Milgromian dynamics (MOND):
Theoretical motivations & cosmological context

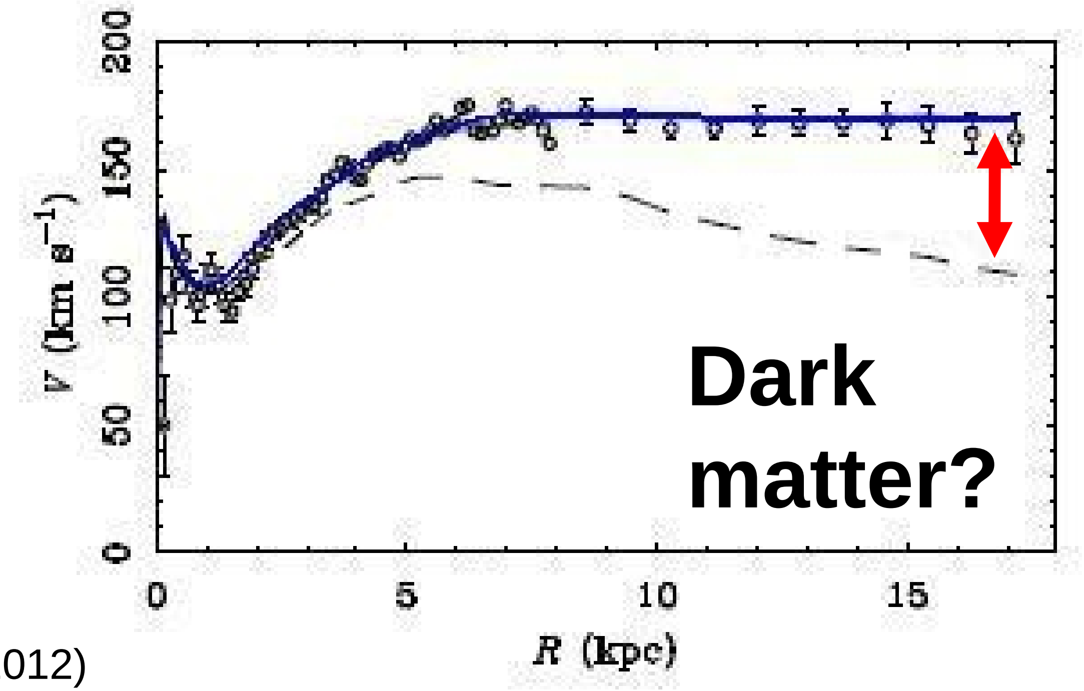
Authors: Moritz Haslbauer, **Indranil Banik** & Pavel Kroupa

Publication: The KBC void and Hubble tension contradict ΛCDM on a Gpc scale – Milgromian dynamics as a possible solution (MNRAS, 499, 2845)

Galaxies

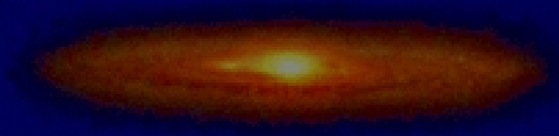
Visible mass X Newtonian Gravity = Acceleration

- **The observed acceleration is discrepant with this prediction**



Living Reviews in Relativity, **15**, 10 (2012)

No direct evidence



Newton gravity

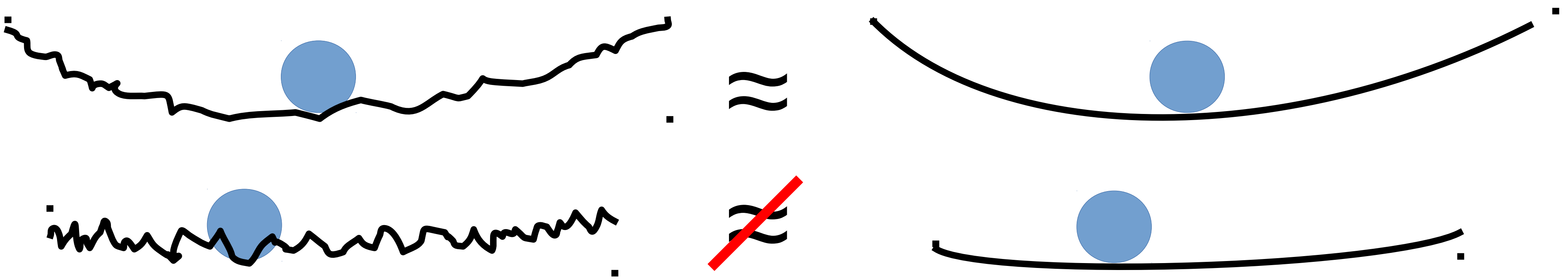
- The circular rotation speed around a mass is given by

$$g = \frac{GM}{r^2} = \frac{v^2}{r}$$

- If $v \approx c$, relativity becomes important and the theory fails (e.g. near black holes)
- What about quantum mechanics?

The classical approximation

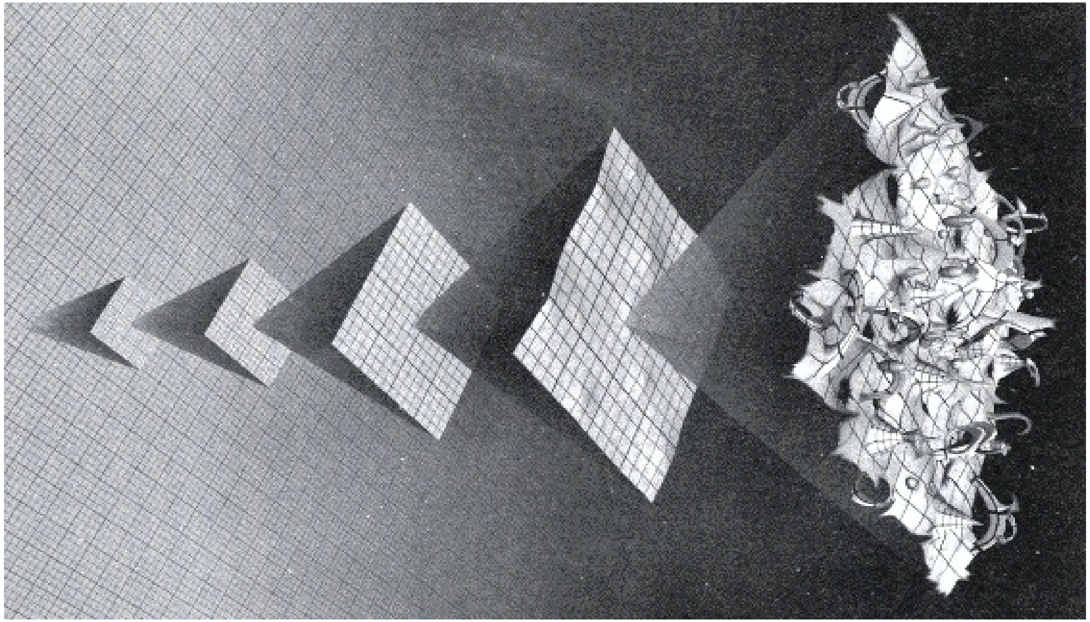
- **Arbitrarily high accuracy in position and velocity measurements impossible**
- **Classical theories assume this is possible**



- **Curvature (i.e. acceleration) so small in second panel that ignoring fluctuations is questionable**

Quantum effects

- **Curvature uncertain and not constant, like gravitational waves: vacuum carries energy**

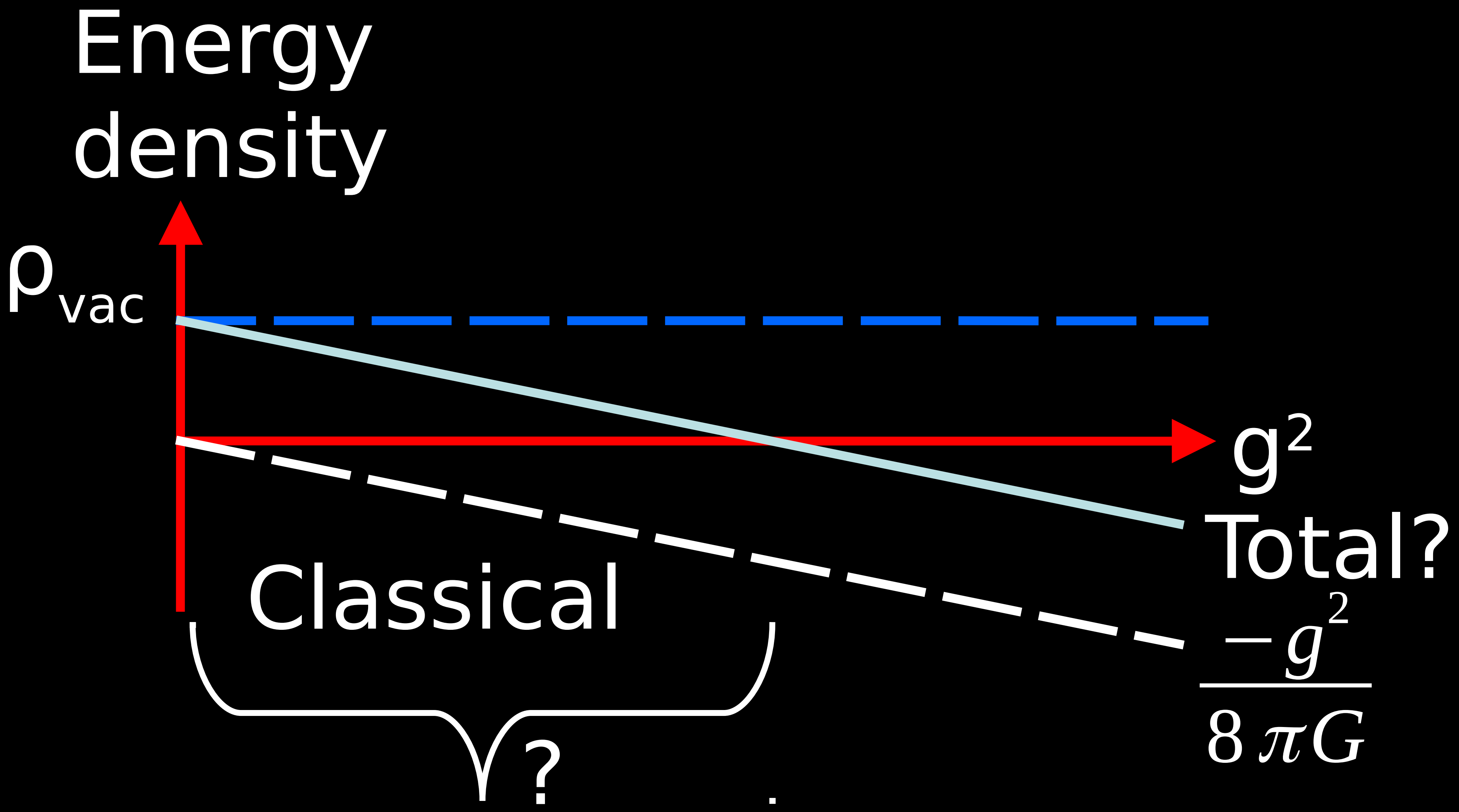


Quantum spacetime

- Empty space has small but non-zero minimum energy
- On large scales, this causes the Universe to accelerate apart – dark energy (measurable)
- Use dark energy density to estimate when quantum effects overwhelm classical (mean) gravitational field

(e.g. Milgrom 1999, Pazy 2013, Verlinde 2016, Smolin 2017, Bagchi & Fring 2019)

$$\frac{g^2}{8\pi G} = \rho_{vac} \quad \Leftrightarrow \quad g = 9\times10^{-10} m/s^2$$



Energy density
ρ_{vac}
g^2
Total?
$\frac{-g^2}{8\pi G}$
Classical
?

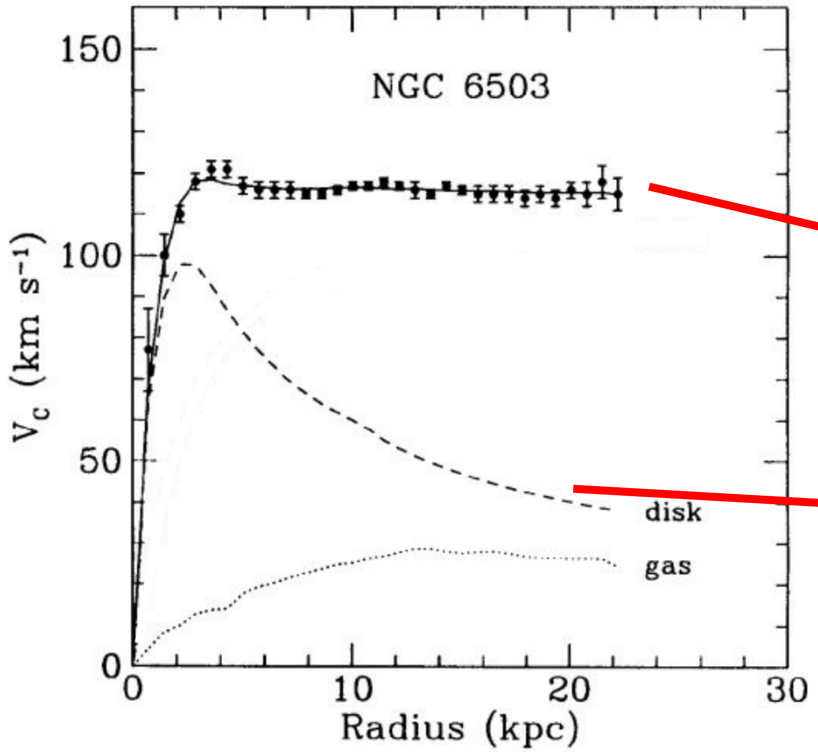
What if $g < 9e\text{-}10\ m/s^2$?

