Euclidean Wormholes and their implications for the Axion Quality Problem

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Short review including new results from:

2009.03917 [JHEP] with James Alvey

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Global Symmetries

Approximate and Exact Global Symmetries are widely used in Particle Physics

Dark Matter

Dark Energy

Flavor

B-L

In particular:

The Peccei-Quinn solution to the Strong CP problem relies on a new global, chiral and anomalous $U(1)_{\mathrm{PQ}}$ symmetry

CP  

Gravity

Global Symmetries are expected to be explicitly broken by Gravity

Black Hole Arguments

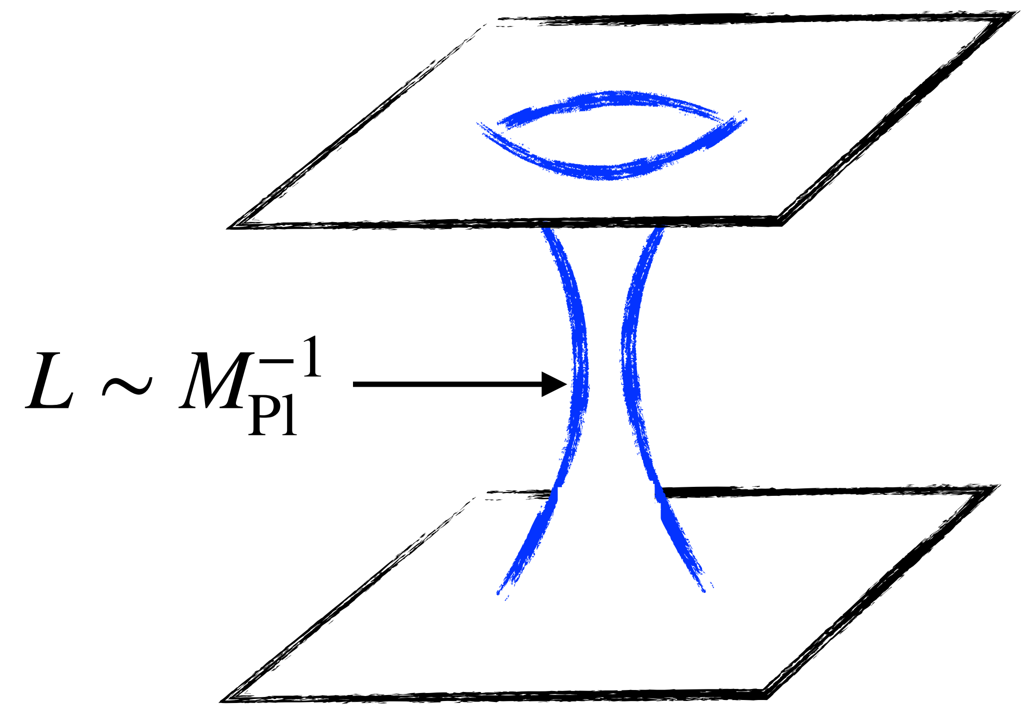
No Global Symmetries in String Theory

No Global Symmetries in AdS/CFT

But by how much?

Wormholes allow to estimate the effects

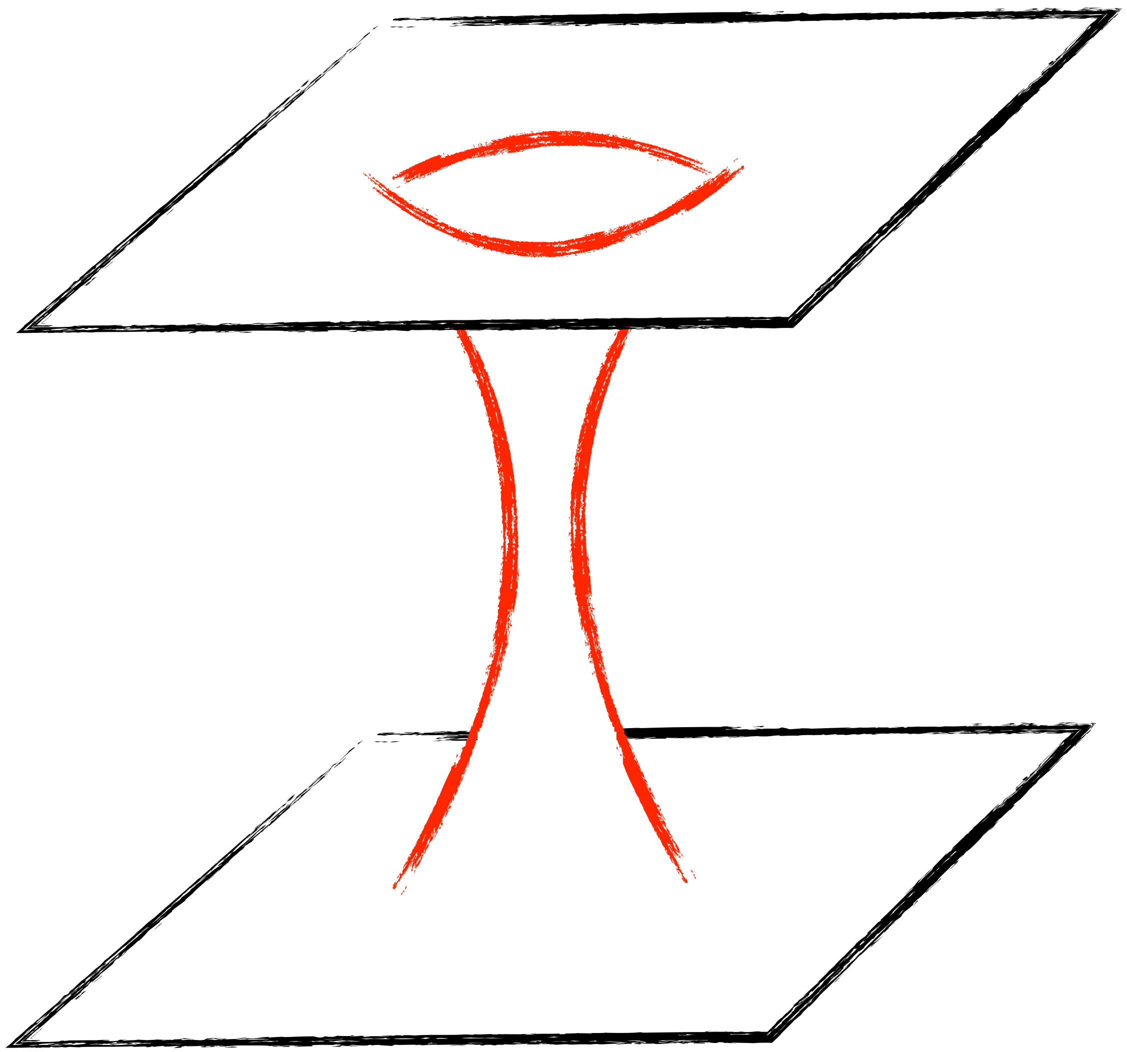
Wormholes are gravitational instantons that contribute non-perturbatively to processes violating global symmetries

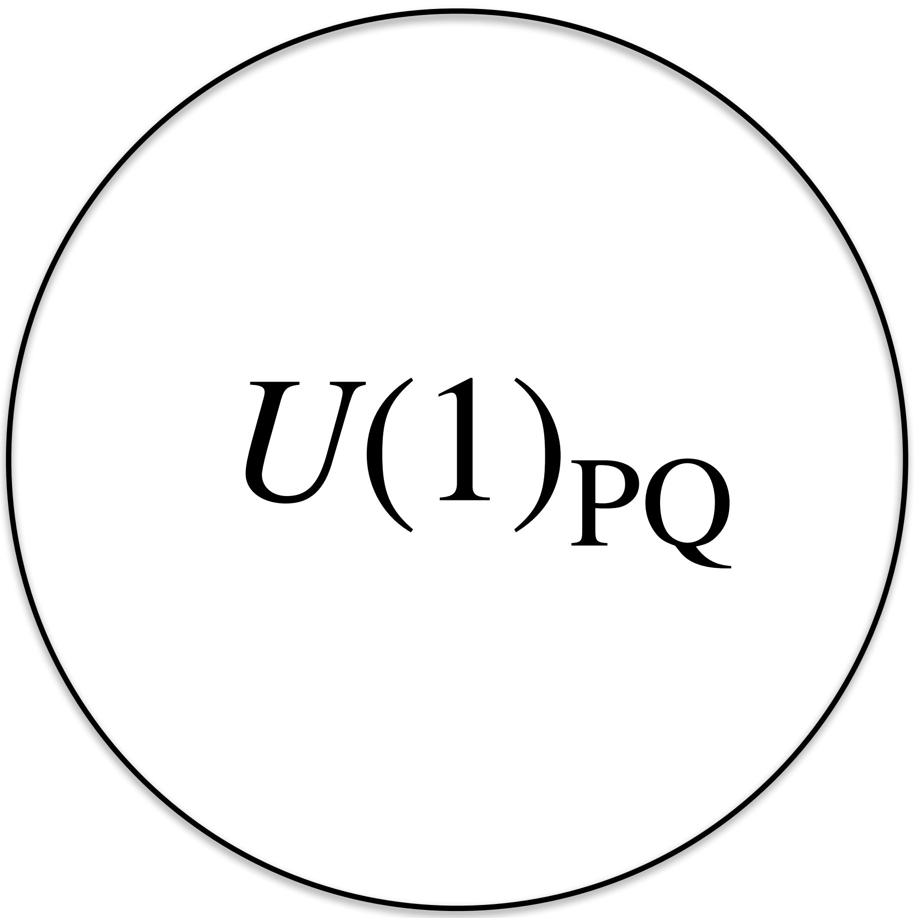


Lead to: $$\Delta V = \sum_n \lambda_n \alpha_n M_{\rm Pl}^4 \left[\frac{\Phi}{M_{\rm Pl}}\right]^n + {\rm h.c.}$$ **with** $$\lambda_n \simeq e^{-S_n}$$

The Plan

Gravity against **Global Symmetries**





Wormholes

The QCD Axion

Outline

1) The Axion Quality Problem

2) Global Symmetry breaking induced by Wormholes
   1) Wormhole dynamics
   2) Scaling of the action in different theories and models
   3) Implications for axion model building

3) Wormhole Caveats

4) Outlook

5) Summary and Conclusions

The Strong CP problem

CP violation in QCD: $\mathscr{L}^{QCD}_{CPV} = \bar{\theta}\frac{\alpha_s}{8\pi}G\tilde{G}$ **with:** $\bar{\theta} = \theta + Arg|M|$

Observationally: $\bar{\theta} < 10^{-10}$ **nEDM Collaboration 2001.11966**

Strong CP problem, why is $\bar{\theta} < 10^{-10}$?

