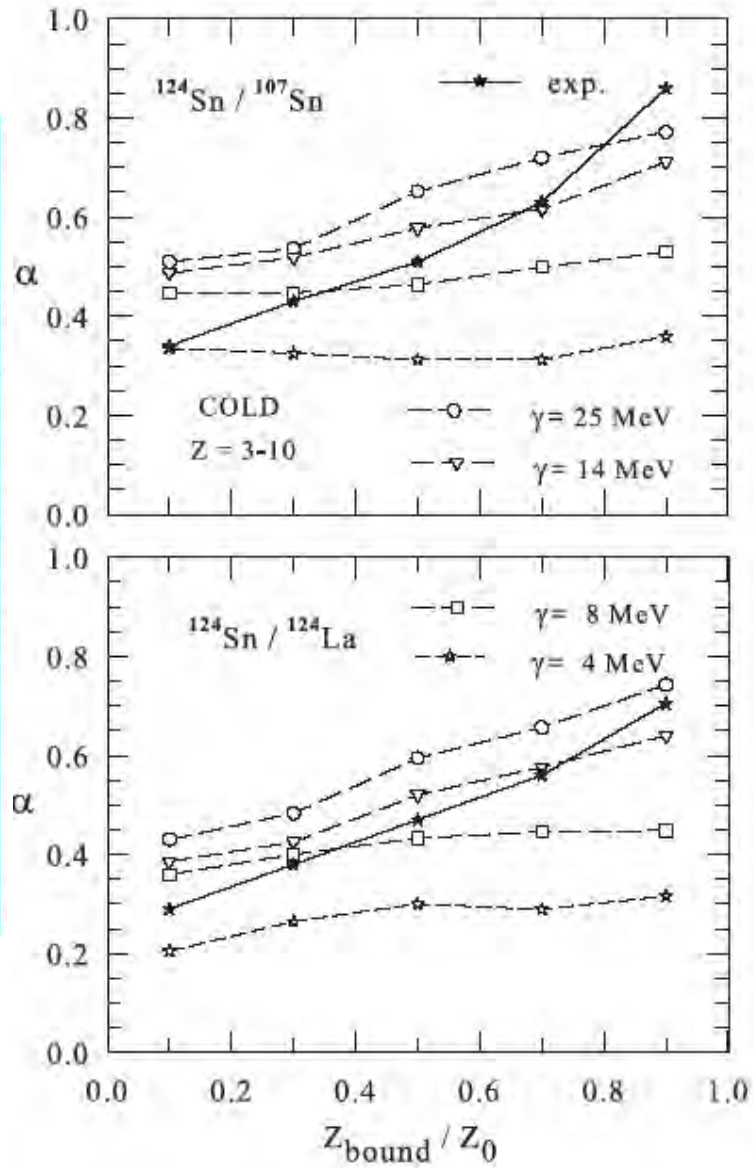
B.A. Li et al., PRC 74 (2006) 034610



Evolving Csym

iwnd2009 Trautmann

arXiv:1001.3867v1 [nucl-ex] 21 Jan 2010



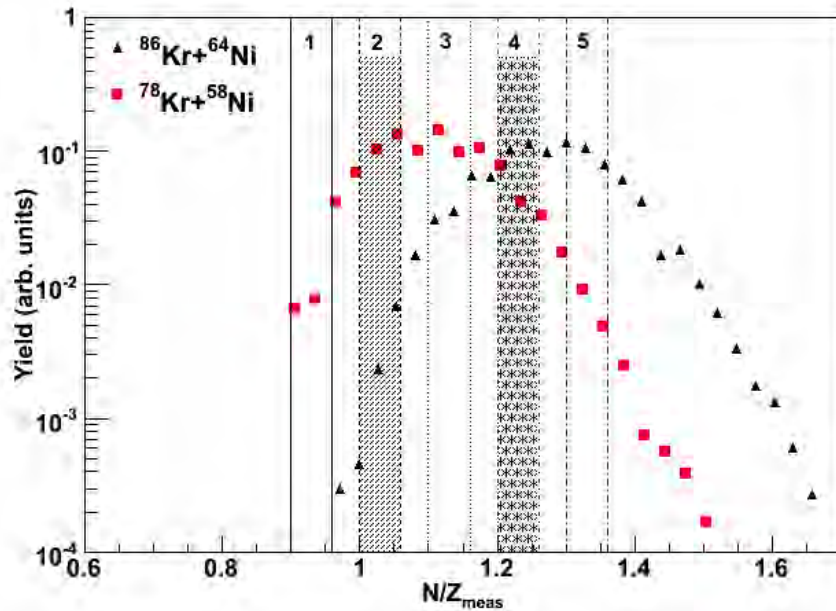
DIT/SMM05 Calculations:

- $C_{\text{sym}} = 25$ MeV
- $C_{\text{sym}} = 20$ MeV
- $C_{\text{sym}} = 15$ MeV

* G.A. Souliotis et al. Phys. Rev. C 75, 011601 (2007)



Reconstructed sources

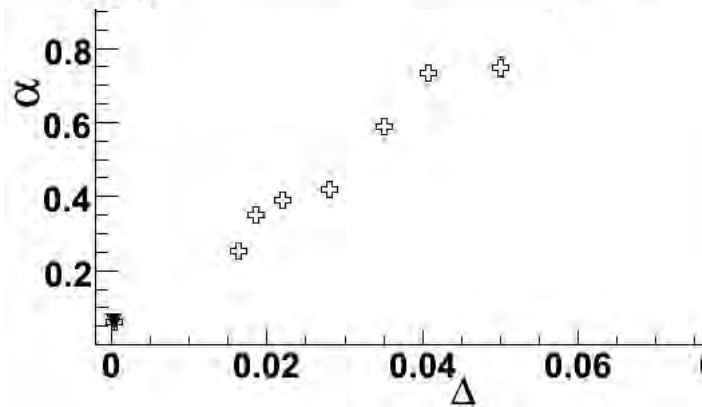


^{78}Kr , $^{86}\text{Kr} + ^{58}\text{Ni}$, ^{64}Ni

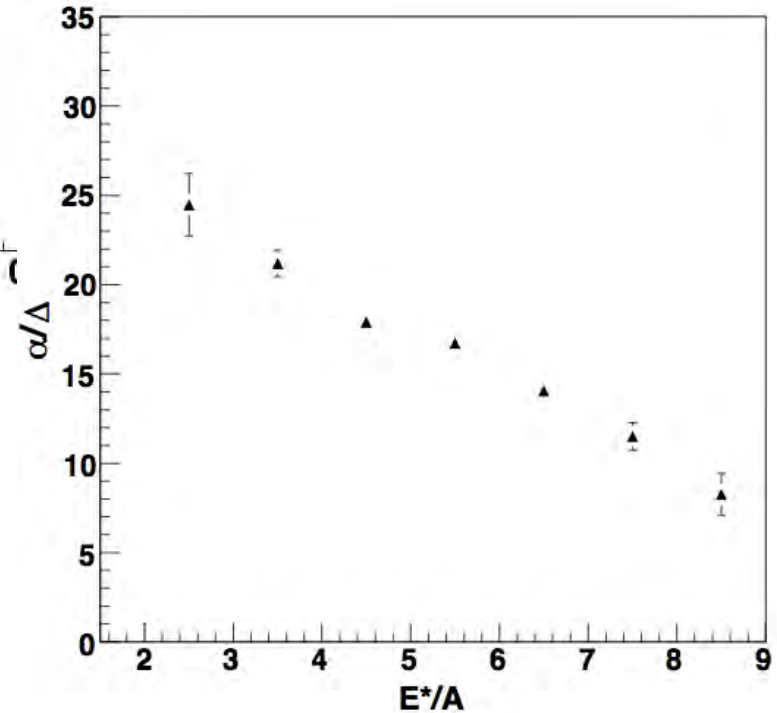
35 MeV/nucleon

NIMROD-ISiS

(includes free neutrons)



$$\frac{\alpha}{\Delta} = \frac{4C_{sym}}{T}$$



Mirror nuclei ratios in well defined systems

Landau expansion of free energy near the critical point

$$\frac{F}{T} = \frac{1}{2}am^2 + \frac{1}{4}bm^4 + \frac{1}{6}cm^6 - \frac{H}{T}m$$

m (=N-Z)/A: Order parameter
H : External field

$$\frac{H}{T} = am_s (1 + \text{Higher order terms in 'm' and 'm}_s')$$

For mirror nuclei yield ratio (ignoring the higher order terms)

$$\frac{1}{2} \ln \left(\frac{Y_2}{Y_1} \right) = am_s$$

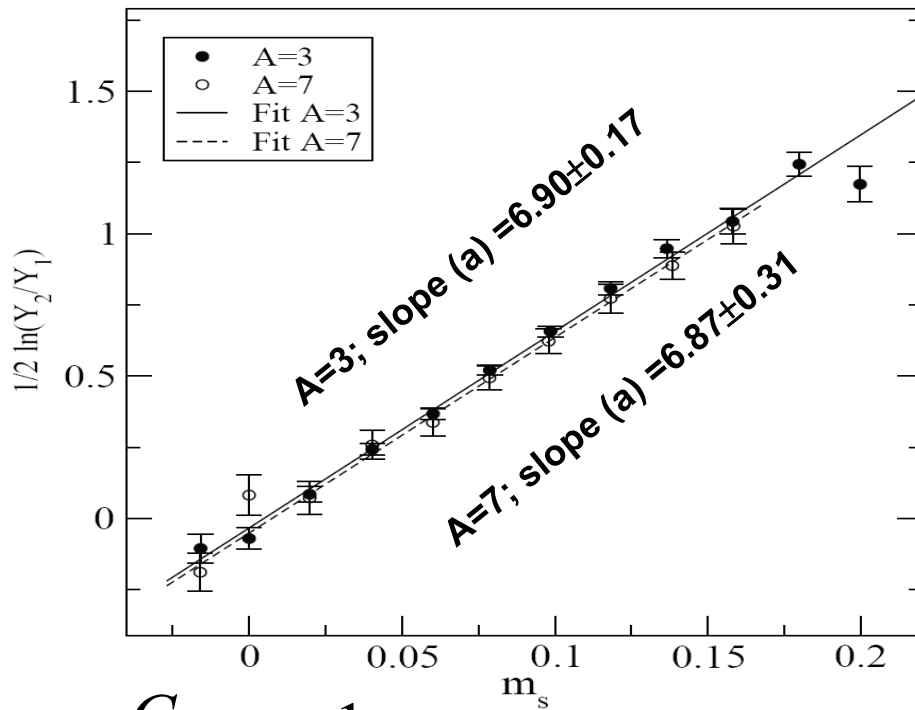
$$\frac{C_{Sym}}{T} \approx \frac{1}{2}a$$

Source Isospin

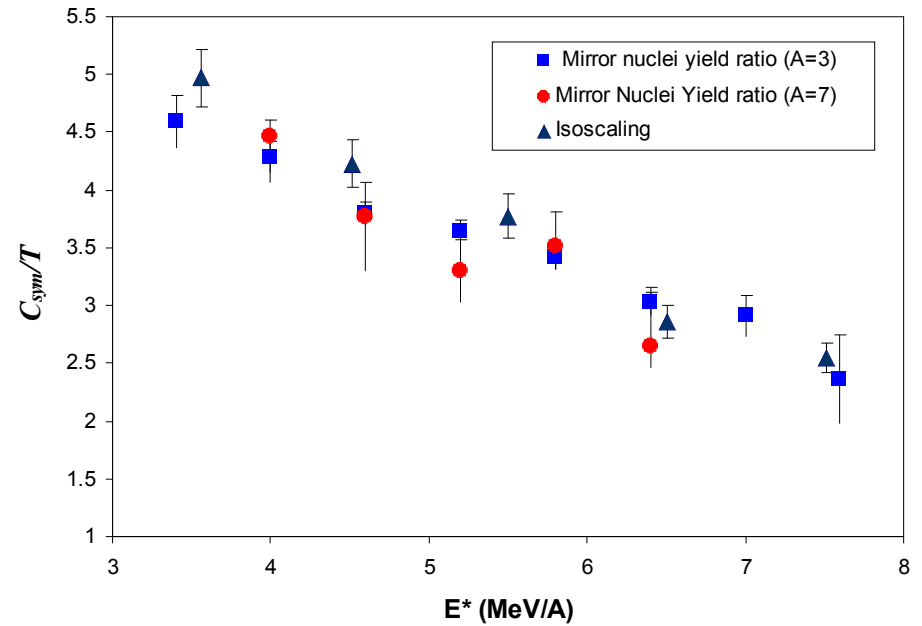


Average ratios (A=3 and A=7) from four different reaction systems

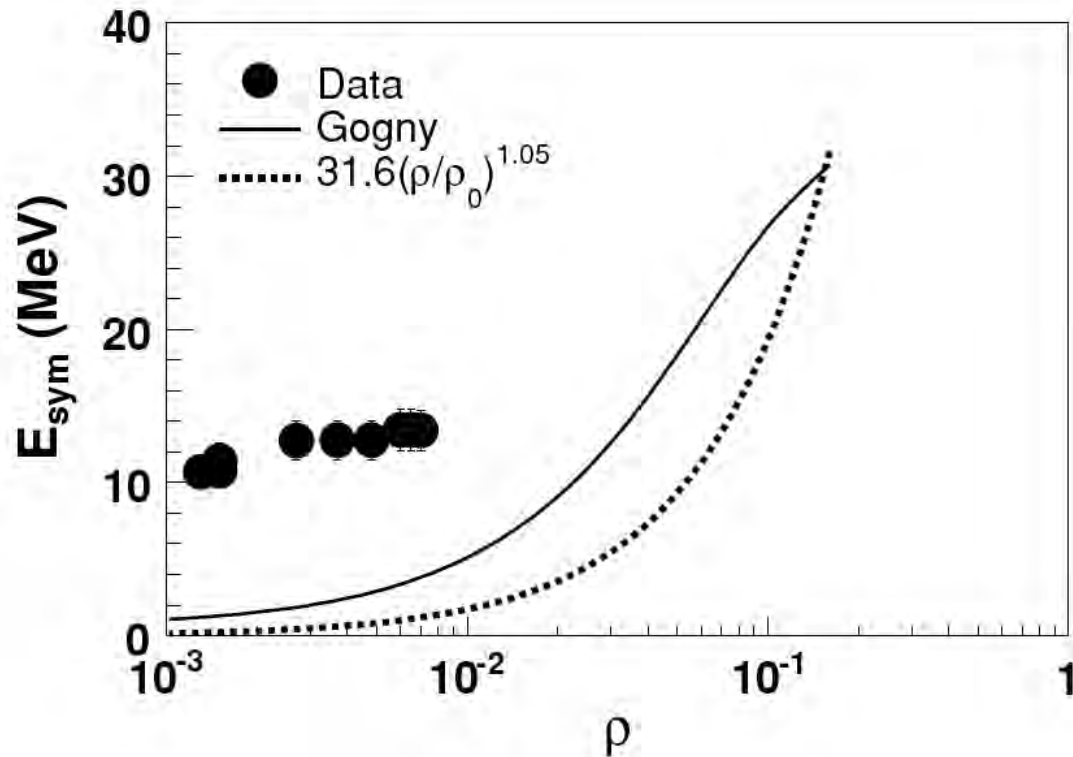
$^{78,86}\text{Kr} + ^{58,64}\text{Ni}$ ($E_{\text{lab}} = 35 \text{ MeV/A}$)



$$\frac{C_{\text{Sym}}}{T} \approx \frac{1}{2} a$$



Low density



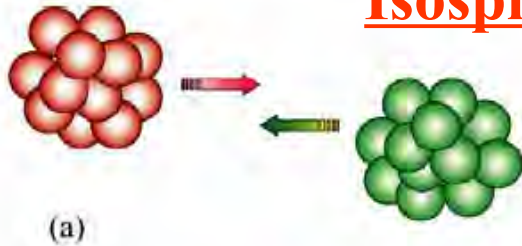
clusters matter
Entropy

Kowalski, C 75, 014601 (2007)



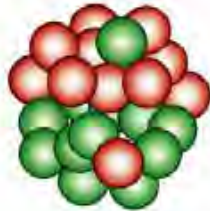
Isospin Equilibration / Diffusion

Isospin Diffusion/Transport



Particle Flux:

$$\Gamma_i = n_i (\underline{v}_i - \underline{v}),$$



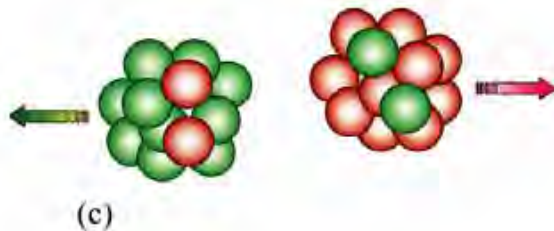
Isospin Flow:

$$\Gamma_I = \Gamma_n - \Gamma_p = -n D_I \frac{\partial \delta}{\partial \mathbf{r}}.$$

Isospin diffusion coefficient D_I depends on the symmetry potential

L. Shi and P. Danielewicz,

Phys. Rev. C **68**, 017601 (2003).