- Quantum gravity effects should become important (ignoring ‘roughness’ is a bad idea)
- Classical theories like General Relativity (leading to Newtonian dynamics) can’t really be expected to work any more

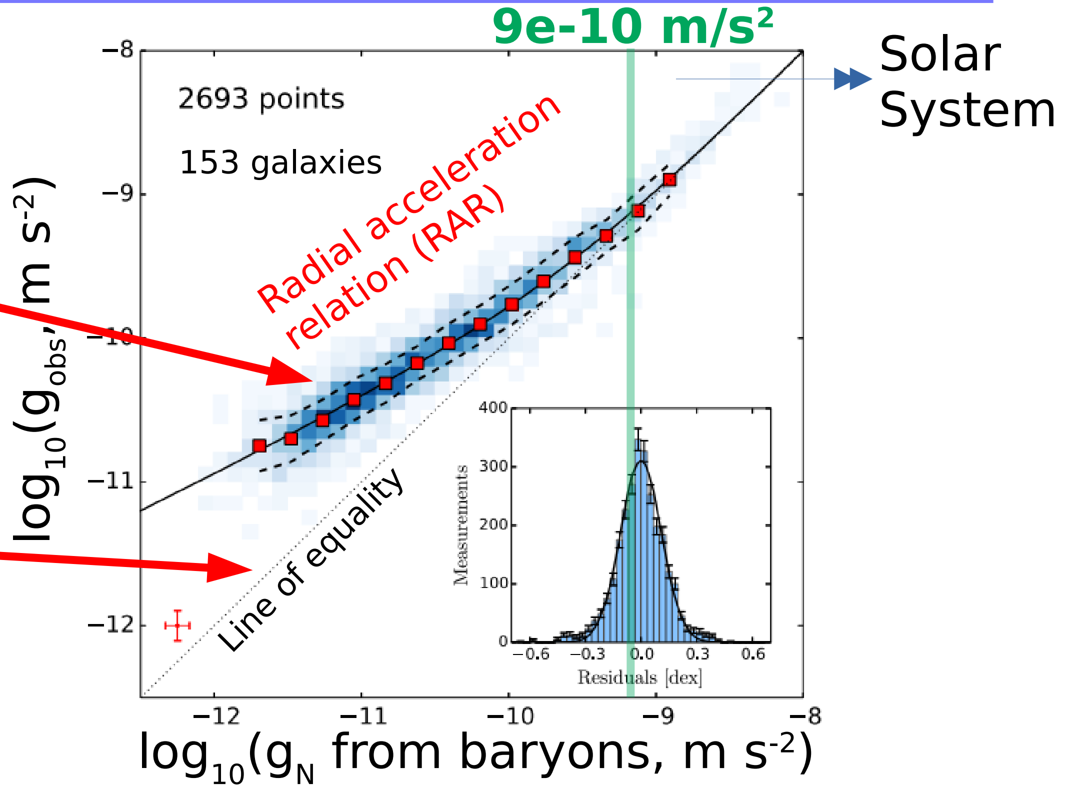
  ⇒ Newtonian gravity likely fails, with the discrepancy larger at smaller accelerations

  ⇒ Acceleration (energy density)-dependent discrepancy with classical theory may be signature of quantum effects

Constraints from galaxies



Freese 2008



McGaugh, Lelli, Schombert 2016

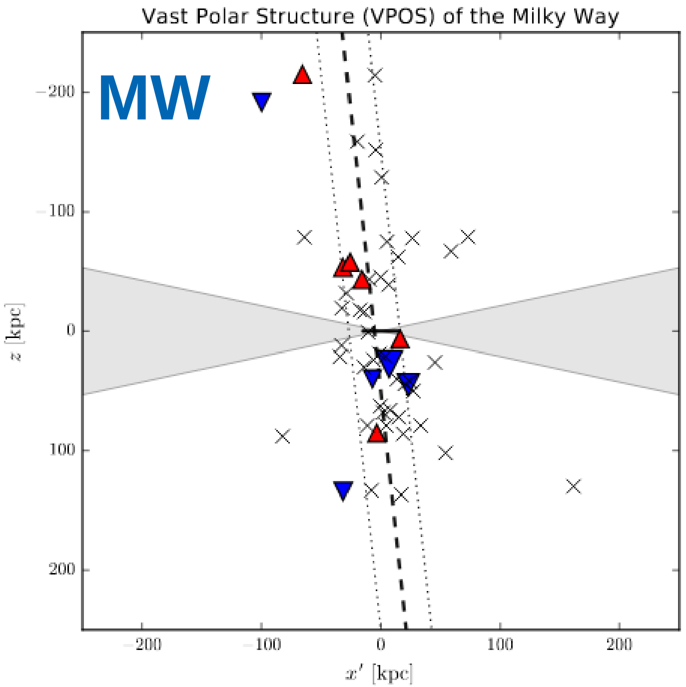
Local Group satellite planes

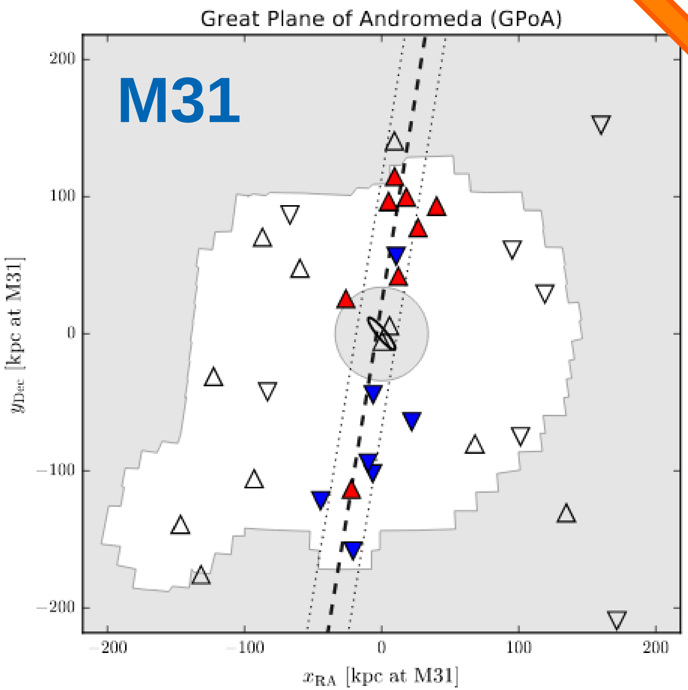
MW satellite galaxies lie within a thin plane (Pawlowski & Kroupa 2013, 2020). Analogous situation for M31 (Ibata+ 2013)

Galaxies observed forming within tidal tails (Mirabel+ 1992)

Satellites were formed from tidal debris. Alternatives not very likely (Pawlowski+ 2014, and references therein)

Should only contain baryons as DM can't cool and form dense tidal tails (Wetzstein+ 2007)





MW and M31 satellite galaxies have high internal velocity dispersions, requiring strong self-gravity (McGaugh & Wolf, 2010; McGaugh & Milgrom 2013)

Internal dynamics can't be explained by Newtonian gravity (Kroupa, 2015)

Leibniz Institute for Astrophysics Potsdam (AIP)
dwarf galaxies/near-field cosmology

- **Leibniz Junior Research Group led by Dr Marcel S. Pawlowski (mpawlowski@aip.de) at Potsdam**
- **"Cosmic Choreographies – Studying Systems of Satellite Galaxies and Their Phase-Space Correlations"**
  - **1 postdoctoral position (up to 5 years)**
  - **2 PhD students**
- **Application deadline: Thursday, December 31, 2020 (bewerbung_2020-20@aip.de)**
- **More information:**

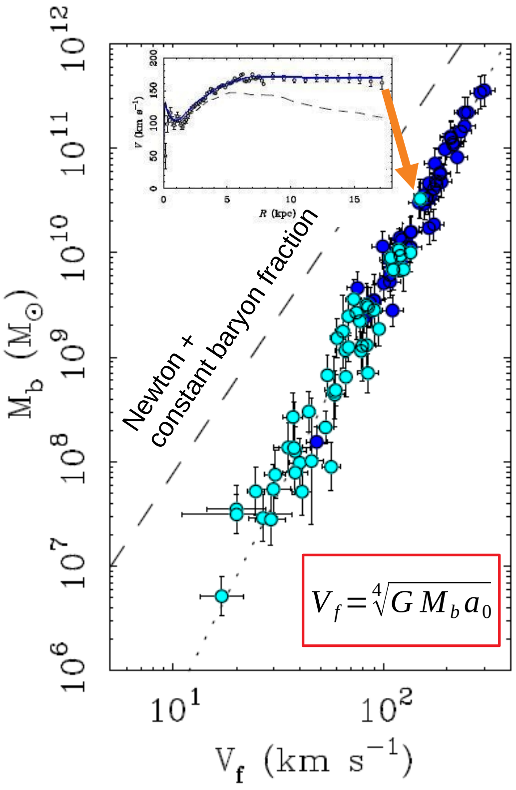
  https://jobregister.aas.org/ad/66c4e4a6

Milgromian dynamics (MOND)

- **Newton gravity/GR developed using Solar System constraints**
- **Developed by M. Milgrom (1983) to address rotation curves without cold dark matter by going beyond Newton**
- **Lagrangian formalism** $$L = L_K - L_P = \rho\left(\frac{1}{2}v^2 - \Phi\right) - \frac{1}{8\pi G}\left(2\boldsymbol{g}\cdot\boldsymbol{g}_N - a_0^2 f[g_N]\right)$$
  - Milgrom 1983

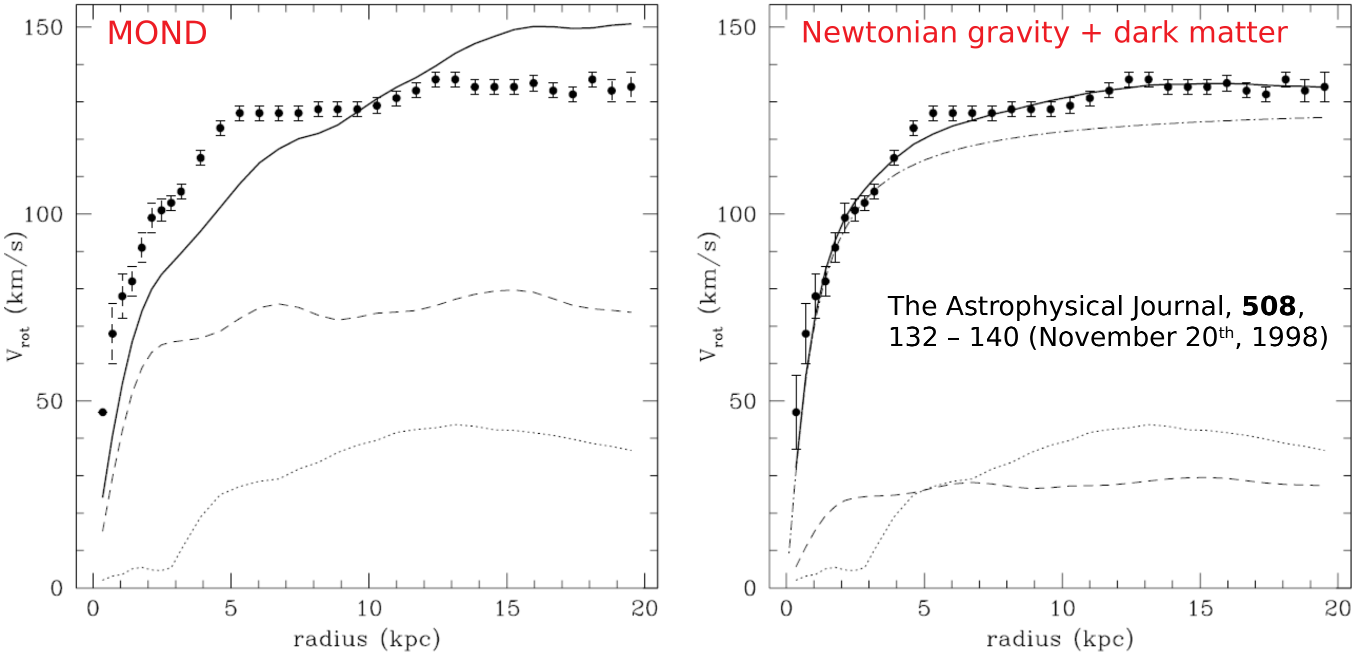
Extremize action

- **Non-linear generalization of the Poisson eqn.:** $$\nabla\cdot\boldsymbol{g} = \nabla\cdot\left(\nu\left(\frac{g_N}{a_0}\right)\boldsymbol{g}_N\right), \quad f \Leftrightarrow \nu$$
  - external field effect (EFE, Milgrom 1986)
  - breaks strong equivalence principle (as observed by Chae+ 2020)
- **Milgrom's constant (from RAR):** $a_0 = 1.2\times10^{-10} m/s^2$
- **Asymptotic limits in spherical symmetry:** $g_N \ll a_0:\ g = \sqrt{a_0 g_N},\quad g_N \gg a_0:\ g = g_N$
- **Relativistic MOND theory where gravitational waves travel at *c* (Skordis & Zlosnik 2019)**



Dark matter can fit anything

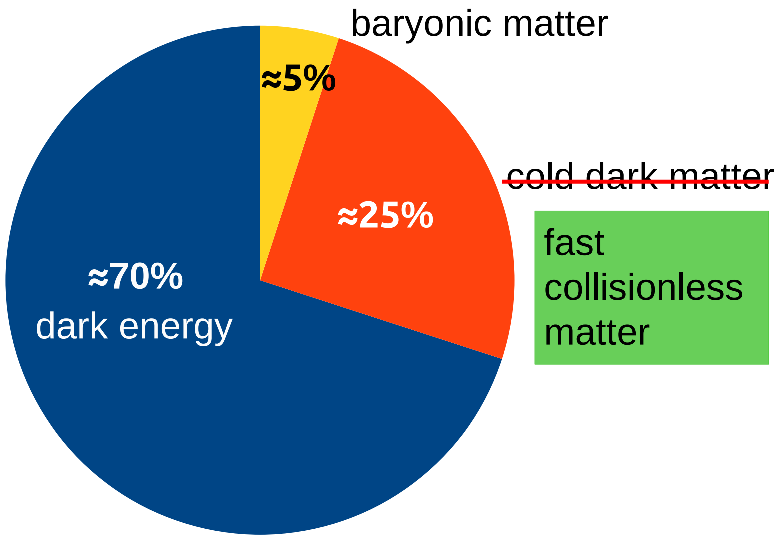
- **Unwary astronomers were given a rotation curve & image and asked to fit the curve**
- **Catch: the image was of the wrong galaxy…**



The Astrophysical Journal, **508**, 132 – 140 (November 20th, 1998)

Cosmological MOND framework (vHDM): overview

- **Proposed by Angus 2009 (MNRAS, 394, 527)**
- **Cold dark matter (CDM) replaced by fast collisionless matter (FCM)**
  - e.g. 11 eV/c^2 sterile neutrinos (e.g. Angus+2007)
  - same overall mass-energy budget as in ΛCDM
- **Standard background cosmology a(t)**
  → Nucleosynthesis (BBN)
  - e.g. Skordis 2006 (Phys. Rev. D, 74, 103513)
- **MOND is applied only to perturbations**
  - e.g. Nusser 2002, Llinares+ 2008, Angus+ 2013, Katz+ 2013, Candlish 2016
- **External field effect from surrounding structures**
  - consequence of the non-linearity of MOND (e.g. Banik+ 2018, ArXiv1808.10545)



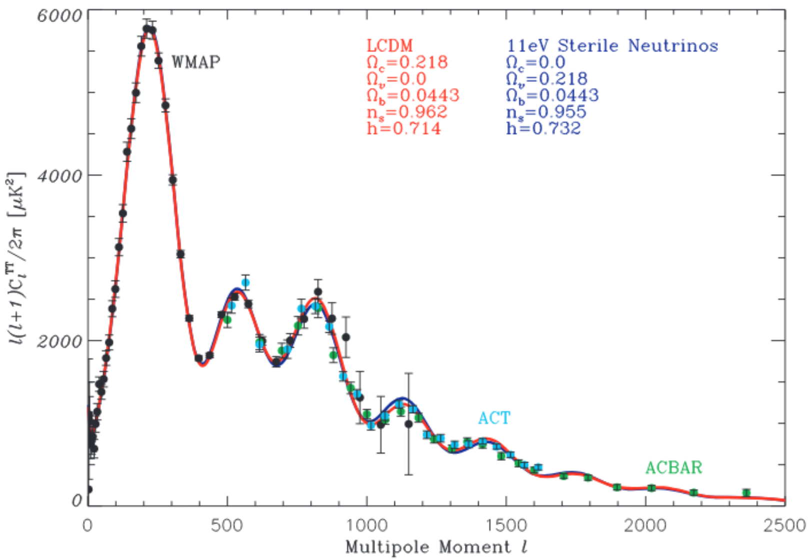
νHDM framework: Impact on CMB

- **Standard expansion history**
  → same angular diameter distance to CMB
- **MOND is sub-dominant at time of recombination (z = 1100) because g ≈ 20 a_0**
- **Free streaming effects negligible if m_ν > 10 eV/c^2**

We impose a prior on the physical thermal mass, $m_{\rm sterile}^{\rm thermal} < 10$ eV, when generating parameter chains, to exclude regions of parameter space in which the particles are so massive that their effect on the CMB spectra is identical to that of cold dark matter.

