PARTICLE SOURCES: INTRODUCTION.

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PARTICLE SOURCE COURSE TOPICS

1) INTRODUCTION

2) Why sources are important
3) Electron sources
4) Polarized electron sources
5) Ions - Protons sources (L.Celona)
6) Antimatter sources – INTRODUCTION
7) Positron sources, antiprotons
8) Polarized positrons
9) Muons (D.Neuffer)
10) Antimatter source engineering
11) Photon sources : INTRODUCTION
12) Synchrotron radiation
13) Compton, Cerenkov, Transition, Diffraction, Smith Purcell Radiation

WHAT IS A SOURCE?

something or someone that causes or produces something (particles), or is the origin of it:

In accelerators -> particles (charged or neutral) to provide 'particles beams'. This means that the particles needs to be 'prepared' before to be used.

Sources can be primary (particles are produced from a 'system') or secondary (particles are produced from other particles….a primary beam is needed)

WHAT IS A BEAM?

- Difficult => No 'quantitative' definition
- What characteristics we expect?

1) Composed by particles (usually charged)
2) Propagating along a 'direction' -> (design)
3) Geometrically confined
4) Showing collective behavior
5) The statistical transverse thermal spread << $\langle v_{propagation} \rangle$
6) If we want that there is not transition (beam also at $t = \infty$) => in respect to 2,3,4 => providing confining forces (E/B fields for charged particles)

HOW TO MANIPULATE A BEAM? (CHARGED) ACCELERATION

Lorentz force $\vec{F}$

$$\vec{F} = q \cdot \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

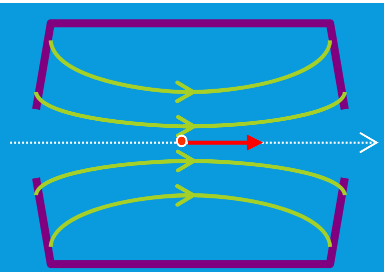
$$\frac{d\vec{p}}{dt} = \vec{F}$$

The particle **total energy** *W* is:

$$\begin{aligned} W^2 &= W_0^2 + (\vec{p} \cdot \vec{p}) \cdot c^2 \\ &= \left(\gamma \cdot mc^2 \right)^2 \\ &= (W_0 + T)^2 \end{aligned}$$

LONGITUDINAL- CAVITIES

Energy gained by a particle in a cavity of length L :



$$\Delta W = \int qEz(s) \cdot \cos(\phi(s)) \cdot ds$$

with : $$\phi(s) = \phi_0 + \omega \cdot t = \phi_0 + \frac{\omega}{c}\int_{s_0}^{s} \frac{ds}{\beta_z(s)}$$

Assuming a *constant velocity* : $\overline{\beta}$

$$\Delta W = \int qEz(s) \cdot \cos\left(\phi_0 + \frac{\omega}{\overline{\beta}c}(s - s_0)\right) \cdot ds \quad \Rightarrow \quad \boxed{\Delta W = qV_0 \cdot T\left(\overline{\beta}\right) \cdot \cos \phi_p}$$

with : $$V_0 = \int |Ez(s)| \cdot ds$$

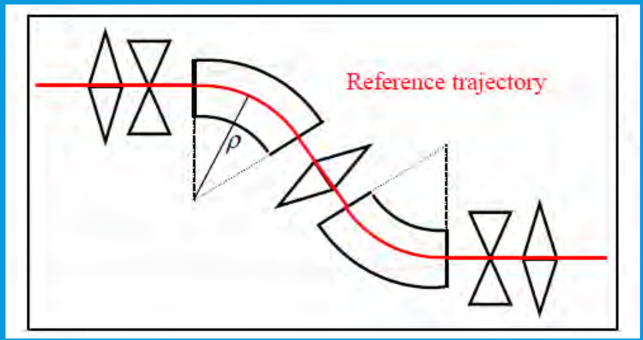
Cavity Voltage

$$T = \frac{1}{V_0}\int Ez(s) \cdot \cos\left(\phi(s) - \phi_p\right) \cdot ds$$

Transit-time factor : $\boxed{0 < T < 1}$

HOW TO MANIPULATE A BEAM?
PARTICLE TRANSVERSE MOTION

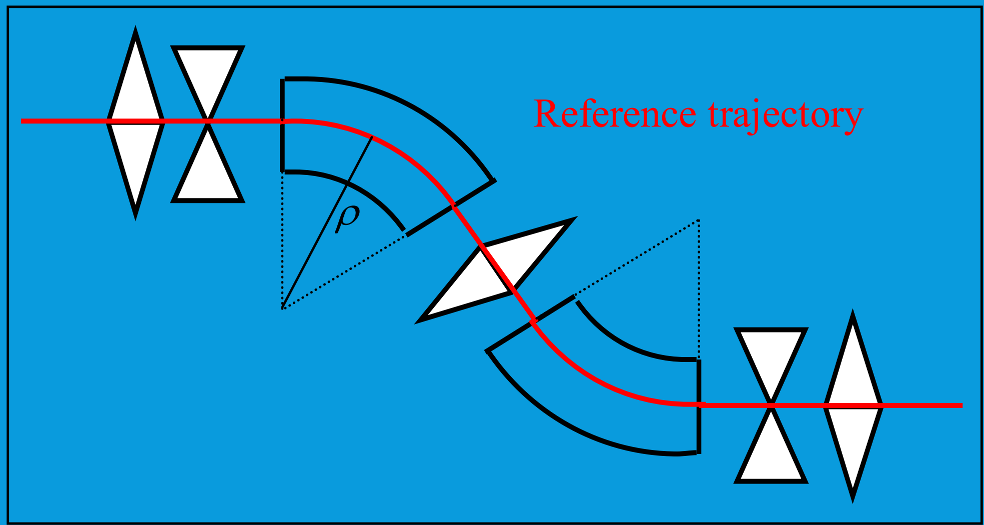
- 1^{st} => chose the good reference system (fundamental)
- For accelerator can be very complex (due to the design geometry) but
- We can define a design trajectory
- We obtain a reference trajectory.
- The goal is to keep all the particle 'confined' in respect to the reference trajectory.



Reference trajectory

An accelerator is designed around a reference trajectory (design orbit in circular accelerators), which is :

1. Straight line in drift and focusing element (no field on the axe)
2. Arc of circle in dipole magnet, horizontal or vertical (Transfer lines)
3. r is the local radius of curvature



On this trajectory, a particle is represented by a curved abscissa : *s*

Reference trajectory

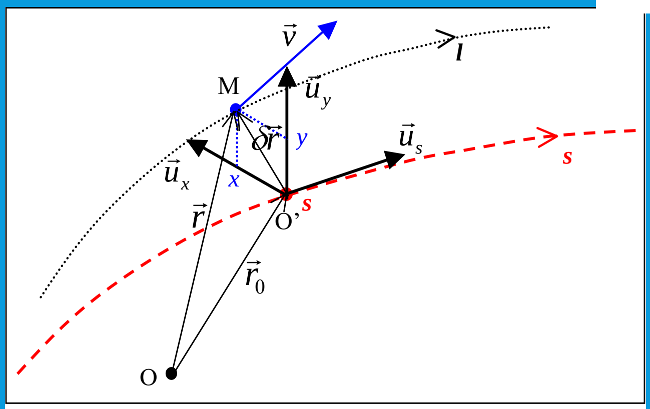
So we will describe the particle motion as a little deviation form the reference particle, moving on the reference trajectory in s coordinate: travelling reference system! => Frenet-Serret System

$\vec{u}_s$: tangent to the reference trajectory

$$\left(\vec{u}_x, \vec{u}_y, \vec{u}_s\right)$$

$\vec{u}_y$: vertical

$\vec{u}_x$: in horizontal plan



RECALL FORCES

- Why magnets ?:
- 1) Not needed to be integrated under vacuum
- 2) Efficiency given by the technological limits:

In theory the magnetic force is efficient at relativistic velocity : $F_E = \beta F_B$

But let's take into account the technological limits. Iron saturation is 2T, Electric field breakdown threshold in vacuum ~ 10 MV/m. So also taking into account a b of 0.1 we have:

$$\frac{F_E}{F_B} = \frac{10\ MV/m}{2T\ * 3\ 10^8} = 0.1 \Rightarrow \frac{F_E}{F_B} = \frac{10\ MV/m}{2T\ * 0.1 * 3\ 10^8} = \frac{1}{6}$$

So already at non relativistic energy B field win!!!