How to measure Isospin Diffusion?

F. Rami et al. (FOPI/GSI) **PRL**84, 1120 (2000)

A measure of isospin transport in the reaction A+B using any isospin tracer X

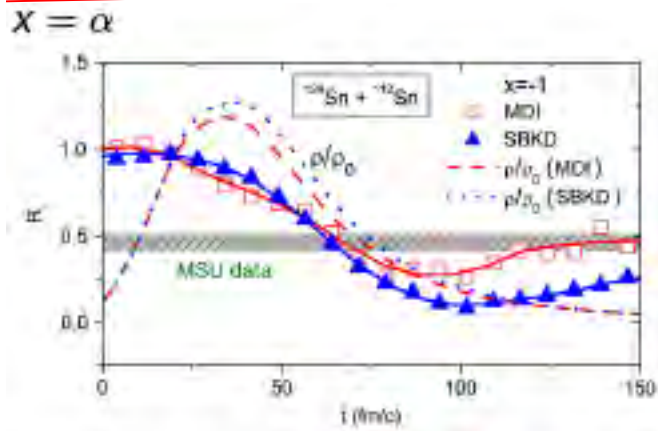
A+A, B+B, A+B

X: isospin tracer

$$R_X = \frac{2X^{A+B} - X^{A+A} - X^{B+B}}{X^{A+A} - X^{B+B}}$$

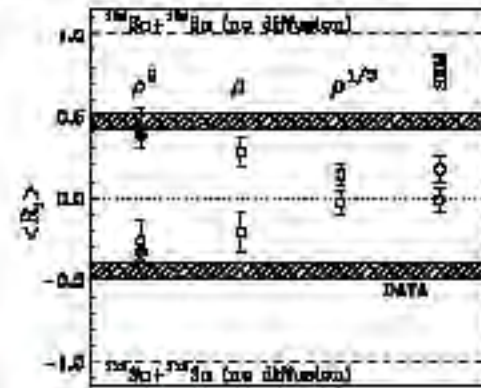


MSU data



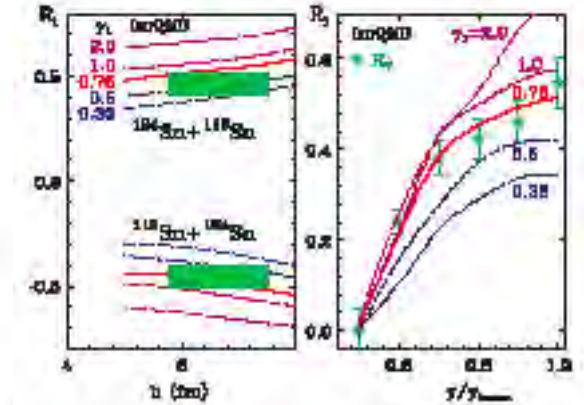
Comparison to IBUU04, $b=6$ fm (B.A. Li)
Best agreement for **asy-stiff**: $x=-1 \equiv \gamma=1$

$X = \alpha$ (Tsang PRL 92 (2004) 062701)

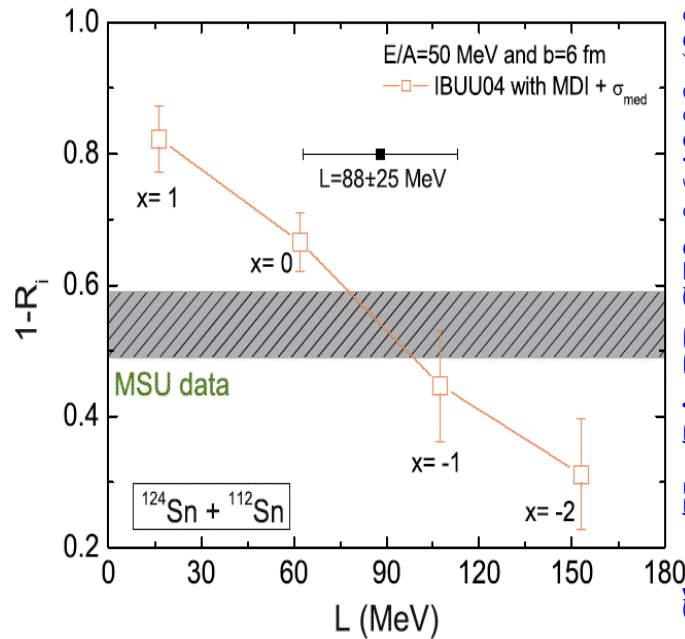


Comparison to BUU97 (B. A. Li)
asy-stiff $\gamma \sim 2$

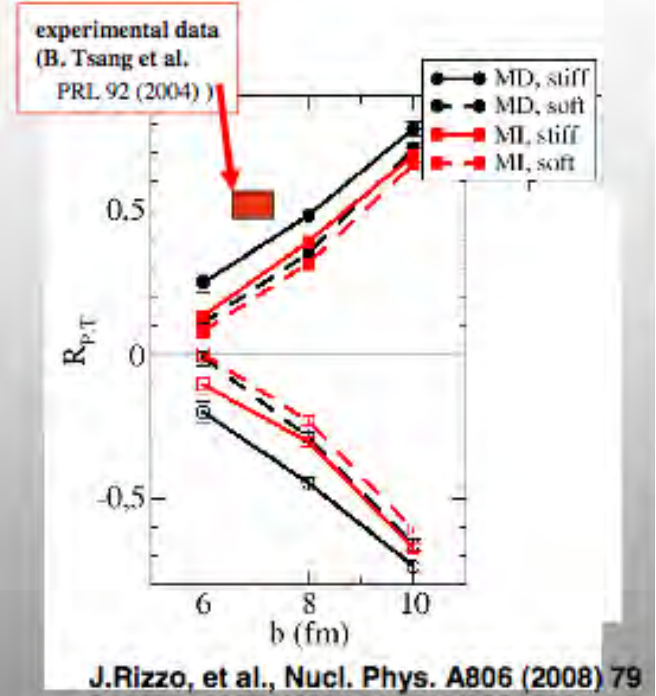
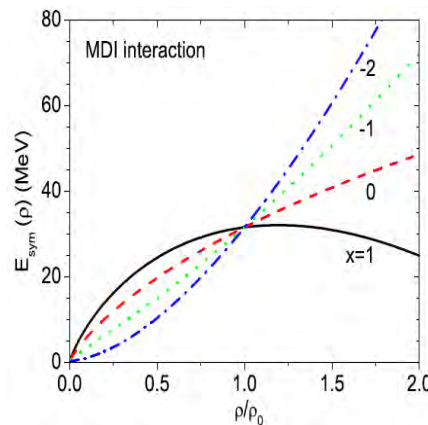
$x = \alpha$ (left) and R_7 (right) both agree in QP region
 $y/y_{beam} > 0.7$ (Tsang PRL 102 (2004) 122701)



Comparison ImQMD (Z. Li):
asy-soft $\gamma \sim 0.7$

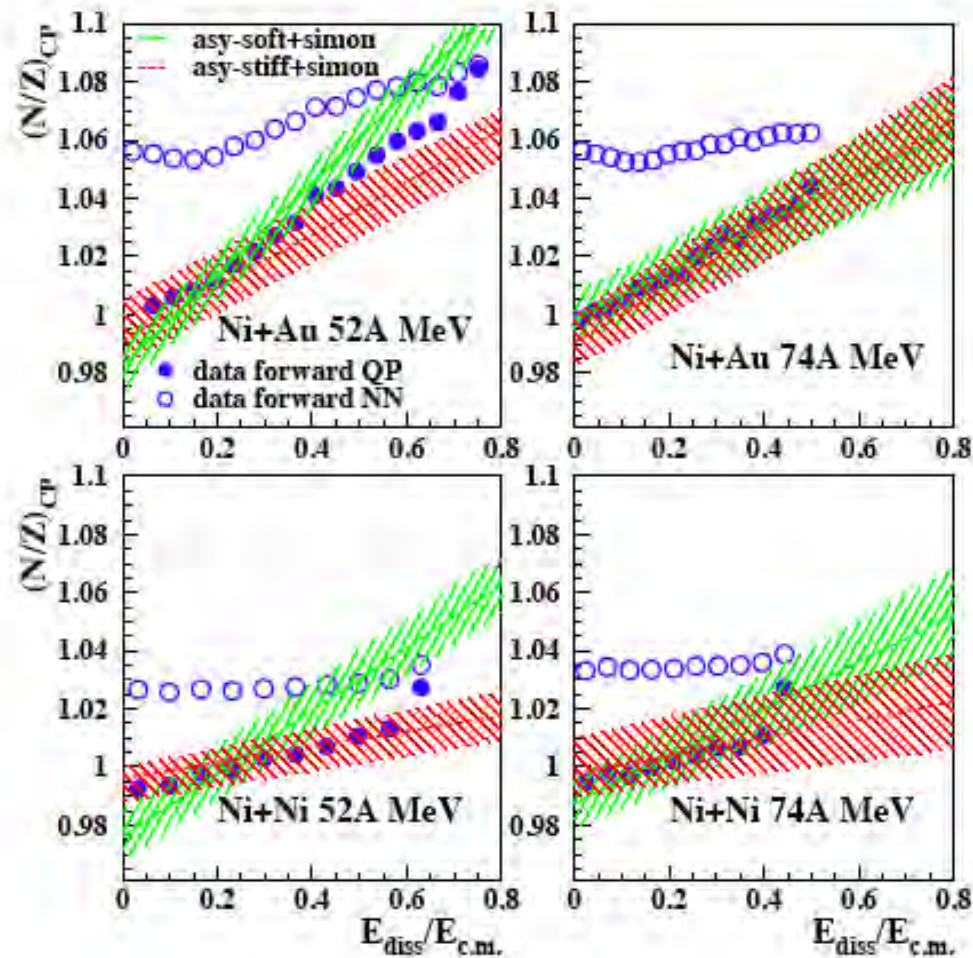


Chen/Ko/Li, PRC72,064309 (2005)

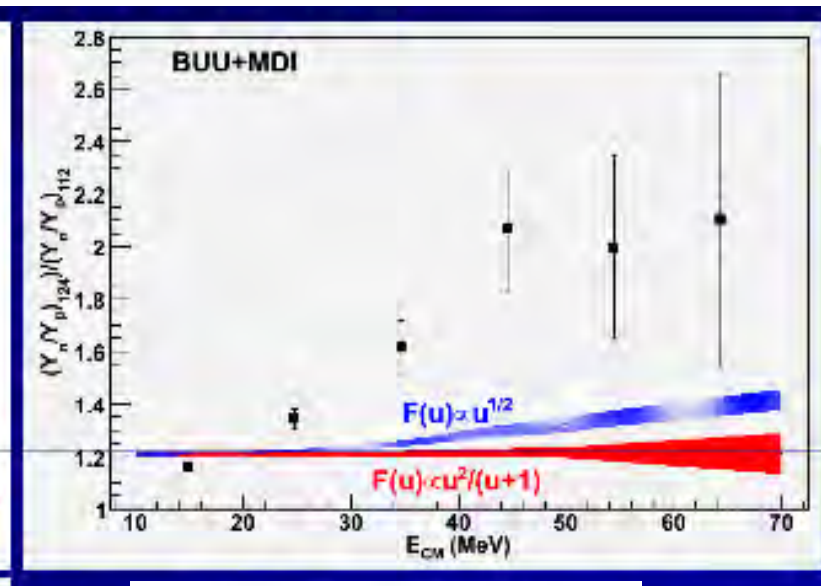
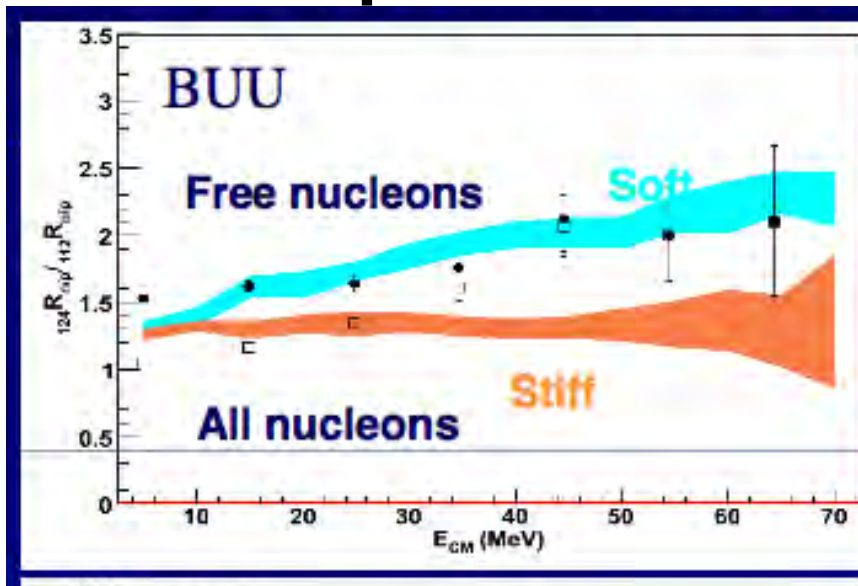


J.Rizzo, et al., Nucl. Phys. A806 (2008) 79

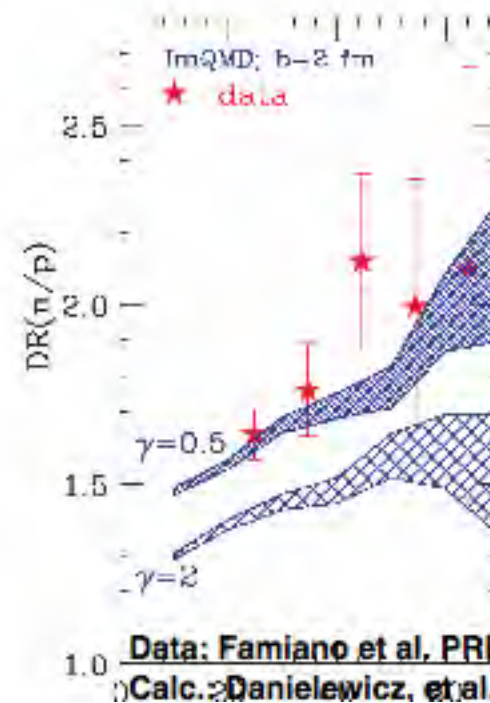
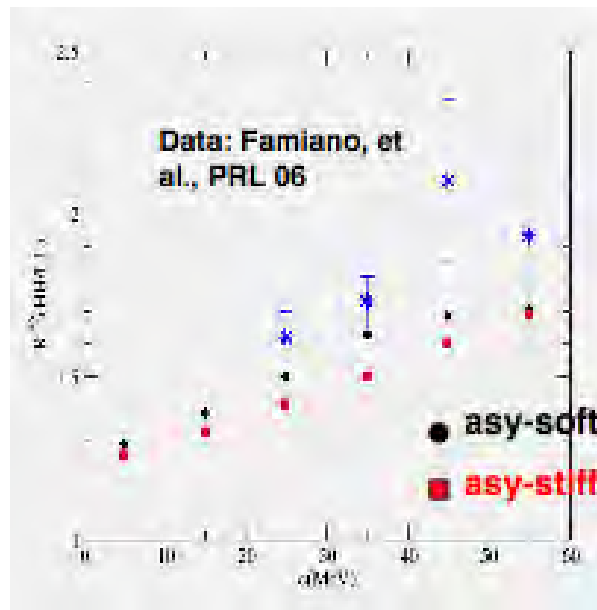
INDRA Data



n/p ratios $^{124}\text{Sn} + ^{124}\text{Sn}, ^{112}\text{Sn} + ^{112}\text{Sn}$



Famiano, 2006 (Li, 1997)

