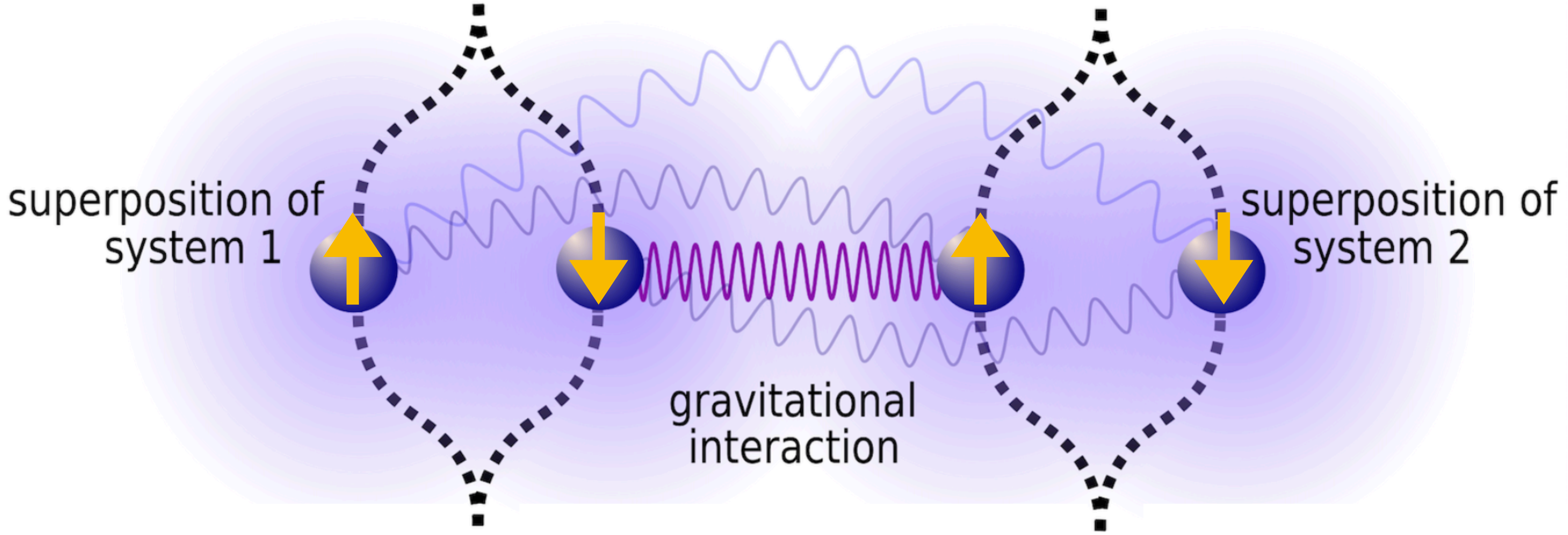
Quantum Gravity witness via Entanglement of Masses: QGEM protocol

rijksuniversiteit groningen

Anupam Mazumdar



Quantum Superpositions of Geometries

Bose + Mazumdar + Morley + Ulbricht + Toros + Paternostro + Geraci + Barker + Kim + Milburn, Phy. Rev. Lett. [ArXiv: 1707.06050]

Marshman +Mazumdar+Bose, [ArXiv: 1907.01568] Phys. Rev. A.

Van de Kamp+Marshman +Bose+Mazumdar [2006.06931 [quant-ph]

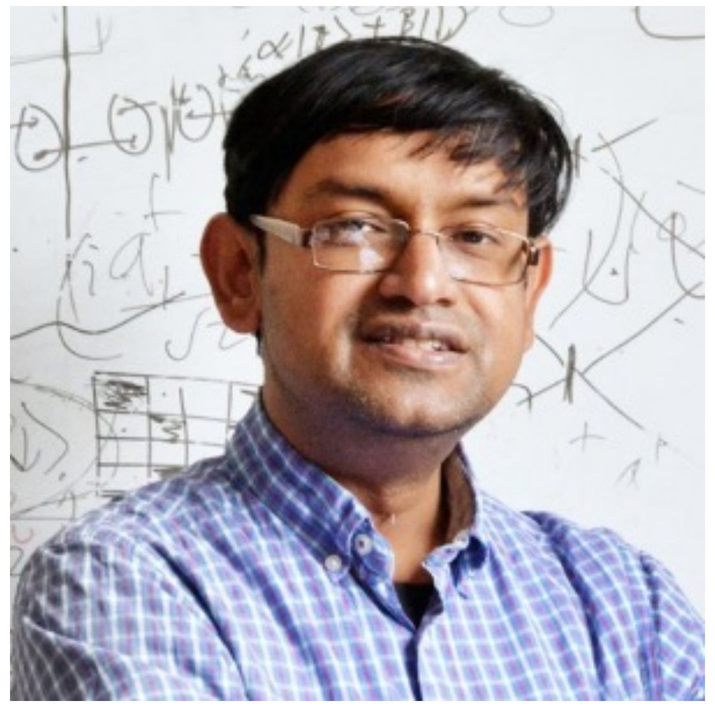
Toros + Van de Kamp + Marshman + Kim + Mazumdar + Bose [arXiv:2007.15029 [gr-qc]