ACCELERATOR AND PARTICLES BEAMS (AND SOURCES) QUALITY

- x,y transverse coordinates in respect the reference particle
- x', y' can be angles defining the TRACE SPACE (x,x') or momenta defining the PHASE SPACE (x,P_x)
- The vector (x,x') represent the dynamical state of one particle
- In complex transport system, in linear approximation, we can solve the equations for each part of the trajectory obtaining M_1, M_2…M_n.
- The final particle state will be represented by $r_f = M_1 \cdot M_2 \cdot \ldots M_n \cdot r_0$ with r=(x,x')
- A beam has MANY particles
- IN A FULL ACCELERATOR DESIGN the r_{0n} are given by THE SOURCE

HILL'S EQUATION

- After the substitution (ref) in the equations we get to the linear equations of motion

$$x'' - \left(k - \frac{1}{\rho^2}\right)x = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

$$y'' + ky = 0$$

- Harmonic oscillators but k and r are k(s) and r(s)
- In general the Hill's equation will be

$$\mathrm{x}'' + k(s)\mathrm{x} = a(s)$$

BEAMS REPRESENTATIONS

- A beam can be described particle by particle starting form the initial source state vector
- We can take Macro particles
- We can use parametrization (Courant Snyder parameters). One particle motion.

PARAMETRIC

COURANT-SNYDER INVARIANT

Hill's equations have an invariant

$$\frac{d^2y}{ds^2} + K_y(s)y = 0 \longrightarrow A(s) = \beta\, y'^2 + 2\alpha y y' + \gamma^2 y^2 = \text{const.}$$

This invariant is the area of the ellipse in phase space (y, y') multiplied by π.

This can be easily proven by substituting the solutions y, y'

$$y(s) = \sqrt{\varepsilon\beta(s)}\cos(\varphi(s) - \phi)$$

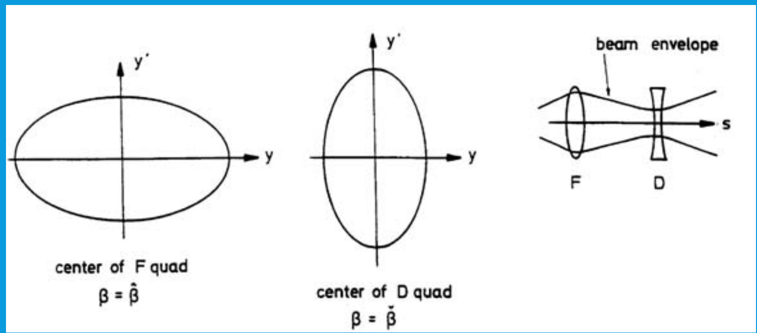
$$y'(s) = -\sqrt{\frac{\varepsilon}{\beta(s)}}\left[\sin(\varphi(s)-\phi) + \alpha(s)\cos(\varphi(s)-\phi)\right]$$

into A(s). You will get the constant ε

A(s) is called ***Courant-Snyder invariant***

COURANT-SNYDER INVARIANT

- Whatever the magnetic lattice, the area of the ellipse stays constant (if the Hill's equations hold)
- *At each different sections s, the ellipse of the trajectories may change orientation shape and size but the area is an invariant.*



This is true for the motion of a single particle !

COURANT-SNYDER PARAMETERS

α, β and γ are the Courant-Snyder parameters of the motion

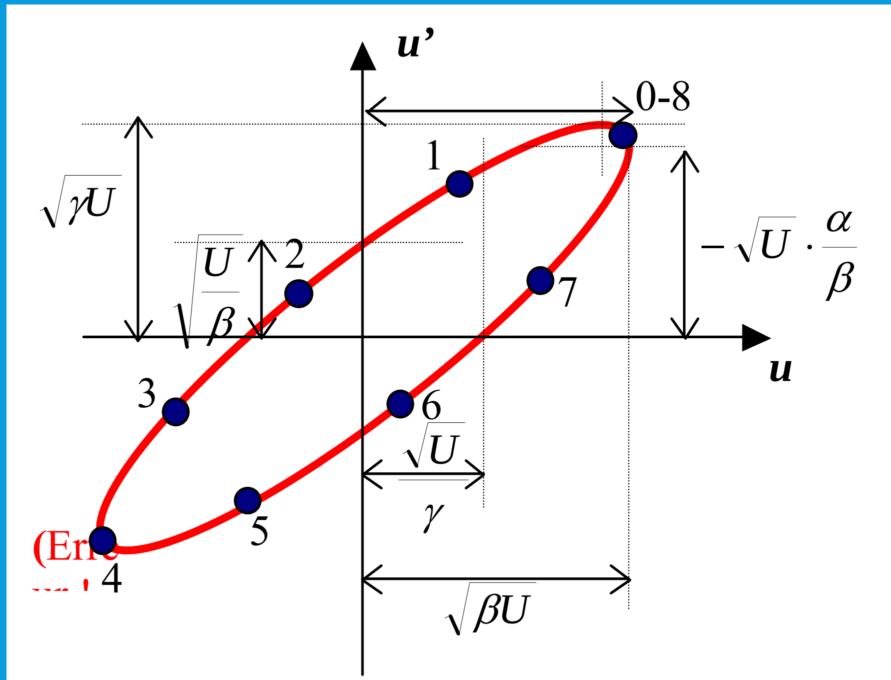
It is easy to show that :

$$\gamma \cdot u^2 + 2 \cdot \alpha \cdot u \cdot u' + \beta \cdot u'^2 = U$$

This is an ellipse equation.

Particle is moving on an ellipse whose shape is given by Courant-Syder parameters.

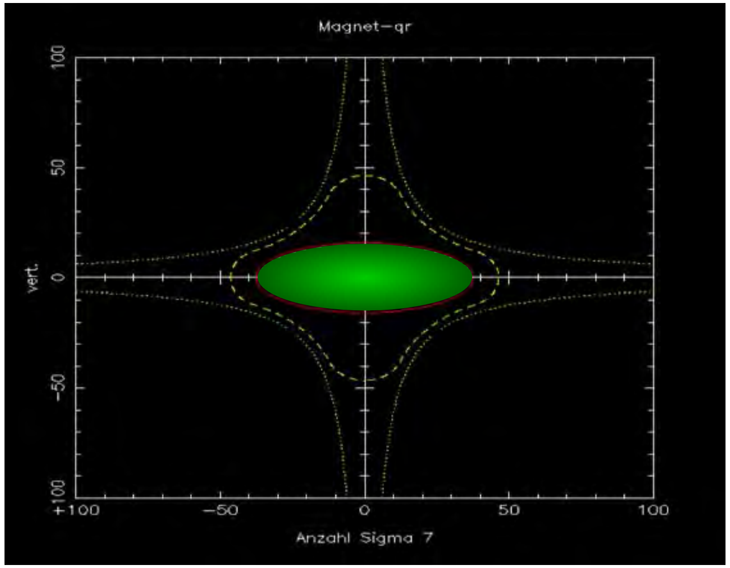
U is the courant-Snyder invariant linked to a particle



The Beta Function

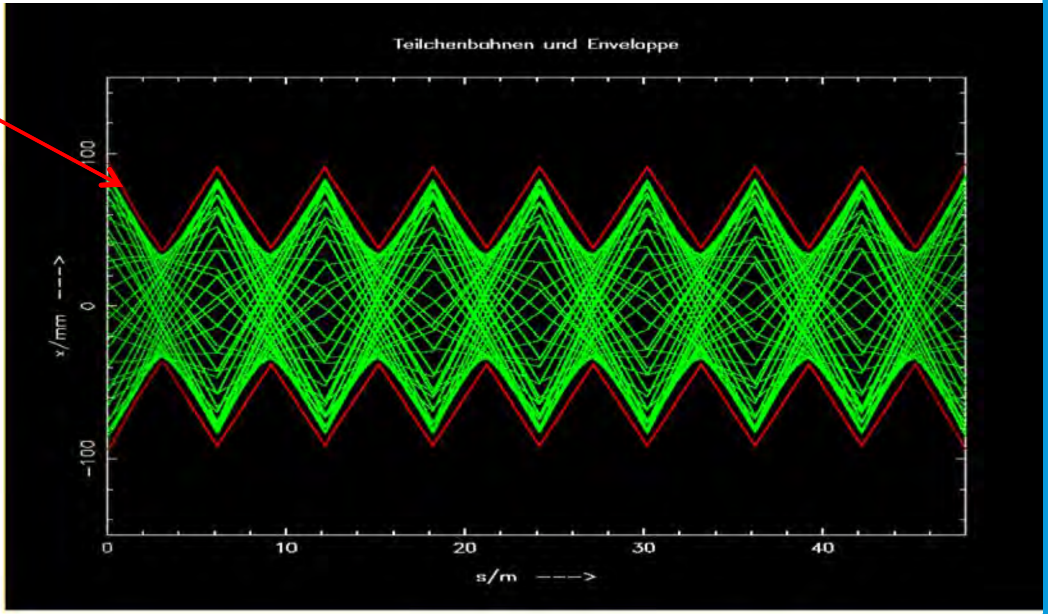
Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$



