Electroweak skyrmions through the Higgs

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The Emergence of Electroweak Skyrmions through Higgs Bosons

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- theory
- numerical calculation
- pheno

Skyrmions as baryons

Skyrme (1961)

Witten (1979)

Adkins, Nappi, Witten (1983)

$SU(2)$ matrix $U = e^{i\pi^a\tau^a/f_\pi}$

Non-trivial static and stable field configurations

Lagrangian

$$\text{Solitons in } \mathcal{L} = \mathcal{L}_{\partial^2} + \mathcal{L}_{\partial^4} + \ldots\ ?$$

Derrick's theorem:

$$\mathcal{L}_{\partial^2} \sim \text{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) \implies \text{no solitons}$$

Skyrme term:

$$\mathcal{L}_{\partial^4} \sim \text{tr}\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

Topology

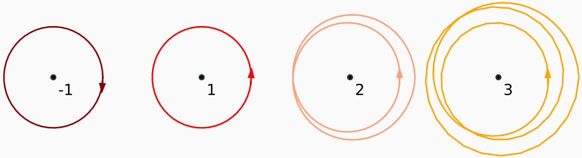
Static solutions with $U =$ constant at spatial infinity:

$$U : \mathbb{R}^3 \cup \{\infty\} \cong S^3 \to S^3$$

Homotopy class characterized by the **winding number**

$$n_U = \epsilon_{ijk} \int d^3x \; U^\dagger \partial_i U \, U^\dagger \partial_j U \, U^\dagger \partial_k U \in \mathbb{Z}$$

For $S^1 \to S^1$:



Global $SU(2)$ skyrmion

$$\mathcal{L} = -\frac{f_\pi^2}{16} \operatorname{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{1}{32e^2} \operatorname{tr}\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

- **Skyrmion**: Topologically protected local minimum of the static-field configuration energy with $n_U = 1$.
- **Vacuum**: $n_U = 0$
- **Antiskyrmion**: $n_U = -1$
- **Multiskyrmions**: $|n_U| > 1$

Gauge skyrmion

Ambjorn, Rubakov (1985)

$$\mathcal{L} = \frac{1}{2g^2} \operatorname{tr}\left(W_{\mu\nu} W^{\mu\nu}\right) - \frac{f_\pi^2}{16} \operatorname{tr}\left(D_\mu U^\dagger D^\mu U\right) + \frac{1}{32e^2} \operatorname{tr}\left[U^\dagger D_\mu U, U^\dagger D_\nu U\right]^2$$

- local minimum still exits for $e < e_{\text{crit}}$
- no longer topologically protected

Introducing the Higgs field

$$\Phi = \begin{pmatrix} \phi_0^* & \phi_1 \\ -\phi_1^* & \phi_0 \end{pmatrix} = sU$$

Lowest-dim. embedding of the Skyrme term in the SMEFT:

$$\boxed{\mathcal{O}_{\mathsf{Sk}} = \frac{1}{8}\,\mathrm{tr}\left[D_\mu \Phi^\dagger, D_\nu \Phi\right]^2}, \qquad \mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \frac{1}{\Lambda^4}\mathcal{O}_{\mathsf{Sk}}$$

Numerical calculation

Ansatz

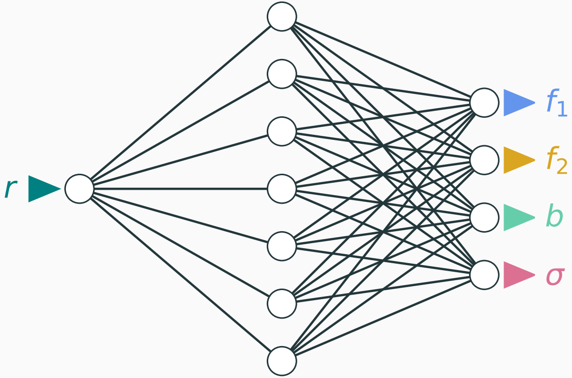
Unitary gauge

$$\phi(x) = \frac{v\sigma(r)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Spherical ansatz:

$$W_i = \frac{\Lambda^2}{v}\tau_a \left(\epsilon_{ija} n_j \frac{f_1(r)}{r} + (\delta_{ia} - n_i n_a)\frac{f_2(r)}{r} + n_i n_a \frac{b(r)}{r} \right)$$

Method: neural net



$$(f_1(r), f_2(r), b(r), \sigma(r)) = \sum_{i=1}^{30} \left[\mathbf{b}_i^{(2)} + \frac{\mathbf{w}_i^{(2)}}{1 + \exp\left(-b_i^{(1)} - w_i^{(1)} r\right)} \right],$$