Planck Collaboration XIII (2016), section 6.4.3

- **MOND effects become important only at z < 50**

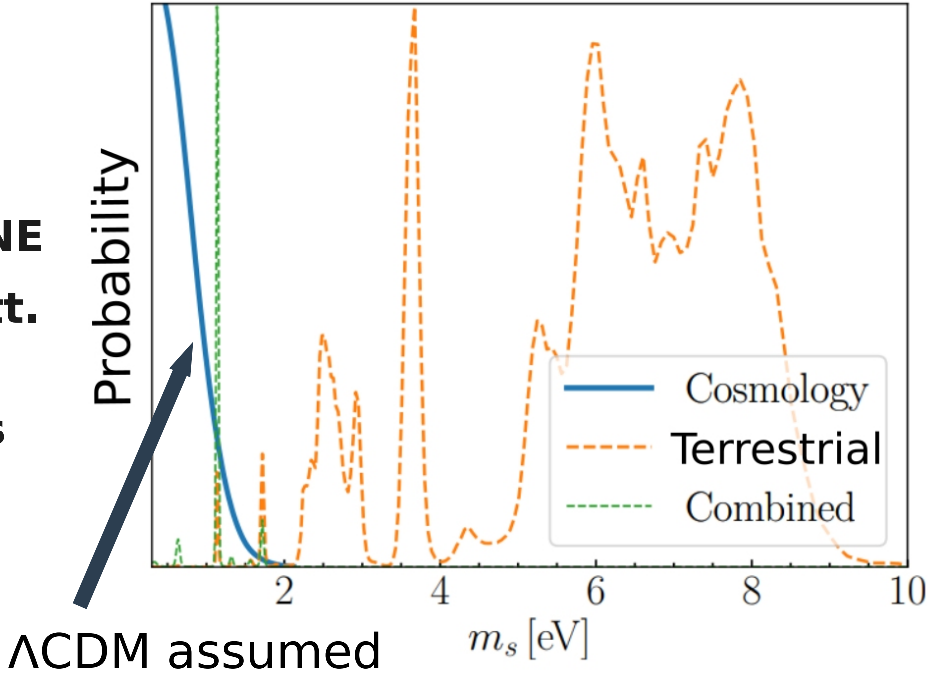


Angus & Diaferio (2011)

Terrestrial evidence for sterile neutrinos

- **Sterile neutrinos proposed to explain ordinary neutrino oscillations & their non-zero rest mass (seesaw mechanism)**
- **Hints of sterile neutrino found by MiniBooNE**

  (Aguilar-Arevalo et al. 2018, Phys. Rev. Lett. 121, 221801)
- **Must avoid prior limit that $m_\nu < 10$ eV/c² as terrestrial experiments are so far quite compatible with slightly larger mass**

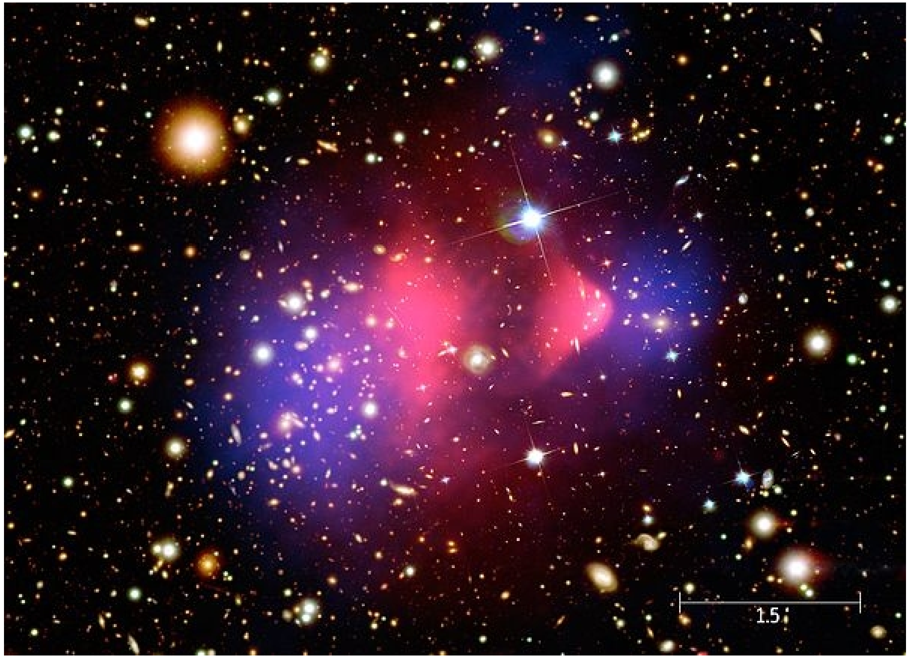


Archidiacono et al. 2020 (ArXiv 2006.12885)

Astronomical evidence for fast collisionless matter

- **Offset between X-ray and weak lensing peaks**
- **$g > a_0$: MOND effects small**

  → Collisionless matter required
- **Tremaine-Gunn limit: $m_\nu > 2\ eV/c^2$ (Angus+ 2007, ApJ, 654, L13)**
- **Current constraints imply collisionless particle mass >10 eV/c^2 (strongest limits from CMB)**



Bullet Cluster, credits: NASA/CXC/M. Weiss

νHDM framework can explain:

- **Expansion history a(t) → BBN**
- **CMB**
- **Bullet Cluster and 30 virialized clusters (Angus+ 2010, MNRAS, 402, 395)**
- **Galaxy rotation curves**
  - unaffected by neutrinos if $m_\nu < 100$ eV/c^2 (Angus+ 2010)
- **νHDM solves problems with ΛCDM on galaxy scales**
  - plane of satellites with high internal σ around MW (Pawlowski & Kroupa 2020), M31 (Ibata+ 2013, Sohn+ 2020), Centaurus A (Müller+ 2018)
  - ΛCDM explanations rejected (Pawlowski+ 2014, MNRAS, 442, 2362)
  - other small scale failures (e.g. Kormendy 2010, Peebles & Nusser 2010, Kroupa 2015, Algorry+ 2017)
- **Large-scale structure?**

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The KBC void & Hubble tension in standard cosmology & Milgromian dynamics

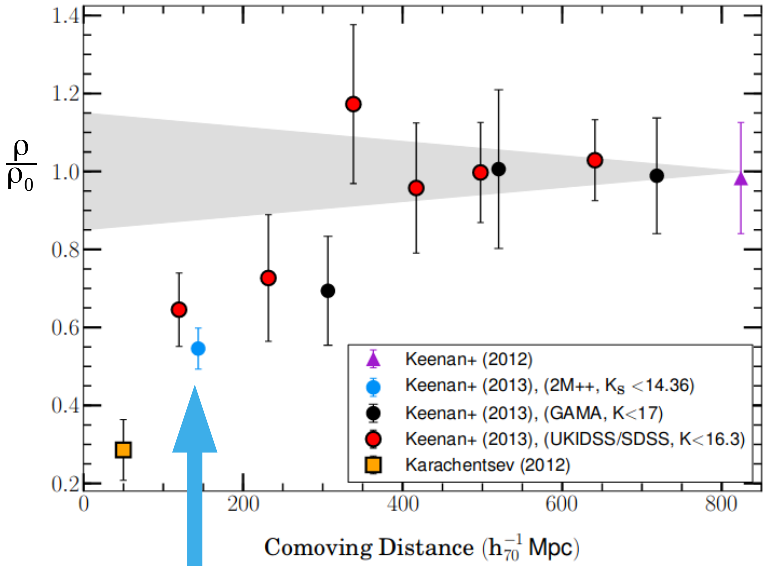
Authors: **Moritz Haslbauer**, Indranil Banik & Pavel Kroupa

Publication: The KBC void and Hubble tension contradict ΛCDM on a Gpc scale – Milgromian dynamics as a possible solution (MNRAS, 499, 2845)

The Keenan-Barger-Cowie (KBC) void

- **A local underdensity is evident across the entire electromagnetic spectrum, ranging from radio to X-ray**
  - Optical: Maddox+1990, Zucca+1997
  - Radio: Rubart & Schwarz 2013, Rubart, Bacon & Schwarz 2014, Secrest+ 2020
  - X-ray: Böhringer+2015, Böhringer, Chan, Collins 2020
- **NIR: Keenan, Barger, Cowie 2013, ApJ, 775, 62**
  - 2M++ galaxy catalog with spectroscopic redshift
  - void evident in number counts (luminosity function)
  - density about 0.5x cosmic mean between 40 and 300 Mpc over 90% of the sky

$$\delta_{obs} \equiv 1 - \frac{\rho}{\rho_0} \approx 0.46 \pm 0.06$$



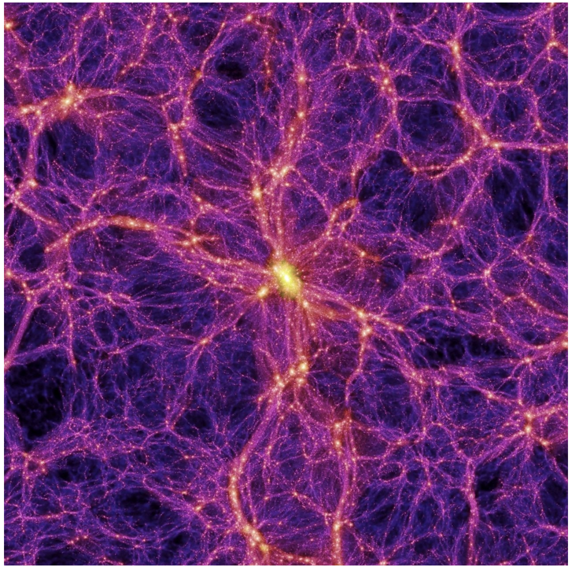
Credits: Kroupa (2015)

Keenan+ (2013): 2M++ with K_s < 14.36

The KBC void in ΛCDM

- **Millennium (MXXL) simulation (Angulo+ 2012)**
  - ΛCDM simulation consistent with WMAP-7 parameters
  - biggest suitable simulation (box size of 4.1 Gpc)
  - Stellar masses assigned semi-analytically

- **Mimic observations (2M++ survey)**
  - select subhaloes with $M_* > 1e10\ M_\odot/h$ at $z = 0$
  - calculate luminosity density contrasts over (40 – 300) Mpc for 1e6 vantage points



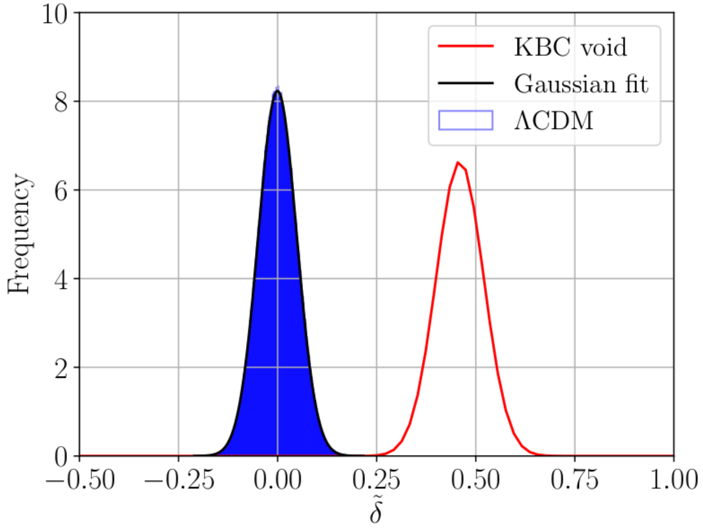
Source: Millennium simulation

⇒ Expected rms density fluctuations for scale-invariant spectrum: 0.032

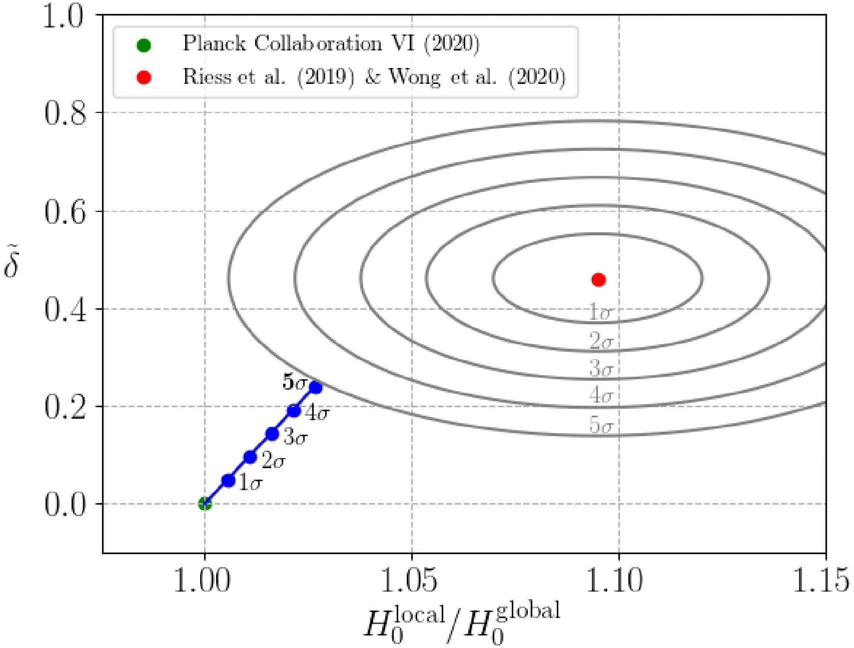
Observed density fluctuation: 0.46±0.06

The KBC void and Hubble tension in ΛCDM

Spheres with an inner radius of 40 Mpc and an outer radius of 300 Mpc



Allowance made for redshift space distortion (RSD): Higher local *H*



The KBC void falsifies ΛCDM at 6.04σ

Combined, the KBC void + Hubble tension falsify ΛCDM at 7.09σ

Large scale failure of ΛCDM

- **The KBC void is physically impossible in a ΛCDM universe**

  It is impossible to get from the $z = 1100$ (CMB) to the $z < 0.2$ (observed nearby Universe) boundary conditions