Maximum size of a particle amplitude

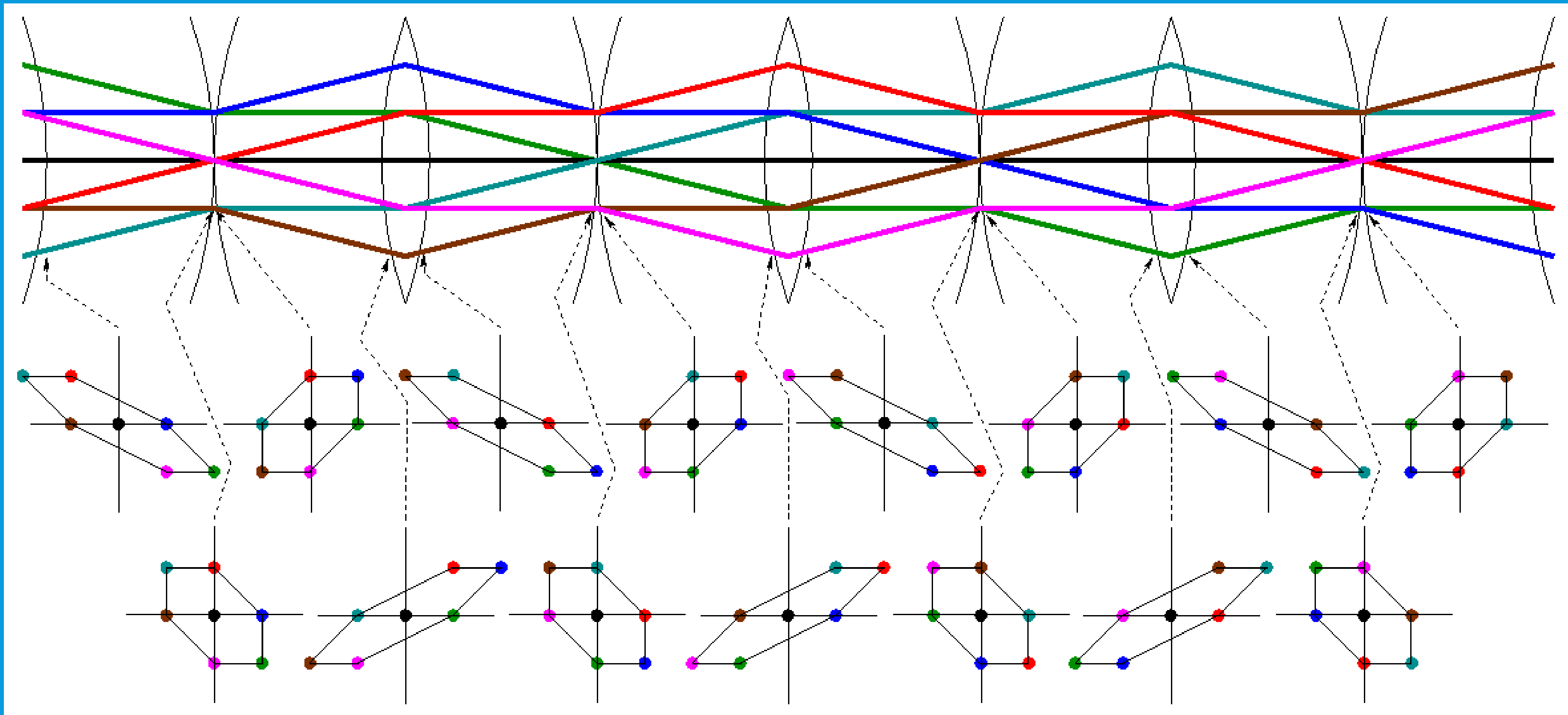
$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



β determines the beam size
(... the envelope of all particle trajectories at a given position "s" in the storage ring.

It reflects the periodicity of the magnet structure.

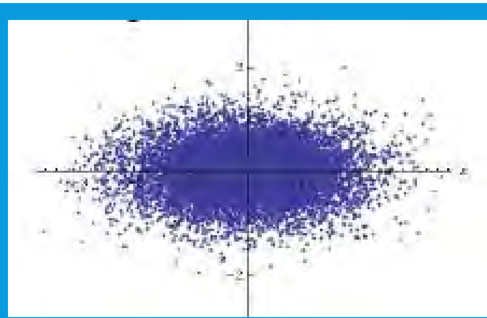
MOTION IN PHASE SPACE



STATISTICAL - MORE PARTICLES

REAL BEAMS – DISTRIBUTION FUNCTION IN PHASE SPACE

A beam is a collection of many charged particles



The beam occupies a finite extension of the phase space and it is described by a distribution function ψ such that

$$\int \psi(\bar{x}, s) d^6\bar{x} = 1 \qquad \bar{x} = (x, p_x, y, p_y, z, \delta)$$

The beam distribution is characterised by the momenta of various orders

$$<\bar{x}>_j (s) = \int x_j \psi(\bar{x}, s) d^6\bar{x}$$

Average coordinates (usually zero)

$$R_{ij}(s) = <(\bar{x} - <\bar{x}>)_i (\bar{x} - <\bar{x}>)_j> = \int (x_i - <\bar{x}>_i)(x_j - <\bar{x}>_j) \psi(\bar{x}, s) d^6\bar{x}$$

The R-matrix also called *σ-matrix* describes the equilibrium properties of the beam giving the second order momenta of the distribution

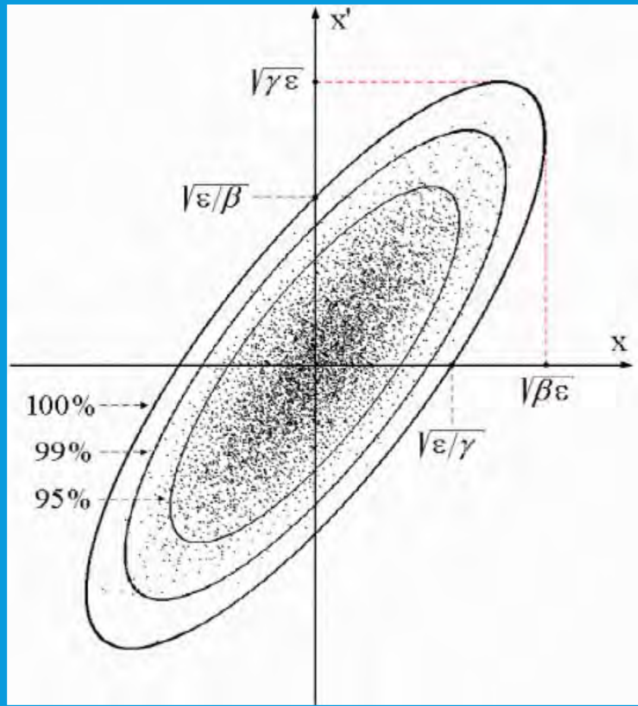
R_{11} = bunch H size; R_{33} = bunch Y size; R_{55} bunch Z size; R_{66} = energy spread

GAUSSIAN BEAMS

In many cases the equilibrium beam distribution is a Gaussian distribution

$$\psi = \psi(\bar{x}, s) = \frac{1}{(2\pi)^3\sqrt{\det R}} e^{-\frac{1}{2} R_{ij}^{-1} (\bar{x} - <\bar{x}>)_i (\bar{x} - <\bar{x}>)_j}$$

$$\rho(x, x') = \frac{1}{2\pi\sqrt{\det R_{xx'}}} e^{-\frac{\beta x'^2 + 2\alpha x x' + \gamma x^2}{2\varepsilon}}$$



The isodensity curves are ellipses

SIGMA MATRIX : DEFINITION

$$\gamma_t \cdot u^2 + 2 \cdot \alpha_t \cdot u \cdot u' + \beta_t \cdot u'^2 = \varepsilon_{u,rms}$$

It can be also written : $U^T \cdot [\sigma]^{-1} \cdot U = I$

$U = \begin{pmatrix} u \\ u' \end{pmatrix}$ is a vector given the particle position in phase-space

$[\sigma] = \begin{pmatrix} \sigma_u & \sigma_{uu'} \\ \sigma_{uu'} & \sigma_{u'} \end{pmatrix} = \begin{pmatrix} \beta_{tu} & -\alpha_{tu} \\ -\alpha_{tu} & \gamma_{tu} \end{pmatrix} \cdot \varepsilon_{u,rms}$ is a beam ***sigma matrix***