Proposed Solutions:

1) An additional Chiral symmetry, Peccei & Quinn (1977)

Discussed in what follows

2) $Arg|M| < 10^{-10}$**, Nelson & Barr (1983-1984)**

Viable possibility but not free from issues, see Dine & Draper [1506.05433]

3) $m_u < 10^{-10}\, m_d$

Highly disfavoured by Lattice calculations, see Funcke et al. [2002.07802]

4) This is not a problem, see e.g. Senjanovic & Tello [2004.04036]

5) Maybe there is no problem, see Ai, Cruz, Garbrecht & Tamarit [2001.07152]

The Peccei-Quinn Mechanism

The Idea: Dynamically relax $\bar{\theta}$ to 0

Addition of a new global chiral symmetry: $U(1)_{\rm PQ}$ $\quad \Phi = f\, e^{ia/f_a}$

Relevant Lagrangian is $\mathscr{L}_a \supset \left(\bar{\theta} + \frac{a}{f_a}\right)\frac{\alpha_s}{8\pi} G\tilde{G}$

But the $U(1)_{\rm PQ}$ symmetry is explicitly broken by QCD instantons that generate a potential for the axion:

$$V_{\rm QCD} \simeq \Lambda_{\rm QCD}^4 \cos\left[\bar{\theta} + \frac{a}{f_a}\right]$$

Minimization yields:

$\langle a \rangle = -\,\bar{\theta} f_a$ **Strong CP problem solved, no CP violation in the vacuum**

$m_a \simeq \Lambda^2_{\rm QCD}/f_a$ **Massive pseudogoldstone boson, *the axion***
Weinberg (1978) and Wilczek (1978)

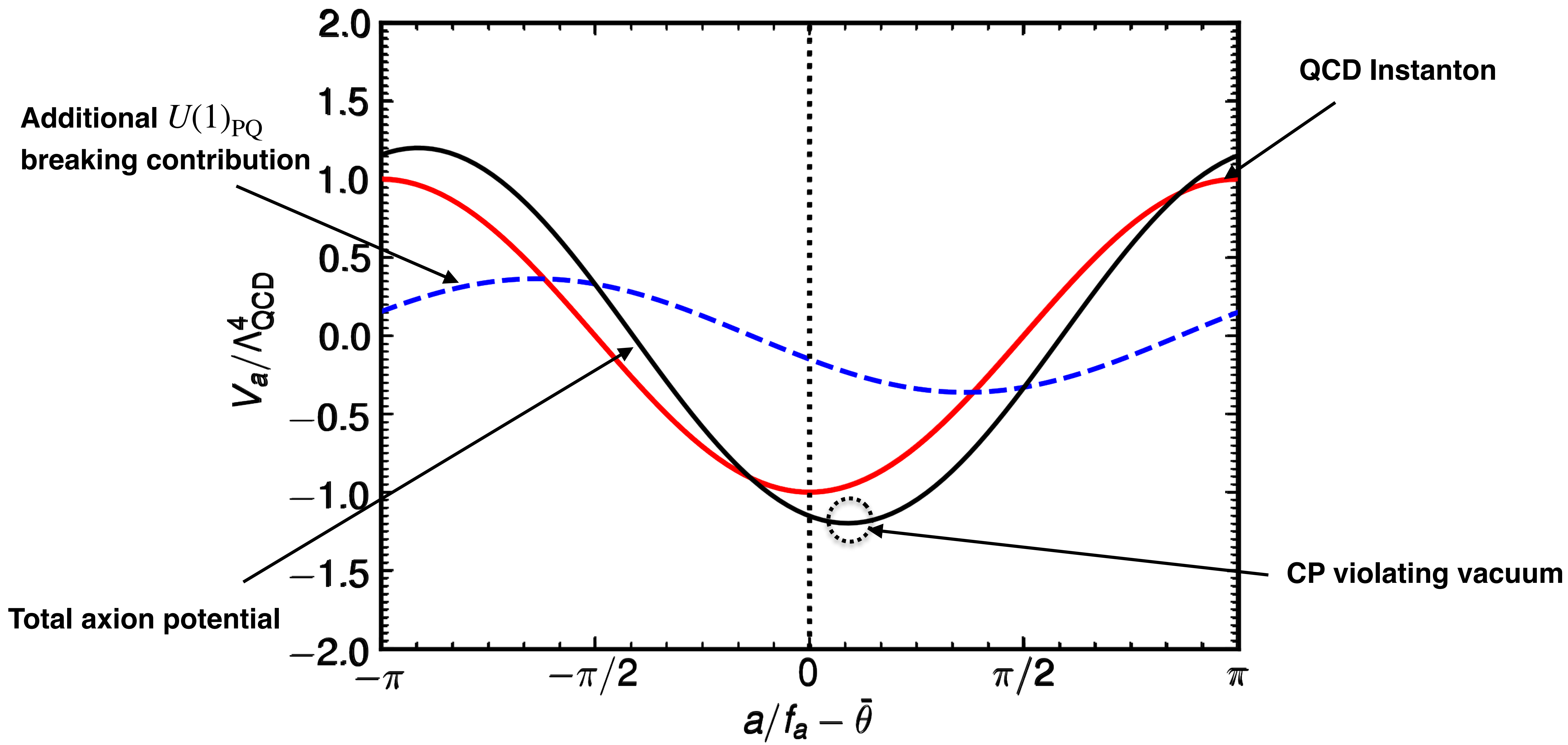
Dark Matter candidate

Preskill, Wise & Wilczek
Abbot & Sikivie
Dine & Fischler (1983)

$m_a \sim 10\,\mu{\rm eV} \quad f_a \sim 10^{12}\,{\rm GeV}$

The Axion Quality Problem

The PQ solution is strongly sensitive to the explicit breaking of $U(1)_{\rm PQ}$



To maintain the efficiency one requires: $\Delta V < 10^{-10}\, V_{\rm QCD}$

The Axion Quality Problem

How restrictive is $\Delta V < 10^{-10}\, V_{\rm QCD}$ **?**

Example:

$$\Delta V = \lambda\, |\Phi|^4 \frac{\Phi}{M_{\rm Pl}} + {\rm h.c.}$$

$$\lambda f_a^5 / M_{\rm Pl} < \Lambda_{\rm QCD}^4$$

$$\downarrow$$

$$\lambda < 10^{-40} \quad \textbf{for} \quad f_a = 10^{12}\,{\rm GeV}$$

This amount of tuning is the Axion Quality Problem!

Kamionkowski & March-Russell [hep-th/9202003]
Holman, Hsu, Kephart, Kolb, Watkins & Widrow [hep-ph/9203206]
Barr & Seckel PRD (1992)

The Axion Quality Problem

Three ways out:

1) Protect the axion with gauge symmetries

$$\Delta V = \lambda_n M_{\rm Pl}^4 \left[\frac{|\Phi|}{M_{\rm Pl}}\right]^{d-n} \left[\frac{\Phi}{M_{\rm Pl}}\right]^{n} + \mathrm{h.c.}$$

$$d > 12$$

(for $\lambda \sim \mathcal{O}(1), n = 1, f_a = 10^{12}\,\mathrm{GeV}$)

Intensive model building in this direction:

see e.g.: Hook, Kumar, Liu, Sundrum [1911.12364]
Cox, Gherghetta, Nguyen [1911.09385], Yin [2007.13320]
Ardu, Di Luzio, Landini, Strumia, Teresi, Wang [2007.12663]
see review: Di Luzio, Giannotti, Nardi, Visinelli [2003.01100]

- But, prototypical axion models (DFSZ & KSVZ) feature a gauge singlet scalar and are not protected by gauge symmetries

2) $\lambda \sim \mathcal{O}(1)$ but phases are aligned with the QCD instanton

This is highly tuned and in addition the axion would be ultraheavy

3) Maybe the additional contributions are only non-perturbative and exponentially small

$\lambda \sim e^{-S}$ $\quad S > 190 \quad$ **No quality problem**

The Global Symmetry Quality Problem

This is relevant for any pseudo-Goldstone Boson

Light Scalar Fields as Dark Energy — see e.g. Frieman, Hill, Stebbins & Waga [astro-ph/9505060]

Light scalar particles as Dark Matter — see e.g. Hu, Barkana & Gruzinov [astro-ph/0003365]

Light scalars related to neutrinos — see e.g. Chikashige, Mohapatra & Peccei [PLB 1981]

Light scalars in the dark sector — see e.g. Weinberg [1305.1971]

Axions-like particles in String Theory — see e.g. Arvanitaki, Dimopoulos, Dubovsky Kaloper & March-Russell [0905.4720]

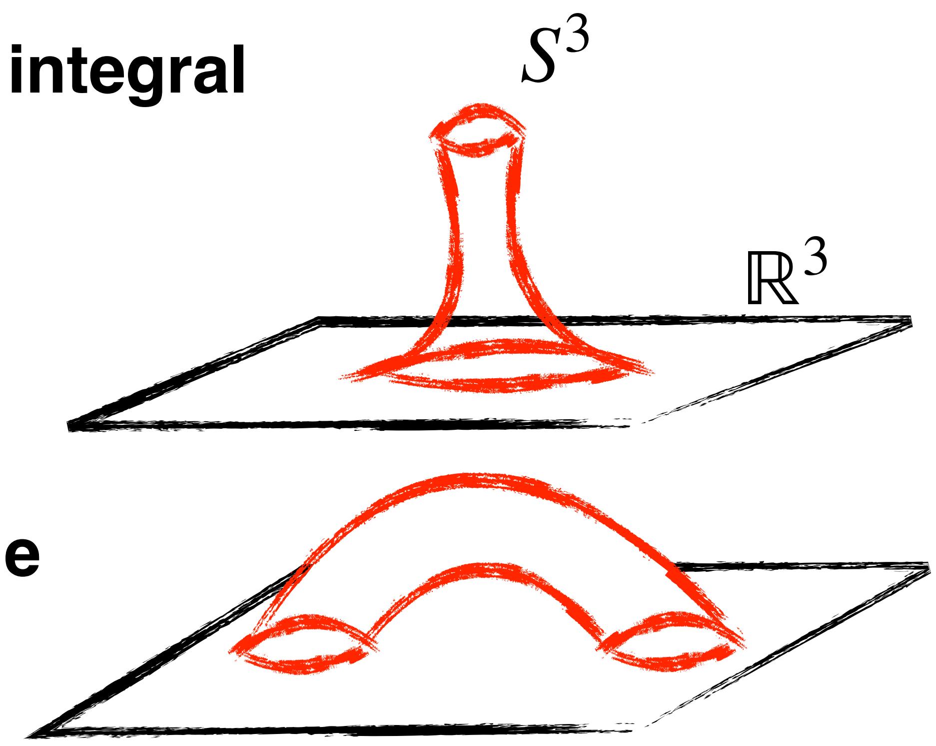
Wormholes and Global Symmetries

Wormholes allow for a quantitative assessment of the issue

Euclidean Wormhole basics

See review of Hebecker, Mikhail & Soler [1807.00824] and also Alonso & Urbano [1706.07415]
Pioneering references include: Hawking (1987), Giddings & Strominger (1987), Coleman (1988), Lee (1988), Abbott & Wise (1989), Coleman & Lee (1990), Kallosh, Linde, Linde & Susskind (1995)