  - Problem arises on 300 Mpc scale, so independent of galaxy-scale baryonic physics

⇒ **To get from the CMB ($z = 1100$) to the $z = 0$ observations, need effectively stronger gravity to grow structures faster**

Applying MOND to KBC void

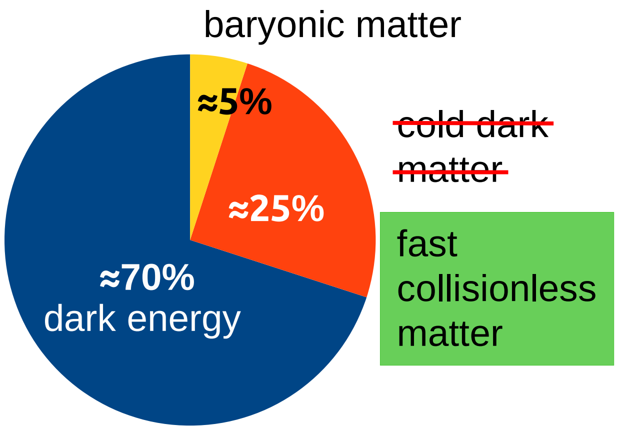
Model assumptions

- **Semi-analytical model starting from z = 9**
- **Initial Maxwell-Boltzmann underdensity profile**
  - amplitude consistent with CMB
- **Standard expansion history & overall mass budget (CDM → sterile neutrinos)**
- **MOND applied only to density perturbations**

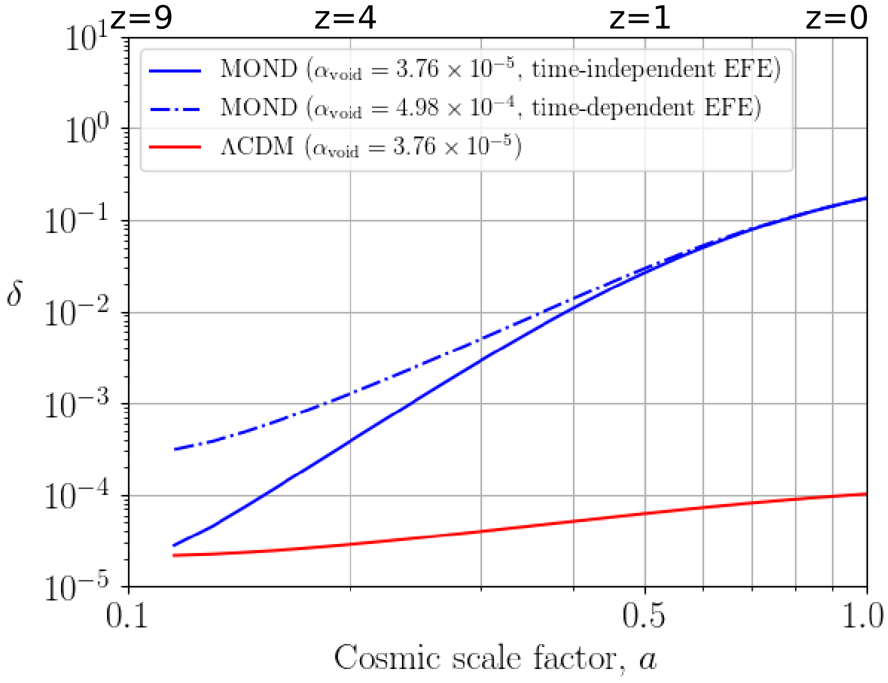
e.g. Nusser 2002, Llinares+ 2008, Angus+ 2013, Katz+ 2013, Candlish 2016

- **External field effect from large scale structure (Milgrom 1986)**
  - gravity from beyond the void

    ⇒ void as a whole moves



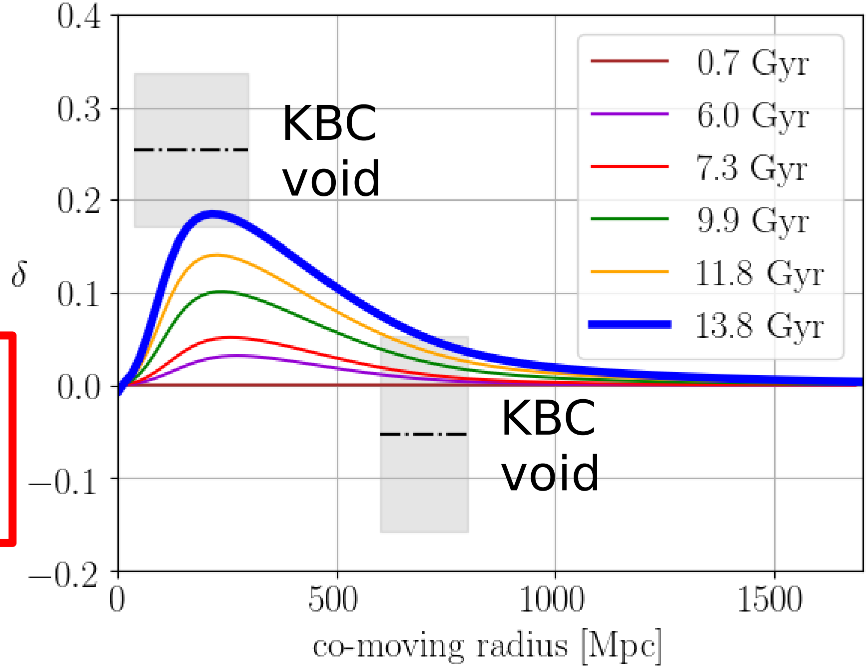
Growth of structure



300 cMpc sphere

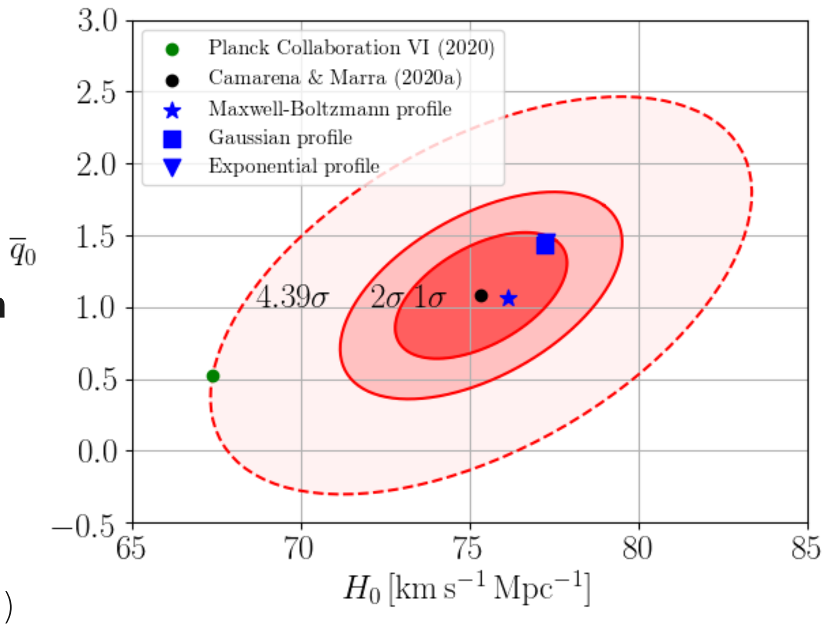
At 300 Mpc:
$g = 0.1a_0$
$g_{ext} = 0.055a_0$

Redshift space distortion (RSD) correction applied



Local Hubble diagram

- **Effects of a local void on cosmological parameters:**
  - local expansion rate is increased: $H_0^{local} \equiv \frac{\dot{a}}{a}(today)$
  - apparent expansion rate appears to accelerate at late times: (extra curvature $\overline{q}_0$ of Hubble diagram) $\overline{q}_0^{local} \equiv \frac{\ddot{a}\,a}{\dot{a}^2}(today)$
- **Camarena & Marra 2020a,b jointly derived H_0 and $\overline{q}_0$ from SNe at redshifts 0.023 – 0.15**
  - $\overline{q}_0$ is 2x standard value of 0.55 → suggestive of a local void
  - high local $\overline{q}_0$ missed in Kenworthy+ 2019 ($\overline{q}_0$ fixed at 0.55)
  - hint of dipole in Hubble diagram (Colin+ 2019, Migkas+ 2020)



$H_0^{obs,local} = 75.35 \pm 1.68\, km/s/Mpc$

$\overline{q}_0^{obs,local} = 1.08 \pm 0.29$

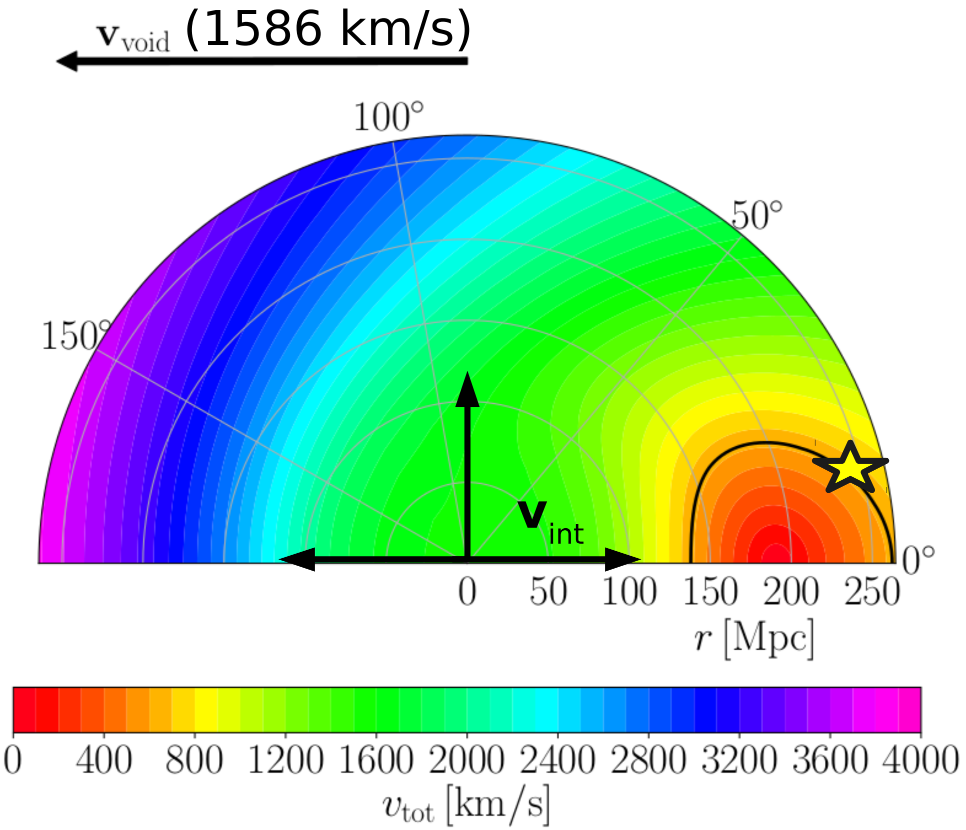
$H_0^{model} = 76.15\, km/s/Mpc$

$\overline{q}_0^{model} = 1.07$

High local H_0 and $\overline{q}_0$ are explained naturally in MOND by outflow from a KBC-like void

Peculiar velocity field

- **Only half of the void rms size is shown**
- **The entire void is moving due to gravity from beyond the void**
- **Partial cancellation between void motion and internal velocities**
- **Large region with peculiar velocity $v_{tot} < v_{LG}$ = 627 km/s (≈0.015 a_0)**
  - Local Group (LG) is off-centered
  - LG not at a special position
- **High peculiar velocities towards void edge consistent with kinematic Sunyaev-Zel'dovich effect (Hoscheit & Barger 2018, Ding et al. 2020)**



Comparison with ΛCDM & νHDM

ΛCDM model

Observational constraints	Level of tension
KBC void (40 – 300 Mpc, 90% of sky)	6.04σ
H_0 (Riess+ 2019 & Wong+ 2020)	5.3σ

Parameters fixed by CMB

Combined tension: 7.09σ

Recent worsening of H_0 tension:

Early: Aiola+ 2020 (ACT)

Late: Pesce+ 2020 (Megamasers)

νHDM model

Observational constraints	Level of tension
KBC void (40 – 300 Mpc)	0.99σ
KBC void (600 – 800 Mpc)	0.97σ
H_0 and $\overline{q}_0$ from SNe data	0.20σ
H_0 from 7 strong lens time-delays	2.05σ
Motion of the LG	2.34σ

3 free parameters (void size & strength, g_{ext})

12 data points

Combined tension: 2.53σ

Outlook: Stellar wide binary (WB) test

- **WBs with separation *r* > 3 kAU (like Proxima Centauri)**
- **Orbital accelerations < a_0 (MOND regime)**
  - MOND boosts orbital velocities by 20% in Solar neighbourhood
- **Accurate observations of about 500 WBs necessary (Banik & Zhao 2018)**
  - e.g. Gaia data release 3, Theia mission
- **Statistical treatment of undetected close companions to WBs necessary**
  - WBs with $r < 3$ kAU should be similar to WBs with $r > 3$ kAU in Newton
- **MOND without EFE is ruled out by the WB test**
  - Pittordis & Sutherland 2019
  - other tests also rule out MOND without EFE (e.g. Chae+ 2020)

Summary & Conclusions (MNRAS, 499, 2845):

- **KBC void falsifies ΛCDM at 6.04σ**
- **KBC void + Hubble tension falsify ΛCDM at 7.09σ**
  - failures of ΛCDM model cover all scales from dwarf galaxies (e.g. disk of satellites, Pawlowski+ 2014) to Gpc scales (this work, see also Kroupa et al. 2010, Kroupa 2012, 2015, Sellwood+ 2019, Asencio+, MNRAS, 2021)
  - matter distribution on a Gpc scale requires enhanced growth of structure

    e.g. Milgromian gravitation (MOND) would also explain galaxy dynamics (RAR), M33 disk stability (Banik+, ApJ, 2021)
- **MOND cosmology with fast collisionless matter (e.g. 11eV/c^2 sterile neutrinos, Angus 2009)**
  - standard expansion history, BBN, CMB anisotropies, and Bullet Cluster + 30 virialized clusters
  - structure growth enhanced and self-regulated by external fields from surrounding structures
- **MOND describes the local observations at 2.53σ**
  - enhanced growth of structure allows the formation of KBC-like voids
  - outflow from large void explains high local H_0 and $\overline{q}_0$
  - common objections to void scenario addressed in Section 5.3 of our paper
  - blogs describing paper: **tritonstation.com** & **darkmattercrisis.wordpress.com**