σ-MATRIX FOR GAUSSIAN BEAMS

The 6x6 σ-matrix can be partitioned into nine 2x2 submatrices

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

with (assuming <x> = 0 and <x'> = 0)

$$\sigma_{xx} = \begin{pmatrix} <x^2> & <x\,x'> \\ <x\,x'> & <x'^2> \end{pmatrix}$$

Direct computation using

$$\rho(x, x') = \frac{1}{2\pi\varepsilon} e^{-\frac{\beta x'^2 + 2\alpha x x' + \gamma x^2}{2\varepsilon}}$$

yields

$$\sigma_{xx} = \begin{pmatrix} \beta_x \varepsilon_x & -\alpha_x \varepsilon_x \\ -\alpha_x \varepsilon_x & \gamma_x \varepsilon_x \end{pmatrix}$$

$$\det(\sigma_{xx}) = (\beta_x \gamma_x - \alpha_x^2)\varepsilon_x^2 = \varepsilon_x^2$$

We can associate an ellipse with the Gaussian beam distribution. The evolution of the beam is completely defined by the evolution of the ellipse

The ellipse associated to the beam is chosen so that its Twiss parameters are those appearing in the distribution function, hence, e.g.

$$\varepsilon_x = \sqrt{<x^2><x'^2> - <xx'>^2}$$

$$\sigma_x = \sqrt{\varepsilon_x \beta_x}$$

$$\sigma_{x'} = \sqrt{\varepsilon_x \gamma_x}$$

GENERIC BEAMS – RMS EMITTANCE

For a generic beam described by a distribution functions ρ we can still compute the average size and divergence and the whole σ-matrix

$$\sigma_{xx} = \begin{pmatrix} < x^2 > & < x\,x'> \\ < x\,x'> & < x'^2 > \end{pmatrix}$$

$$\sigma_{xx} = \begin{pmatrix} \beta_x \varepsilon_x & -\alpha_x \varepsilon_x \\ -\alpha_x \varepsilon_x & \gamma_x \varepsilon_x \end{pmatrix}$$

we associate to this distribution the ellipse which has the same second order momenta R_{ij} and we deal with this distribution ***as if it was a Gaussian distribution***

and since $$\det(\sigma_{xx}) = (\beta_x \gamma_x - \alpha_x)\varepsilon_x^2 = \varepsilon_x^2$$

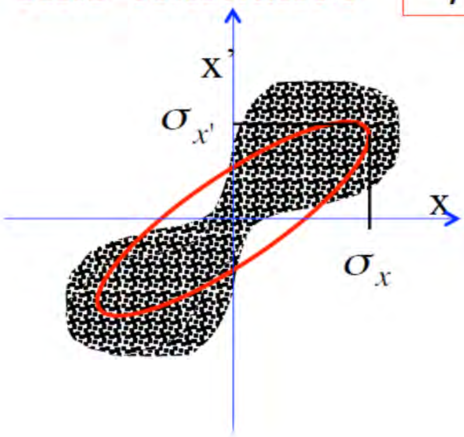
The invariant of the ellipse will be $$\varepsilon_x = \sqrt{< x^2 >< x'^2 > - < xx'>^2}$$

which is the ***rms emittance of the beam***

The sigma matrix is like the covariance matrix taking as distribution average the barycentre, and the phase/trace space coordinates of the particles as ‘deviations’

RMS EMITTANCE

rms emittance $\boxed{\varepsilon_{rms}}$



$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,x')\,dx\,dx' = 1 \qquad \boxed{f'(x,x')=0}$$

rms beam envelope:

$$\sigma_x^2 = \langle x^2 \rangle = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^2 f(x,x')dxdx'$$

Define rms emittance:

$$\boxed{\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon_{rms}}$$

such that:

$$\sigma_x = \sqrt{\langle x^2 \rangle} = \sqrt{\beta \varepsilon_{rms}}$$

$$\sigma_{x'} = \sqrt{\langle x'^2 \rangle} = \sqrt{\gamma \varepsilon_{rms}}$$

Since: $\alpha = -\dfrac{\beta'}{2}$

it follows: $\alpha = -\dfrac{1}{2\varepsilon_{rms}}\dfrac{d}{dz}\langle x^2 \rangle = -\dfrac{\langle x x' \rangle}{\varepsilon_{rms}} = -\dfrac{\sigma_{xx'}}{\varepsilon_{rms}}$

ONE PARTICLE EMITTANCE?

$$\sigma_x \sigma_{p_x} \geq \frac{\hbar}{2}$$

Heisemberg Principle

$$\lambdabar = \frac{\hbar}{mc}$$

Compton Wavelength

$$\epsilon_{n,rms}^{1e^-} \geq \frac{1}{2}\frac{\hbar}{mc} = \frac{\lambdabar}{2}$$

$$\epsilon_{n,rms}^{1e^-} \geq 1.9 \times 10^{-13} m$$

To normalize for the trace space we have to divide for mc

ADIABATIC INVARIANT AND ADIABATIC DAMPING

The slope of the trajectory is $z' = p_z/p_s$.
Accelerate the particle: p_s increases to $p_s+\Delta p_s$, but p_z doesn't change => slope changes.

$$\begin{cases} z' = \dfrac{p_z}{p_s} \\ z' + \Delta z' = \dfrac{p_z}{p_s + \Delta p_s} = p(1-\delta) \end{cases}$$

$$\varepsilon = \beta z'^2 + \gamma z^2 \Rightarrow \Delta\varepsilon = 2\beta z'\Delta z' + 2\gamma z \Delta z \quad (\text{assuming } \alpha = 0)$$

$$\Delta z' = -z'\delta \Rightarrow \Delta\varepsilon = 2\beta z'\Delta z' = -2\beta z'^2\delta = -2\varepsilon \sin^2(\phi + \phi_0)\delta$$

β -> Courant Snyder parameter

$$\langle \Delta\varepsilon \rangle = -\varepsilon\delta \Rightarrow \frac{d\varepsilon}{\varepsilon} = -\frac{dp}{p} = -\delta$$

$$\varepsilon(p) = \varepsilon(p_0) \cdot \frac{p_0}{p}$$

normalized emittance

$$\varepsilon_n = \varepsilon\beta\gamma$$

Invariant of the motion

- In a stationary Poincaré section -> ε
- In an accelerating Poincaré section -> $\varepsilon\beta\gamma$

β = the relative speed
γ = the relativistic factor

$$\begin{cases} \beta = \dfrac{v}{c} \\ \gamma^2 = \dfrac{1}{1-\beta^2} \end{cases}$$

NORMALIZED EMITTANCE

The emittance can be normalized by multiplying the emittance with the dimensionless particle-velocity to speed-of-light ratio:

$$\varepsilon_{norm} = \beta \cdot \gamma \cdot \varepsilon \quad \text{with} \quad \beta = v/c \quad \text{and} \quad \gamma = 1/\sqrt{1-\beta^2}$$

The normalized emittance stays roughly the same throughout most parts of most accelerators!

RMS AND COURANT SNYDER EMITTANCE

BEAM EMITTANCE AND COURANT-SNYDER INVARIANT

We have seen that the beam distribution can be associated to an ellipse containing 66% of the beam (one r.m.s.)

In this way the beam rms emittance is associated with the Courant Snyder invariant of the betatron motion

This links a statistical property of the beam (rms emittance) with single particle property of motion (the Courant-snyder invairnat)

In this way the Courant-Snyder invariant acquires a statistical significance as rms emittance of the beam. Hence the beam rms emittance is a conserved quantity also for generic beams.

This is valid as long as the Hill's equations are valid or more generally the system is Hamiltonian. As such the conservation of the emittance is a manifestation of the general theorem of Hamiltonian system and statistical mechanics known as the **Liouville theorem**

LIOUVILLE'S THEOREM

Liouville's theorem: *In a Hamiltonian system, i.e. n the absence of collisions or dissipative processes, the density in phase space along the trajectory is invariant'.*

$$\frac{df}{dt}=\frac{\partial f}{\partial t}+\sum_{k=1}^{n}\frac{\partial f}{\partial q_k}\frac{\partial q_k}{\partial t}+\sum_{k=1}^{n}\frac{\partial f}{\partial p_k}\frac{\partial p_k}{\partial t}=\frac{\partial f}{\partial t}+\left[H,f\right]=0$$

Liouville theorem states that volume of 6D phase are preserved during the beam evolution (take f to be the characteristic function of the volume occupied by the beam). However if the Hamiltonian can be separated in three independent terms

$$H=H(x,p_x,y,p_y,\tau,\delta)=H(x,p_x)+H(y,p_y)+H(\tau,\delta)$$

The conservation of the phase space density occurs for the three projection on the

(x, p_x) plane (Horizontal emittance)
(y, p_y) plane (Vertical emittance)
(z, p_z) = (τ, δ) plane (longitudinal emittance)

BEAM EMITTANCE AND LIOUVILLE'S THEOREM

The Courant-Snyder invariant is the area of the ellipse phase space.