- **They represent saddle points in the Euclidean path integral**
- **As such they mediate topology change transitions:**
- **These wormholes are supported because a given amount of global charge flows through their throat**
- **They have Planck scale sizes, and their effect for the low energy observer is as the explicit breaking of global symmetries**



- **In particular, they generate a potential of the form:**

EFT $E \ll M_{\rm Pl}$

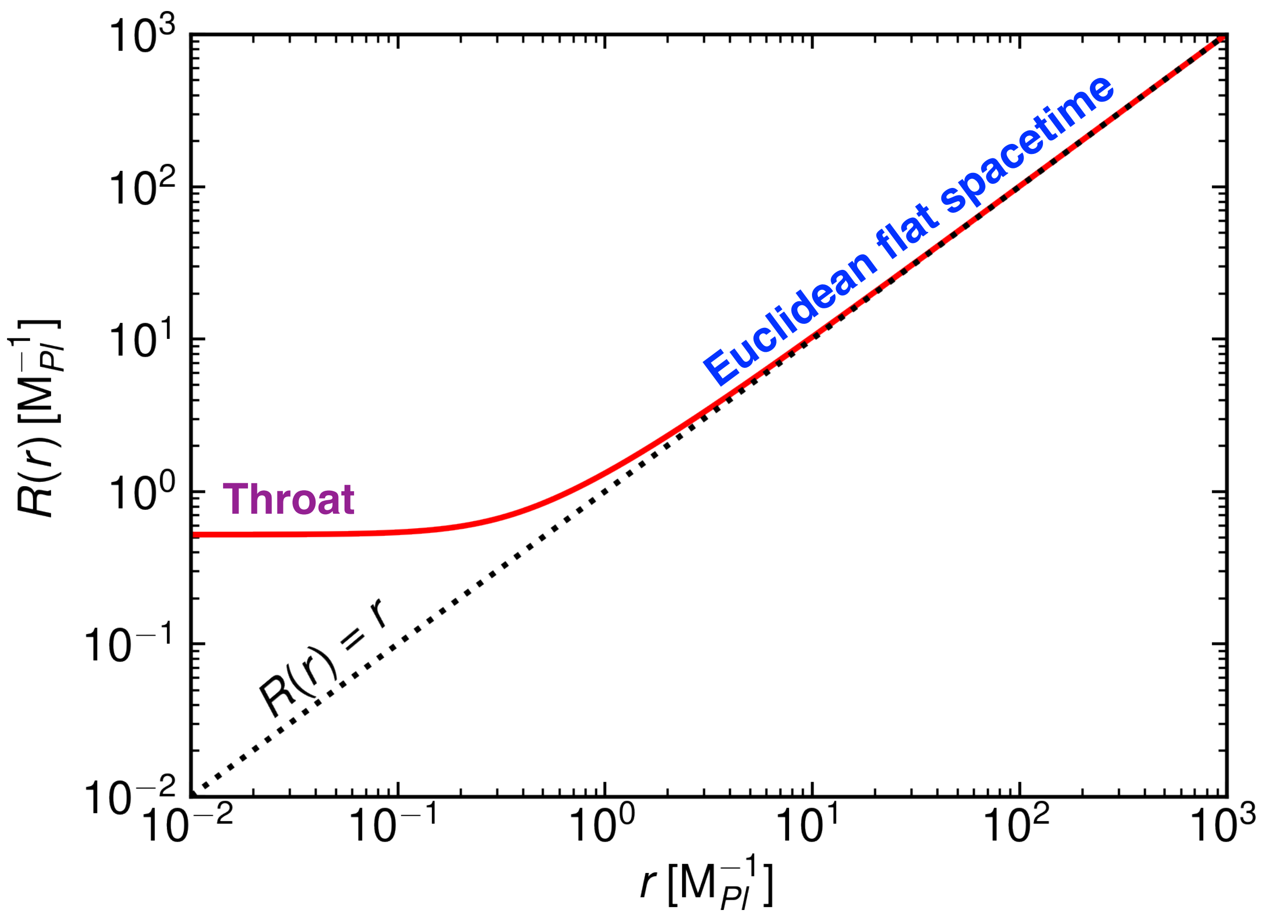
$$\Delta V = \sum_{n=1}^{\infty} \alpha_n |\lambda_n| e^{i\beta_n} M_{\rm Pl}^{4-n} \Phi^n + \mathrm{h.c.}, \qquad \lambda_n = e^{-S_n}$$

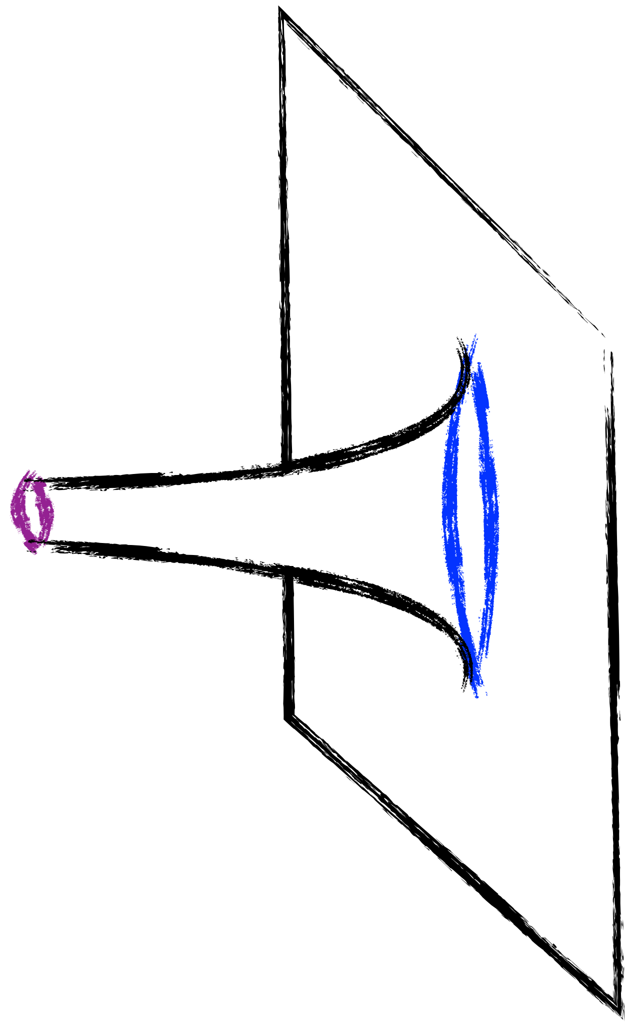
$$n = 1 \to \Delta V_1 \sim M_{\rm Pl}^4 e^{-S_1} \cos\left(a/f_a + \beta\right)$$ **(Planck Enhanced)** $S > 190$

Wormhole Dynamics

A Wormhole: $$ds_E^2 = dr^2 + R(r)^2 d^2\Omega_3$$

(closed FLRW metric but in Euclidean spacetime)





The Action: $$S_E = \int_{\mathcal{M}} d^4x \sqrt{g} \left[-\frac{M_{\rm Pl}^2}{16\pi} \mathcal{R} \right] + S_{\rm Matter}(\phi_i, \lambda_i) + S_G^{\rm high}(\mathcal{R}^2, \dots) + \int_{\delta\mathcal{M}} dS_3$$

Leading approximation: $$S_E \simeq M_{\rm Pl}^2 L^2 + \text{dynamics} + \text{surface}$$

Wormhole Actions: A short timeline in 4D

Scenario:	Action Scaling	Quality Problem?
An axion minimally coupled to gravity Giddings & Strominger [NPB 1988]	$S \sim M_{\rm Pl}/f_a$	No
An axion+the dilaton Giddings & Strominger [NPB 1988]	$S \sim \frac{1}{g_s} M_{\rm Pl}/f_a$	No
An unbroken U(1) symmetry in GR Abbott & Wise [NPB 1989] Coleman & Lee [NPB 1990]	$S = \infty$ $S \sim \log(M_{\rm Pl}/m)$	Yes
An SSB U(1) symmetry in GR Abbott & Wise [NPB 1989] Kallosh, Linde, Linde & Susskind [PRD 1995]	$S \sim \log(M_{\rm Pl}/f_a)$	Yes
An SSB U(1) within many set ups Kallosh, Linde, Linde & Susskind [PRD 1995]	$S \sim \log(M_{\rm Pl}/f_a)$	Yes
An SSB U(1)+the dilaton Alvey & Escudero (2020)	$S \sim \frac{1}{g_s} \log(M_{\rm Pl}/f_a)$	Yes (unless $g_s < 0.1$)

The Giddings & Strominger Wormhole

Action: $$S_E = \int_{\mathscr{M}} \mathrm{d}^4x\,\sqrt{g}\left[-\frac{M_{\rm Pl}^2}{16\pi}\mathscr{R} + \frac{1}{2}f_a^2\partial_\mu\theta\partial^\mu\theta\right] - \frac{M_{\rm Pl}^2}{8\pi}\int_{\delta\mathscr{M}} dS_3\sqrt{g^{(3)}}\,(K-K_0)$$