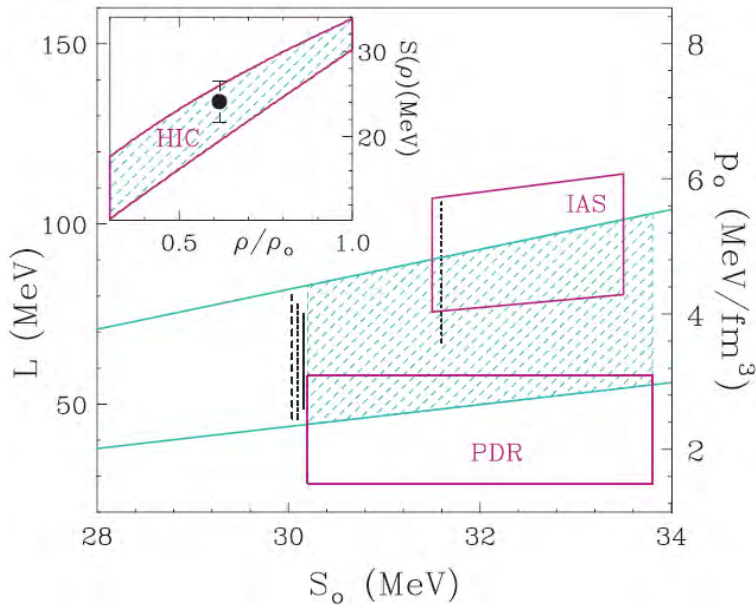
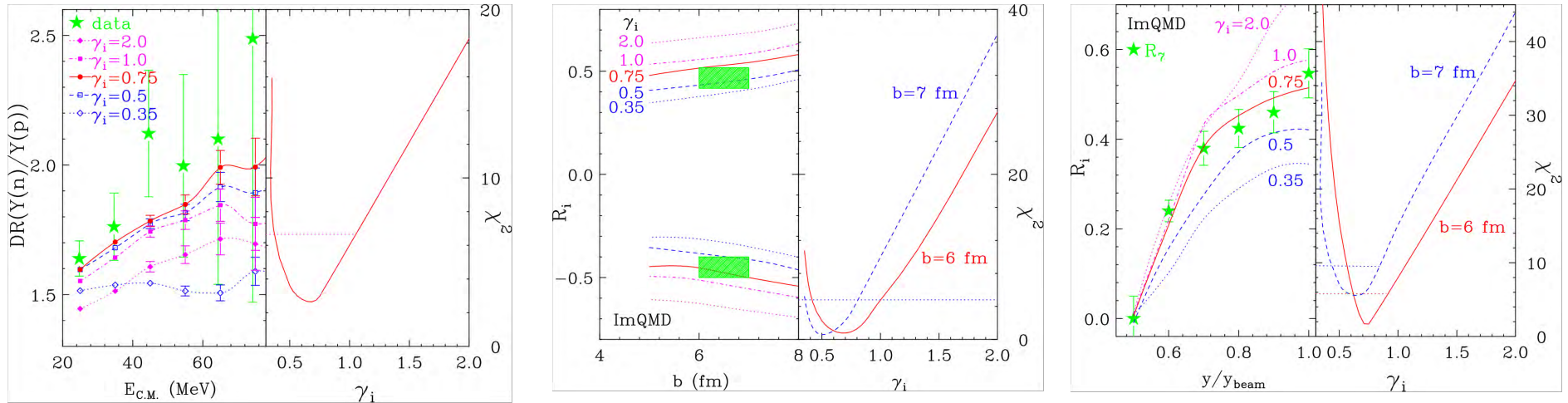


SMF simulations V. Baran 07

Isospin diffusion and double n/p ratio

ImQMD: n/p ratios and two isospin diffusion measurements

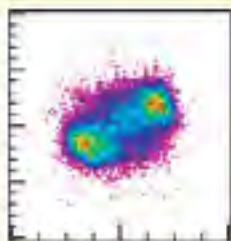
Tsang/Zhang/Danielewicz/Famiano/Li/Lynch/Steiner, PRL 102, 122701 (2009)



ImQMD: Isospin Diffusion and double n/p ratio →
 $E_{sym}(\rho_0)=28 - 34$ MeV
 $L=38 - 103$ MeV



Isospin migration in neck fragmentation

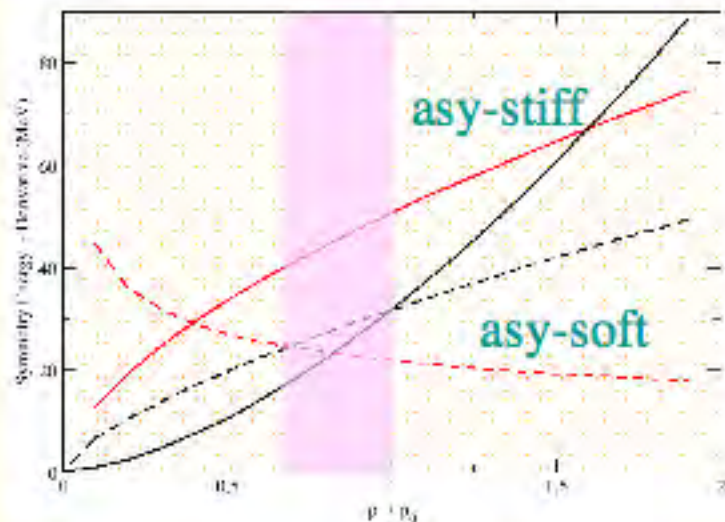


Asymmetry flux



$$\rho_{IMF} < \rho_{PLF(TLF)}$$

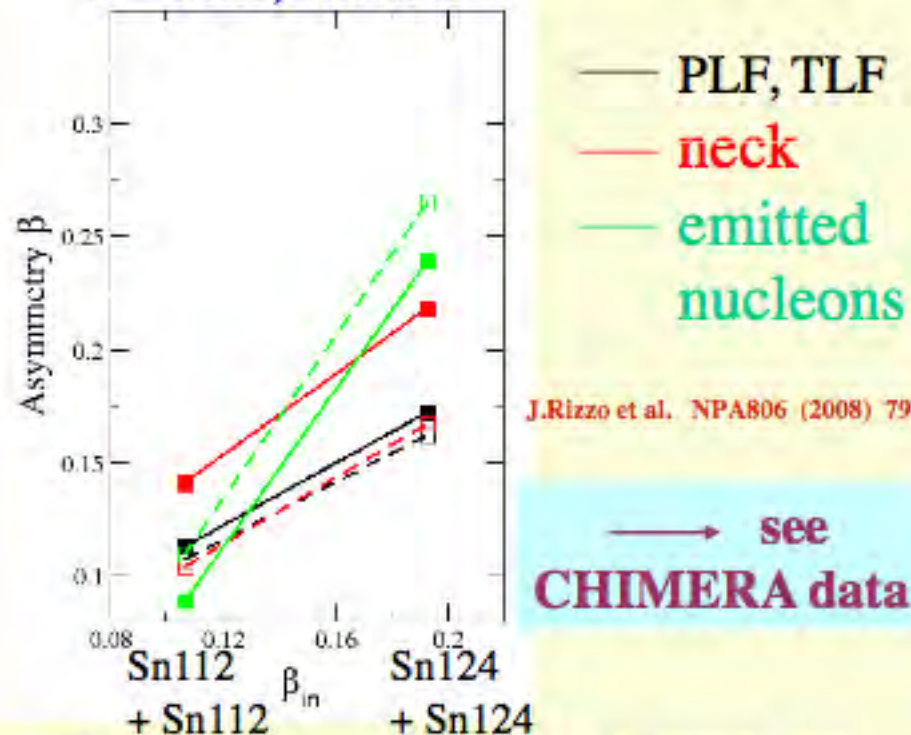
Density gradients \rightarrow derivative of E_{sym}



Larger derivative with asy-stiff \rightarrow larger isospin migration effects

- Transfer of asymmetry from PLF and TLF to the low density neck region: **neutron enrichment of the neck region**
- Effect related to the derivative of the symmetry energy with respect to density

$b = 6 \text{ fm}, 50 \text{ A MeV}$



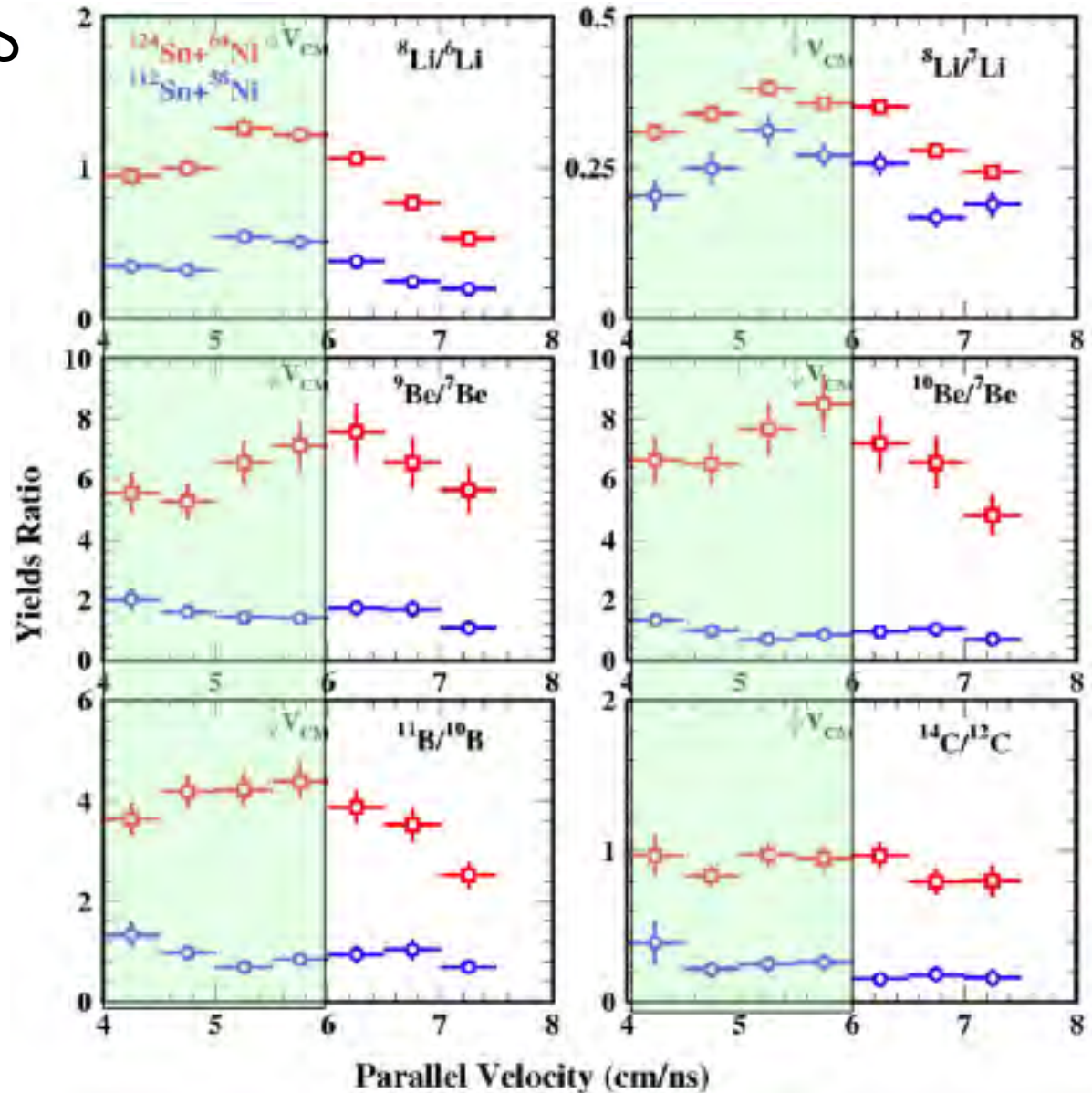
J.Rizzo et al. NPA806 (2008) 79

\rightarrow see CHIMERA data

The asymmetry of the neck is larger than the asymmetry of PLF (TLF) in the stiff case

ISOTOPIC RATIOS FOR LIGHT FRAGMENTS
 (Z=3-6) IN THE 35/A.MeV
 NEUTRON RICH $^{124}\text{Sn}+^{64}\text{Ni}$ AND
 NEUTRON POOR $^{112}\text{Sn} + ^{58}\text{Ni}$
 REACTIONS

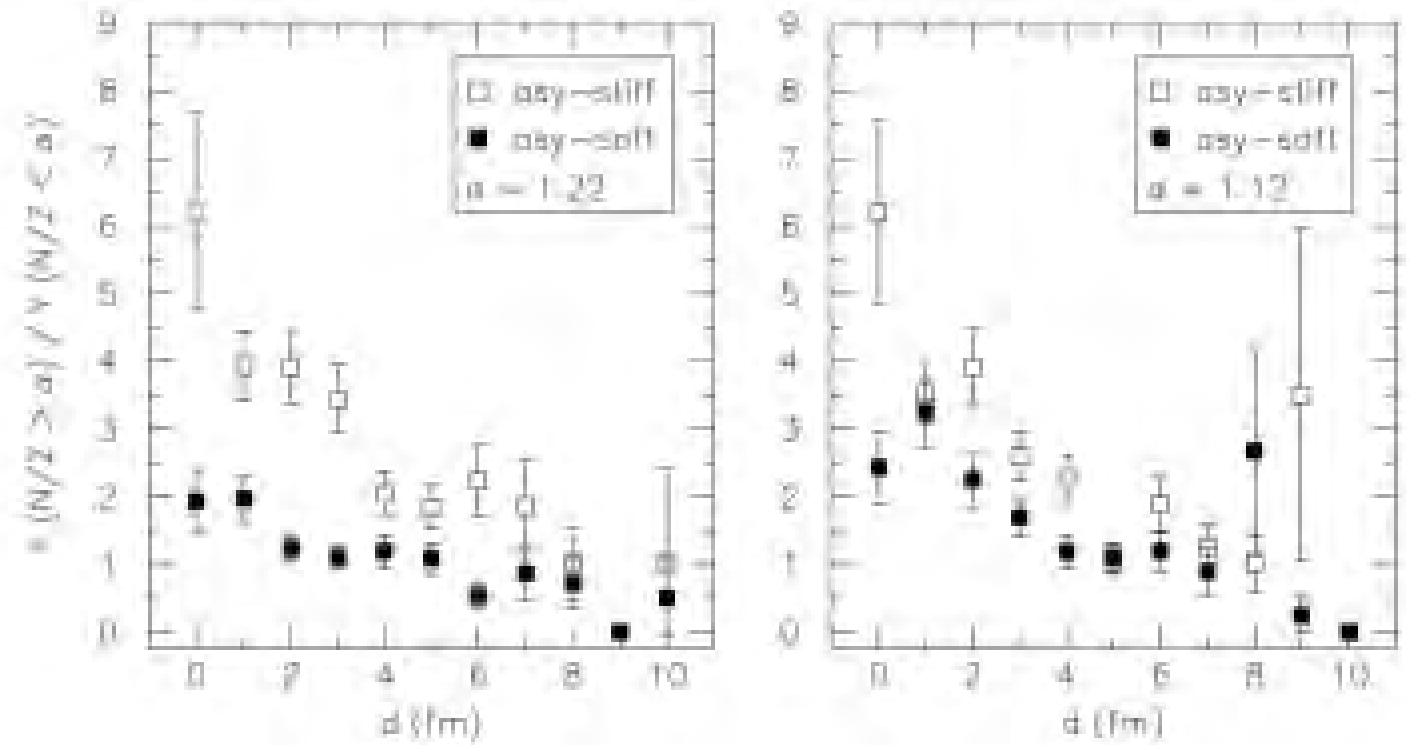
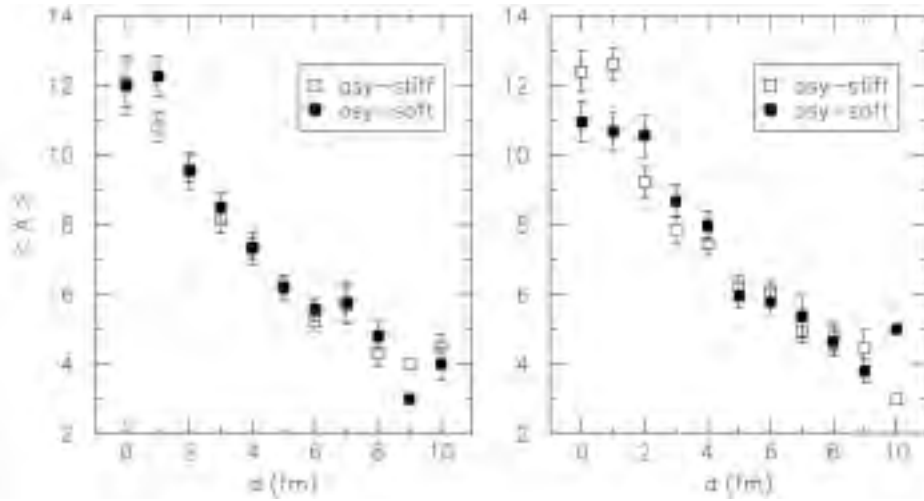
Enhancement
 In n-rich fragments
 for mid velocity