Appendix

Leibniz Institute for Astrophysics Potsdam (AIP)
dwarf galaxies/near-field cosmology

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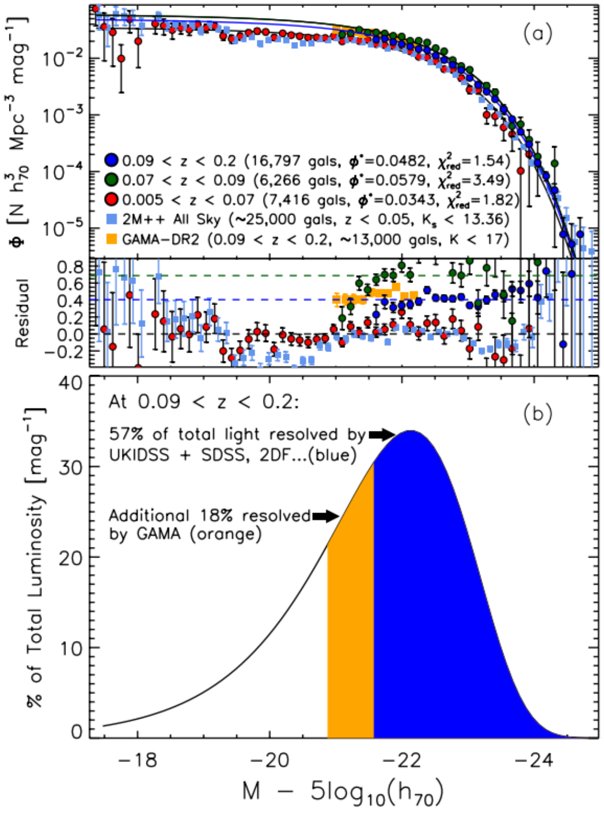
KBC (2013): luminosity function

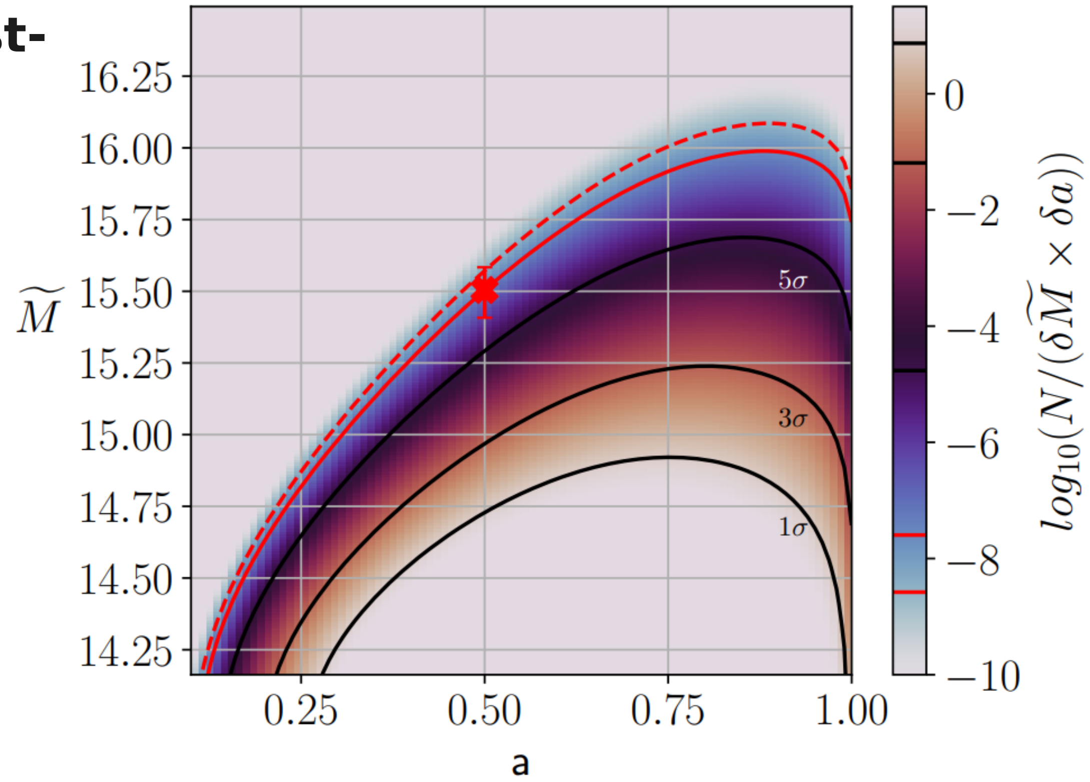
- **Shape of luminosity function clearly determined based on 57-75% of luminosity function**
- **Normalization systematically lower at low z**
- **Density contrast similar between magnitude bins**



El Gordo Galaxy Cluster Interaction
Asencio, Banik, Kroupa 2021 (DOI: 10.1093/mnras/staa3441)

- **Mass-redshift distribution of fast-interacting clusters along past lightcone in ΛCDM simulation**

- **El Gordo inconsistent at 6.16σ**



Outlook

- **Enable standard RAMSES hydrodynamics and cosmology to study galaxy formation with Phantom of RAMSES (PoR MOND patch, Lüghausen+ 2015)**
  - Simulation by Nils Wittenburg
  - SPODYR group (University of Bonn)

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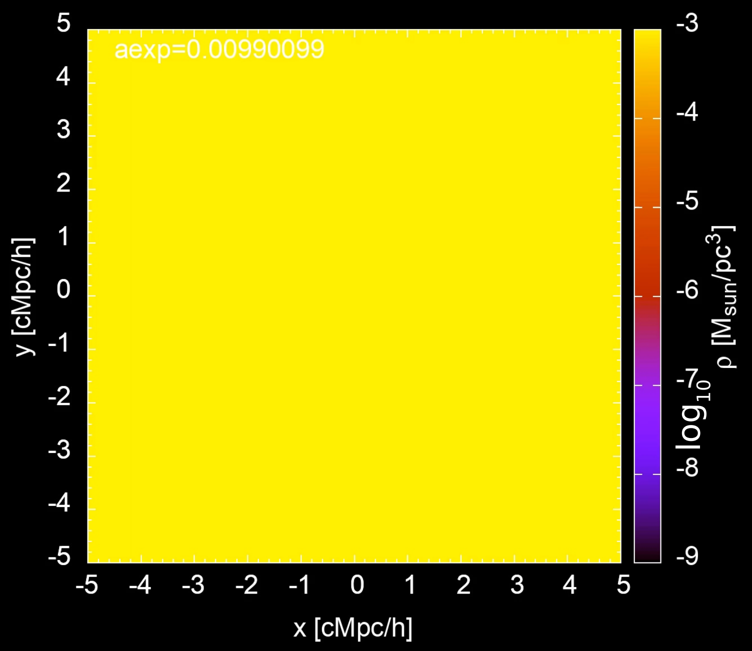
Origin of galaxies in a MOND cosmology



Wittenburg+ (in prep.)

Outlook

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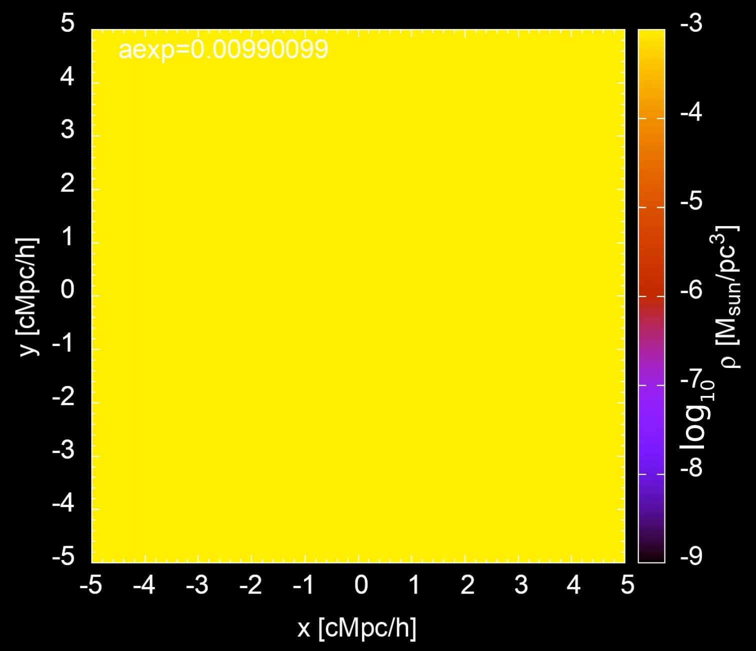
  - Simulation by Nils Wittenburg
  - SPODYR group (University of Bonn)
- **Self-consistent cosmological MOND simulation on much larger scales**
  - study formation of voids in a large simulation box
  - What does a typical KBC-like void profile look like?
  - cosmic variance

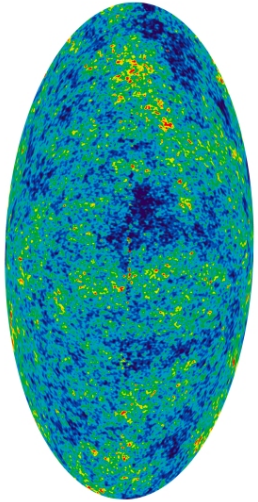
Origin of galaxies in a MOND cosmology



Wittenburg+ (in prep.)

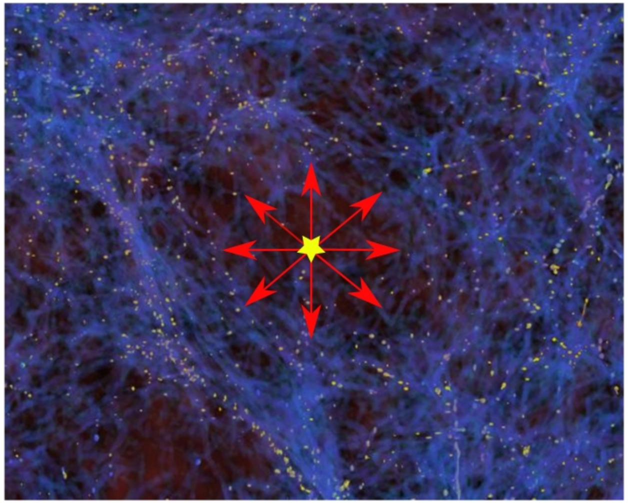
Overview: Keenan-Barger-Cowie void and H_0 tension

Cosmic microwave background
(CMB) at z = 1100



Keenan-Barger-Cowie (KBC) void

diameter: ≈1 Gpc
density contrast: ≈50%
observed



+ Hubble tension solved ✓✓

Take home message

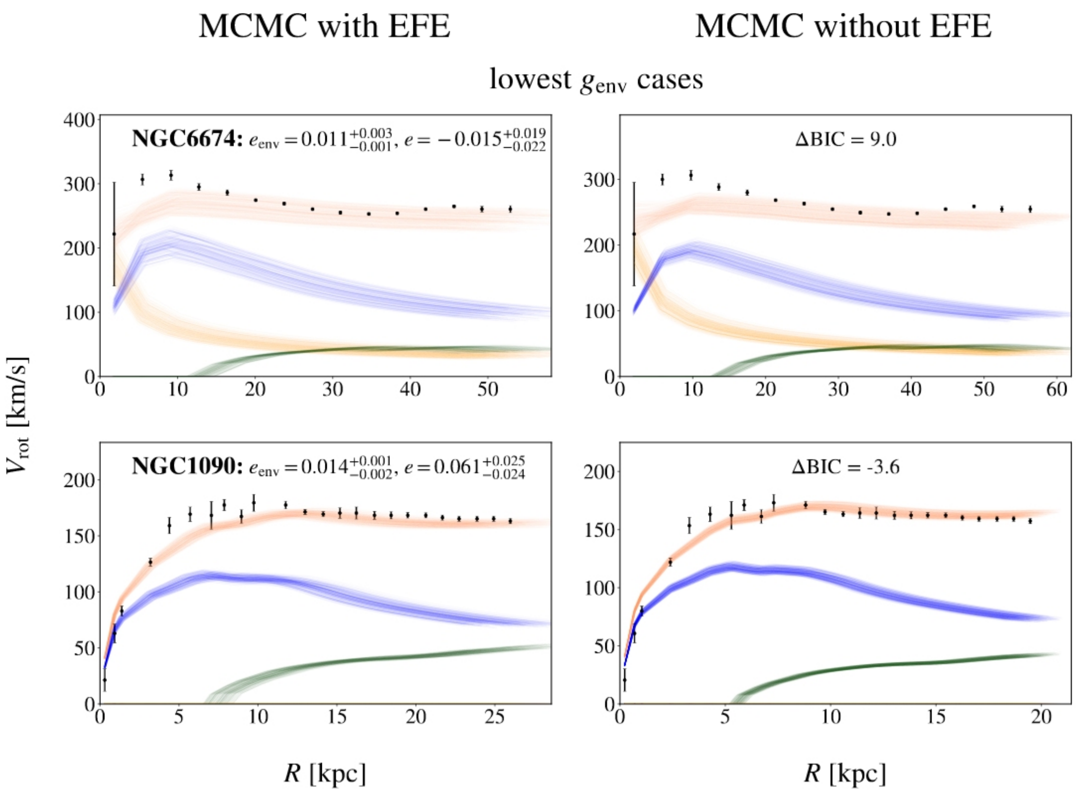
1a) The observed KBC void falsifies ΛCDM at 6.04σ