where $\theta = a/f_a$

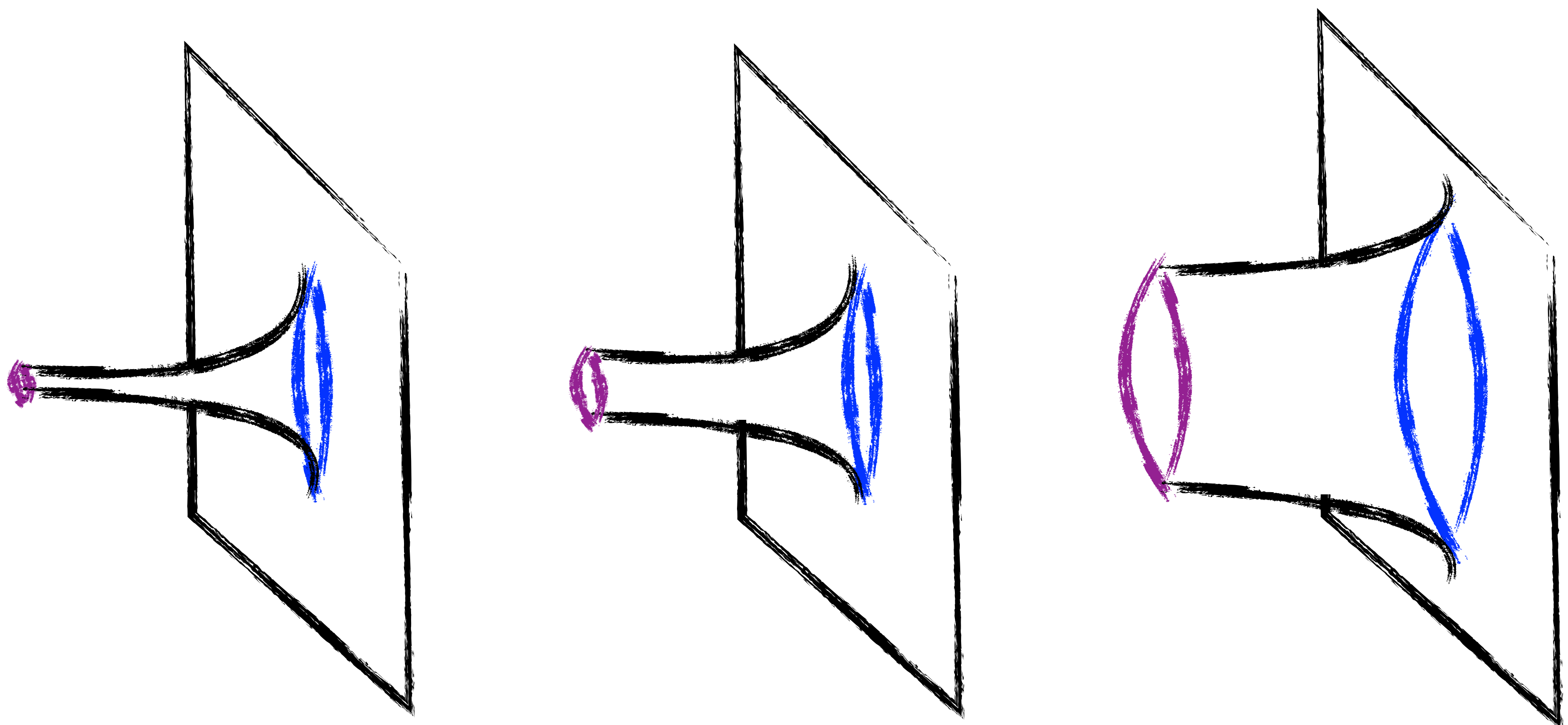
Charge conservation: $R(r)^3\, f_a^2\,\theta'(r) = \dfrac{n}{2\pi^2}$

EOM $R'^2 = 1 - \dfrac{L^4}{R^4}$

$$L \simeq \frac{1}{\sqrt{M_{\rm Pl} f_a}}$$

$$S \simeq M_{\rm Pl}^2 L^2 \simeq M_{\rm Pl}/f_a$$

$f_a = 10^{16}\,\mathrm{GeV}$ $f_a = 10^{15}\,\mathrm{GeV}$ $f_a = 10^{14}\,\mathrm{GeV}$



Interpretation:
The flow of global charge requires a larger wormhole as fa is reduced.
This generates a large value for the action because the surface of the throat is larger.

SSB U(1) in Einstein Gravity

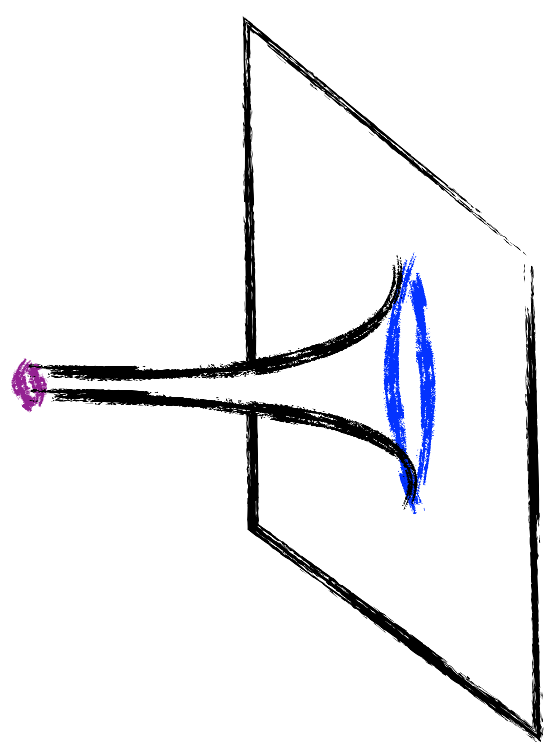
Action: $S_E = \int_{\mathcal{M}} \mathrm{d}^4x\sqrt{g}\left[-\frac{M_{\rm Pl}^2}{16\pi}\mathcal{R} + \frac{1}{2}\partial_\mu f\partial^\mu f + \frac{1}{2}f^2\partial_\mu\theta\partial^\mu\theta + V(f)\right] - \frac{M_{\rm Pl}^2}{8\pi}\int_{\delta\mathcal{M}} dS_3\sqrt{g^{(3)}}\,(K-K_0)$

Potential: $V(f) = \frac{\lambda_\Phi}{4}\left(f^2 - f_a^2\right)^2$ **Charge conservation:** $R(r)^3\, f(r)^2\,\theta'(r) = \frac{n}{2\pi^2}$

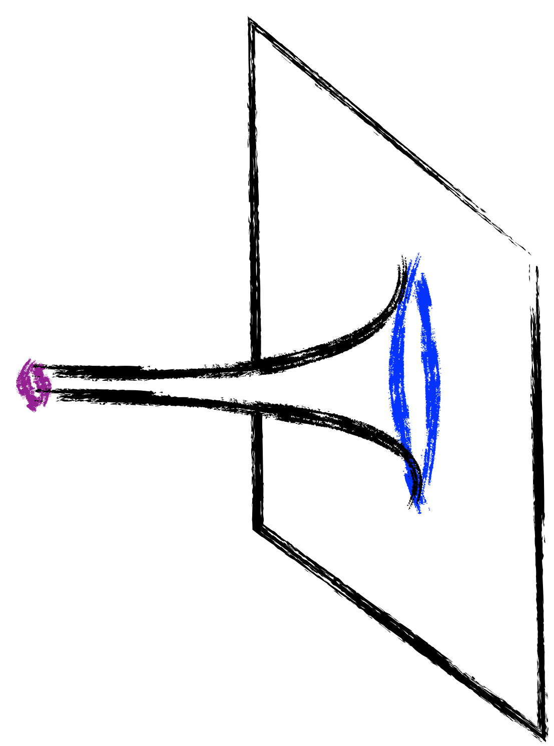
EOM: $R'^2 = 1 - R^2\left(\frac{8\pi}{3M_{\rm pl}^2}\right)\left(V(f) - \frac{1}{2}f'^2 + \frac{n^2}{8\pi^4 f^2 R^6}\right)$ $f'' + \frac{3R'f'}{R} = \frac{\mathrm{d}V}{\mathrm{d}f} - \frac{n^2}{4\pi^4 f^3 R^6}$

at r = 0 we have $R'(0) = f'(0) = 0$ $\longrightarrow$ $f(0) \simeq M_{\rm Pl}\,,\; L \sim 1/M_{\rm Pl}$ $\longrightarrow$ $S \sim M_{\rm Pl}^2 L^2 \sim \mathcal{O}(1)$

$f_a = 10^{16}\,\mathrm{GeV}$



$f_a = 100\,\mathrm{GeV}$



Key result of Kallosh et al.:

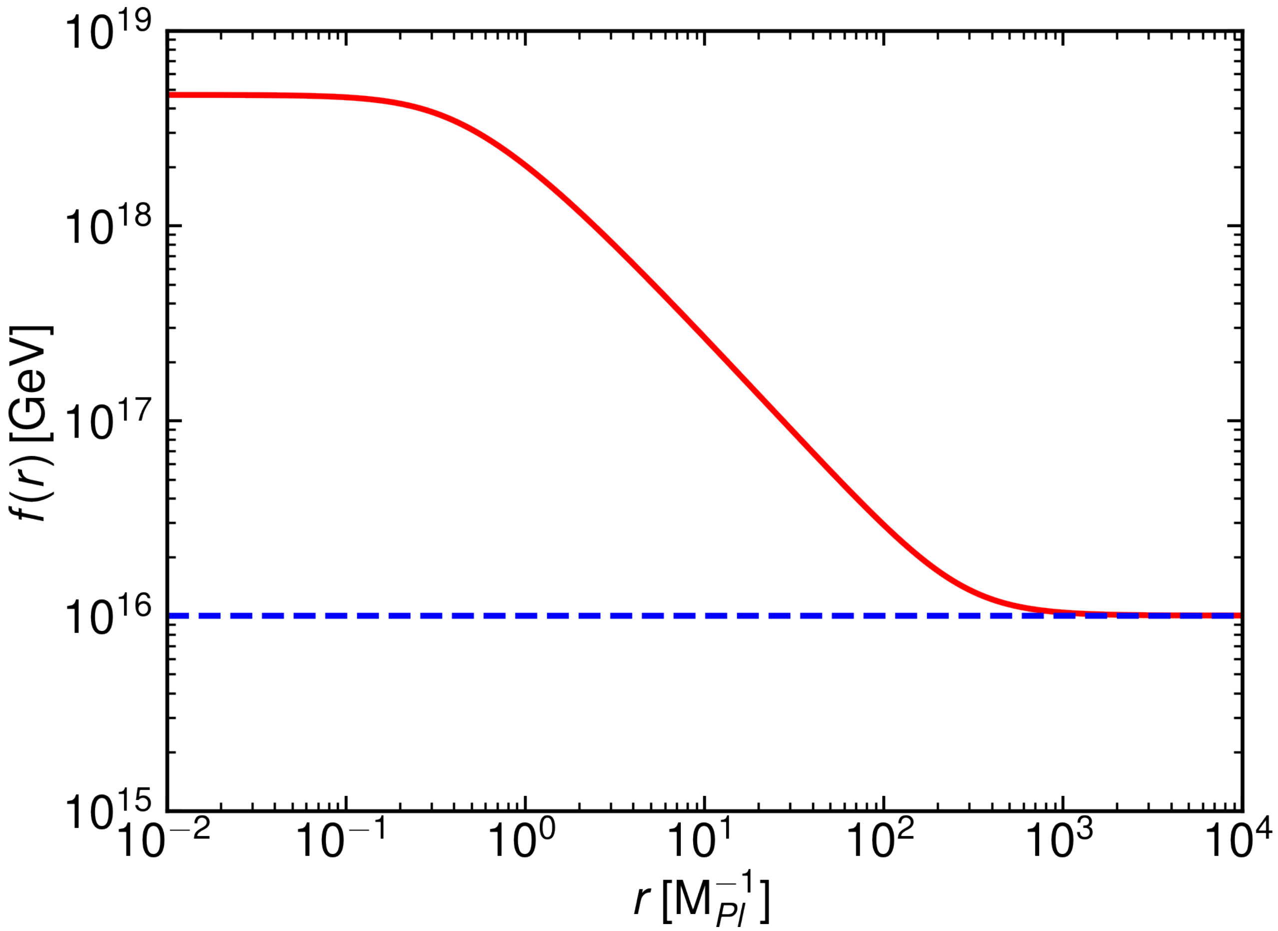
If the radial field is allowed to vary, it is virtually impossible to make the wormhole larger than $L \sim 1/M_{\rm Pl}$ and hence $S \sim \mathcal{O}(1)$.