E. De Filippo et al., Acta Physica Polonica B37, 199, 2006;
P. Russotto, Procs of IWM 05, pp. 13, ed. SIF, 2006.

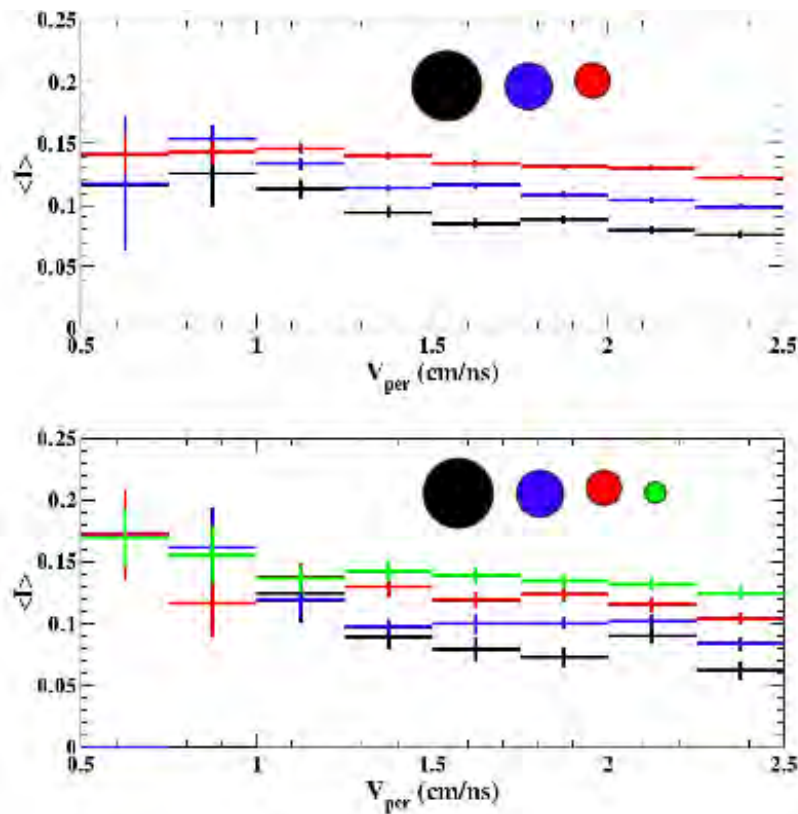


58 Fe + 58 Fe (N/Z = 1.23) and
 58Ni + 58Ni (N/Z = 1.07), at 47A MeV



Isospin and fragment hierarchy

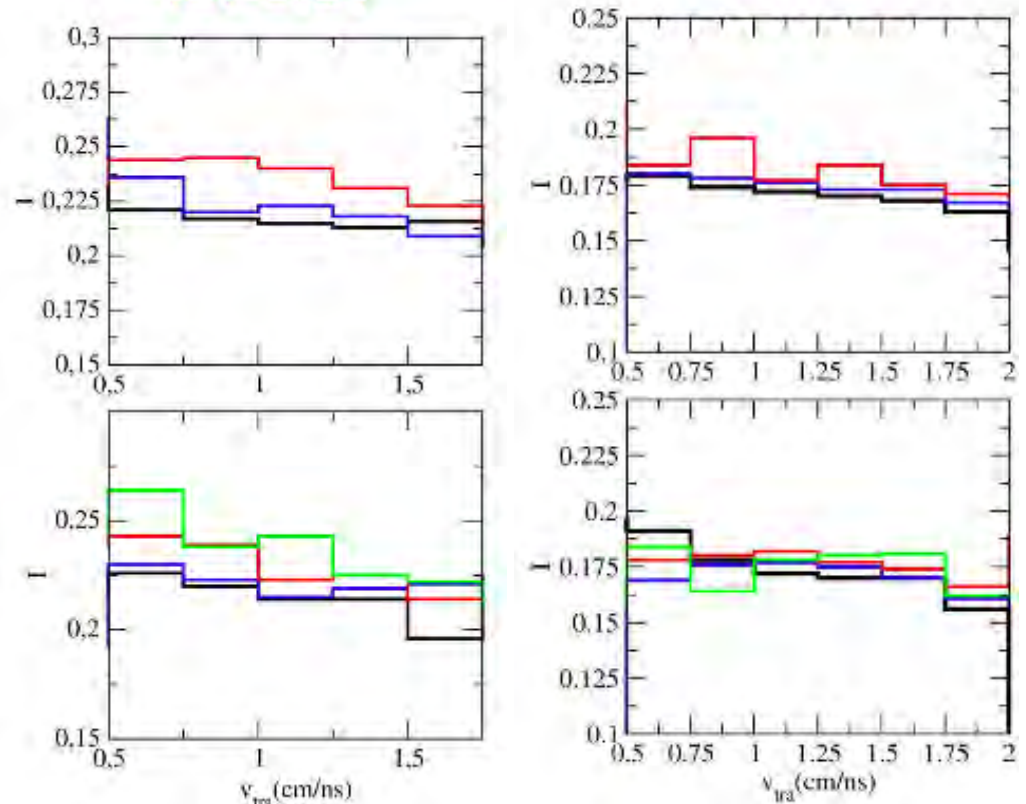
exp for $^{124}\text{Sn}+^{64}\text{Ni}$ 35 AMeV



theoretical BNV but for
 $^{124}\text{Sn}+^{124}\text{Sn}$ 50 AMeV

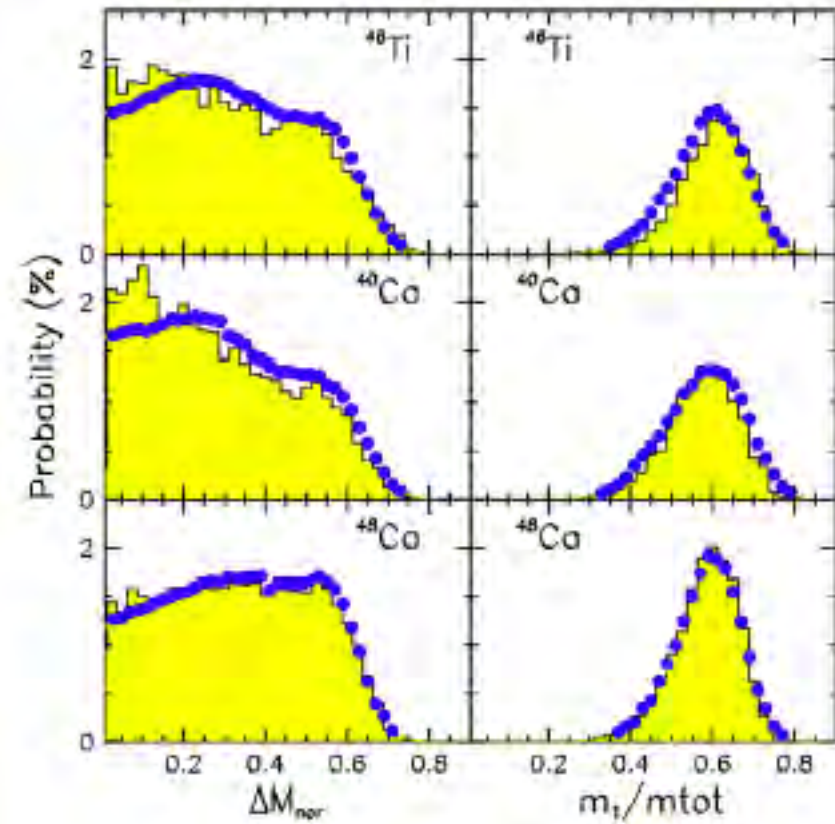
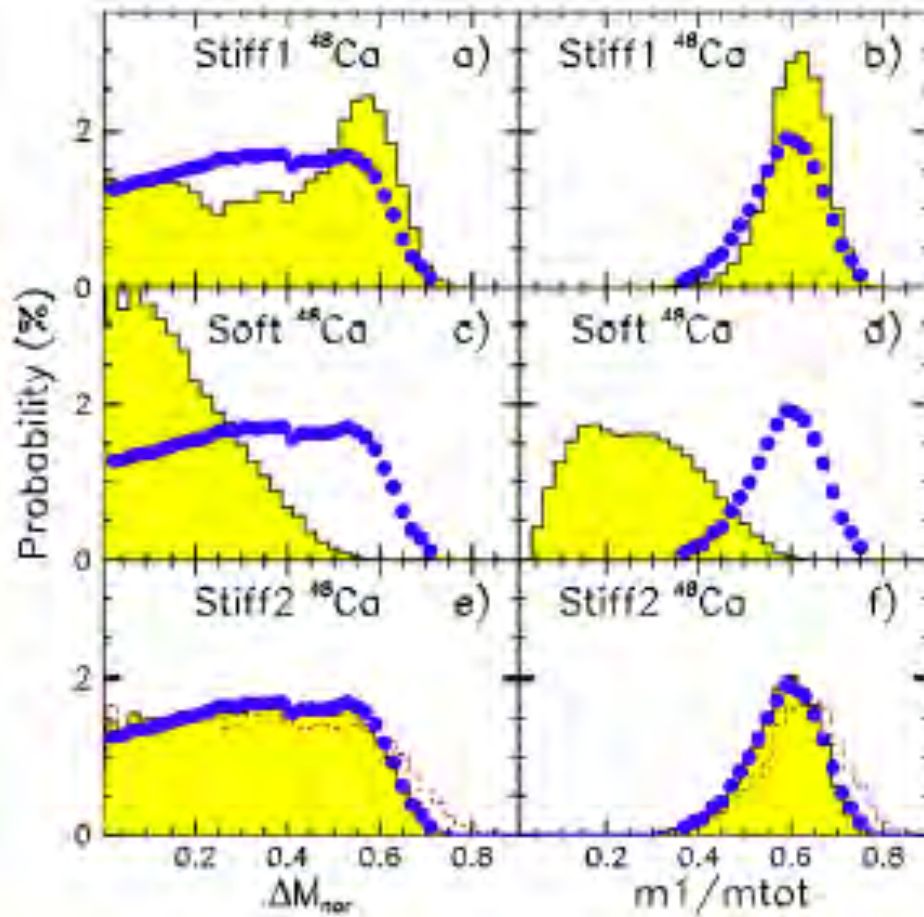
ASY-STIFF

ASY-SOFT



Fusion vs Deep Inelastic in Central collisions

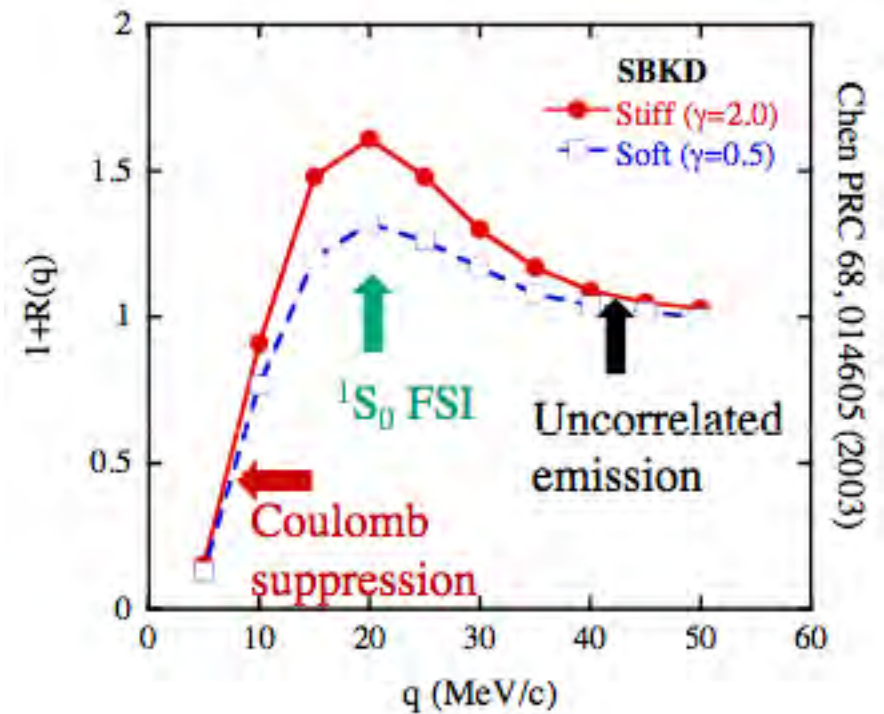
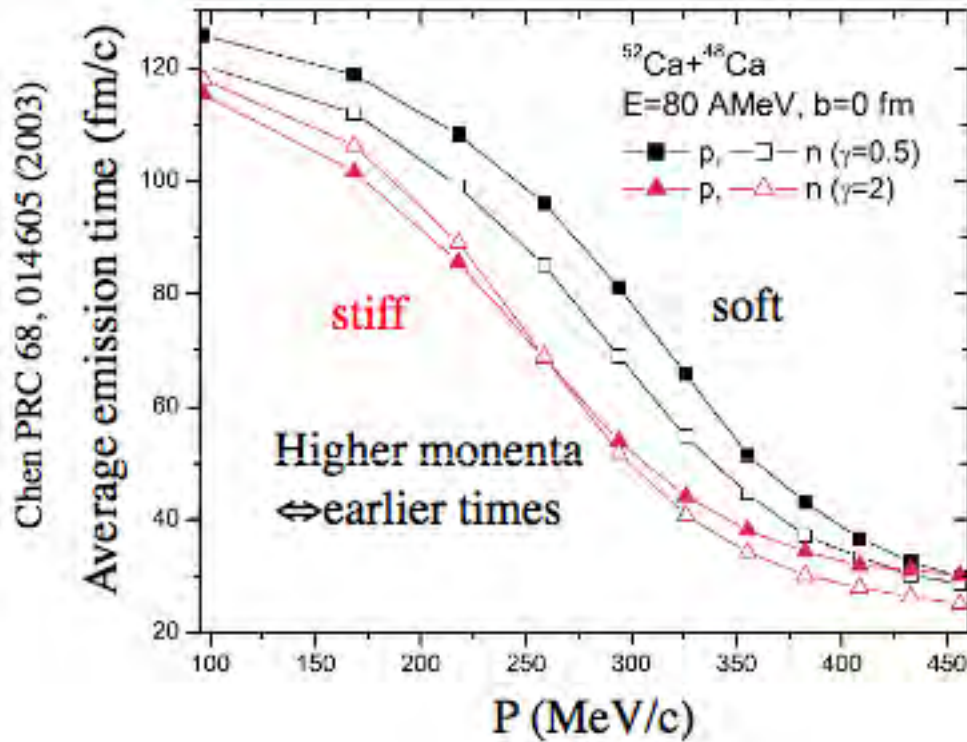
$^{40}\text{Ca} + ^{40}\text{Ca}$, ^{48}Ca , ^{46}Ti at 25 MeV/A