The conservation of the area is a general property of Hamiltonian systems (any area not only ellipses !)

The invariance of the rms emittance is the particular case of a very general statement for Hamiltonian systems (*Liouville theorem*)

This is valid as long as the motion is Hamiltonian, i.e.

No damping effects, no quantum diffusion, due to emission of radiation

no scattering with residual gas, no beam beam collisions

no collective effects (e.g. interaction with the vacuum chamber, no self interaction)

BRIGHTNESS

- But a lot of experiment does not need only phase space chracteristics, but also intensity…
- BRIGTHNESS

$$B = \frac{dI}{dS d\Omega}$$

$$\overline{B} = \frac{2I}{\pi^2 \varepsilon_x \varepsilon_y}$$

[A/(m-rad)²]

$$\overline{B}_n = \frac{2I}{\pi^2 \varepsilon_{nx} \varepsilon_{ny}}$$

Normalized

$$B_{6D} \propto \frac{Ne}{\varepsilon_{nx}\varepsilon_{ny}\sigma_t\sigma_\gamma}$$

EMITTANCE

- Invariant
- RMS -> Many particles
- Describe the beam 'occupied area' in the TRACE or PHASE SPACE
- Provide A FIGURE OF MERIT (quality factor) of a beam.
- Introduce the Brightness
- WHY?
- 1) Many applications (synchrotrons, Colliders, Medical facilities….) needs small and low divergence beams. Diferent shapes can be provided with the optical elements……but….
- 2) Very important -> Quality of experimental activity is linked to the possibility to define the initial state. 6D emittance represent the spread!

SOURCE

- A good Source is capable to produce low emittance, high current beams (high brightness) in the limit required by the users.

- Emittance and Number of particles are the QUALITY FACTOR of the sources

WHY SOURCES ARE IMPORTANT

Emittance defines the beam quality and so the performance of
the machine in respect to the users requirements

Emittance is produced at the source. Source in a certain way takes into account
also the trip from A (source) to B (user)

In fact:

1) The machine has memory – the final emittance can only be 'worst'
2) The machine has not memory – > Cooling. The emittance can be reduced but the full facility scheme must takes into account the cooling constraints.

COOLING

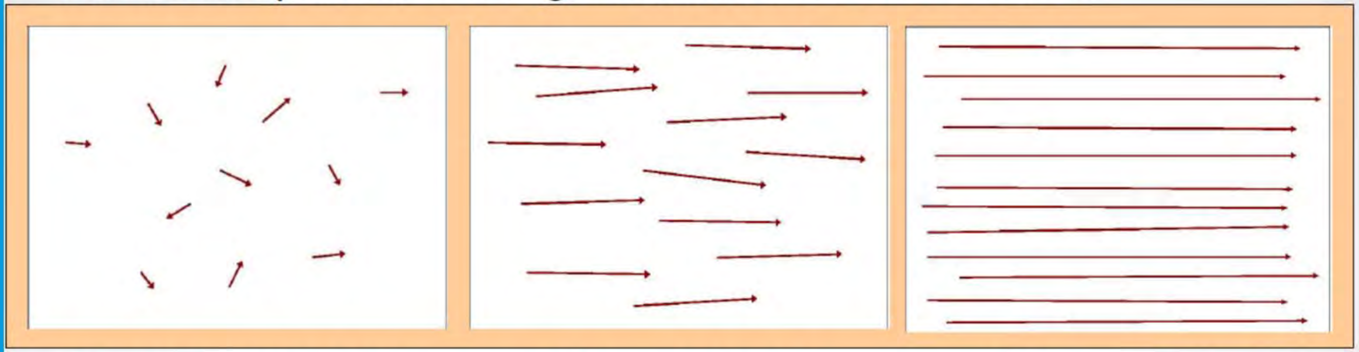
Leptons : radiative cooling
Hadrons : Stochastic cooling, electron cooling, Laser cooling

COOLING

Beam Temperature

Where does the beam temperature originate from?

The beam particles are generated in a 'hot' source



at rest (source) **at low energy** **at high energy**

In a standard accelerator the beam temperature is not reduced
(thermal motion is superimposed the average motion after acceleration)

but: many processes can heat up the beam

e.g. heating by mismatch, space charge, intrabeam scattering, internal targets, residual gas, external noise

COOLING

Beam Temperature Definition

Longitudinal beam temperature

$$\boxed{\frac{1}{2}k_B T_\parallel = \frac{1}{2}mv_\parallel^2 = \frac{1}{2}mc^2\beta^2(\frac{\delta p_\parallel}{p})^2}$$

Transverse beam temperature

$$\boxed{\frac{1}{2}k_B T_\perp = \frac{1}{2}mv_\perp^2 = \frac{1}{2}mc^2\beta^2\gamma^2\theta_\perp^2} \qquad \theta_\perp = \frac{v_\perp}{\beta c}, \quad \theta_\perp(s) = \sqrt{\frac{\epsilon}{\beta_\perp(s)}}$$

Distribution function

$$f(v_\perp, v_\parallel) \propto \exp(-\frac{mv_\perp^2}{2k_B T_\perp} - \frac{mv_\parallel^2}{2k_B T_\parallel})$$

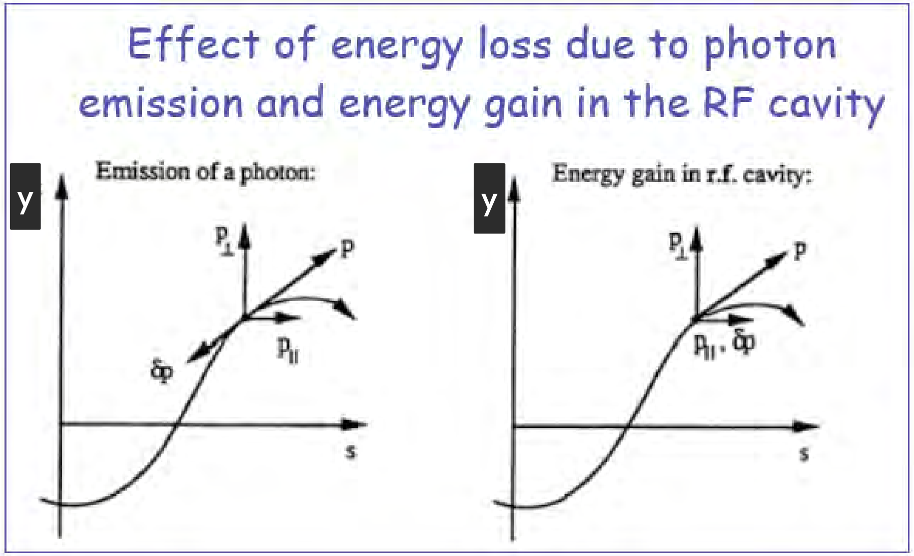
Particle beams can be anisotropic: $k_B T_\parallel \neq k_B T_\perp$

e.g. due to laser cooling or the distribution of the electron beam

Don‘t confuse: beam energy $\leftrightarrow$ beam temperature

(e.g. a beam of energy 100 GeV can have a temperature of 1 eV)

TRANSVERSE RADIATIVE DAMPING ….ANALOGOUS FOR LONGITUDINAL



In the vertical plane the particle undergoes Betatron oscillations. So the vertical displacement will be $y = A\sqrt{\beta}cos\theta, y' = -\frac{A}{\sqrt{\beta}}sin\theta$ where the amplitude A is given by the Courant Snyder invariant $A^2 = \gamma y^2 + 2\alpha yy' + \beta y'^2$

For every turn the lost energy (SR with $1/\gamma$ angle) is restored by the RF cavity (zero angle).This change the longitudinal momentum. For the zero synchrotron amplitude particle : $\frac{\Delta p}{p} = \frac{\Delta y\prime}{y\prime} = \frac{U}{E}$ with U -> energy lost by SR and E -> nominal energy.

Using the Amplitude definition it is possible to demonstrate that after many kicks, in average, (averaged on all the betatron phases) the amplitude to the first order will vary as : $\frac{\langle \Delta A \rangle}{A} = -\frac{U}{2E}$

So we will have that $\frac{dA}{dt} = -\frac{U}{2ET_0}A$ with a damped solution $Ae^{-\frac{t}{\alpha y}}$ with $\alpha_y = \frac{U}{2ET_0}$ -> damping decrement.

DAMPING RINGS

Reduce the phase space volume of the beams so that the design beam emittances are obtained.