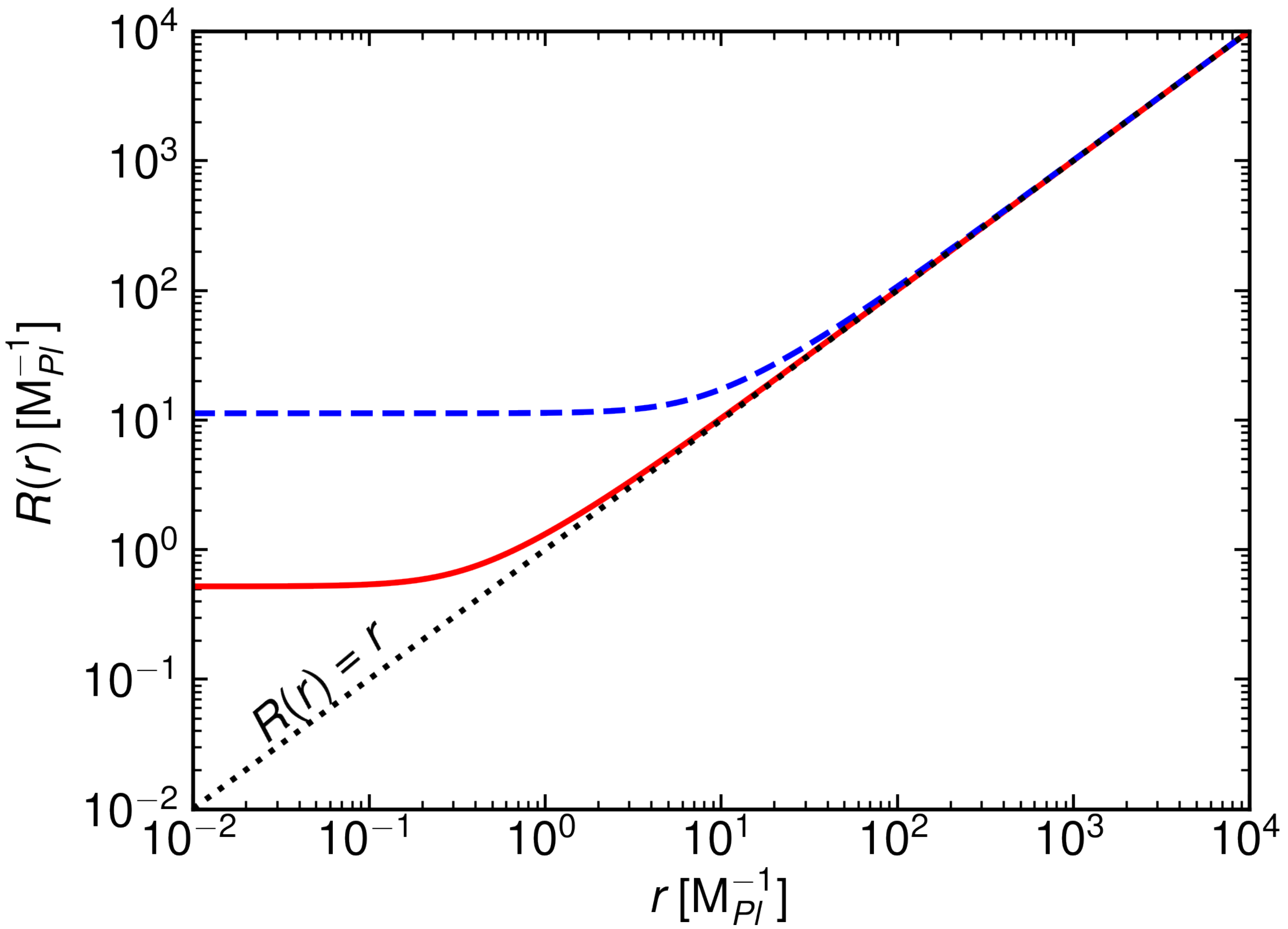
Result holds including $\mathcal{O}(\mathcal{R}^2)$ corrections and with very exotic (and extreme) potentials

A close look at the solutions

Radial Mode dynamical

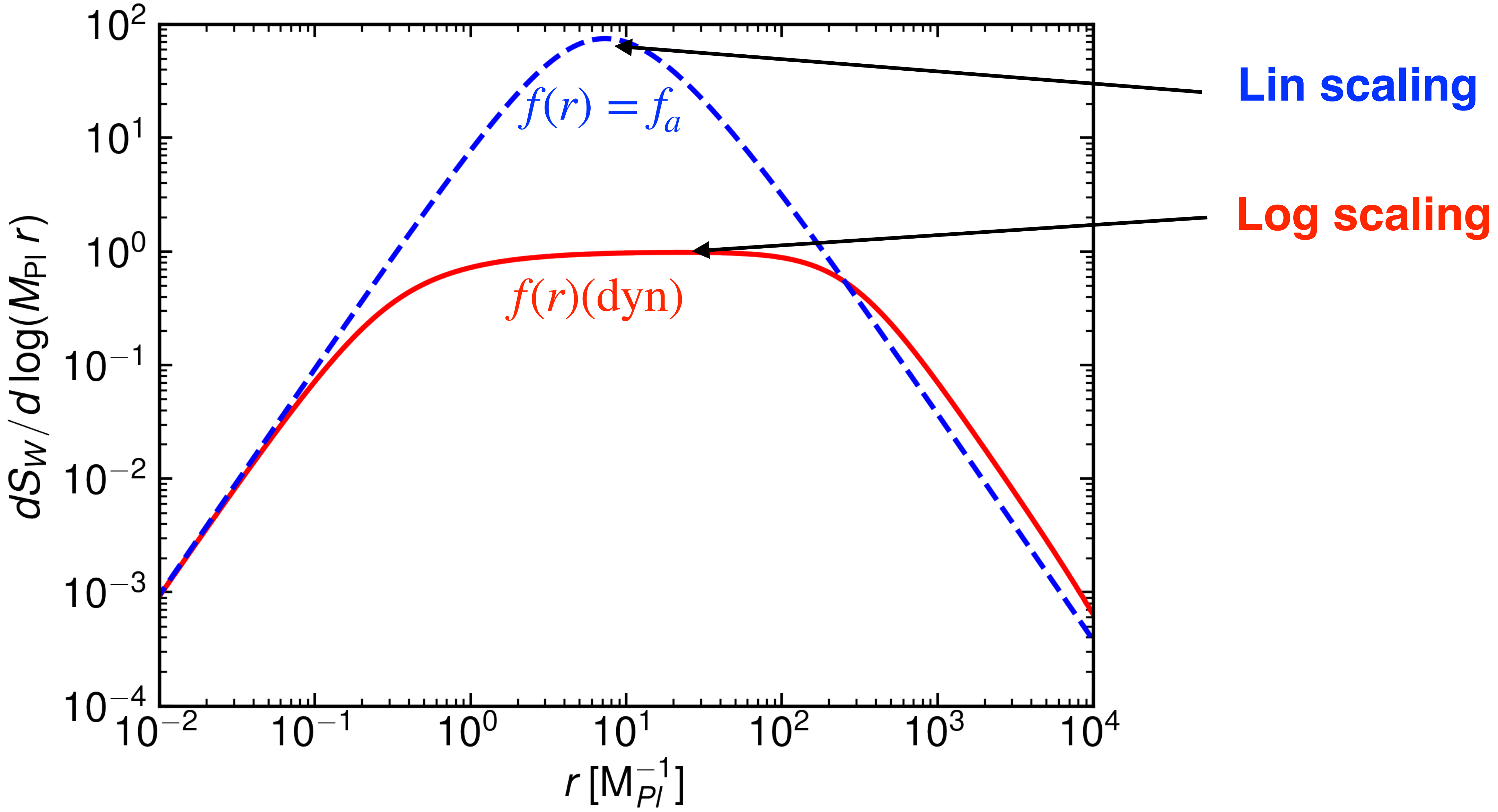
Radial Mode non-dynamical





Example with $f_a = 10^{16}\,\mathrm{GeV}\,,\ \lambda_\Phi = 0.01$

Where is the Log coming from?



Action: $S_E = 2\pi^2 \int_0^\infty dr \left[R^3 (f')^2 + \frac{3M_{\rm pl}^2}{4\pi} R R'(1-R') \right]$ $\quad f'' + \frac{3R'f'}{R} = \frac{dV}{df} - \frac{n^2}{4\pi^4 f^3 R^6} \quad f(r) \simeq \sqrt{n/2\pi^2}/r$

outside the wormhole: $S_E = 2\pi^2 \int_{r_-}^{r^+} dr \left[r^3 (f')^2 \right] \simeq n \int_{r_-}^{r^+} dr \left[r^3 (1/r)^2 \right] = n \log(r_-/r_+) \simeq n \log(M_{\rm Pl}/f_a)$

Summary

- **In the Giddings & Strominger Wormhole** $S \sim M_{\rm Pl}/f_a$ **because the size scales as** $L \sim 1/\sqrt{M_{\rm Pl} f_a}$

- **Including the dynamics of the radial mode makes the wormhole of Planck size and then** $S \sim \mathcal{O}(1)$