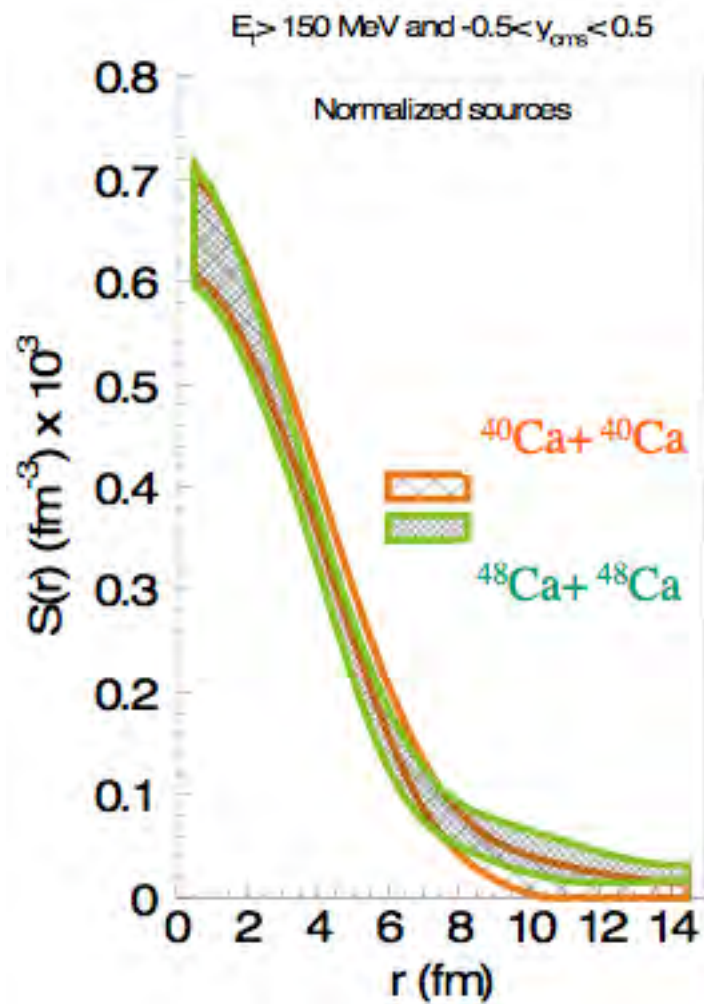
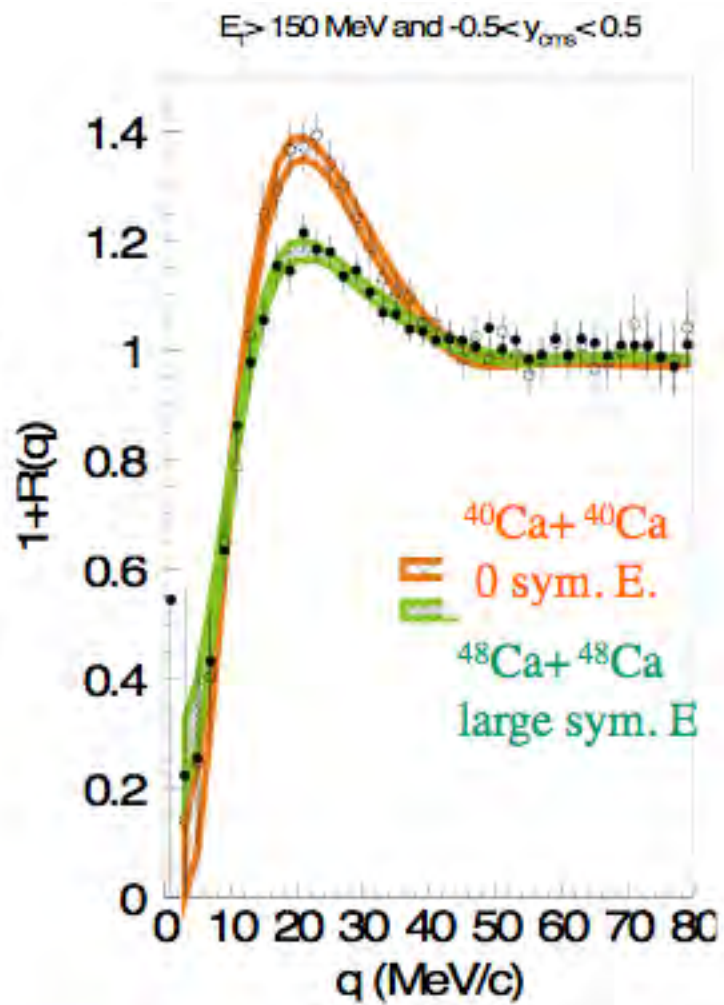
Amorini et al. PRL 102 (2009) 112701



Correlation functions



V.Henzl, et al., (2009)



V.Henzl, et al., (2009)



Nuclear Mass in Thomas-Fermi Model

Myers/Swiatecki, NPA 601, 141 (1996)

Thomas-Fermi Model analysis of 1654 ground state mass of nuclei with $N, Z \geq 8$

- Four Liquid Drop Properties
 - Radius constant of nuclear matter $r_0 = 1.14$ fm
 - Volume binding coefficient $a_1 = 16.24$ MeV
 - Symmetry energy coefficient $J = 32.65$ MeV
 - Surface energy coefficient $a_2 = 18.63$ MeV
- Five Droplet Model Properties [26]
 - Compressibility coefficient $K = 234$ MeV
 - Curvature energy coefficient $a_3 = 12.1$ MeV
 - Neutron skin stiffness coefficient $Q = 35.4$ MeV
 - Density-symmetry coefficient $L = 49.9$ MeV
 - Symmetry anharmonicity coefficient $M = 7.2$ MeV.

Thomas-Fermi Model + Nuclear Mass $\rightarrow E_{\text{sym}}(\rho_0) = 32.65$ MeV $L = 49.9$ MeV

E_{sym} : Pygmy Dipole Resonances

RAPID COMMUNICATIONS

PHYSICAL REVIEW C 76, 051603(R) (2007)

Nuclear symmetry energy and neutron skins derived from pygmy dipole resonances

A. Klimkiewicz,^{1,2} N. Paar,³ P. Adrich,^{1,2} M. Fallot,¹ K. Boretzky,¹ T. Aumann,¹ D. Cortina-Gil,⁴ U. Datta Pramanik,¹
Th. W. Elze,⁵ H. Emling,¹ H. Geissel,¹ M. Hellström,¹ K. L. Jones,¹ J. V. Kratz,⁶ R. Kulessa,² C. Nociforo,⁶ R. Palit,⁵
H. Simon,¹ G. Surówka,² K. Sümmerer,¹ D. Vretenar,³ and W. Walus²
(LAND Collaboration)

using the experimental pygmy strength, parameters of the nuclear symmetry energy ($a_4 = 32.0 \pm 1.8$ MeV and $p_0 = 2.3 \pm 0.8$ MeV/fm³) are deduced as well as neutron-skin thicknesses $R_n - R_\rho$ of 0.24 ± 0.04 fm for ¹³²Sn

Pygmy Dipole Resonances of ^{130,132}Sn → $E_{\text{sym}}(\rho_0) = 32 \pm 1.8$ MeV $L = 43.125 \pm 15$ MeV

RAPID COMMUNICATIONS

PHYSICAL REVIEW C 81, 041301(R) (2010)

Constraints on the symmetry energy and neutron skins from pygmy resonances in ⁶⁸Ni and ¹³²Sn

Andrea Carbone,¹ Gianluca Colò,^{1,2} Angela Bracco,^{1,2} Li-Gang Cao,^{1,2,3,4} Pier Francesco Bortignon,^{1,2}
Franco Camera,^{1,2} and Oliver Wieland²

Pygmy Dipole Resonances of ⁶⁸Ni and ¹³²Sn → $E_{\text{sym}}(\rho_0) = 32.3 \pm 1.3$ MeV, $L = 64.8 \pm 15.7$ MeV



E_{sym} : IAS+LDM

E_{sym} from Isobaric Analog States + Liquid Drop model with surface symmetry energy

tions, especially for the slope scaled with a_a^V . Thus, e.g. the analysis of excitation energies of isobaric analog states [97,98] yields independent values of a_a^V and a_a^S . While the volume symmetry coefficient from this type of analysis, $a_a^V \simeq (31.5-33.5)$ MeV, comes out quite in the middle of values found for the Skyrme interactions, the surface symmetry coefficient, $a_a^S \simeq (9.5-12)$ MeV, comes out right at the lower end of the values encountered for the Skyrme interactions. The coefficient ratio from that analysis is in the range $a_a^V/a_a^S \simeq (2.8-3.3)$. That ratio produces the effective surface displacement in the range of $\Delta_e R = (r_0/3)(a_a^V/a_a^S) \simeq (1.06-1.26)$ fm. Moreover, Figs. 14 and 15 yield the respective ranges of $\Delta R^0 \simeq (0.85-1.05)$ fm and $L/a_a^V \simeq (2.4-3.4)$ or $L \simeq (78-111)$ MeV. The analysis [97,98] is relatively model-independent, provided the curvature effects play little role for heavier nuclei. If the latter were not the case, though, a bit softer symmetry energy would need to be deduced.

IAS+Liquid Drop Model with Surface Esym →

$E_{\text{sym}}(\rho_0) = 32.5 \pm 1$ MeV $L = 94.5 \pm 16.5$ MeV

Danielewicz/Lee, NPA 818, 36 (2009)



Droplet Model Analysis on Neutron Skin

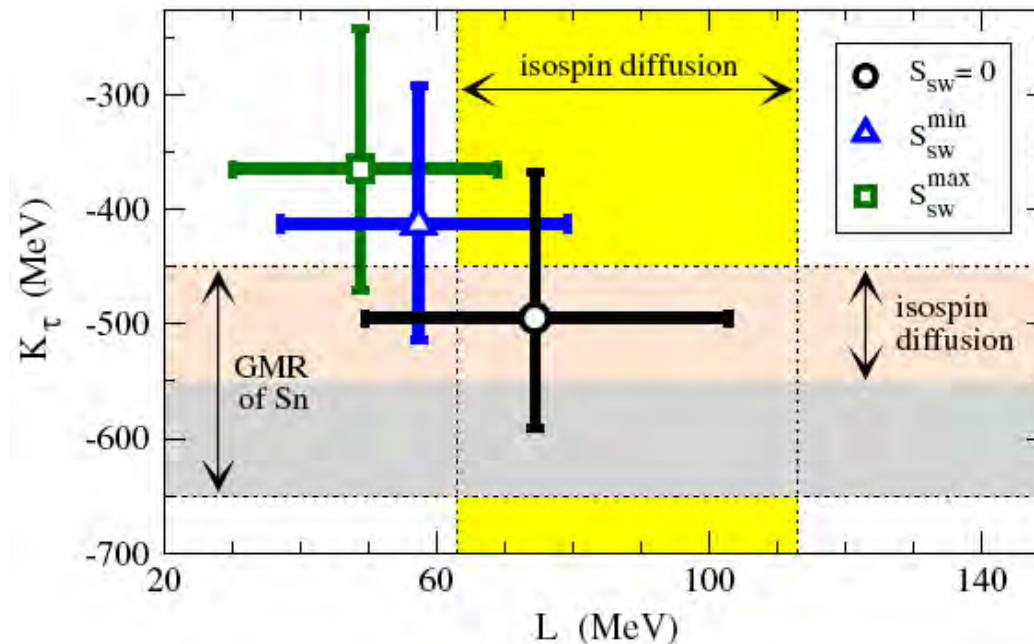
PRL 102, 122502 (2009)

PHYSICAL REVIEW LETTERS

week ending
27 MARCH 2009

Nuclear Symmetry Energy Probed by Neutron Skin Thickness of Nuclei

M. Centelles,¹ X. Roca-Maza,¹ X. Viñas,¹ and M. Warda^{1,2}



meaningful [26]. We first set $b_n = b_p$ (i.e., $S_{sw} = 0$) as done in the DM [12,23,26] and in the analysis of data in Ref. [19]. Following the above, we find $L = 75 \pm 25$ MeV

extremes of S_{sw} according to mean field models. The results are shown in Fig. 3. Our above estimates of L and K_τ could be shifted by up to -25 and $+125$ MeV, respec-

Droplet Model + N-skin $\rightarrow E_{sym}(\rho_0)=31.6$ MeV, $L=66.5 \pm 36.5$ MeV

Droplet Model Analysis on Neutron Skin

PHYSICAL REVIEW C 80, 024316 (2009)

Neutron skin thickness in the droplet model with surface width dependence: Indications of softness of the nuclear symmetry energy

M. Warda,^{1,2,*} X. Viñas,^{1,†} X. Roca-Maza,^{1,‡} and M. Centelles^{1,§}

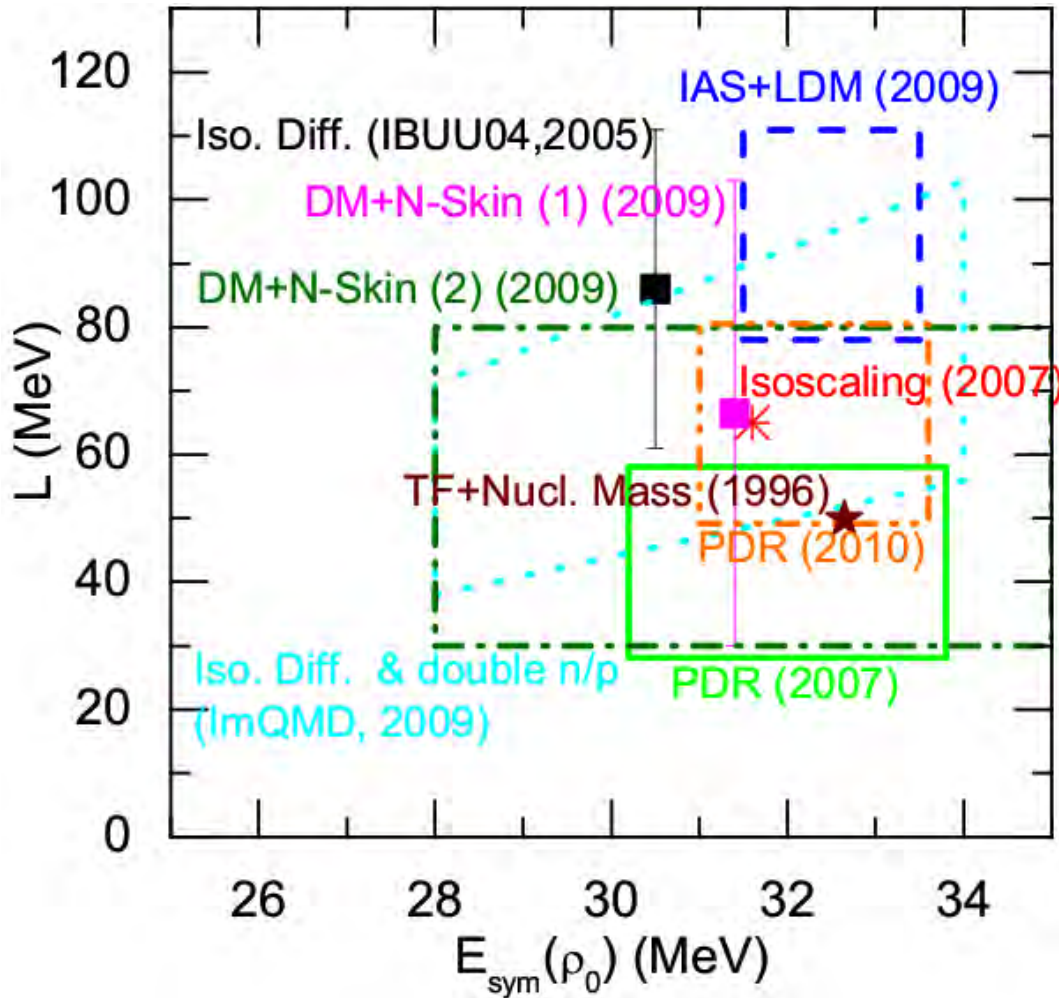
of J/Q values is known, the compatible range of values of the parameter L can be estimated from the linear correlation between L and J/Q shown in Fig. 1. From our analysis we find the constraints $30 \lesssim L \lesssim 80$ MeV.