Toros + Mazumdar + Bose 2008.08609 [gr-qc]

See Also: Marletto and Vedral appeared on the same day [1707.06036] (half of the story only)

Collaborators

Sougato Bose

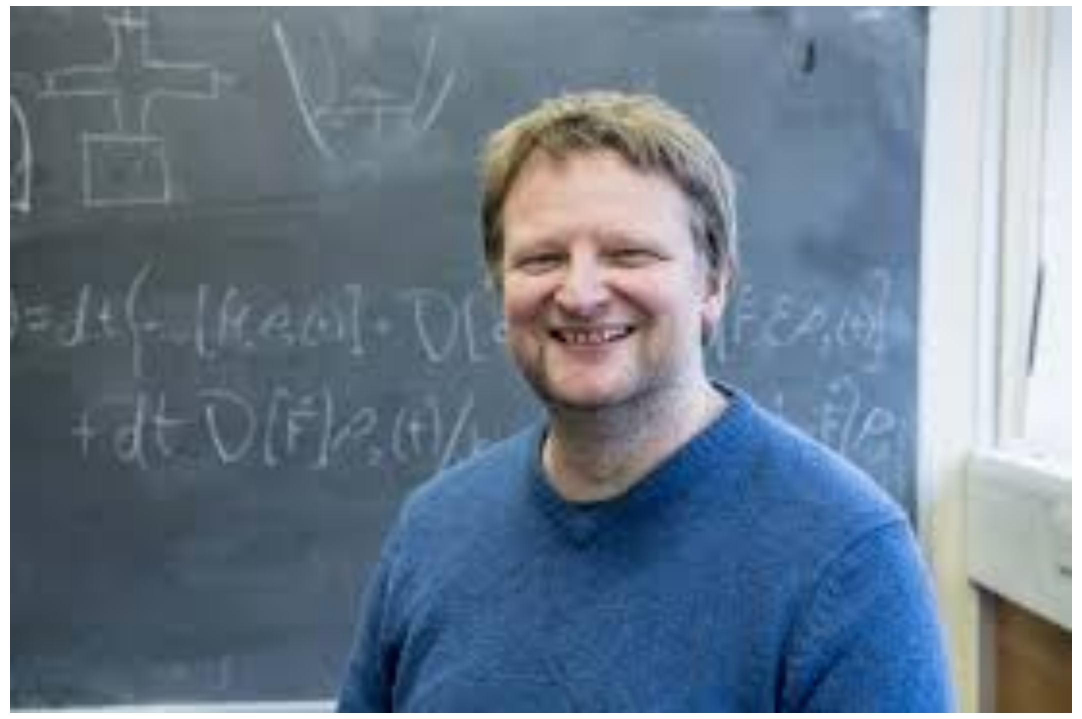


Ryan Marshman

Myungshik Kim

Andrew Geraci

Hendrik Ulbricht