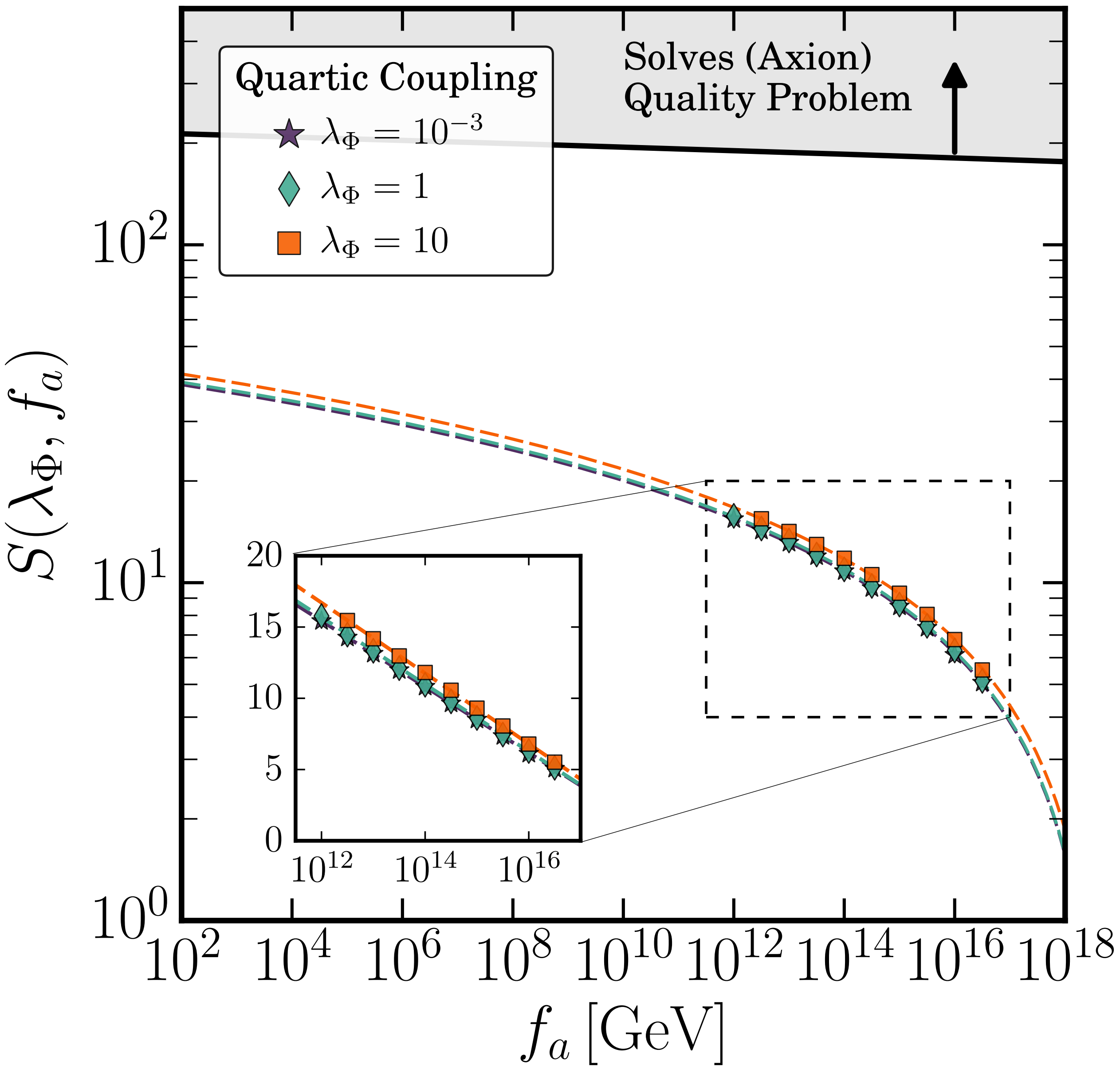
- **The small** $S \sim \log M_{\rm Pl}/f_a$ **correction is due to the work that it takes the radial mode to go from the Planck scale to** f_a **as required by the topological conditions**

Model building implications

Typical axion models do feature a radial mode which means that we should stick to:

$$S \sim \log M_{\rm Pl}/f_a$$

Thus, axions appear to have a quality problem within non-perturbative GR:



Model building implications

This is yet relevant for gauge protected axions:

$$\Delta V = \lambda_n M_{\rm Pl}^4 \left[\frac{|\Phi|}{M_{\rm Pl}}\right]^{d-n} \left[\frac{\Phi}{M_{\rm Pl}}\right]^n + \text{h.c.}$$

$$\lambda_n \sim \mathcal{O}(1)$$

$$\lambda_n \lesssim \left(\frac{f_a}{M_{\rm Pl}}\right)^n$$

$$d > \frac{12}{1 - \frac{1}{17.3}\log\left(\frac{f_a}{4\times 10^{11}\,\text{GeV}}\right)}$$

$$d > \frac{12}{1 - \frac{1}{17.3}\log\left(\frac{f_a}{4\times 10^{11}\,\text{GeV}}\right)} - n$$

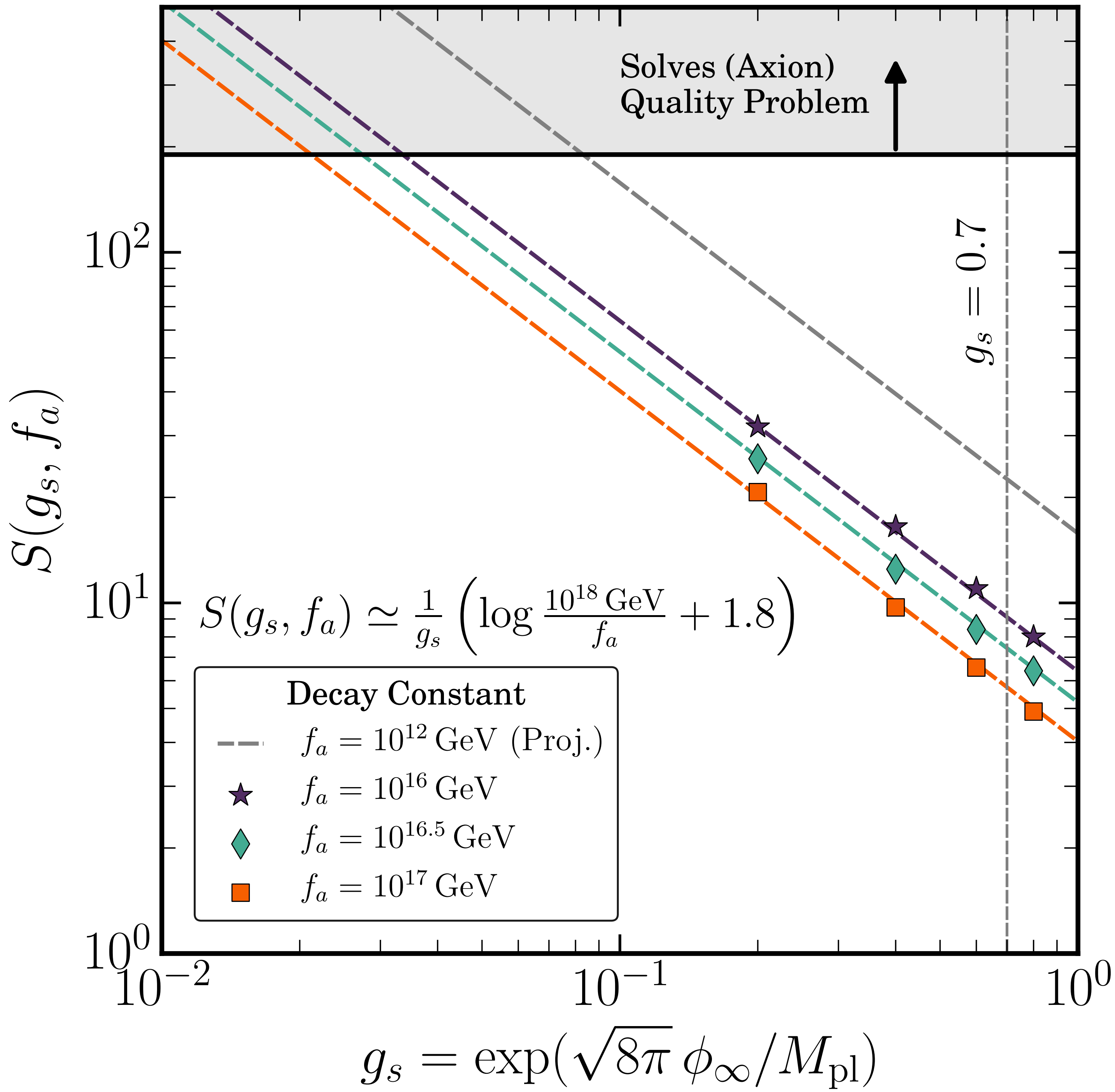
- **Thus, the dimensionality of allowed operators would be lower than expected with** $\lambda_n \sim \mathcal{O}(1)$

Extending Gravity?

Low-energy limit of an open String Theory with the dilaton in 4D:

$$S \simeq \frac{1}{g_s} \log(M_{\rm Pl}/f_a)$$

- **In this very simple set up, unless $g_s < 0.1$ there is an axion quality problem**

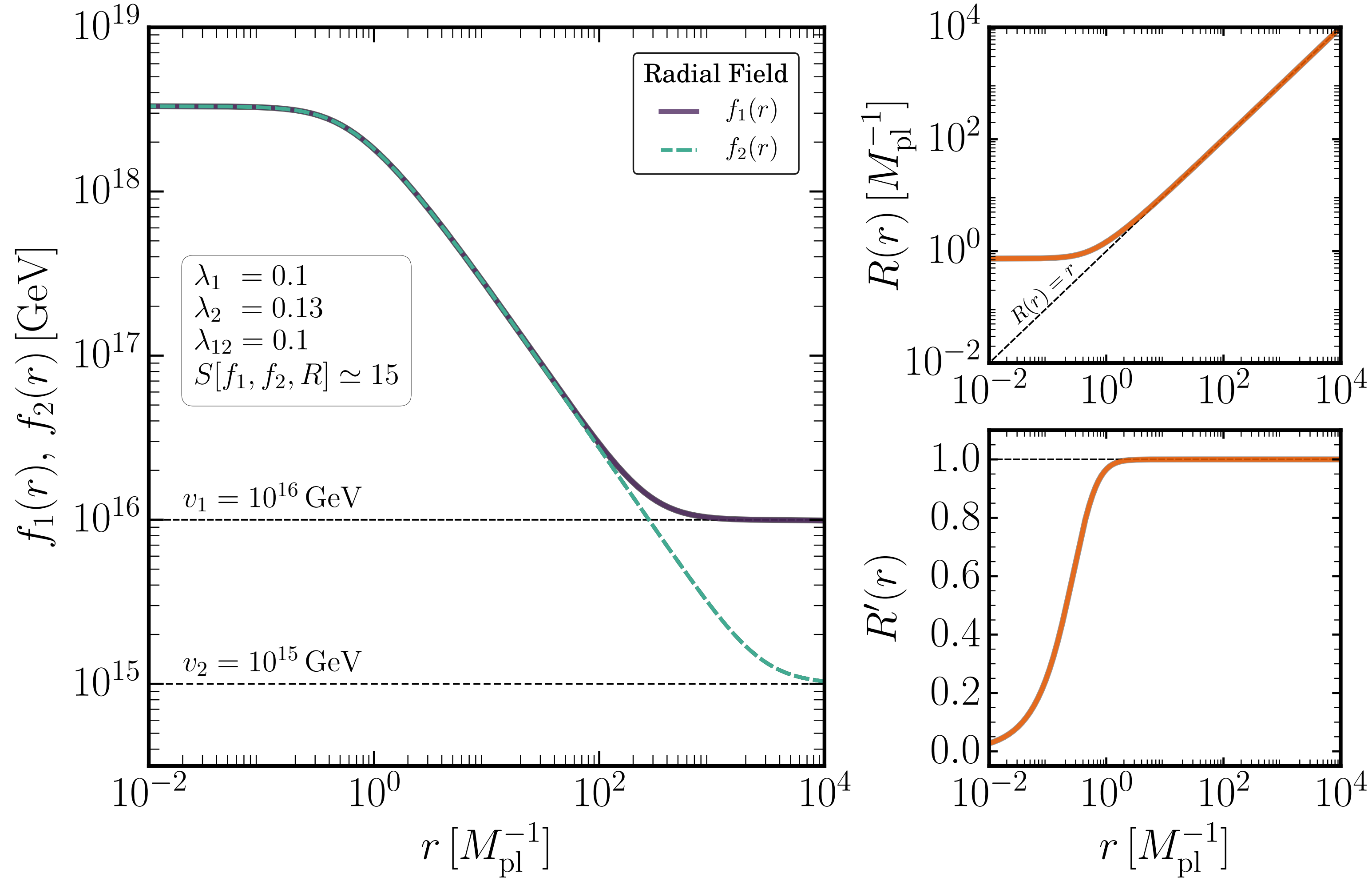


What happens if you add particle content?