Emittance evolution:

initial emittance

$$\varepsilon_f = \varepsilon_{eq} + (\varepsilon_i - \varepsilon_{eq})\, e^{-2T/\tau_D}$$

final emittance

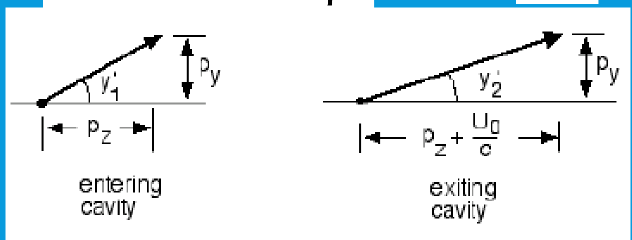
equilibrium emittance

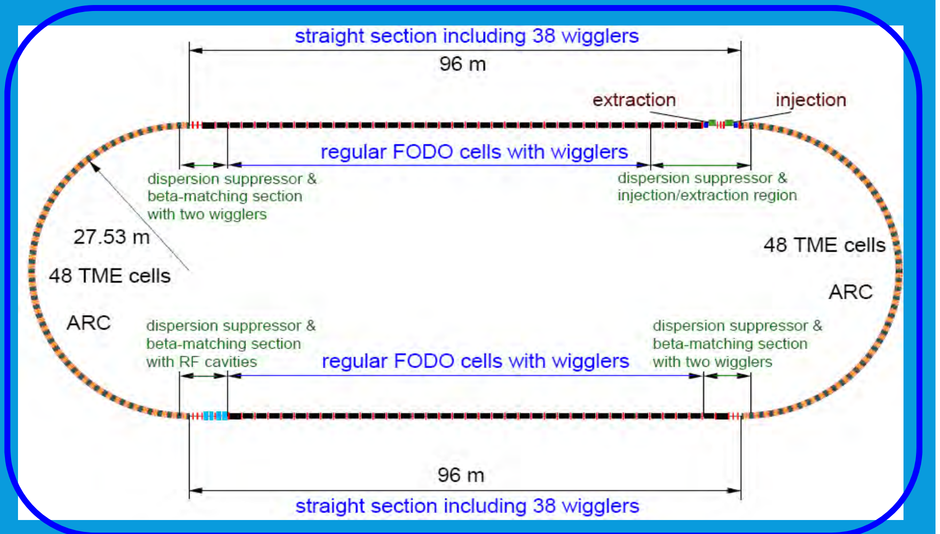
damping time

$$\tau_D \propto \frac{\rho^2}{E^3}$$

• damping driven by incoherent synchrotron radiation + RF gain

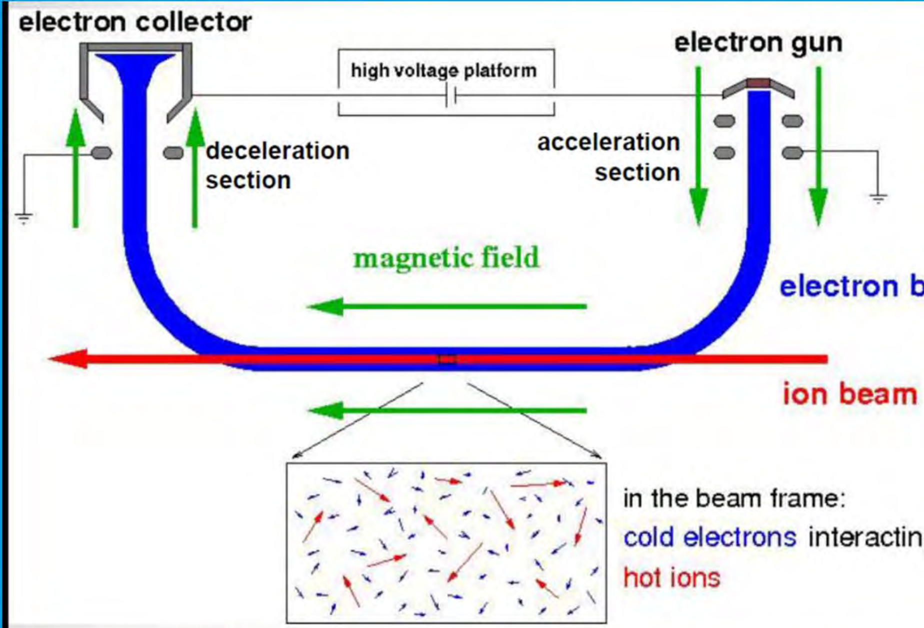
$$U_0 = \frac{4\pi}{3}\alpha\hbar c\frac{\gamma^4}{\rho} \qquad \frac{U_0}{c}$$





• theoretical limit: *quantum* character of the radiation

ELECTRON COOLING



$v_{e\|} = \beta_e c = \beta_i c = v_{i\|}$

$E_e = m_e/M_i \cdot E_i$

e.g.: 220 keV electrons
cool 400 MeV protons

electron temperature
$k_B T_\perp \approx 0.1$ eV
$k_B T_\| \approx 0.1 - 1$ meV

superposition of a cold intense electron beam with the same velocity

momentum transfer by Coulomb collisions

cooling force results from energy loss in the co-moving gas of free electrons

first estimate: (Budker 1967) $\tau = \frac{3}{8\sqrt{2\pi} n_e Q^2 r_e r_i c L_C} \left(\frac{k_B T_e}{m_e c^2} + \frac{k_B T_i}{m_i c^2}\right)^{3/2}$

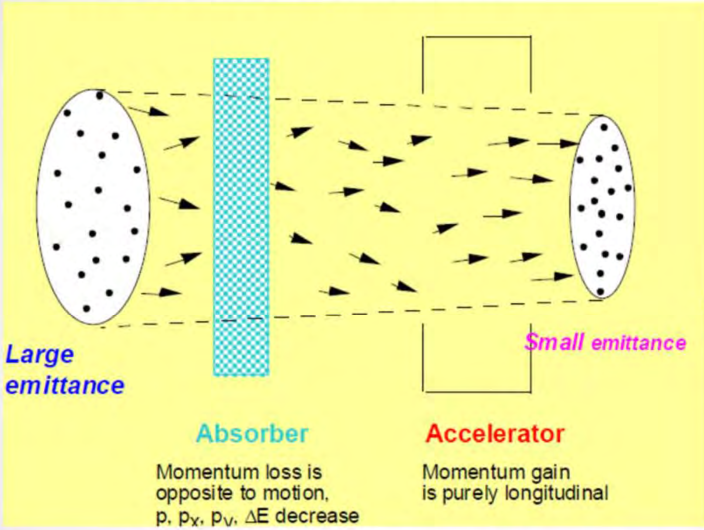
for large relative velocities

cooling time $\tau_z \propto \frac{A}{Q^2} \frac{1}{n_e \eta} \beta^3 \gamma^5 \theta_z^3$ $\left\{ \begin{array}{l} \theta_{x,y} = \frac{v_{x,y}}{\gamma \beta c} \\ \theta_\| = \frac{v_\|}{\gamma \beta c} \end{array} \right.$

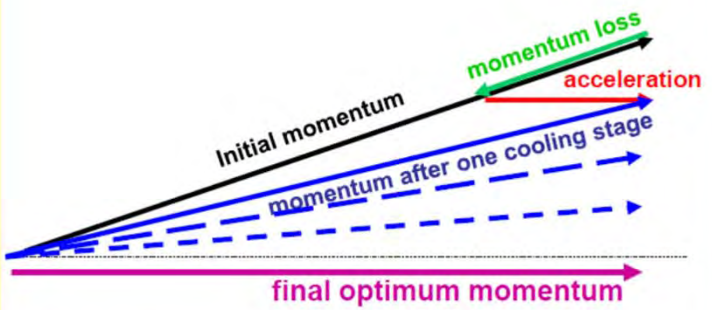
cooling rate (τ^{-1}):

2. Ionization Cooling

energy loss in solid matter



proposed for muon cooling



not useful for heavy particles due to strong interaction with matter

transverse cooling

$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta\gamma\beta_\perp}{2}\frac{\langle\theta^2_{rms}\rangle}{ds}$$