Marko Toros

Mauro Paternostro

Gerard Milburn

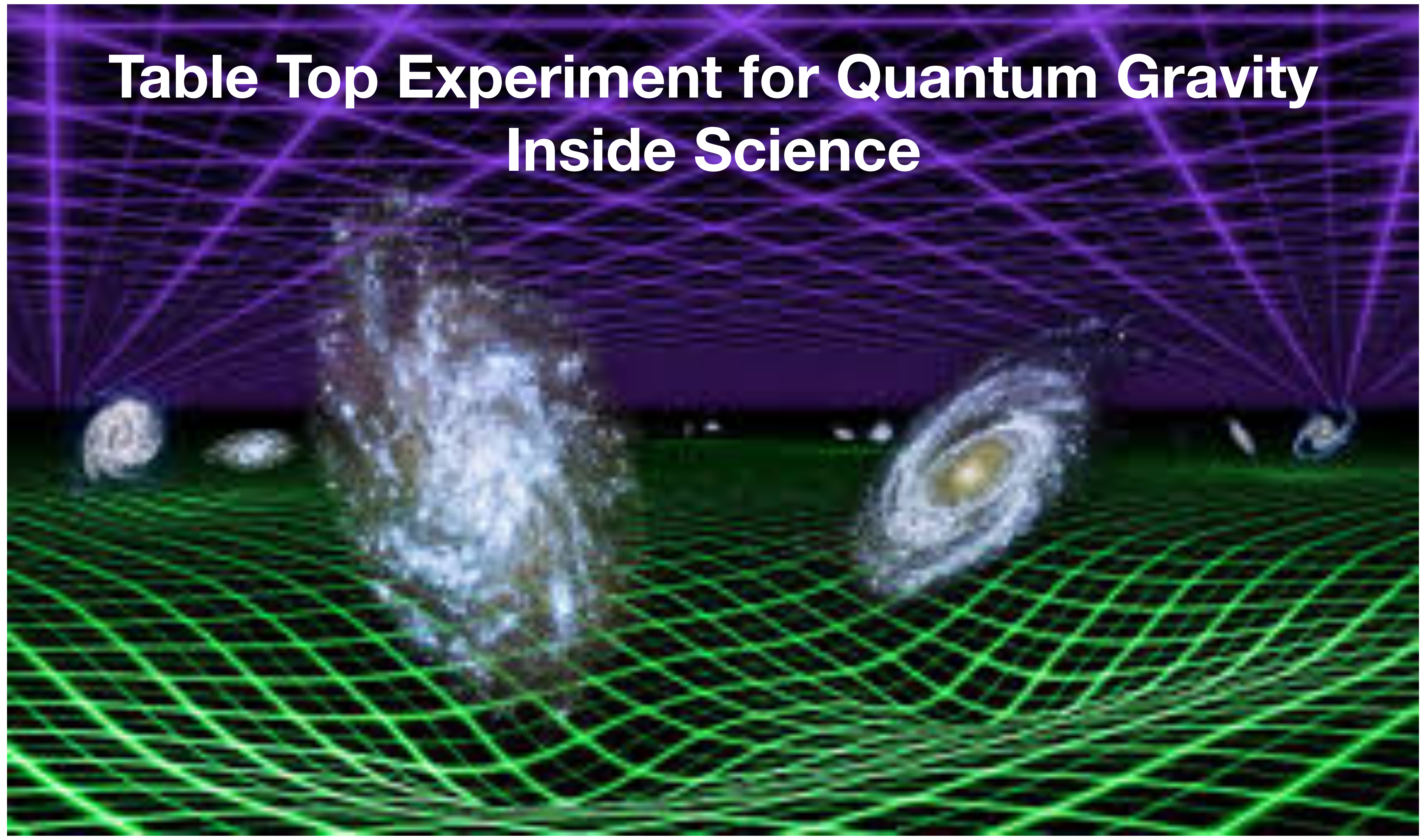
Peter Barker

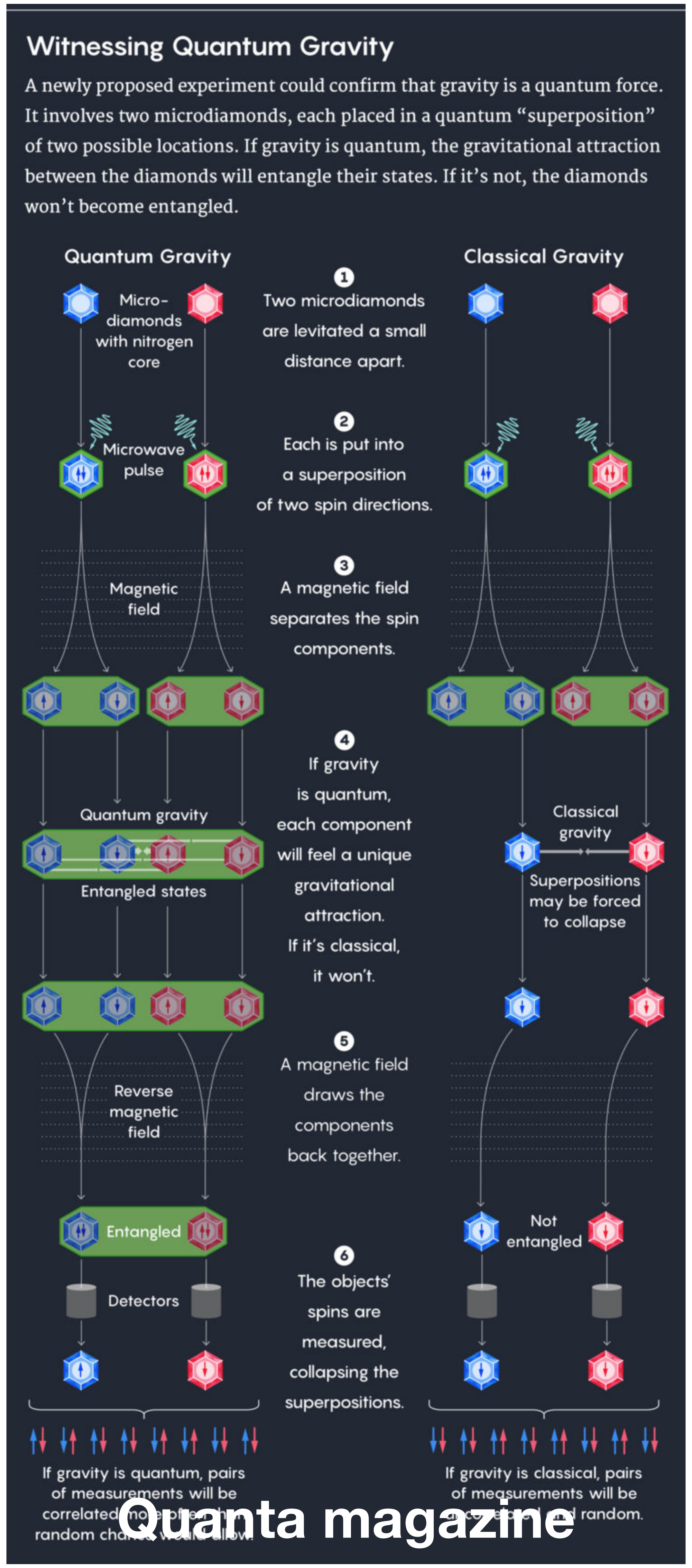