Method: training

Minimize the loss function:

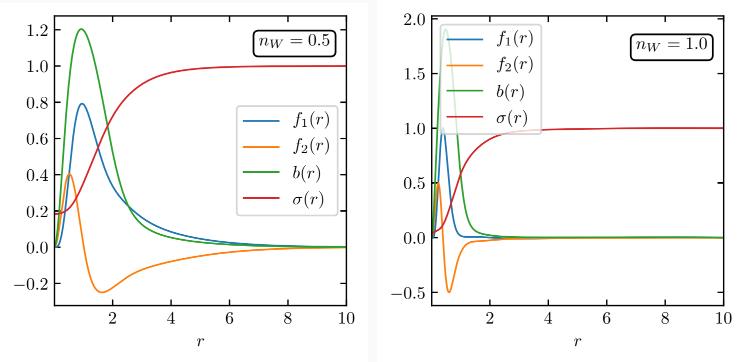
$$L[f_1, f_2, b, \sigma] = E[f_1, f_2, b, \sigma] + \omega_{\mathrm{BC}} \sum_k \mathrm{BC}_k[f_1, f_2, b, \sigma]^2 + \omega_n \left(n_W[f_1, f_2, b, \sigma] - n_W\right)^2$$

over all values of the neural net parameters.

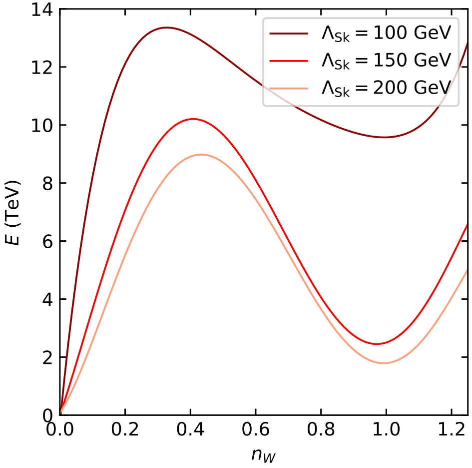
The weigths ω_i are adjusted to $\sim 10^{4\text{–}5}$ so that

(min. L) $\implies$ (min. E while satisfying BC and $n_W =$ fixed value)

Results: profile functions



Results: energy vs n_W



Results: critical mass

$$E = \frac{4\pi v^3}{\Lambda^2} E_{\text{nat}}$$

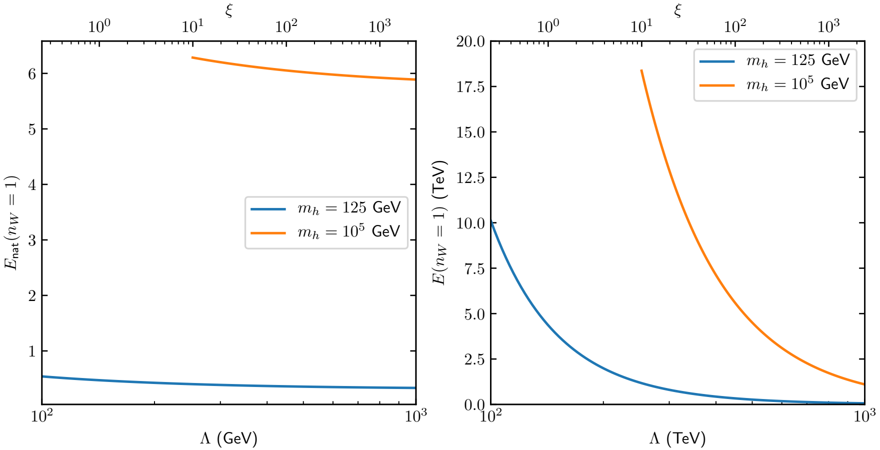
For large Λ:

$$M_{\text{Sk}} \simeq E_{\text{nat}}(n_W = 1)\frac{4\pi v^3}{\Lambda^2} \simeq 0.35\frac{4\pi v^3}{\Lambda^2}$$

Below $\Lambda_{\text{crit}} \simeq 100\,\text{GeV}$, the skyrmion disappears, thus

$$\Lambda > \Lambda_{\text{crit}} \implies M_{\text{Sk}} \lesssim 10\,\text{TeV}.$$