Droplet Model + N-skin $\rightarrow E_{\text{sym}}(\rho_0)=28 - 35$ MeV, $L=55 \pm 25$ MeV



E_{sym} around normal density

9 constraints on $E_{\text{sym}}(\rho_0)$ and L from nuclear reactions and structures



$E_{\text{sym}}(\rho_0) = 28 - 35$ MeV
 $L = 28 - 111$ MeV

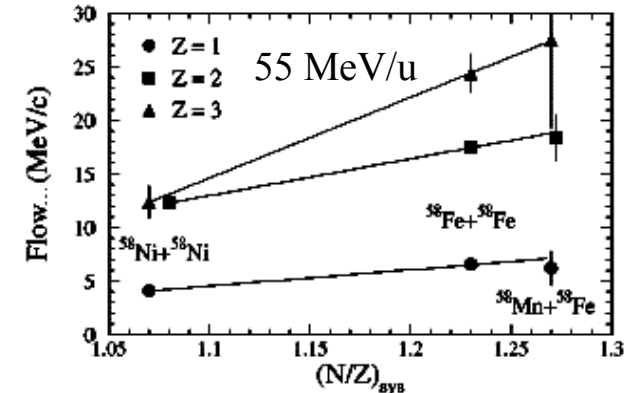
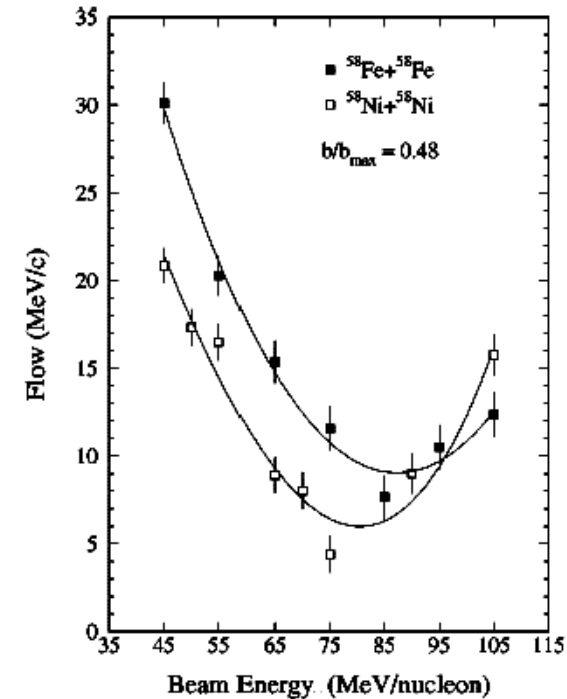
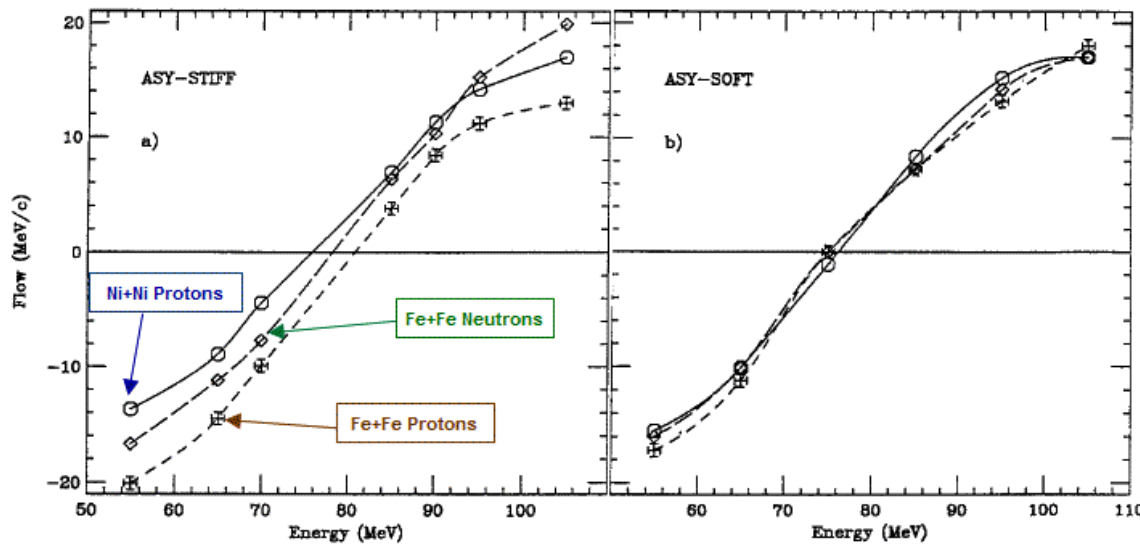
Still within large uncertain region !!

Transverse Collective Flow Iso-spin Effects observed in transverse flow of Z = 1, 2, & 3 fragments

BNV Calculation: 55 MeV/u $^{58}\text{Fe}+^{58}\text{Fe}$

-Asy-Stiff $E_{\text{sym}}(\rho)$ shows 20% increased ^3He flow in comparison to ^3H flow.

-Asy-Soft $E_{\text{sym}}(\rho)$ shows ^3He and ^3H flow are equal.



M. Di Toro *et al.*, Prog. Part. Nucl. Phys. **42**, 125 (1999).

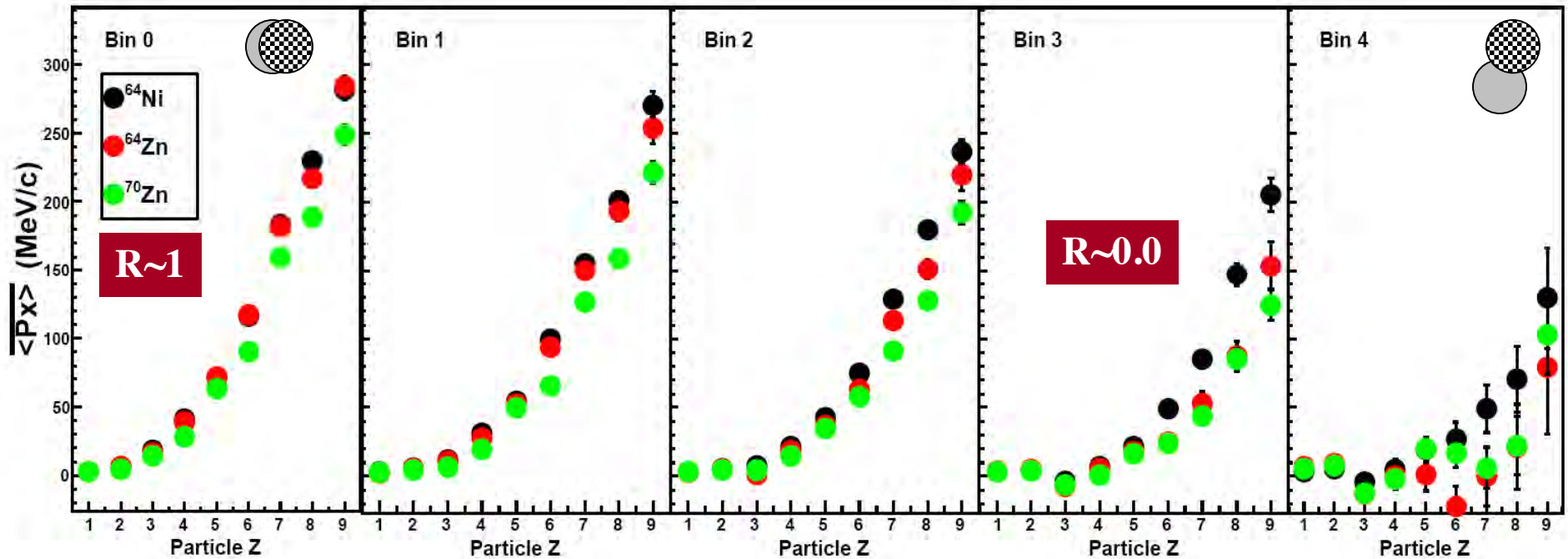
L. Scalone *et al.*, Phys. Lett. B **461**, 9 (1999).

R. Pak *et al.*, Phys. Rev. Lett. **78**, 1026 (1997).

R. Pak *et al.*, Phys. Rev. Lett. **78**, 1022 (1997).



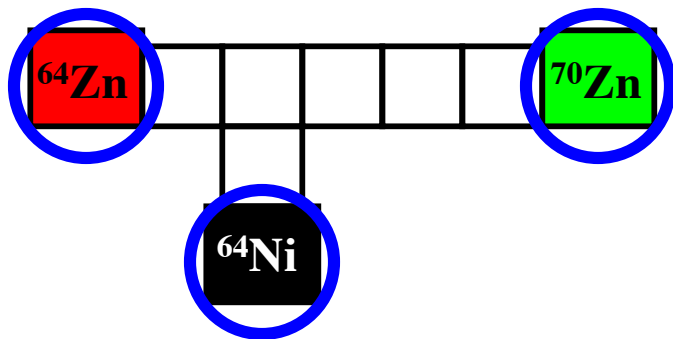
IMF Transverse Flow



Mass Dependence

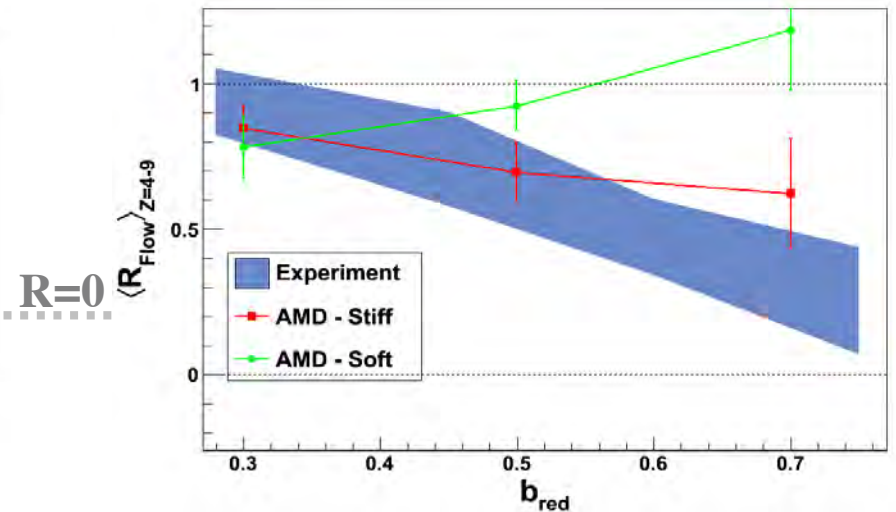
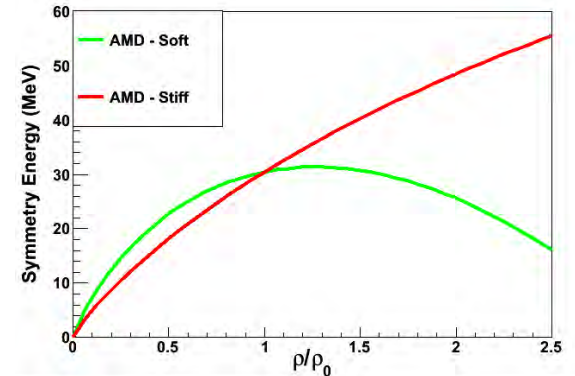
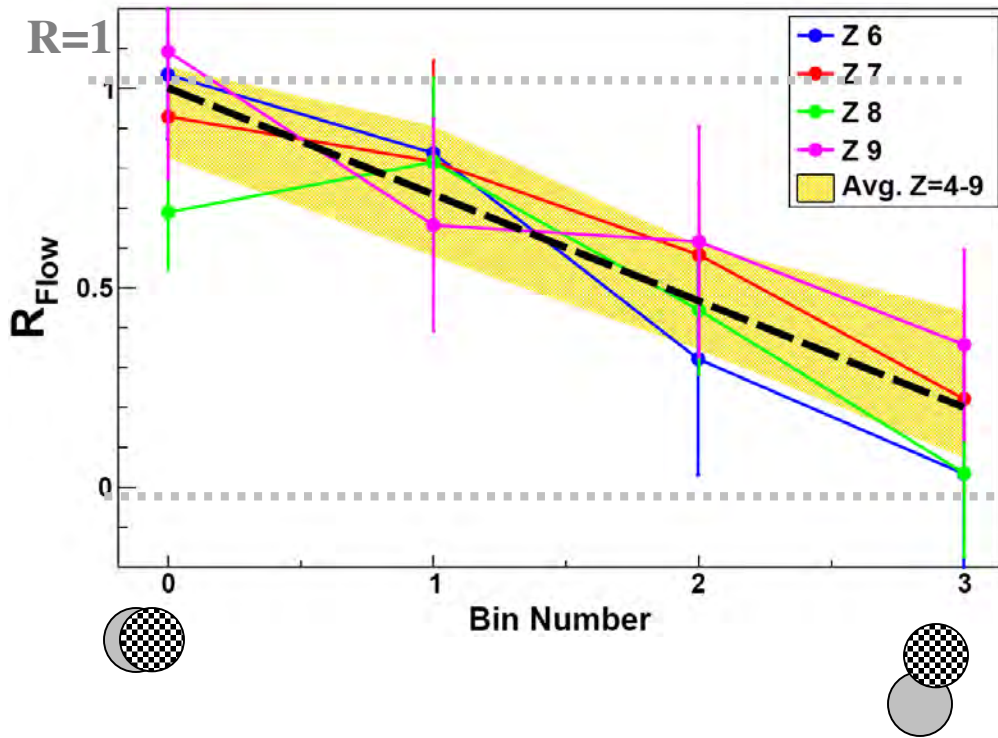


Coulomb Dependence



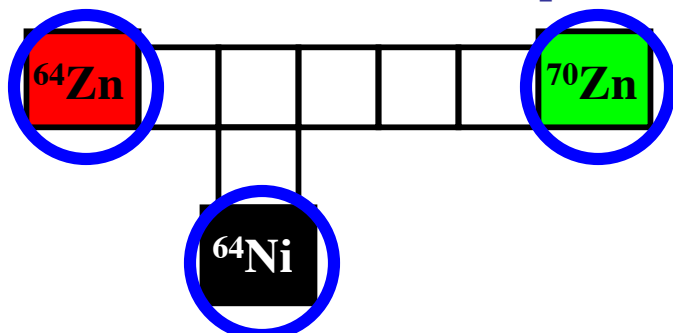
$$R_{Flow} = \frac{\langle Px/A \rangle_{^{64}\text{Zn}} - \langle Px/A \rangle_{^{70}\text{Zn}}}{\langle Px/A \rangle_{^{64}\text{Ni}} - \langle Px/A \rangle_{^{70}\text{Zn}}}$$

IMF Transverse Flow



Mass Dependence

Coulomb Dependence

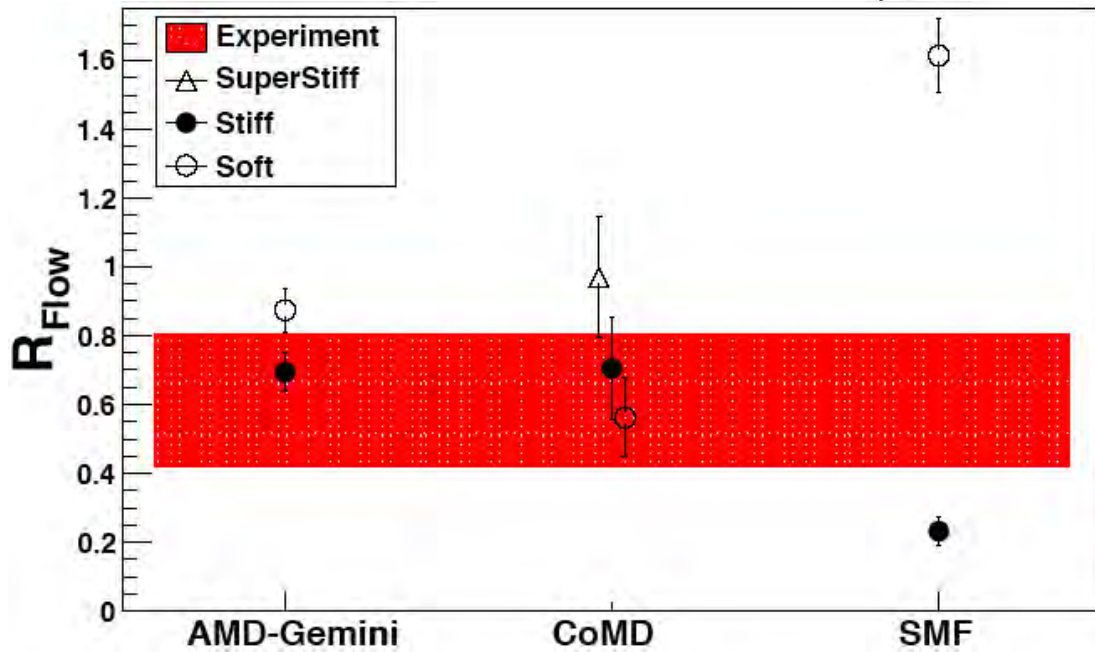


$$R_{Flow} = \frac{\overline{\langle Px/A \rangle_{^{64}\text{Zn}}} - \overline{\langle Px/A \rangle_{^{70}\text{Zn}}}}{\overline{\langle Px/A \rangle_{^{64}\text{Ni}}} - \overline{\langle Px/A \rangle_{^{70}\text{Zn}}}}$$