1b) The observed KBC void + Hubble tension falsify ΛCDM at 7.09σ

2) Neither problem occurs in MOND

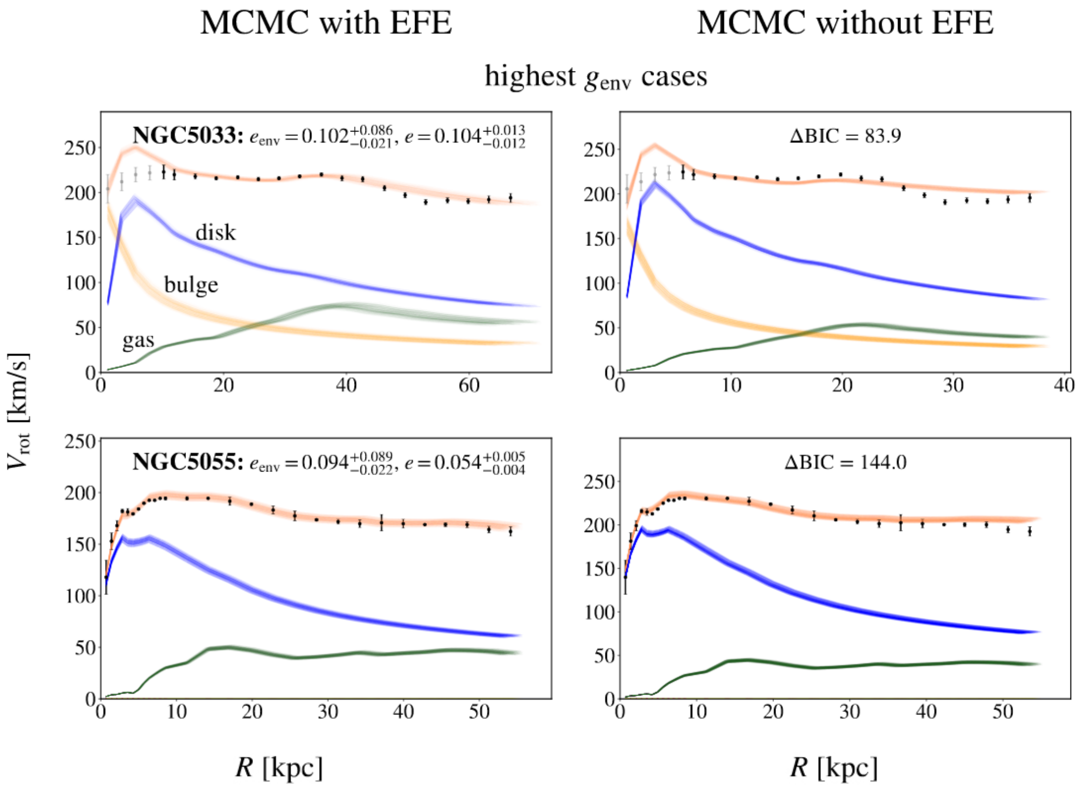
Detection of external field effect (EFE)

- **Chae+2020 (ApJ, 904, 51)**
- **No EFE expected because galaxies isolated**
- **No EFE required in rotation curve fits (|ΔBIC| < 10)**

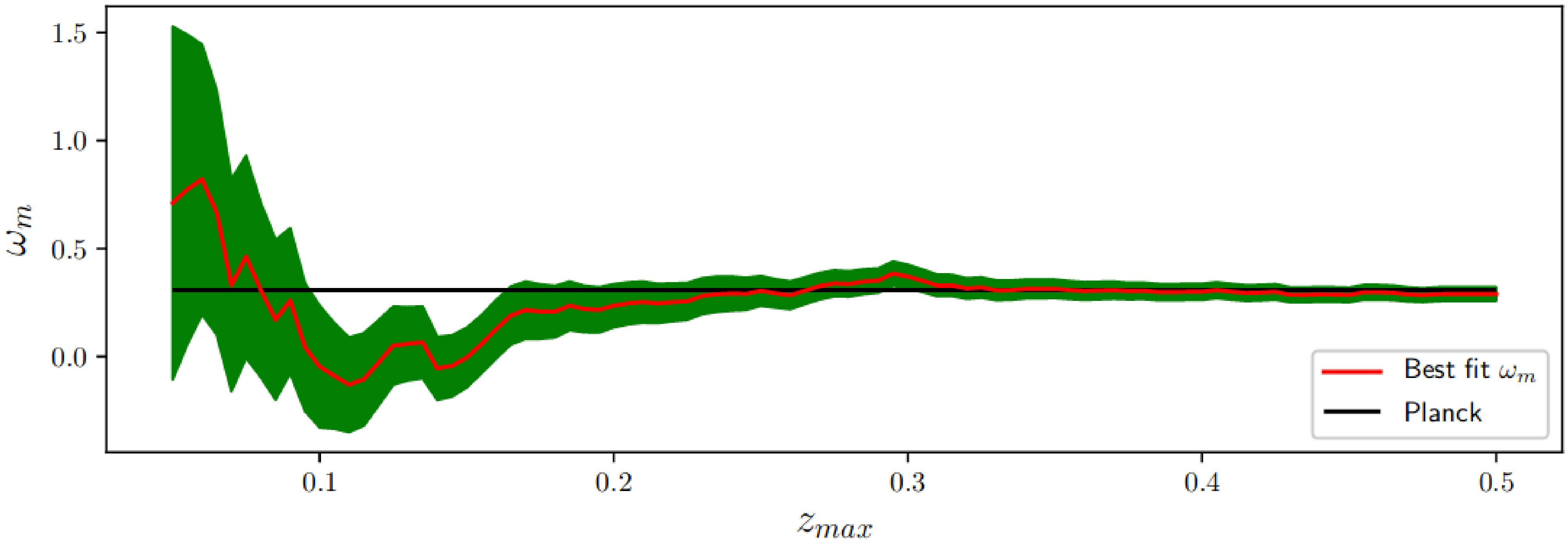


Detection of external field effect (EFE)

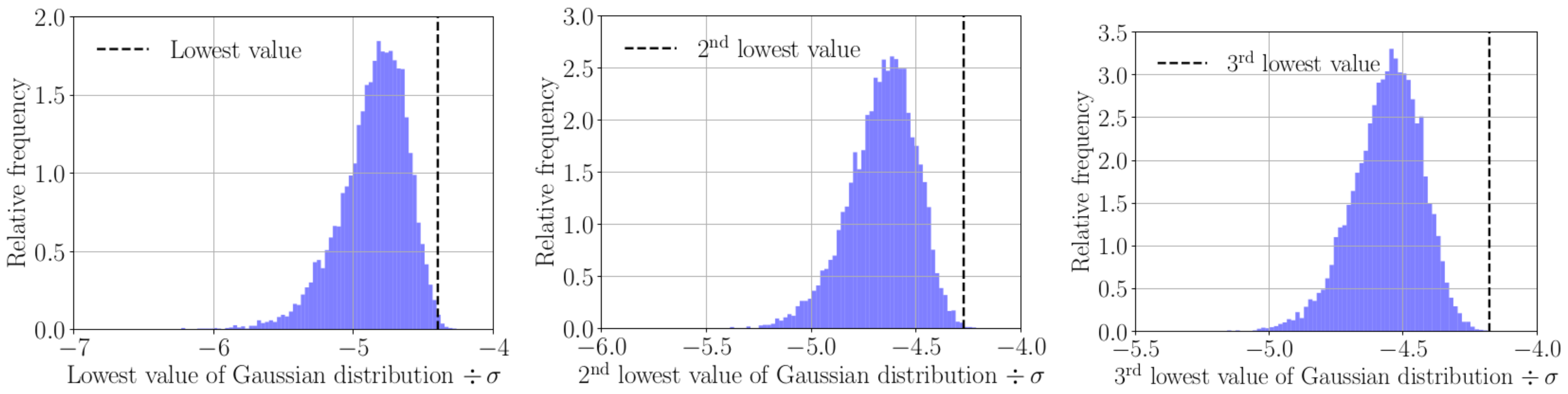
- **Chae+2020 (ApJ, 904, 51)**
- **Strong EFE expected based on environment**
- **"Very strong" evidence for EFE in rotation curves (ΔBIC >> 10)**
- **Inferred g_{ext} from rotation curve agrees with prior estimates**



Colgáin (2019)

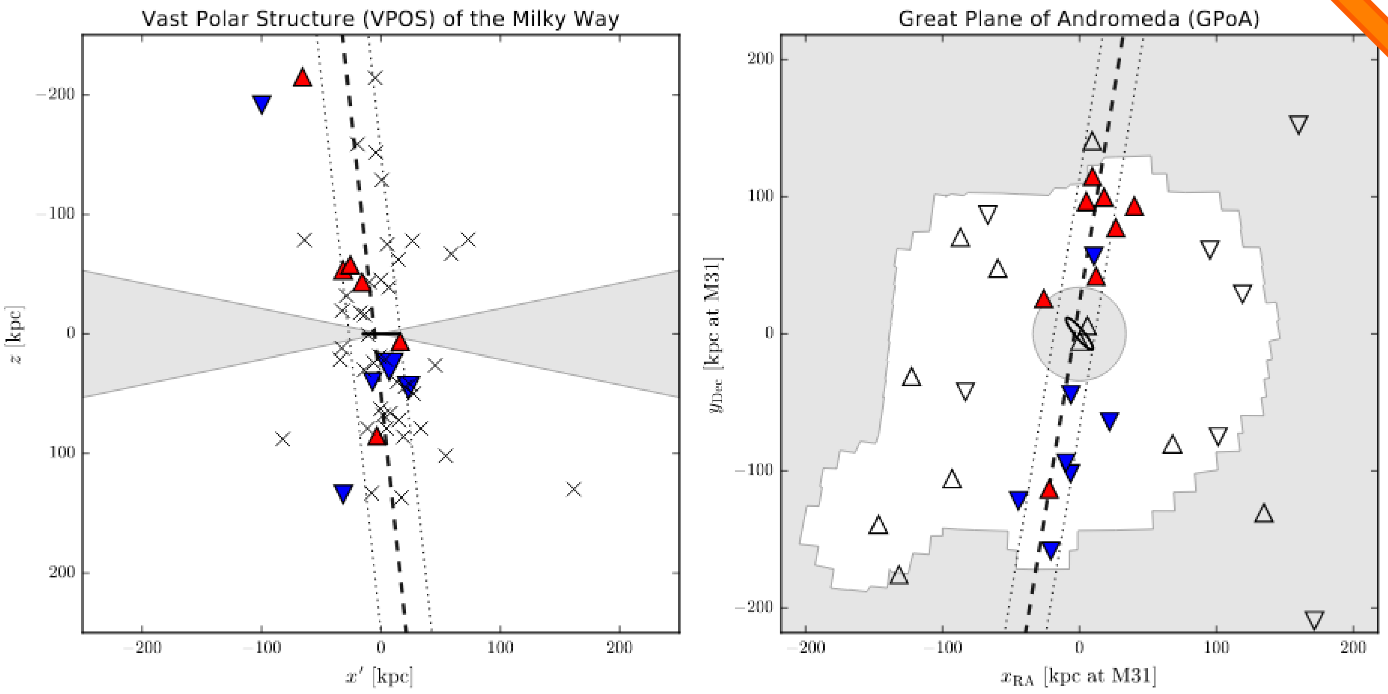


Results: Gaussianity test of the selected density fluctuations in the MXXL simulation



MW satellite galaxies lie within a thin plane (Pawlowski & Kroupa 2013). Analogous situation for M31 (Ibata+ 2013)

Galaxies observed forming within tidal tails (Mirabel+ 1992)



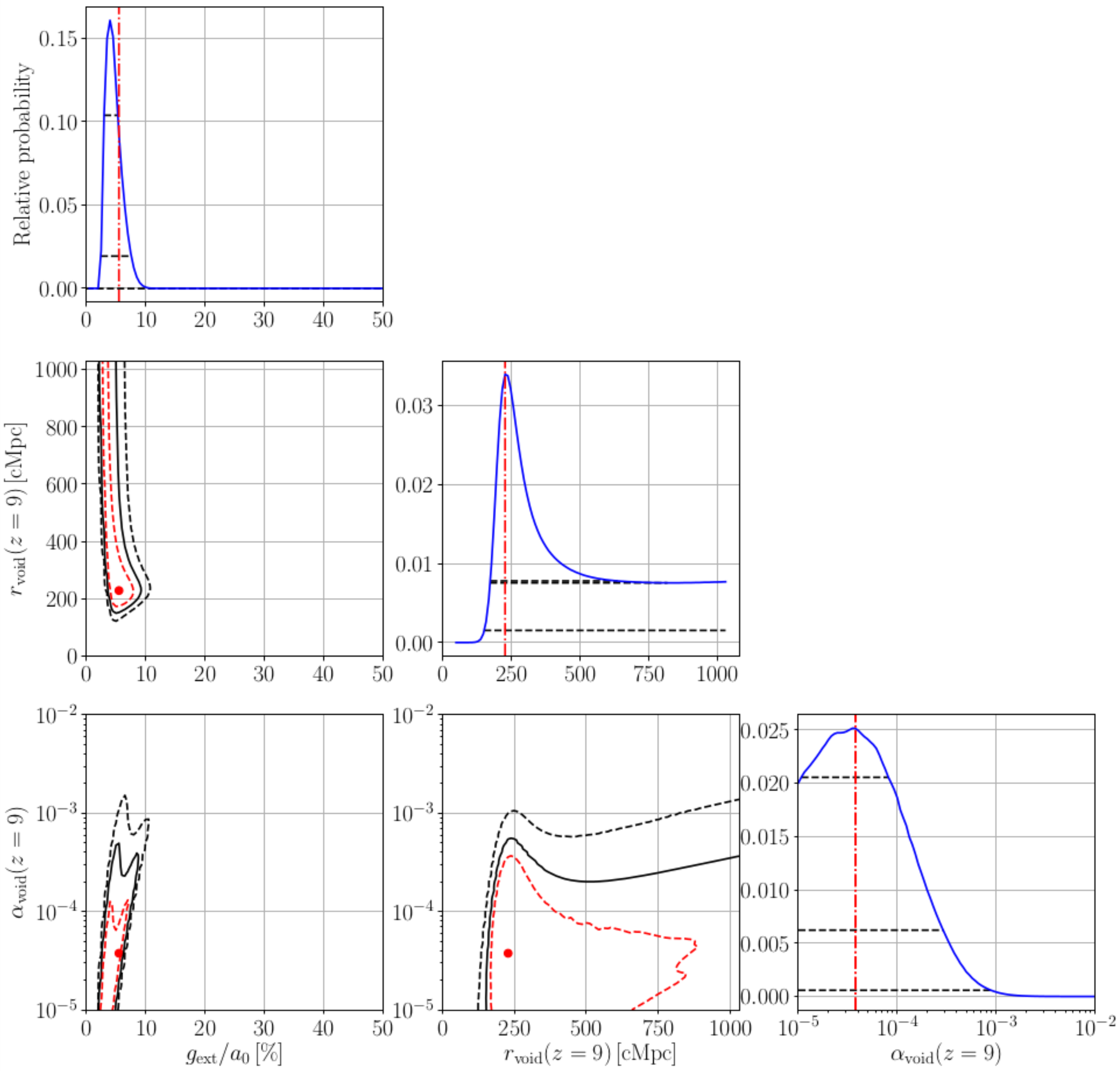
Satellites were formed from tidal debris. Alternatives not very likely (Pawlowski+ 2014 and references therein)

Should only contain baryons as DM can't cool and form dense tidal tails (Wetzstein+ 2007)

MW and M31 satellite galaxies have high internal velocity dispersions, requiring strong self-gravity (McGaugh & Wolf, 2010)