$U(1)_1 \times U(1)_2$

$$S \simeq n_1 \log(M_{\rm Pl}/v_1) + n_2 \log(M_{\rm Pl}/v_2)$$

The action is largely independent on the interactions between the fields



- **The two pseudoGoldstone bosons become super heavy and strongly mixed**

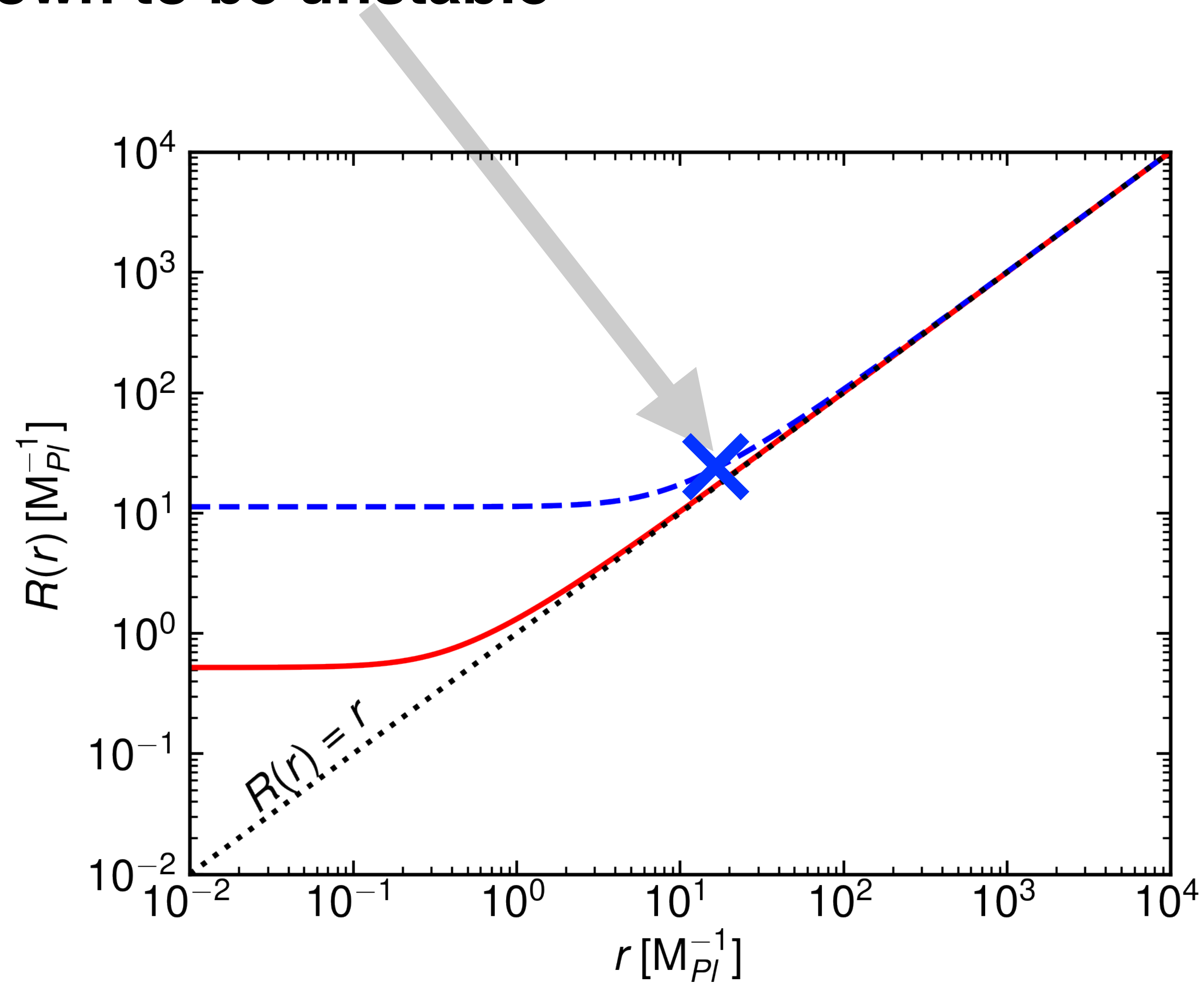
$$\tan 2\gamma = \frac{2 v_1 v_2}{v_1^2 - v_2^2}$$

Wormholes Caveats & Outlook

- **Stability: Are wormholes really saddle points in the Euclidean Action?**
  Rubakov & Shvedov [gr-qc/9604038, gr-qc/9608065], Alonso & Urbano [1706.07415]

In fact, the Giddings and Strominger wormhole has shown to be unstable

Hertog, Truijen & Van Riet [PRL 1811.12690]



- **But the one including a radial field has not been yet studied in the literature**
  In preparation, Alvey & Escudero

Wormholes Caveats & Outlook

High Energies and String Theory

Our results use initial conditions at $r = 0$, i.e. $E = \infty$. Something can clearly break down there

Maybe decompactification changes the picture?

Wormhole Interpretation: Quantum Gravity and Alpha parameters

- **Recent analysis suggests that the interpretation of Coleman and Giddings & Strominger holds within AdS/CFT** — Marolf & Maxfield [2002.08950]
- **In addition, recent studies suggest that wormholes can play an important role in Quantum Gravity (maybe not axionic wormholes)** — Saad, Shenker & Stanford [1903.11115]; Penington, Shenker, Stanford & Yang [1911.11977]; Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini [1911.12333]
- **In that case, we are left with the alpha parameters in front of the operators generated by wormholes. This limits the predictability.**
- **It has been argued that:** $\dim(\mathscr{H}_{WH}) = 1$ — McNamara & Vafa [2004.06738]
- **Perhaps arguments can be made to shape the single wormhole state.**

Wormholes are not the end of the story but may be behind part of it

- **Breaking of global symmetries by Black Holes at finite temperature:** Fichet & Saraswat [1909.02002]
- **Breaking of global symmetries by Black Hole evaporation in quantum gravity**
  Harlow & Shaghoulian [2010.10539], Chen & Lin [2011.06005], Hsin, Iliesiu & Yang [2011.09444]

Summary & Take Away Messages

- We have extensively studied Euclidean wormhole solutions with axions featuring Spontaneous Symmetry Breaking

- The dynamics of wormholes we consider appear to be broadly independent of additional particle content and interactions

- The dynamics of the dilaton does not alter the log Mpl/fa behaviour of the action, it simply modulates the action

Conclusions

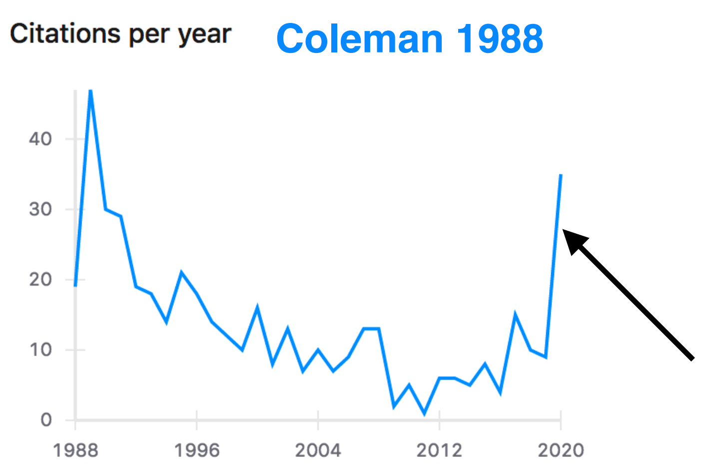
- The efficiency of the Peccei-Quinn solution is strongly dependent upon explicit sources of $U(1)_{\rm PQ}$ symmetry breaking

- Wormholes represent a “controlled” system to investigate gravitational symmetry breaking

- Within this set up, typical axion models appear to have a quality problem within non-perturbative Einstein Gravity