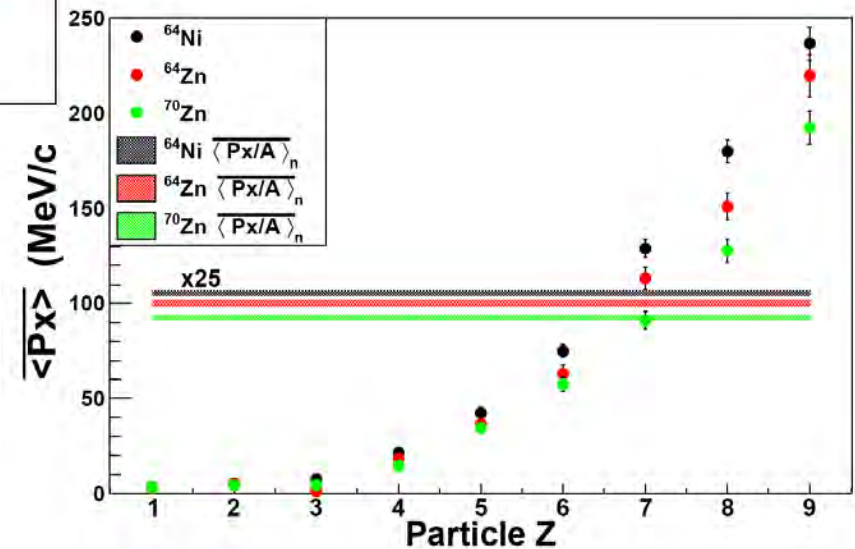
Average Z=4-9

Nucleon weighted flow

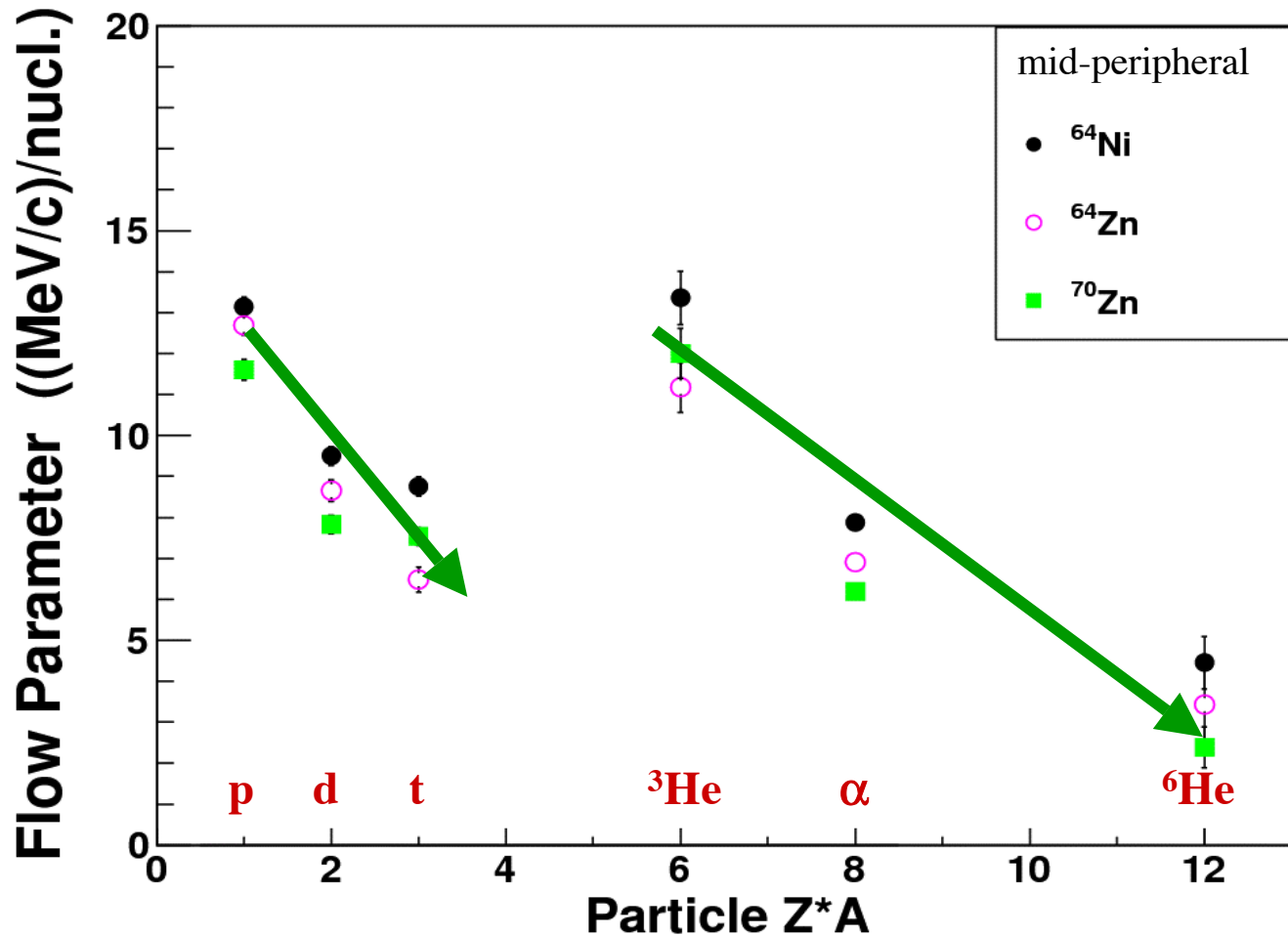
$$\langle Px/A \rangle_{Nucleon} = \left(\sum_{f=0}^{N_{frag}} (Px)_f \right) \div N_{Nucleons} \quad \dots \text{for } Z=3-9$$



Note: SMF nucleon flow calculated for $Z > 3$



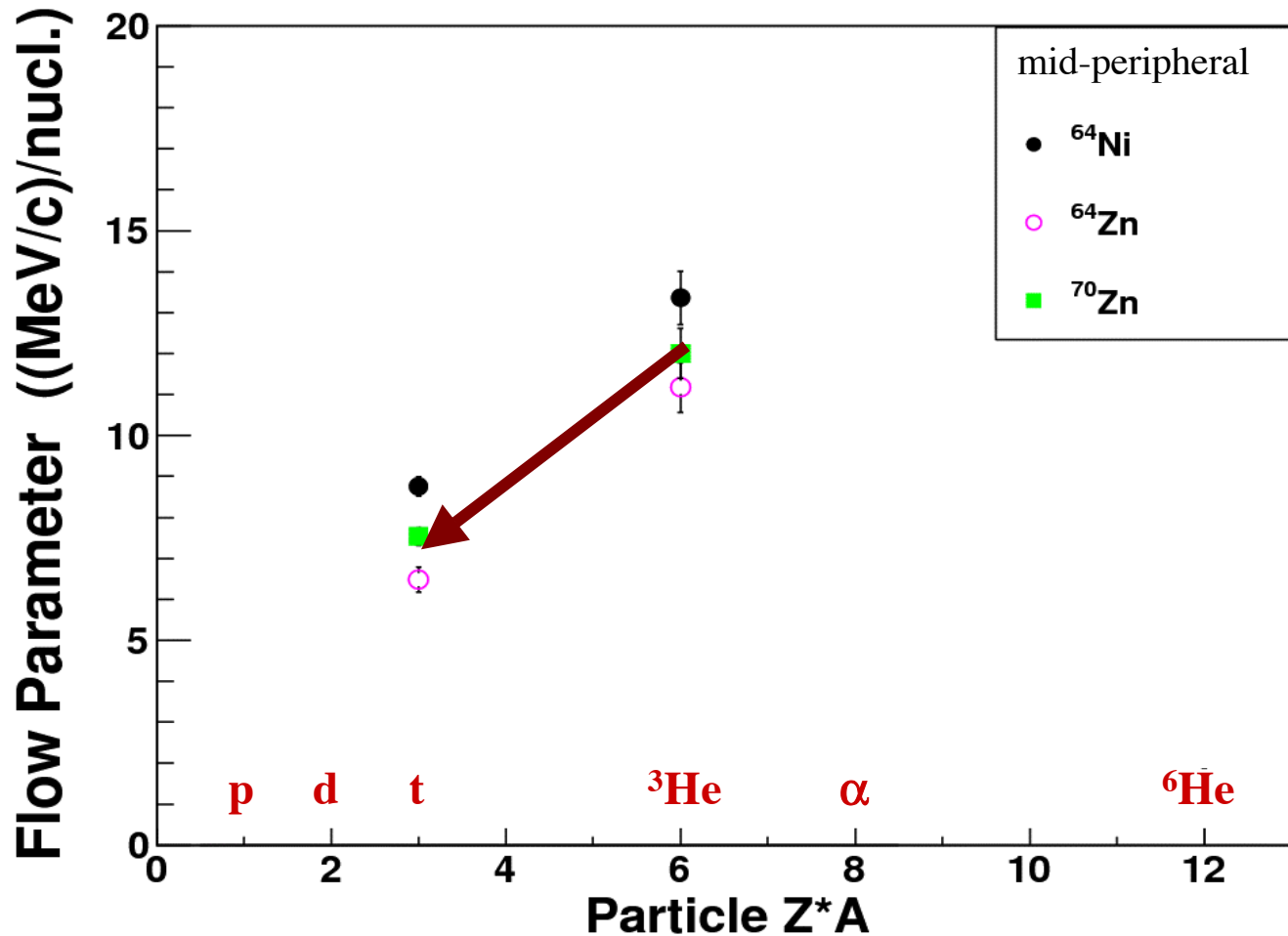
LCP Flow



- Dependence on $(N/Z)_{\text{sys}}$
- Expands on Pak *et al.*

- Strong Isotopic Trends
- ↓ Flow with ↑ n-rich

LCP Flow

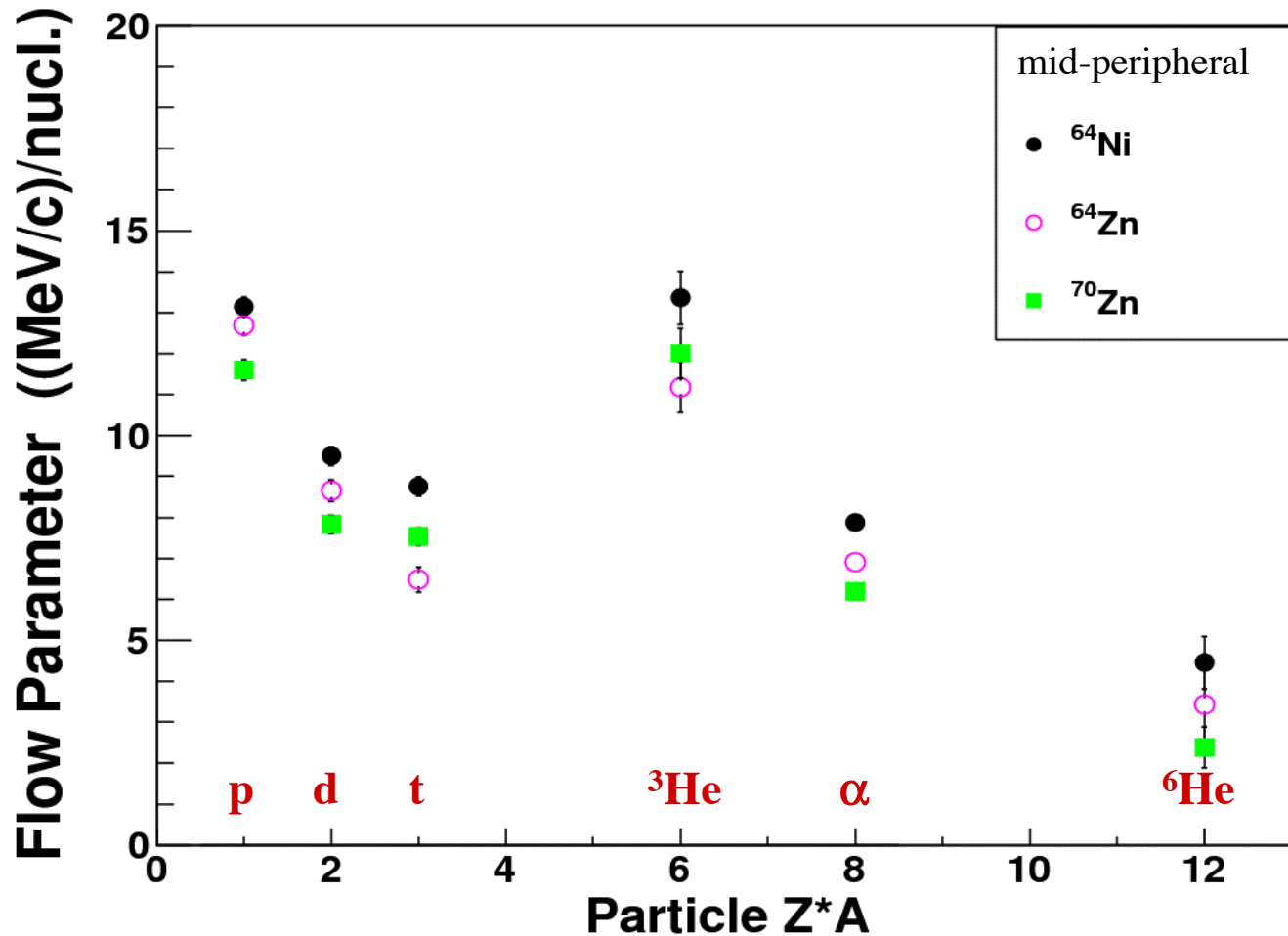


- Dependence on $(N/Z)_{\text{sys}}$
- Expands on Pak *et al.*

- Strong Isotopic Trends
- \downarrow Flow with \uparrow n-rich

- Isobaric Effects ($A=3$)
- Same A , different N/Z
- $E_{\text{sym}}(\rho)$ prediction

LCP Flow



- Dependence on $(N/Z)_{\text{sys}}$
- Expands on Pak *et al.*

- Strong Isotopic Trends
- ↓ Flow with ↑ n-rich

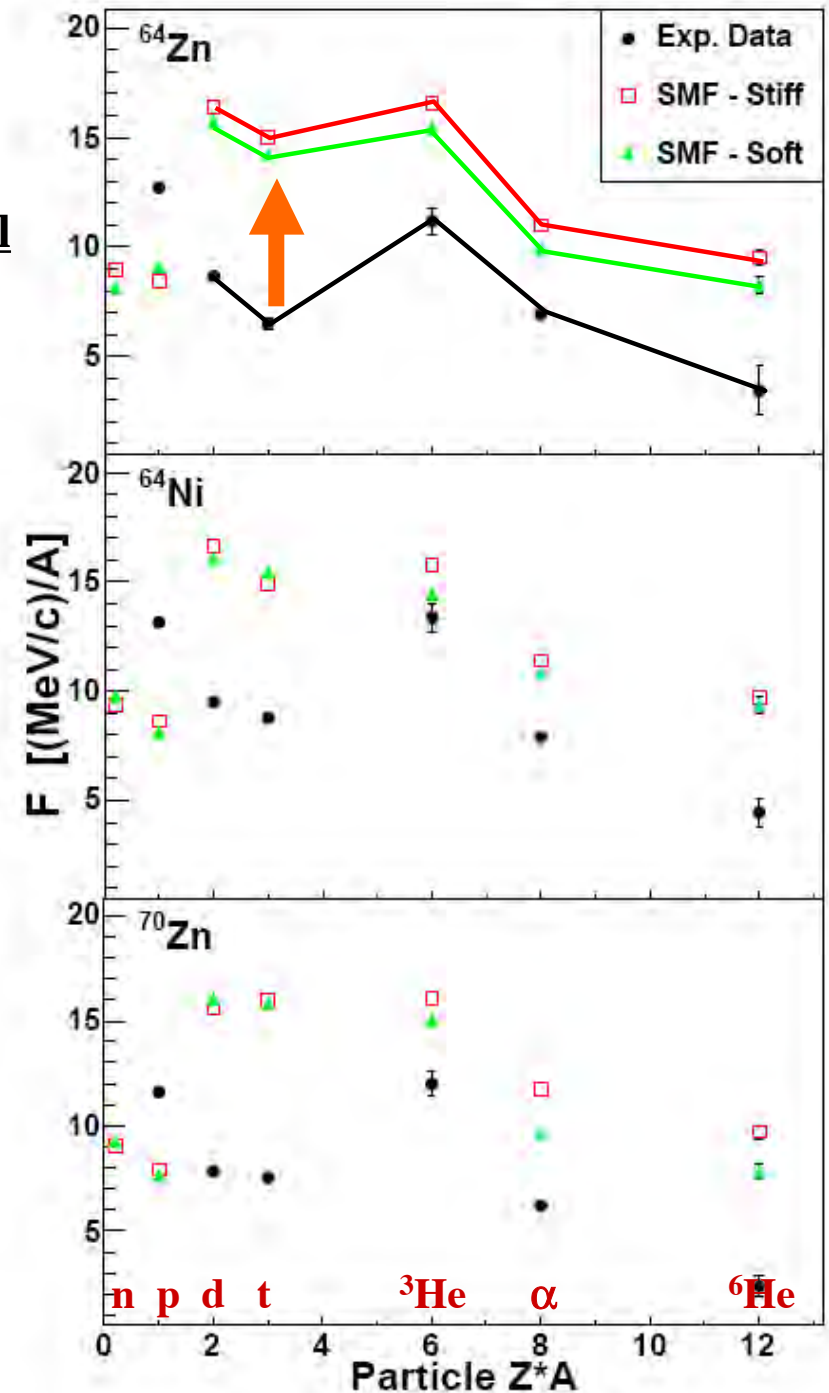
- Isobaric Effects ($A=3$)
- Same A , different N/Z
- $E_{\text{sym}}(\rho)$ prediction

Decreasing flow with increasing N/Z with both const. Z and A for LCPs.