Gavin Morley

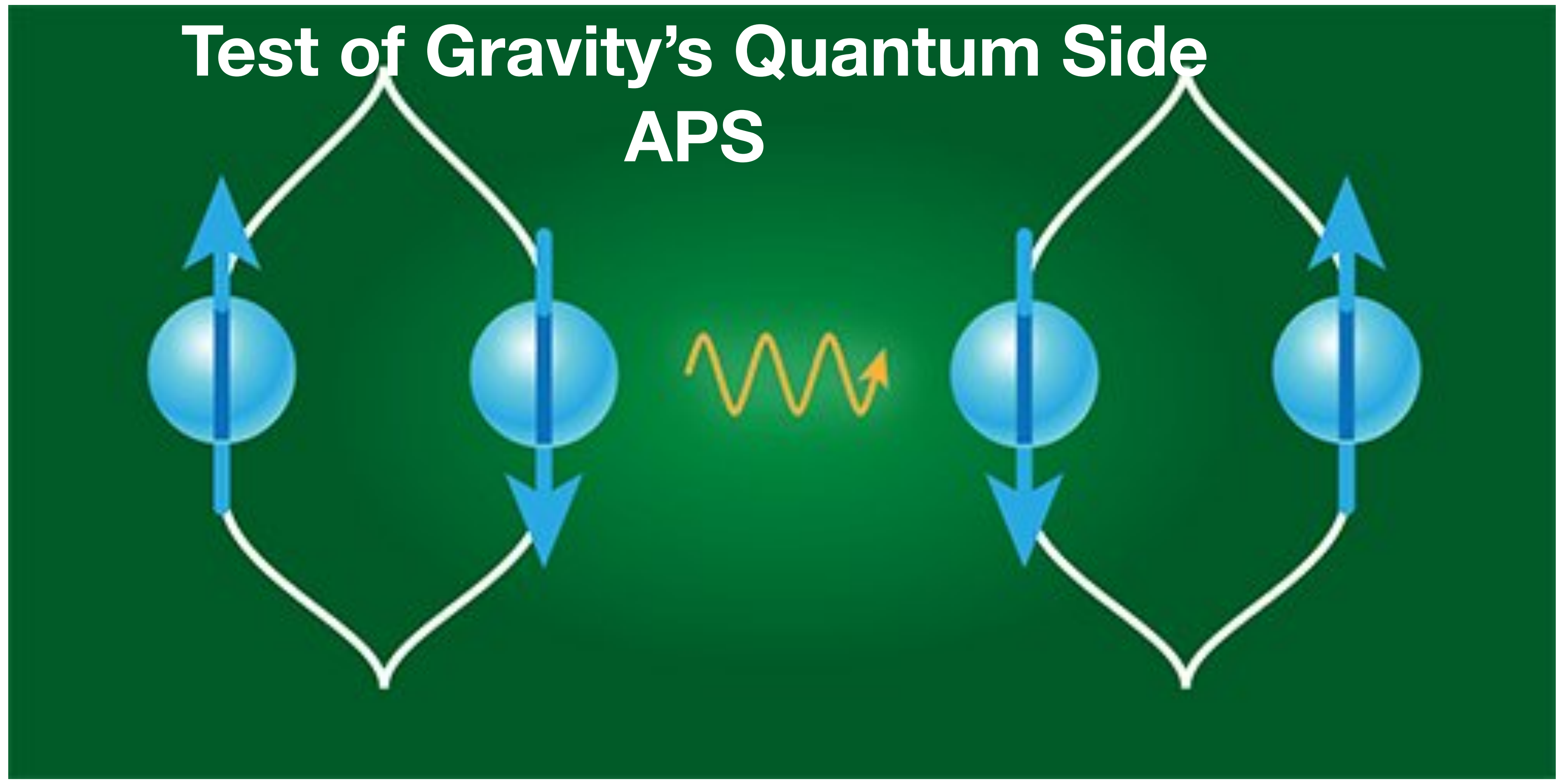
Ron Folman

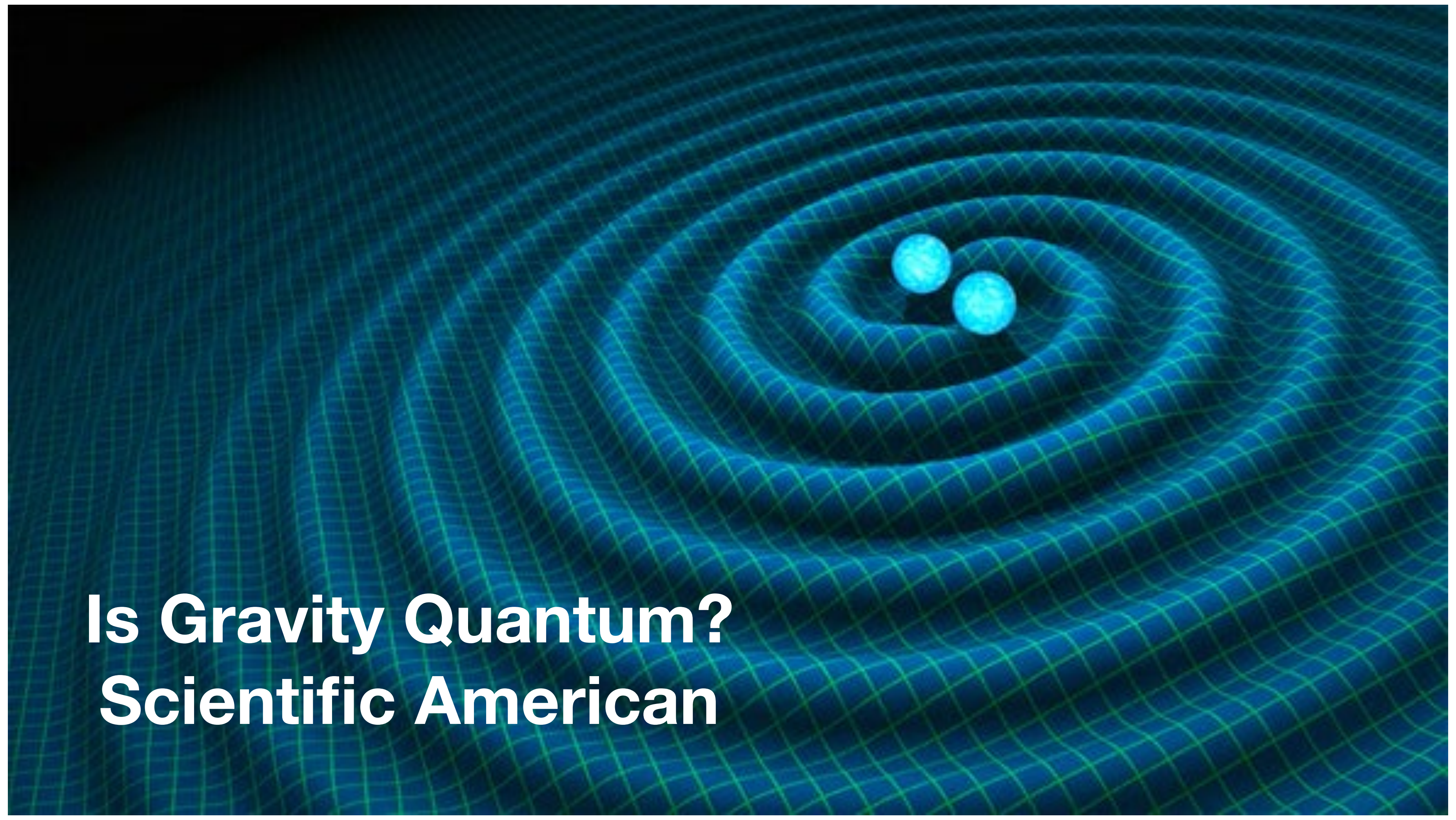
Martine Schut