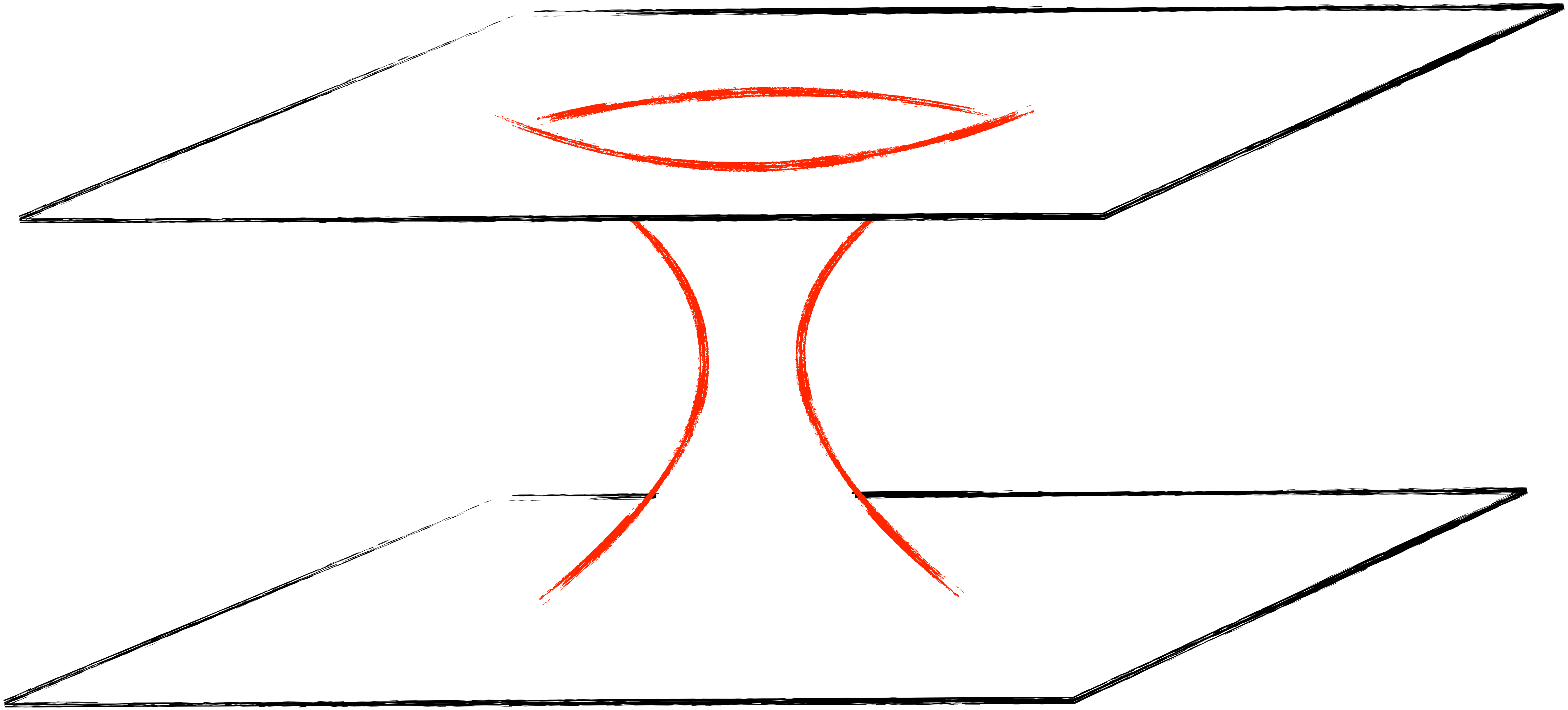
- Wormholes seem to lead to large breaking of global symmetries that would be relevant for model building of gauge protected axions

- Of course, statements based on wormholes are subject to several caveats but that may nonetheless be resolved soon



Time for questions and comments

Thank



You

Back Up

Full Equations U(1)

Action $$S_E = \int_{\mathcal{M}} \mathrm{d}^4x\,\sqrt{g}\left(-\frac{M_{\rm pl}^2}{16\pi}\mathcal{R} + \frac{1}{2}\partial_\mu f \partial^\mu f + \frac{1}{2} f^2 \partial_\mu\theta\partial^\mu\theta + V(f)\right) - \frac{M_{\rm pl}^2}{8\pi}\int_{\partial\mathcal{M}} \mathrm{d}S_3 \sqrt{g^{(3)}}(K - K_0)$$

Charge flow $$\partial_\mu(\sqrt{g} f^2 \partial^\mu\theta) = 0 \qquad R(r)^3 f(r)^2 \theta'(r) = \frac{n}{2\pi^2}, \quad n \in \mathbb{N}$$

Stress-Energy $$T_{\mu\nu} = \partial_\mu f \partial_\nu f - f^2 \partial_\mu\theta\partial_\nu\theta - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma f \partial^\sigma f - \frac{1}{2} f^2 \partial_\sigma\theta\partial^\sigma\theta + V(f)\right)$$

Field eq. $$f'' + \frac{3R'f'}{R} = \frac{\mathrm{d}V}{\mathrm{d}f} - \frac{n^2}{4\pi^4 f^3 R^6}$$

$$V(f) = \frac{\lambda_\Phi}{4}(f^2 - f_a^2)^2$$

Hubble $$R'^2 = 1 - R^2\left(\frac{8\pi}{3M_{\rm pl}^2}\right)\left(V(f) - \frac{1}{2}f'^2 + \frac{n^2}{8\pi^4 f^2 R^6}\right)$$

Acceleration $$\frac{R''}{R} = -\frac{8\pi}{3M_{\rm pl}^2}\left(V(f) + (f')^2 - \frac{n^2}{4\pi^4 f^2 R^6}\right)$$

Action: $$S_E = 2\pi^2 \int_0^\infty \mathrm{d}r \left(R^3 (f')^2 + \frac{3M_{\rm pl}^2}{4\pi} R R'(1 - R')\right)$$

Full Equations U(1)+Dilaton

Action $$S_E = \int d^4x\sqrt{g}\left(-\frac{M_{Pl}^2}{16\pi}\mathcal{R}+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+e^{\beta\phi\frac{\sqrt{8\pi}}{M_{\rm pl}}}\left(\frac{1}{2}\partial_\mu f\partial^\mu f+\frac{1}{2}f^2\partial_\mu\theta\partial^\mu\theta+V(f)\right)\right)$$

Charge flow $$R^3 f^2\exp(l_\alpha\phi)\theta' = n/(2\pi^2) \qquad l_\alpha = \sqrt{8\pi}/M_{\rm pl} \qquad \beta = 1$$

$$V(f) = \frac{\lambda_\Phi}{4}(f^2-f_a^2)^2$$

Dilaton $$\phi'' + 3\frac{R'}{R}\phi' = l_\alpha e^{l_\alpha\phi}\left(V(f)+\frac{1}{2}(f')^2 - \frac{n^2}{8\pi^4 e^{2l_\alpha\phi}f^2R^6}\right)$$

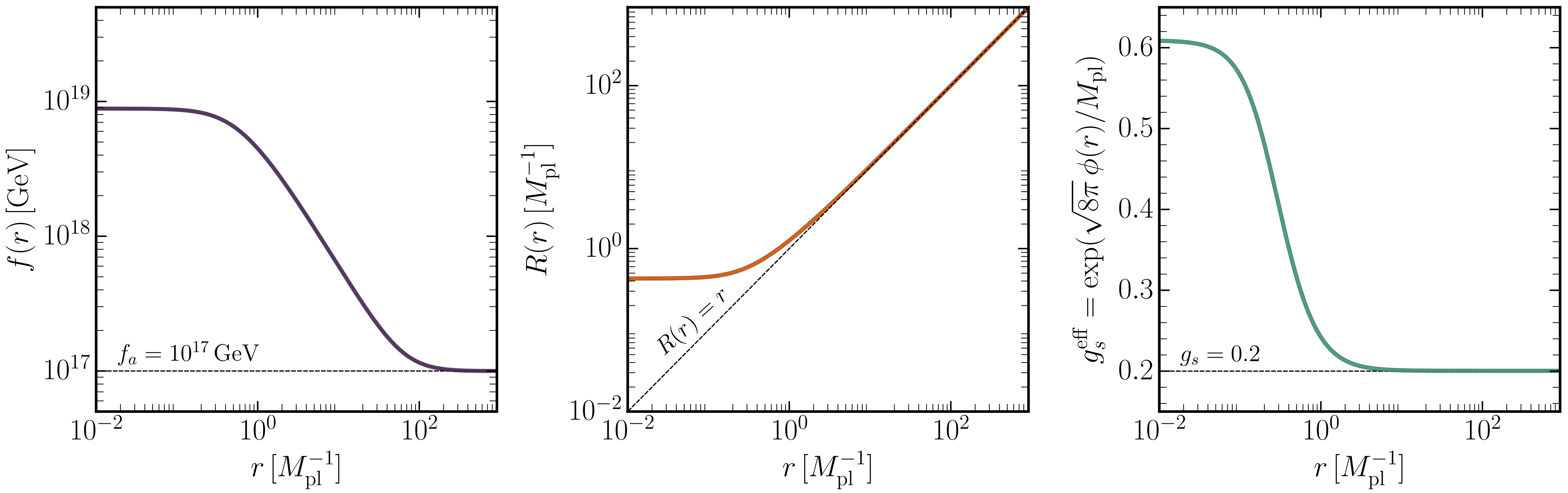
Field eq. $$f''+3\frac{R'}{R}f'+l_\alpha\phi' f' = \frac{\mathrm{d}V}{\mathrm{d}f} - \frac{n^2}{4\pi^4e^{2l_\alpha\phi}f^3R^6}$$

Hubble $$R'^2 = 1 - R^2\left(\frac{8\pi}{3M_{Pl}^2}\right)\left(-\frac{1}{2}\phi'^2+e^{l_\alpha\phi}\left[V(f)-\frac{1}{2}f'^2+\frac{n^2}{8\pi^4e^{2l_\alpha\phi}f^2R^6}\right]\right)$$

Acceleration $$\frac{R''}{R} = -\left(\frac{8\pi}{3M_{Pl}^2}\right)\left[\phi'^2+e^{l_\alpha\phi}\left(V(f)+f'^2-\frac{n^2}{4\pi^4e^{2l_\alpha\phi}f^2R^6}\right)\right]$$

Action $$S_E = 2\pi^2\int_0^\infty \mathrm{d}r\left(R^3\left[e^{l_\alpha\phi}(f')^2+(\phi')^2\right]+\frac{3M_{\rm pl}^2}{4\pi}RR'(1-R')\right)$$

Dilaton Profiles



Action

$$S_E = 2\pi^2 \int_0^\infty \mathrm{d}r \left(R^3 \left[e^{l_\alpha \phi} (f')^2 + (\phi')^2 \right] + \frac{3M_{\rm pl}^2}{4\pi} R R' (1 - R') \right)$$

Outside the wormhole we have the dilaton at gs and gravity can be neglected so that: $S_E \simeq 2\pi^2 \int_{r_-}^{r_+} \mathrm{d}r \left(R^3 e^{l_\alpha \phi} (f')^2 \right)$

The equation for f becomes $f'' + 3\frac{1}{r} f' = \lambda f^3 - \frac{n^2}{4\pi^4 g_s^2 f^3 r^6}$ **which can be solved by:** $f \simeq \sqrt{n/(2\pi^2 g_s)}/r$

The we have $S_E = 2\pi^2 \int_{r_-}^{r^+} \mathrm{d}r \left[r^3 (f')^2 \right] \simeq \frac{n}{g_s} \int_{r_-}^{r^+} \mathrm{d}r \left[r^3 (1/r)^2 \right] \simeq \frac{n}{g_s} \log(r_-/r_+) \simeq \frac{n}{g_s} \log(M_{\rm Pl}/f_a)$

Alpha parameters

$$e^{I} = \int \frac{d\alpha}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\alpha^2 + \alpha\sqrt{\Delta}\int d^4x\sqrt{g}\,\mathcal{O}(x)\right)$$

Some cool cartoons