LCP Flow

Comparison with the Stochastic Mean-Field Model

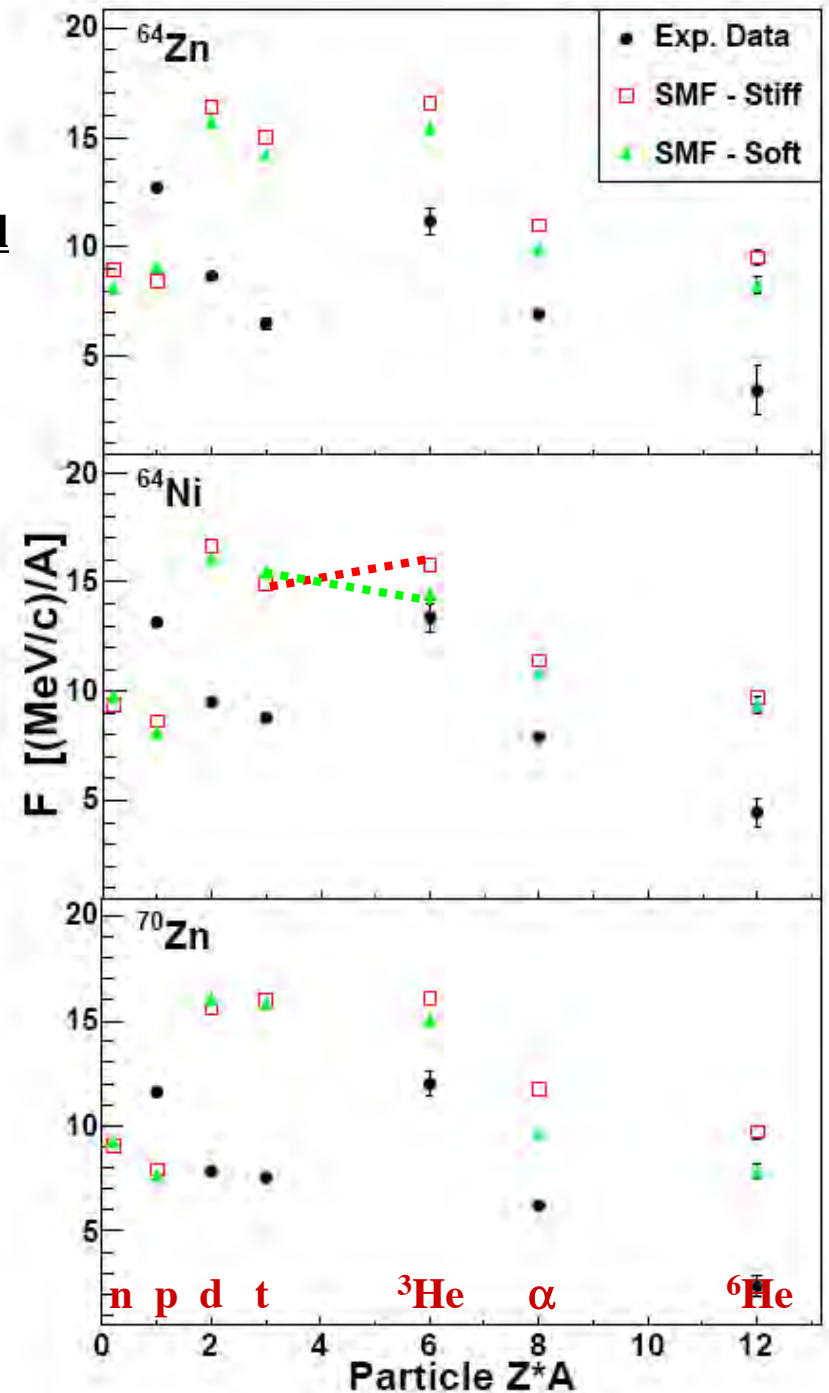
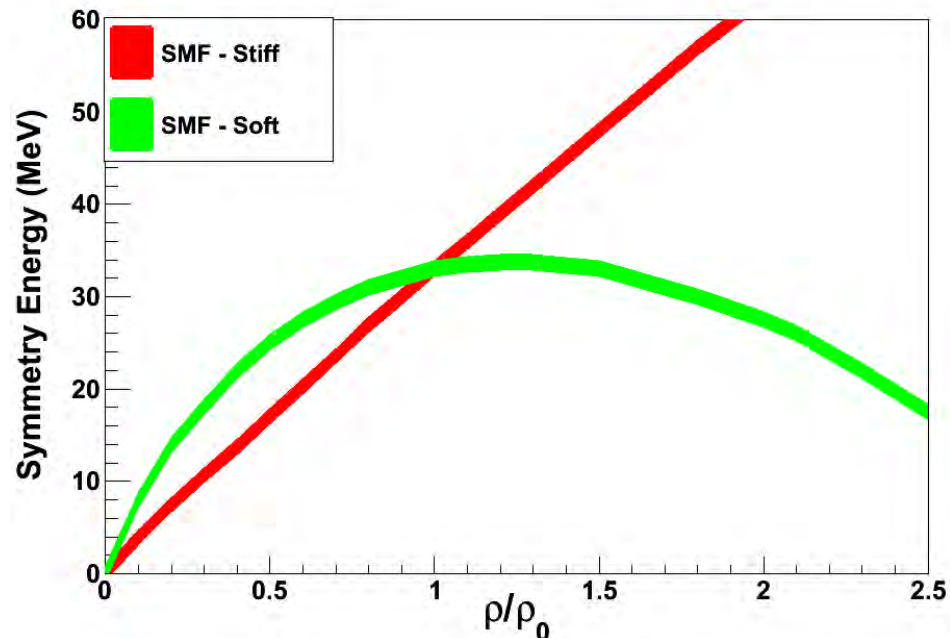
- Overall isotopic flow trend is reproduced, except protons.
- Differential movement of neutrons and protons
- Phase-space coalescence and early emission time.



LCP Flow

Comparison with the Stochastic Mean-Field Model

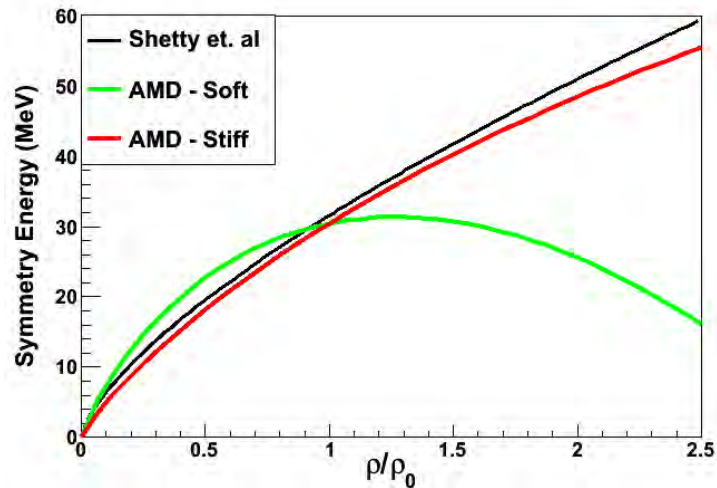
- Overall isotopic flow trend is reproduced, except protons.
- Phase-space coalescence and early emission time.
- Differential movement of neutrons and protons.
- Sensitivity to symmetry energy.



Flow Summary

AMD Model

- Mass-Coulomb Dependence
- R_{Flow} (mid-peripheral)

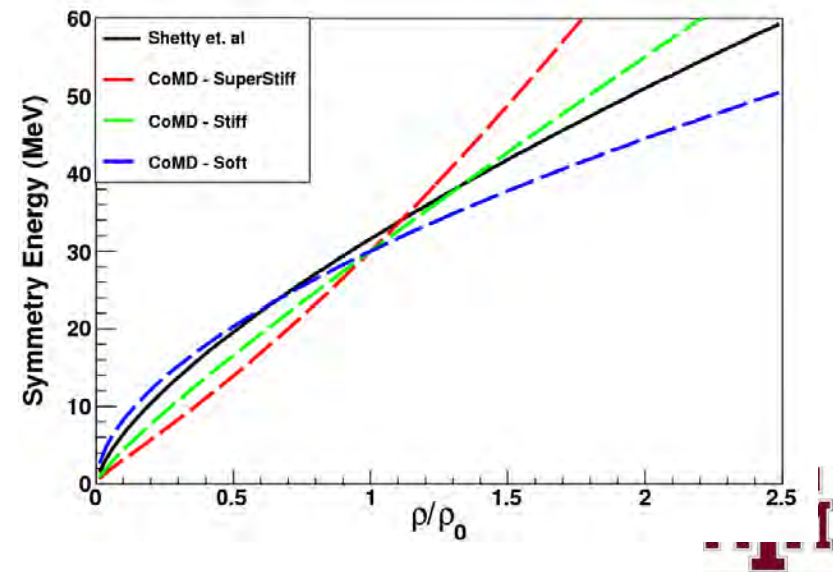
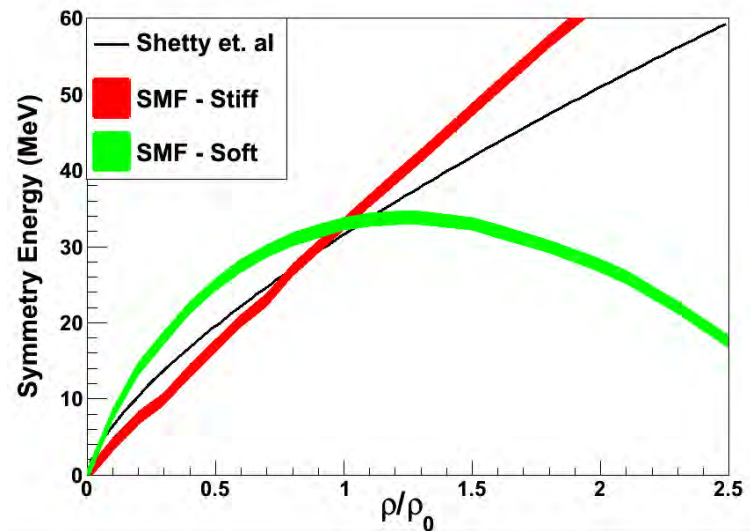


CoMD Model

- R_{Flow} (mid-peripheral)

SMF Model

- triton/³He Flow
- $(N/Z)_{sys}$ Dependence
- R_{Flow} (mid-peripheral)

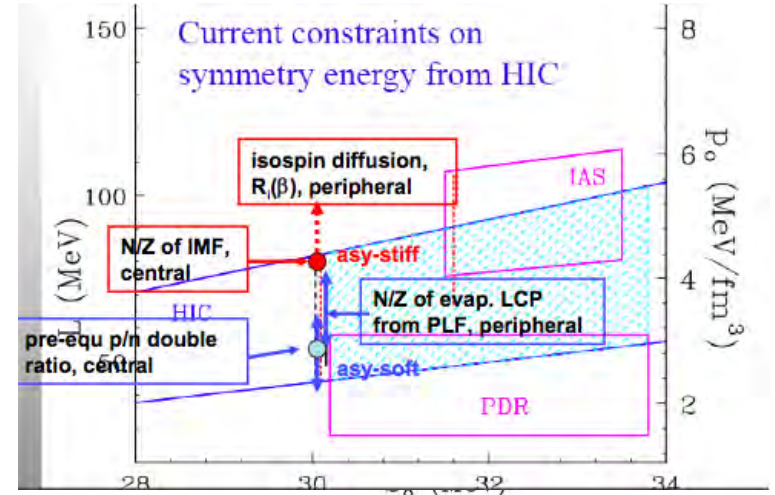


M.B. Tsang *et al.* Phys. Rev. Lett. **102**, 122701 (2009).
 B.A. Li, L.W. Chen and C.M. Ko. Phys. Rep. **464**,113 (2008).
 D.V. Shetty, S.J. Yennello, and G.A. Souliotis. PRC **76**, 24606 (2007).

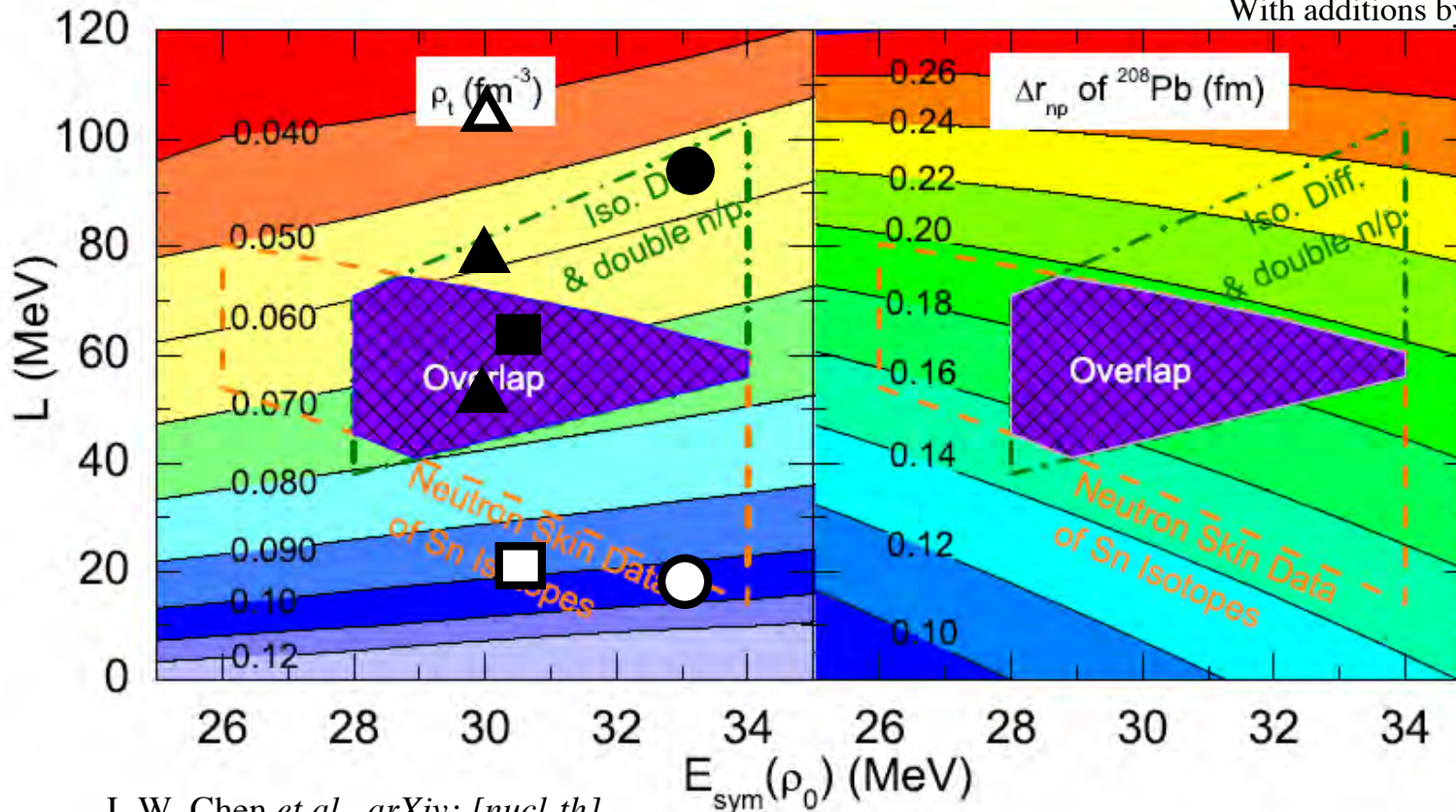
Summary $E_{\text{sym}}(\rho)$

- AMD some agreement
- CoMD did not agree
- SMF

$$E_{\text{sym}} = S_v + \frac{L}{3} \left(\frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left(\frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \dots$$



M.B. Tsang *et al.* PRL. **102**, 122701 (2009).
With additions by H. Wolter, NUFRA209



L.W. Chen *et al.*, arXiv: [nucl-th]



Summary

- Many proposed observables
 - Various data sets
- (apologies to that which I didn't have time to show)
- Some overlap - plenty of room for improvement
 - Need to understand differences in model predictions
 - New observables with increased power to discriminate welcome

Many Thanks

Z. Kohley, L. W. May, S. Wuenschel, R. Tripathi, R. Wada,
K. Hagel, G. A. Souliotis, D. V. Shetty, S. Galanopoulos, M.
Mehlman, W. B. Smith, S. N. Soisson, B. C. Stein, R.
Dienhoffer

M. Di Toro, M. Colonna, M. Zielinska-Pfabe,
A. Bonasera
A. Botvina

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