Anupam Mazumdar









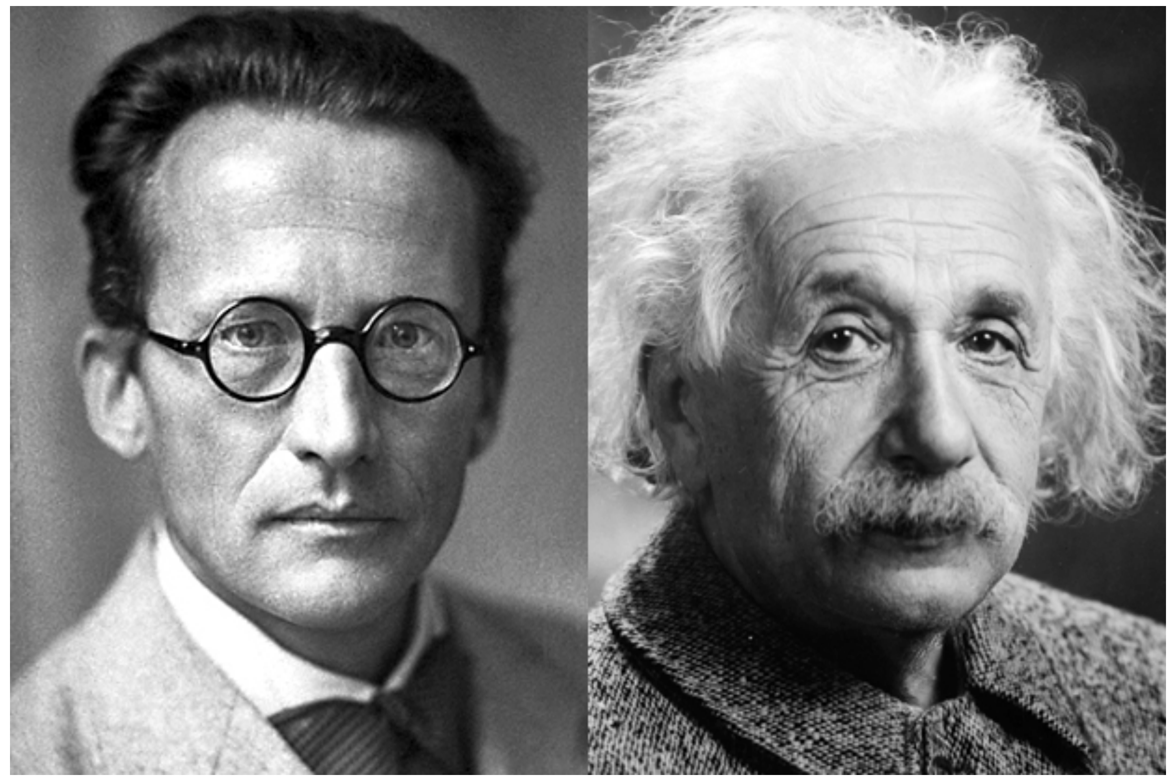
Quantum Gravity Meets Quantum Information & Quantum Technologies