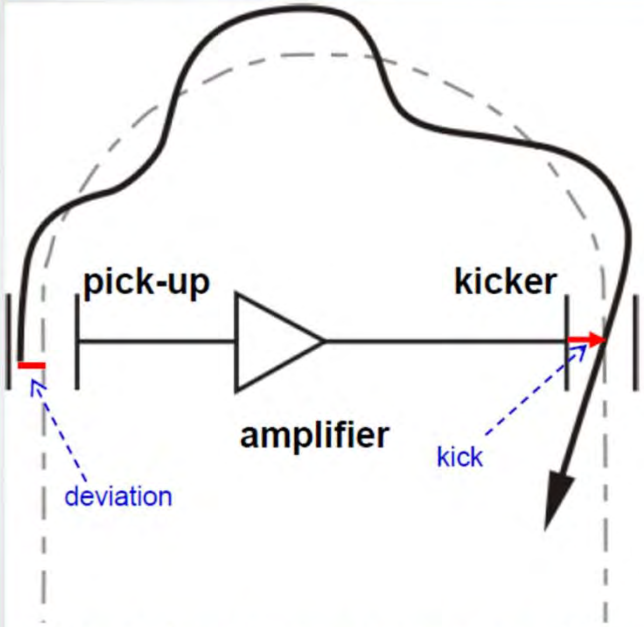
$$= -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E}$$

⇒ small $\beta_\perp$ at absorber in order to minimize multiple scattering

large L_R, (dE/ds) ⇒ light absorbers (H_2)

STOCHASTIC COOLING

First cooling method which was successfully used for beam preparation



Principle of transverse cooling:
measurement of deviation from ideal orbit
is used for correction kick (feedback)

S. van der Meer, D. Möhl, L. Thorndahl et al.
(1925 – 2011) (1936-2012)

Conditions:

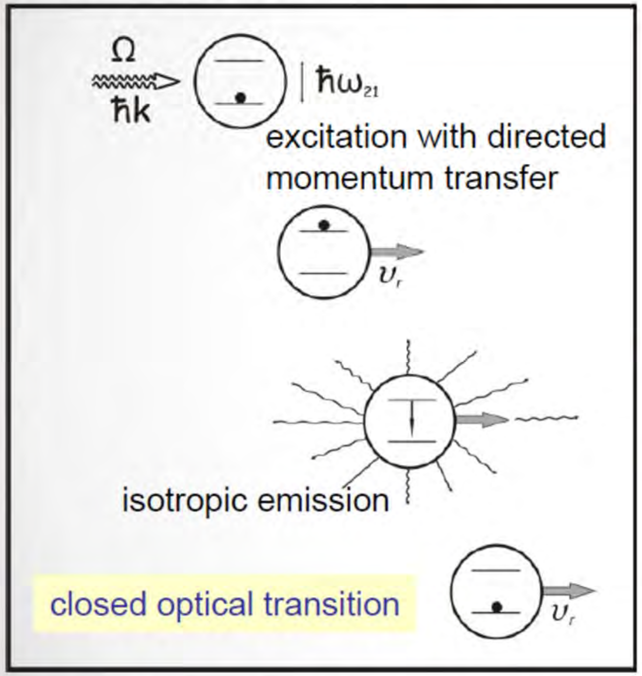
Betatron motion phase advance
(pick-up to kicker): (n + ½) π

Signal travel time = time of flight of particle
(between pick-up and kicker)

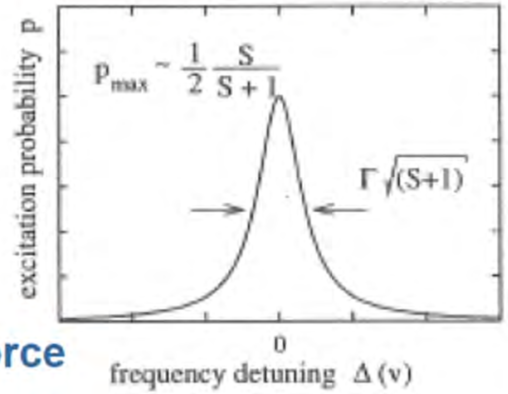
Sampling of sub-ensemble of total beam

LASER COOLING

$$\Omega = \gamma\omega_{21}(1 - \beta\cos\theta)$$



the directed excitation and isotropic emission result in a transfer of velocity v_r



cooling force

$$\vec{F}(\vec{v},\vec{k}) = \frac{\hbar\vec{k}}{2}S\Gamma\frac{(\Gamma/2)^2}{(\omega-\omega_{21}-\vec{v}\,\vec{k}) + (\Gamma/2)^2(1+S)}$$

Lorentzian with width $\Gamma/k \sim 10$ m/s

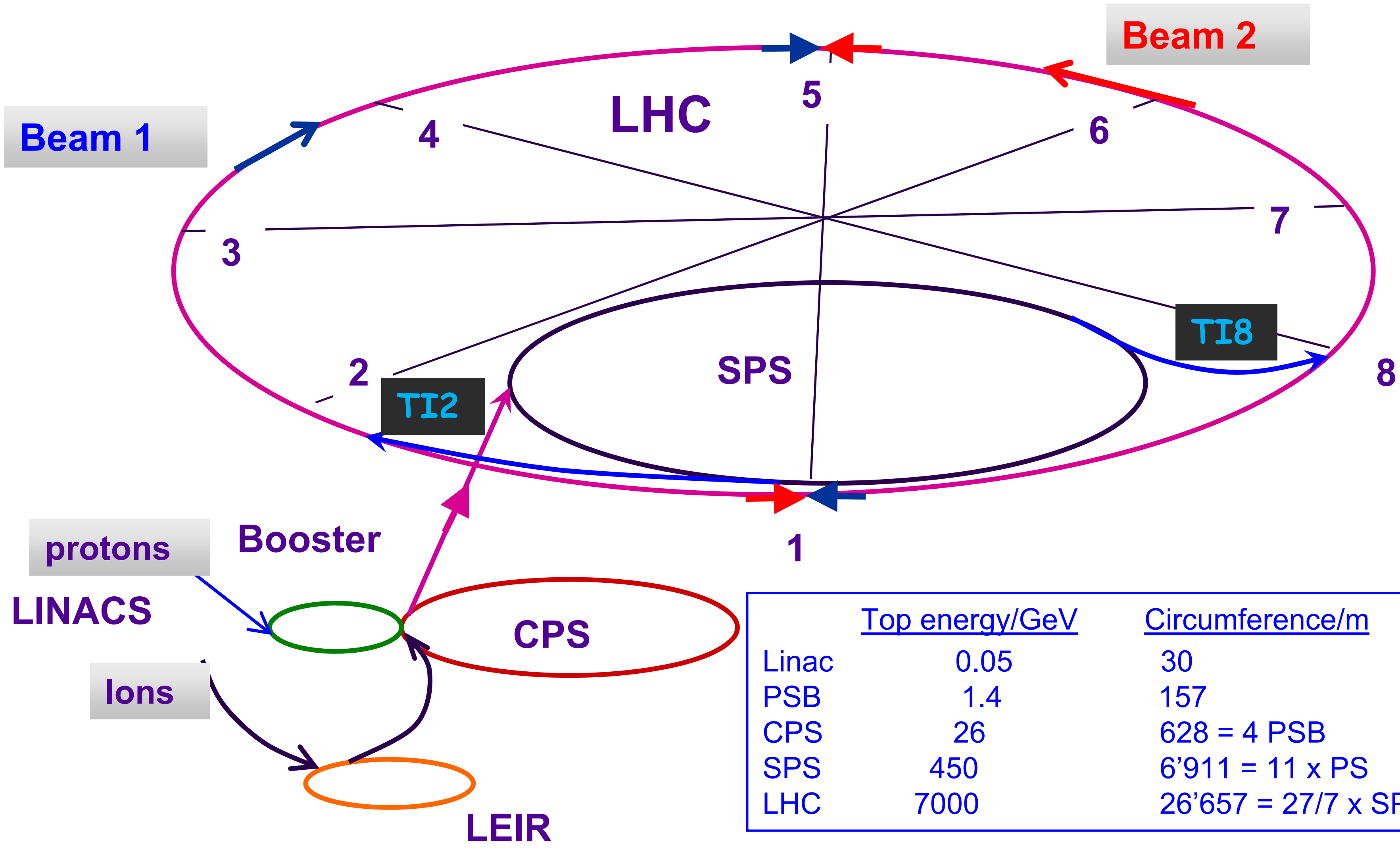
minimum temperature $T_D = \frac{\hbar\Gamma}{2k_B}$ (Doppler limit)
typical $10^{-5} - 10^{-4}$ K

typical cooling time ~ 10 μs

drawback: only longitudinal cooling

- The source quality and the required cooling to match the users required performances 'propagate' togheter in the machine design and definition.
- To improve the source design and quality will relax the constraints on the cooling and machine requirements increasing the global performances
- Luminosity, Brightness, Emittances, Average and Instantaneous current are ALL parameters depending on sources.
- In some way the 'global' photo of the beam in the machine will depend on sources.