Results: mass vs Λ



Results: radius

$$R_{\mathsf{Sk}}^2 = \left(\frac{v}{\Lambda^2}\right)^2 \left\langle r^2 \right\rangle = \left(\frac{v}{\Lambda^2}\right)^2 \frac{1}{24\pi^2} \int d^3x \, r^2 \, \epsilon_{ijk} \, \mathrm{tr}\left(iW_i W_j W_k\right)$$

$$R_{\mathsf{Sk}} \simeq 0.6 \frac{v}{\Lambda^2}$$

Phenomenology

Production and $B+L$ non-conservation

$$n_W = 0 \to n_W = 1 \text{ process}$$

$$\Downarrow$$

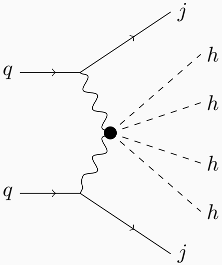
$$\Delta n_{CS} = 1$$

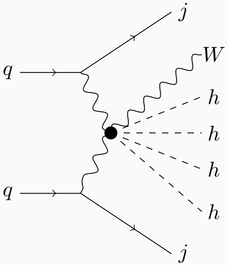
$$\Big\Downarrow \quad \begin{aligned} \partial_\mu J^\mu_{B+L} &= \tfrac{3}{8\pi^2}\,\mathrm{tr}\, W_{\mu\nu}\widetilde{W}^{\mu\nu} \\ &= 6\,\partial_\mu J^\mu_{\mathrm{CS}} \end{aligned}$$

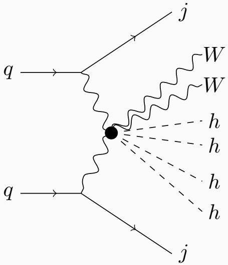
$$\Delta(B+L) = 6$$

Similar to instantons, exponentially suppressed.

Probing $\mathcal{O}_{\mathsf{Sk}}$ at colliders: processes







Probing $\mathcal{O}_{\mathsf{Sk}}$ at colliders: cross section and Λ

$$\sigma = (2.70\,\text{pb}) \left(\frac{\sqrt{s}}{14\,\text{TeV}}\right)^{8.93} \left(\frac{1\,\text{TeV}}{\Lambda}\right)^{8},$$

300 events with final state $b\bar{b}b\bar{b}b\bar{b}\gamma\gamma$ at hadron colliders:

$$\begin{aligned}
\Lambda &< 58\,\text{GeV} && \text{for } \sqrt{s} = 14\,\text{TeV},\ \textstyle\int dt\, L = 300\,\text{fb}^{-1},\\
\Lambda &< 77\,\text{GeV} && \text{for } \sqrt{s} = 14\,\text{TeV},\ \textstyle\int dt\, L = 3000\,\text{fb}^{-1},\\
\Lambda &< 320\,\text{GeV} && \text{for } \sqrt{s} = 50\,\text{TeV},\ \textstyle\int dt\, L = 3000\,\text{fb}^{-1},\\
\Lambda &< 690\,\text{GeV} && \text{for } \sqrt{s} = 100\,\text{TeV},\ \textstyle\int dt\, L = 3000\,\text{fb}^{-1}.
\end{aligned}$$

10 events with final state $b\bar{b}b\bar{b}b\bar{b}\gamma\gamma$ at a muon collider:

$$\Lambda < 650\,\text{GeV} \qquad \text{for } \sqrt{s} = 14\,\text{TeV},\ \textstyle\int dt\, L = 3000\,\text{fb}^{-1}.$$

Dark matter

skyrmions are long-lived

$$\Omega h^2 \simeq \frac{3 \times 10^{-27}\,\mathrm{cm}^3\mathrm{s}^{-1}}{\langle \sigma_{\mathrm{ann}} \mathrm{v} \rangle}$$

$$\Big\downarrow \; \sigma_{\mathrm{ann}} \simeq \pi R_{\mathrm{Sk}}^2 \text{ and } v = 1/2$$

$$\Lambda \lesssim 2\text{–}3\,\mathrm{TeV}$$

$$(\Lambda \propto \sigma^{1/4})$$

Summary

- Neural nets for variational problems

- Skyrmions still present with gauge and Higgs fields
- $M_{\text{Sk}} \sim 1/\Lambda^2$
- $\Lambda > \Lambda_{\text{crit}} \simeq 100\,\text{GeV} \implies M_{\text{Sk}} \lesssim 10\,\text{TeV}$

- $\mathcal{O}_{\text{Sk}}$ generated by many UV models

- Skyrmion production unlikely
- $\mathcal{O}_{\text{Sk}}$ may be probed at colliders
- Viable dark matter candidates