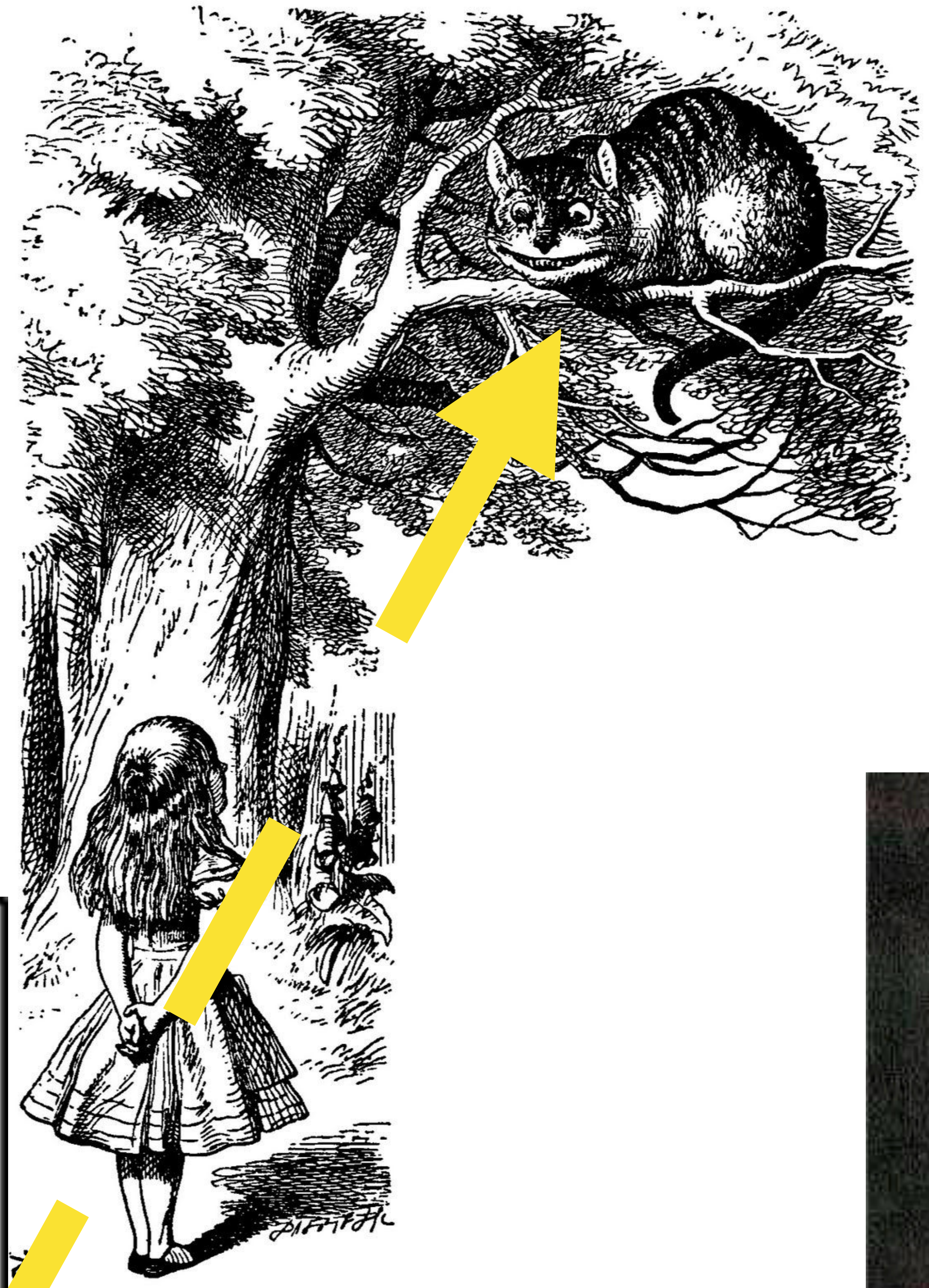
Perfect Avenue for Ground Breaking Contributions