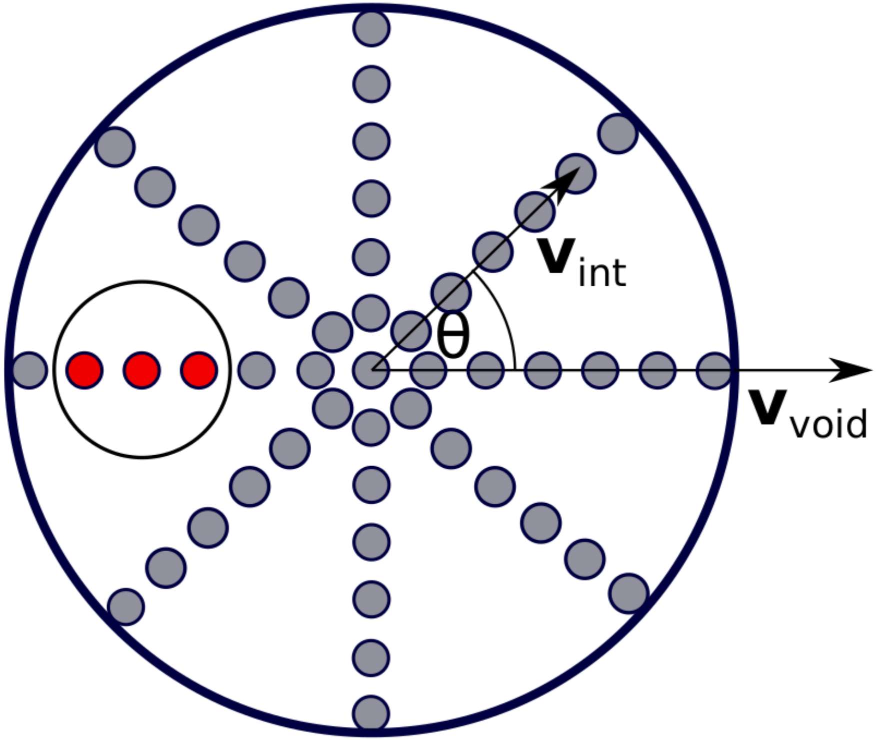
Internal dynamics can't be explained by Newtonian gravity (Kroupa, 2015)

Results:



Relative probability
$r_{\mathrm{void}}(z=9)$ [cMpc]
$\alpha_{\mathrm{void}}(z=9)$
g_{ext}/a_0 [%]
$r_{\mathrm{void}}(z=9)$ [cMpc]
$\alpha_{\mathrm{void}}(z=9)$

Results: Peculiar velocities



Results: H_0 from strong lensed systems

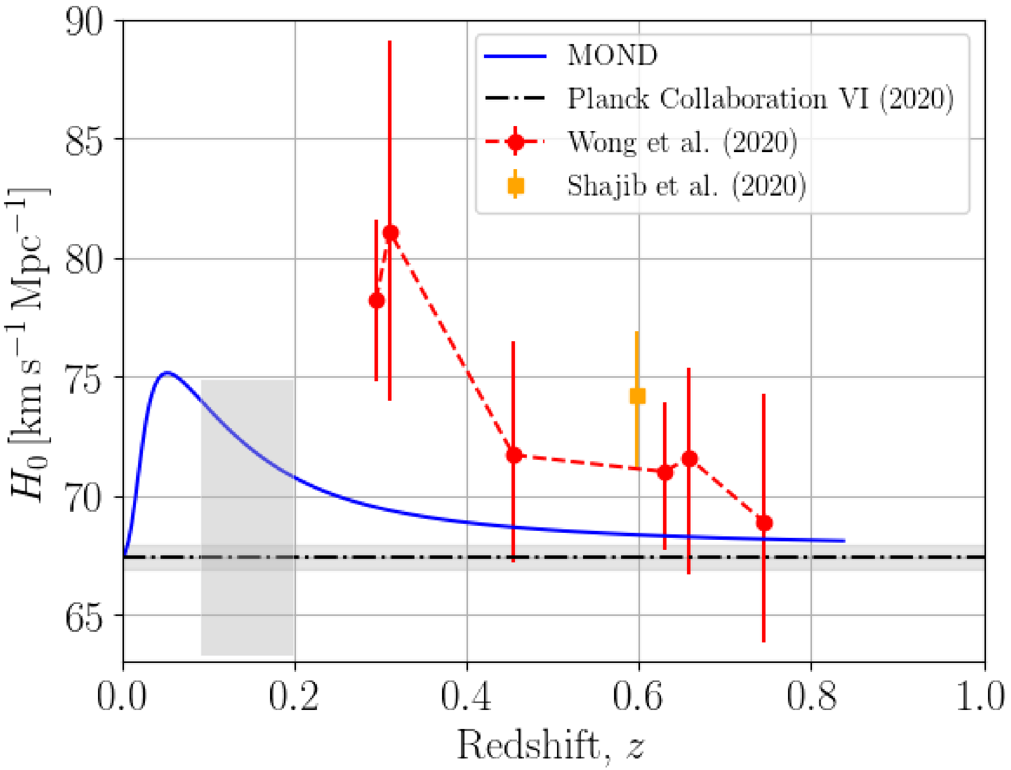
- **Empirically, light deflection in strong lenses works similar to General Relativity for the same non-relativistic** g **(Collett+ 2018)**

- **Relativistic MOND theories exist where gravitational waves travel at the speed of light (Skordis & Złośnik 2019)**

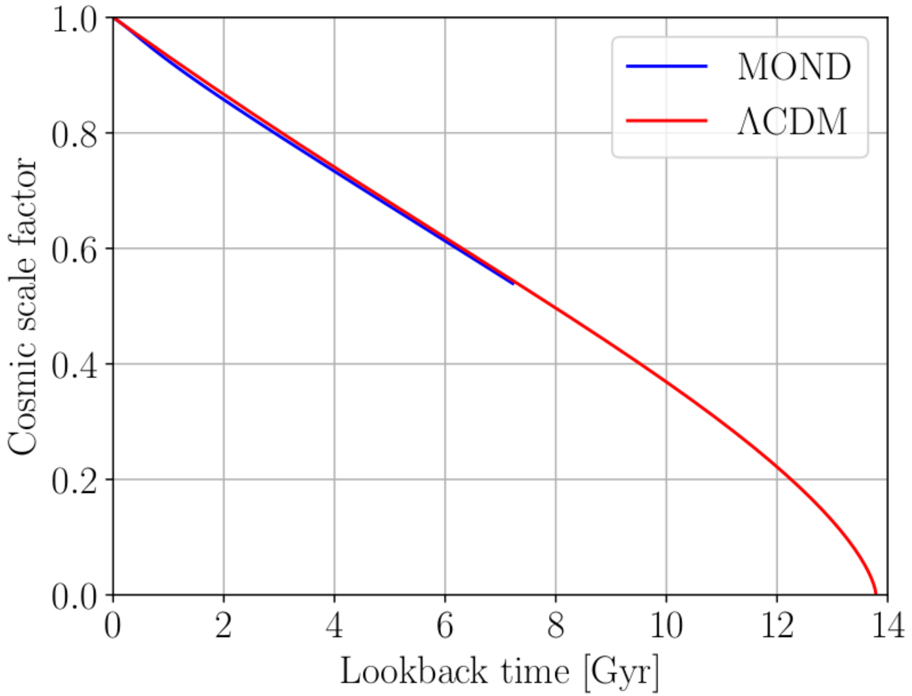
- **Decreasing inferred H_0 with lens redshift (Wong+2020) evident at 1.9σ**

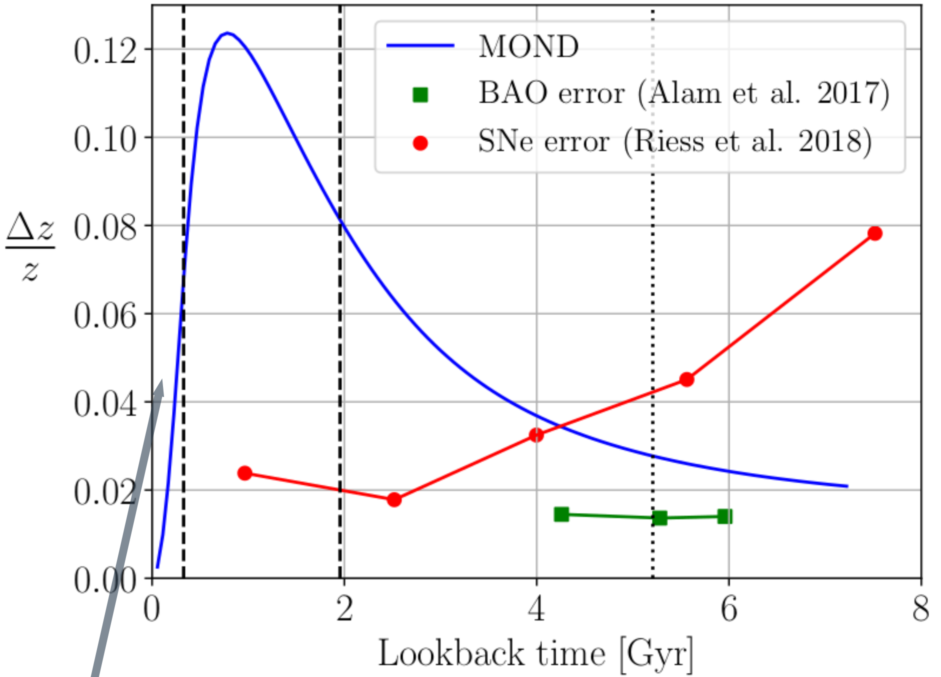
  ⇒ curvature in Hubble diagram (like with SNe)

- **Consistent with MOND model (2.05σ), but suggestive of a larger void than KBC data**



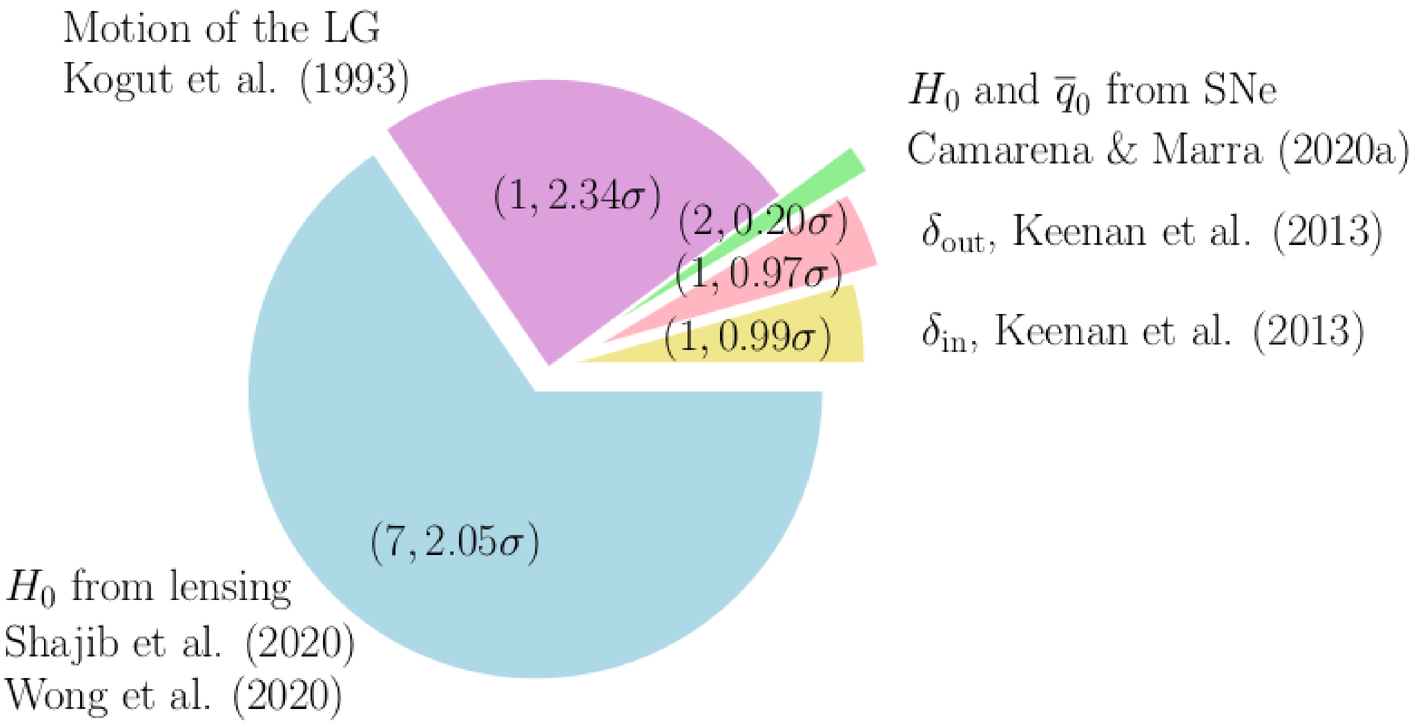
vHDM framework: apparent expansion history





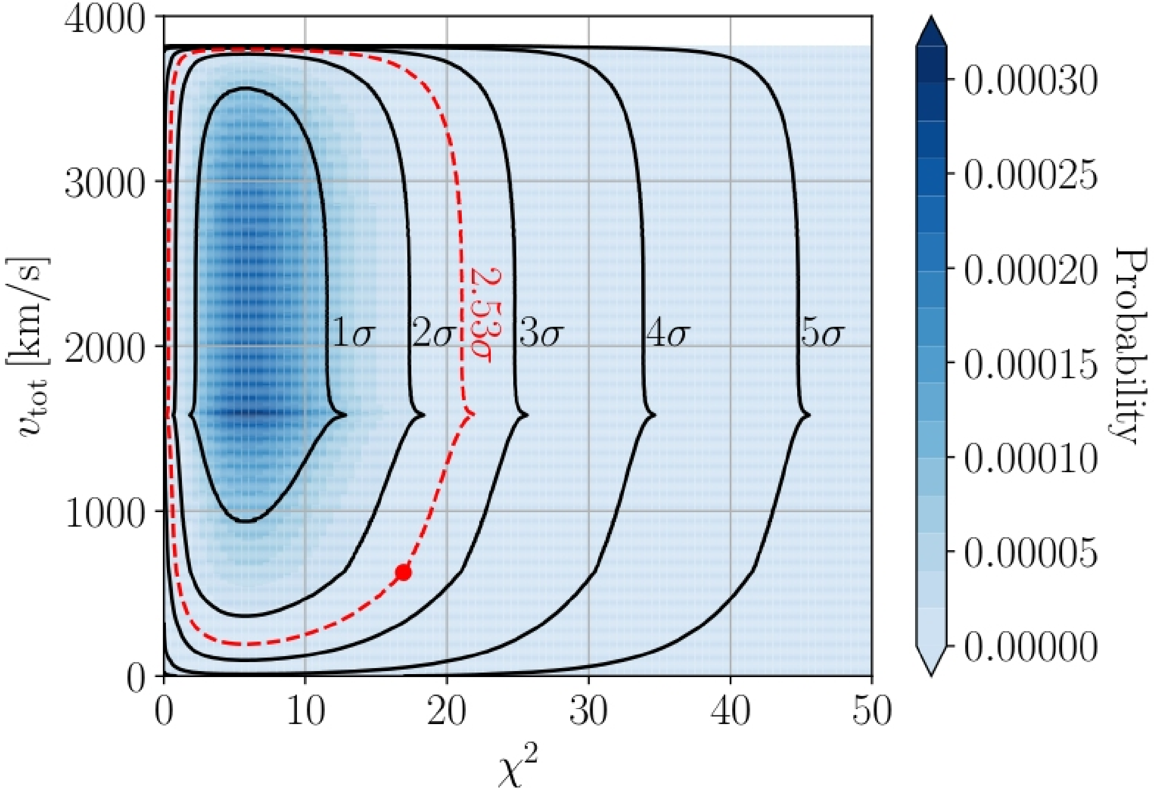
Hubble flow velocity very small: v_{Hubble} ~~>>~~ v_{pec}
(e.g. Kim+2020)