EXAMPLES LHC

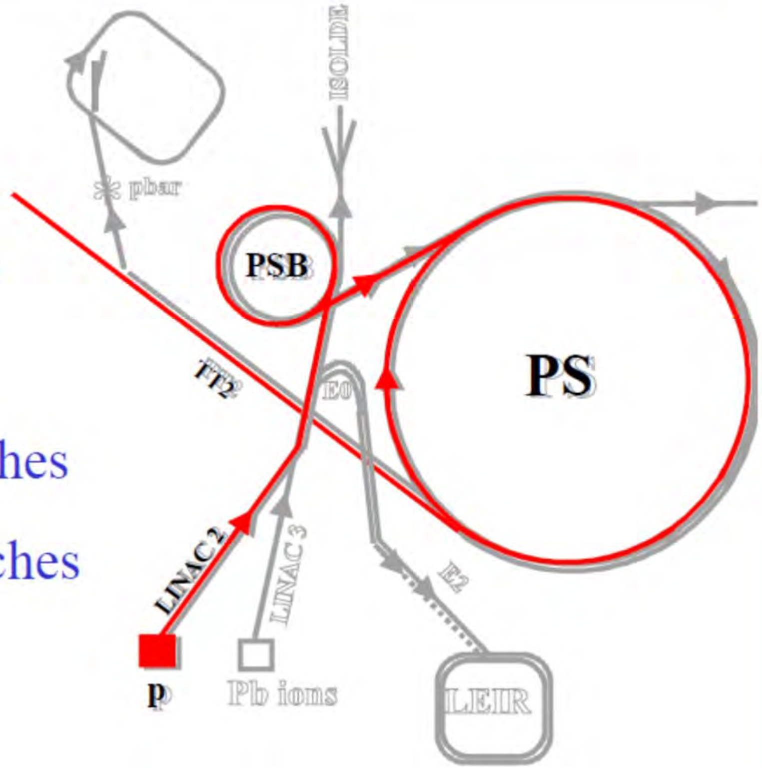


	Top energy/GeV	Circumference/m
Linac	0.05	30
PSB	1.4	157
CPS	26	628 = 4 PSB
SPS	450	6'911 = 11 x PS
LHC	7000	26'657 = 27/7 x SPS

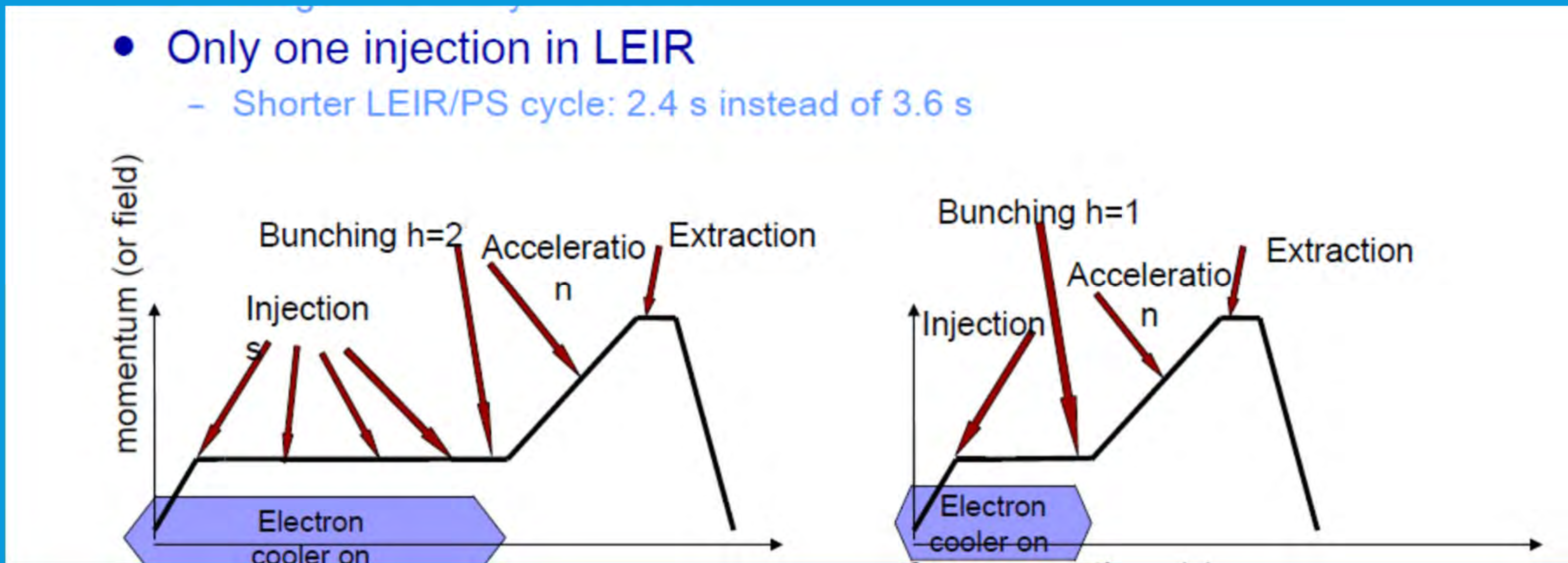
Note the energy gain/machine of 10 to 20.
The gain is typical for the useful range of magnets.

Proton beam production for LHC

- –Linac2 (50MeV)
- –PSB (1.4GeV) 4+2 bunches
- –PS (25GeV) 72 bunches
- –SPS (450 GeV) 4 x 72 bunches
- –LHC (7 TeV) 2 x 2808 bunches

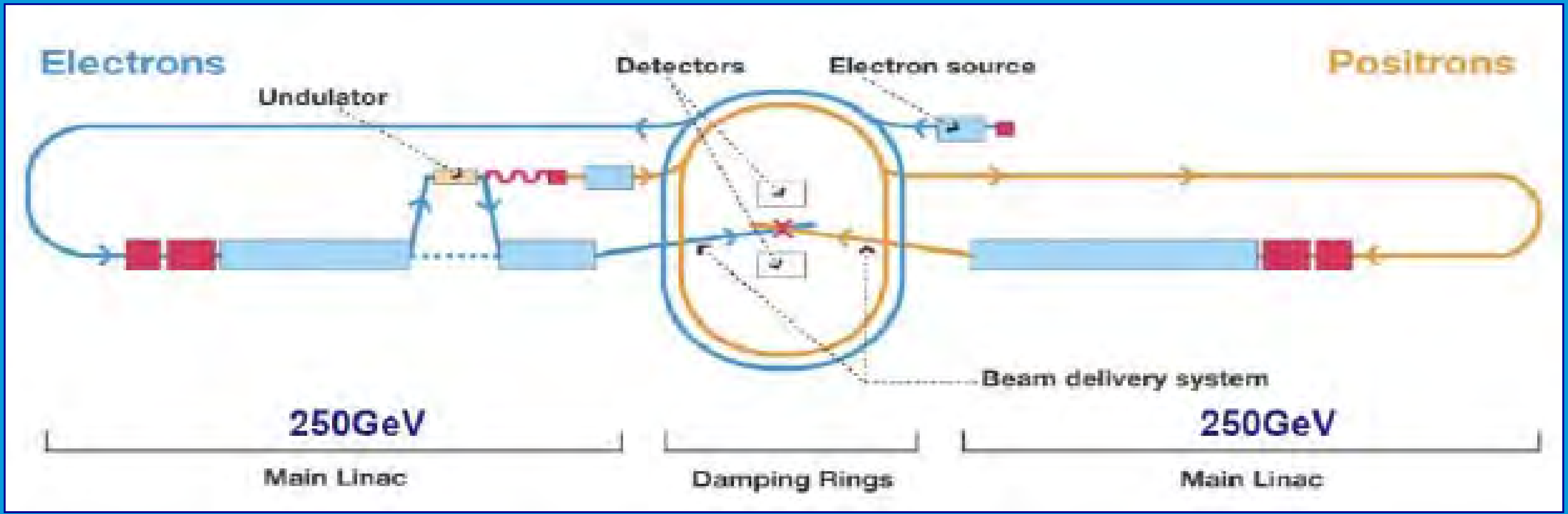


LEIR (HEAVY IONS) OLD CYCLE



ILC

- **International Linear Collider (ILC)**
- **Repetition rate . 5 Hz**



ILC POSITRON SOURCE PARAMETERS

Parameter	Symbol	Value	Units
Bunch Population	N_b	$2x10^{10}$	#
Bunches per pulse	n_b	2625	#
Bunch spacing	t_b	369	ns
Pulse repetition rate	f_{rep}	5	Hz
Injection Energy (DR)	E_0	5	GeV
Beam Power (x1.5)	P_o	300	kW
Polarization e-(e+)	P	80(30)	%

SUMMARY

- Beams
- Beams manipulation
- Beams quality and invariants
- Emittance and Brightness
- Why source are important
- Cooling
- Emittance preservation