Results:



Motion of the LG
Kogut et al. (1993)
$(1, 2.34\sigma)$
H_0 and $\overline{q}_0$ from SNe
Camarena & Marra (2020a)
$(2, 0.20\sigma)$
δ_{out}, Keenan et al. (2013)
$(1, 0.97\sigma)$
δ_{in}, Keenan et al. (2013)
$(1, 0.99\sigma)$
$(7, 2.05\sigma)$
H_0 from lensing
Shajib et al. (2020)
Wong et al. (2020)

Results:



4000
3000
2000
1000
0
$v_{\rm tot}$ [km/s]
0
10
20
30
40
50
χ^2
1σ
2σ
2.53σ
3σ
4σ
5σ
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
0.00000
Probability

Results:

Maxwell-Boltzmann density profile, $g_{\rm ext} = 0.055\,a_0$, $r_{\rm void} = 228.2$ cMpc, $\alpha_{\rm void} = 3.76 \times 10^{-5}$, $v_{\rm void} = 1586\,{\rm km\,s^{-1}}$, $r_{\rm void}^{\rm rms} = 528.7$ Mpc, $n_{\rm EFE} = 0$

Parameter	$H_0^{\rm local}$ [km s^{-1} Mpc^{-1}]	$\overline{q}_0^{\rm local}$	$H_0^{\rm lensing}$ [km s^{-1} Mpc^{-1}]	$v_{\rm LG}$ [km s^{-1}]	$\delta_{\rm in}$	$\delta_{\rm out}$
Observations	75.35 ± 1.68	1.08 ± 0.29	––	627	0.254 ± 0.083	-0.052 ± 0.105
MOND model	76.15	1.07	See Figure 7	See Figure 8	0.172	0.050
χ^2	0.34		14.66	––	0.99	0.94
Degrees of freedom	2		7	––	1	1
χ (1D Gaussian equivalent)	0.20		2.05	2.34	0.99	0.97

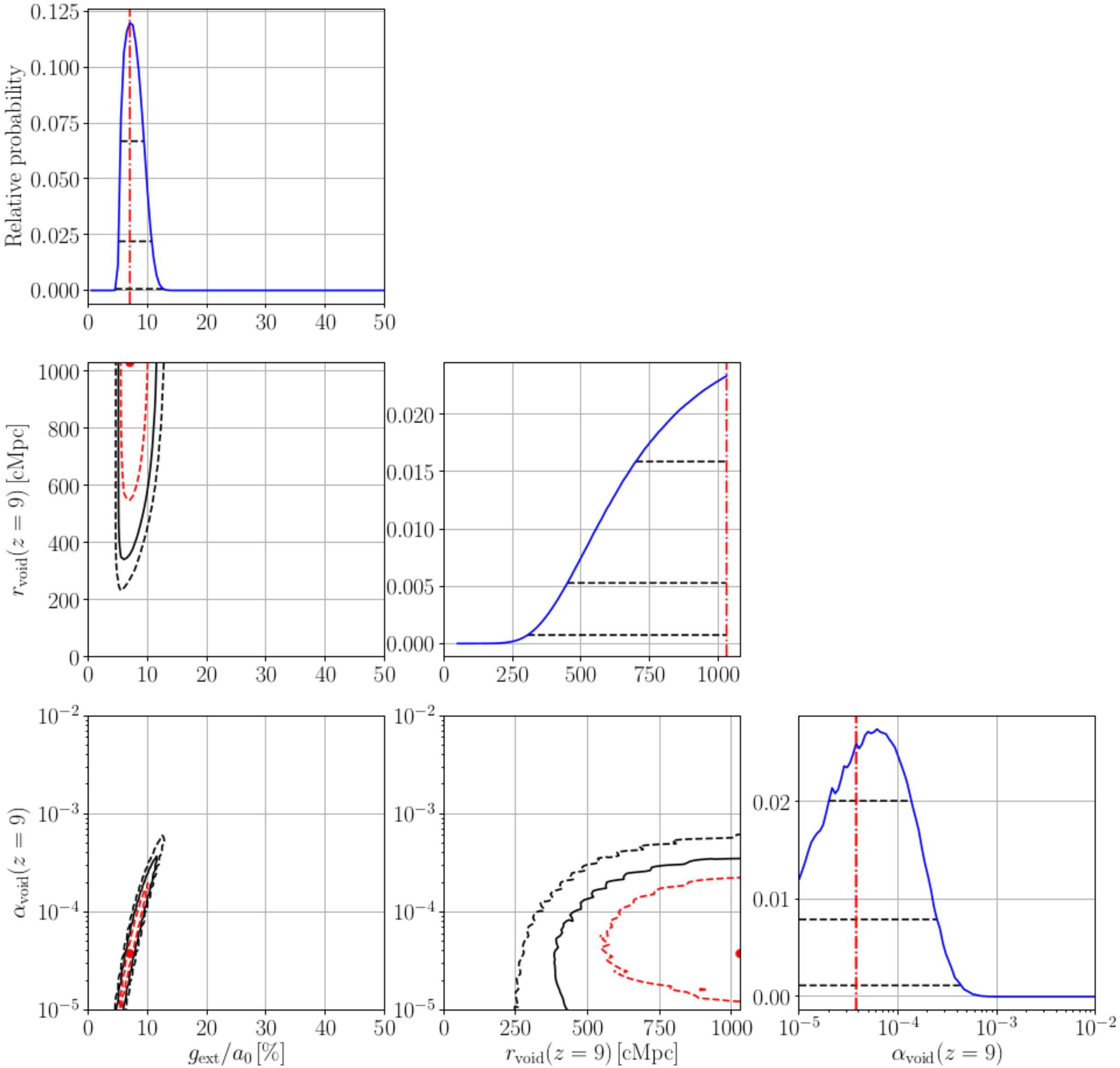
Gaussian density profile, $g_{\rm ext} = 0.070\,a_0$, $r_{\rm void} = 1030.0$ cMpc, $\alpha_{\rm void} = 3.76 \times 10^{-5}$, $v_{\rm void} = 2018\,{\rm km\,s^{-1}}$, $r_{\rm void}^{\rm rms} = 744.7$ Mpc, $n_{\rm EFE} = 0$

Parameter	$H_0^{\rm local}$ [km s^{-1} Mpc^{-1}]	$\overline{q}_0^{\rm local}$	$H_0^{\rm lensing}$ [km s^{-1} Mpc^{-1}]	$v_{\rm LG}$ [km s^{-1}]	$\delta_{\rm in}$	$\delta_{\rm out}$
Observations	75.35 ± 1.68	1.08 ± 0.29	––	627	0.274 ± 0.081	-0.085 ± 0.108
MOND model	77.24	1.43	––	––	0.155	0.078
χ^2	1.79		12.74	––	2.19	2.26
Degrees of freedom	2		7	––	1	1
χ (1D Gaussian equivalent)	0.83		1.76	2.35	1.48	1.50

Exponential density profile, $g_{\rm ext} = 0.080\,a_0$, $r_{\rm void} = 1030.0$ cMpc, $\alpha_{\rm void} = 7.56 \times 10^{-5}$, $v_{\rm void} = 2307\,{\rm km\,s^{-1}}$, $r_{\rm void}^{\rm rms} = 730.4$ Mpc, $n_{\rm EFE} = 0$

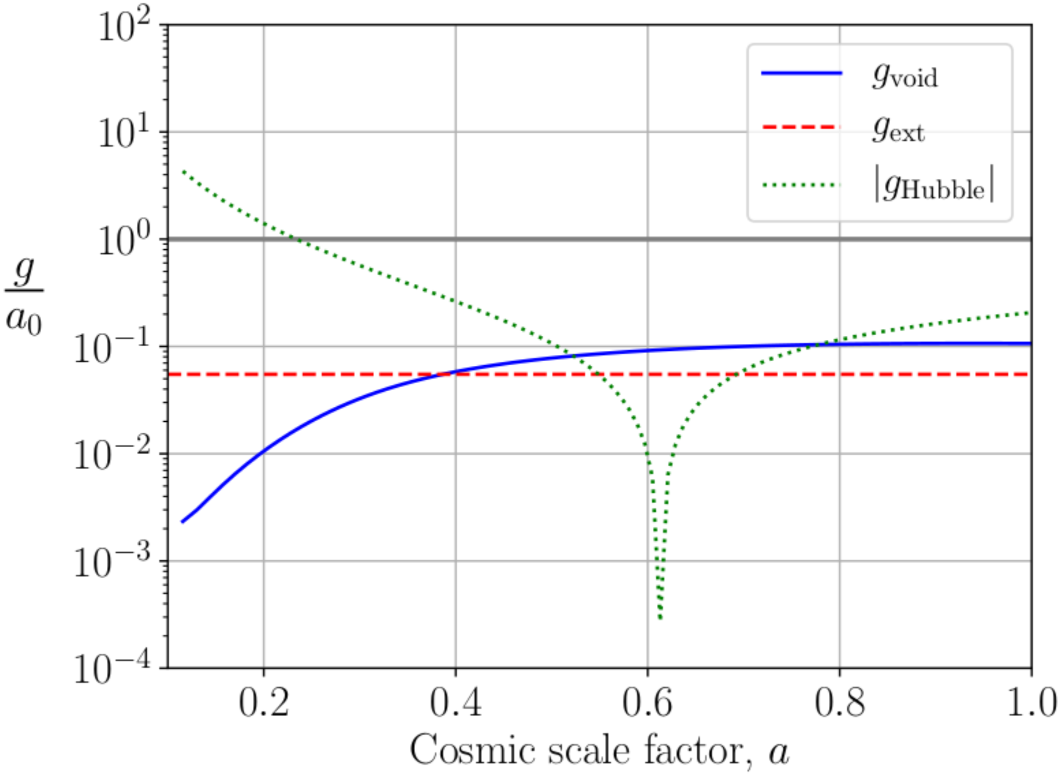
Parameter	$H_0^{\rm local}$ [km s^{-1} Mpc^{-1}]	$\overline{q}_0^{\rm local}$	$H_0^{\rm lensing}$ [km s^{-1} Mpc^{-1}]	$v_{\rm LG}$ [km s^{-1}]	$\delta_{\rm in}$	$\delta_{\rm out}$
Observations	75.35 ± 1.68	1.08 ± 0.29	––	627	0.276 ± 0.080	-0.078 ± 0.108
MOND model	77.25	1.46	––	––	0.158	0.073
χ^2	1.98		13.19	––	2.17	1.97
Degrees of freedom	2		7	––	1	1
χ (1D Gaussian equivalent)	0.89		1.83	2.47	1.47	1.40

Results:



Relative probability
0.125
0.100
0.075
0.050
0.025
0.000
0
10
20
30
40
50
$r_{\rm void}(z=9)$ [cMpc]
1000
800
600
400
200
0
0.020
0.015
0.010
0.005
0.000
250
500
750
1000
$\alpha_{\rm void}(z=9)$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.02
0.01
0.00
$g_{\rm ext}/a_0$ [%]
$r_{\rm void}(z=9)$ [cMpc]
$\alpha_{\rm void}(z=9)$

Results: Hubble field effect



Results: Time-dependent external field effect

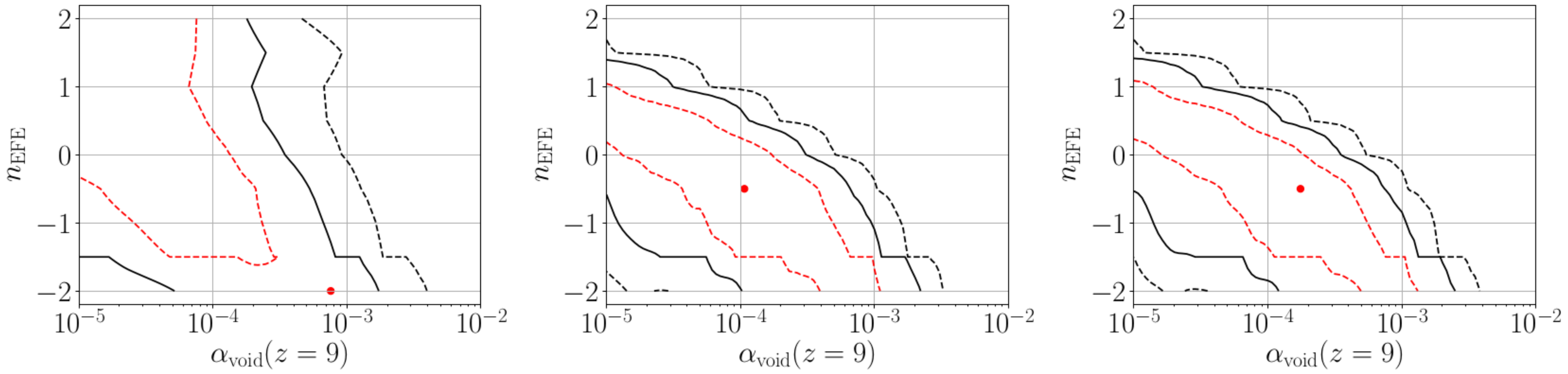


Figure 14. Marginalized posterior distribution of the indicated model parameters based on 9×10^6 MOND models for a Maxwell-Boltzmann (left), Gaussian (middle), and exponential (right) initial profile. The red dashed, black solid, and black dashed lines mark the 1σ, 2σ, and 3σ confidence levels, respectively. A stronger EFE in the past ($n_{\rm EFE} < 0$) requires a stronger initial void strength at $z = 9$. The red dots mark the best-fitting models: $g_{\rm ext} = 0.030\,a_0$, $r_{\rm void} = 218.3\,\text{cMpc}$, $\alpha_{\rm void} = 7.56 \times 10^{-4}$, $n_{\rm EFE} = -2$ (Maxwell-Boltzmann profile, left-hand panel); $g_{\rm ext} = 0.065\,a_0$, $r_{\rm void} = 1030.0\,\text{cMpc}$, $\alpha_{\rm void} = 1.07 \times 10^{-4}$, $n_{\rm EFE} = -0.5$ (Gaussian profile, middle panel); and $g_{\rm ext} = 0.070\,a_0$, $r_{\rm void} = 1030.0\,\text{cMpc}$, $\alpha_{\rm void} = 1.75 \times 10^{-4}$, $n_{\rm EFE} = -0.5$ (exponential profile, right-hand panel).

Results: Time-dependent external field effect