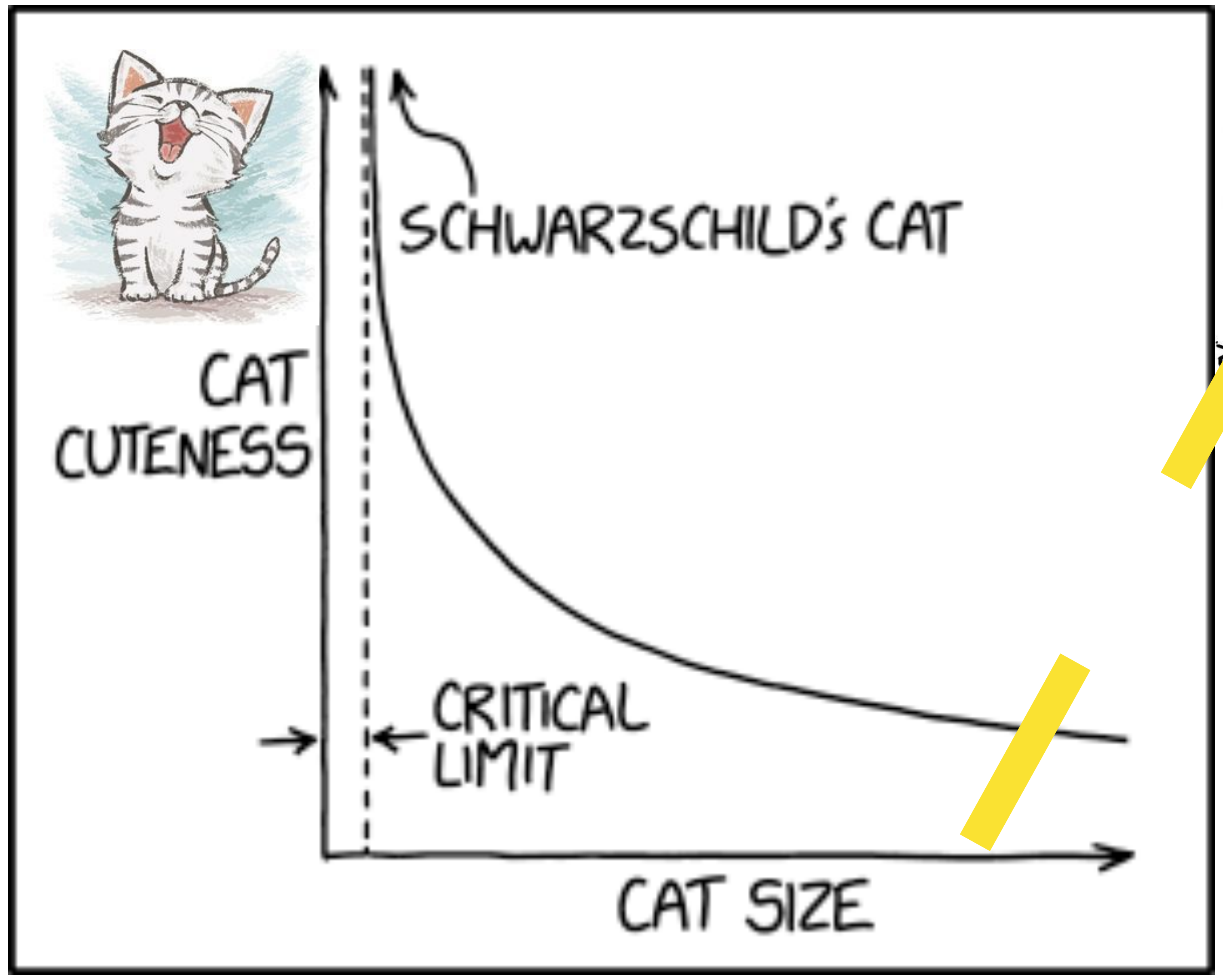
Is Gravity Classical or Quantum?

A very simple question, but extremely hard to answer !

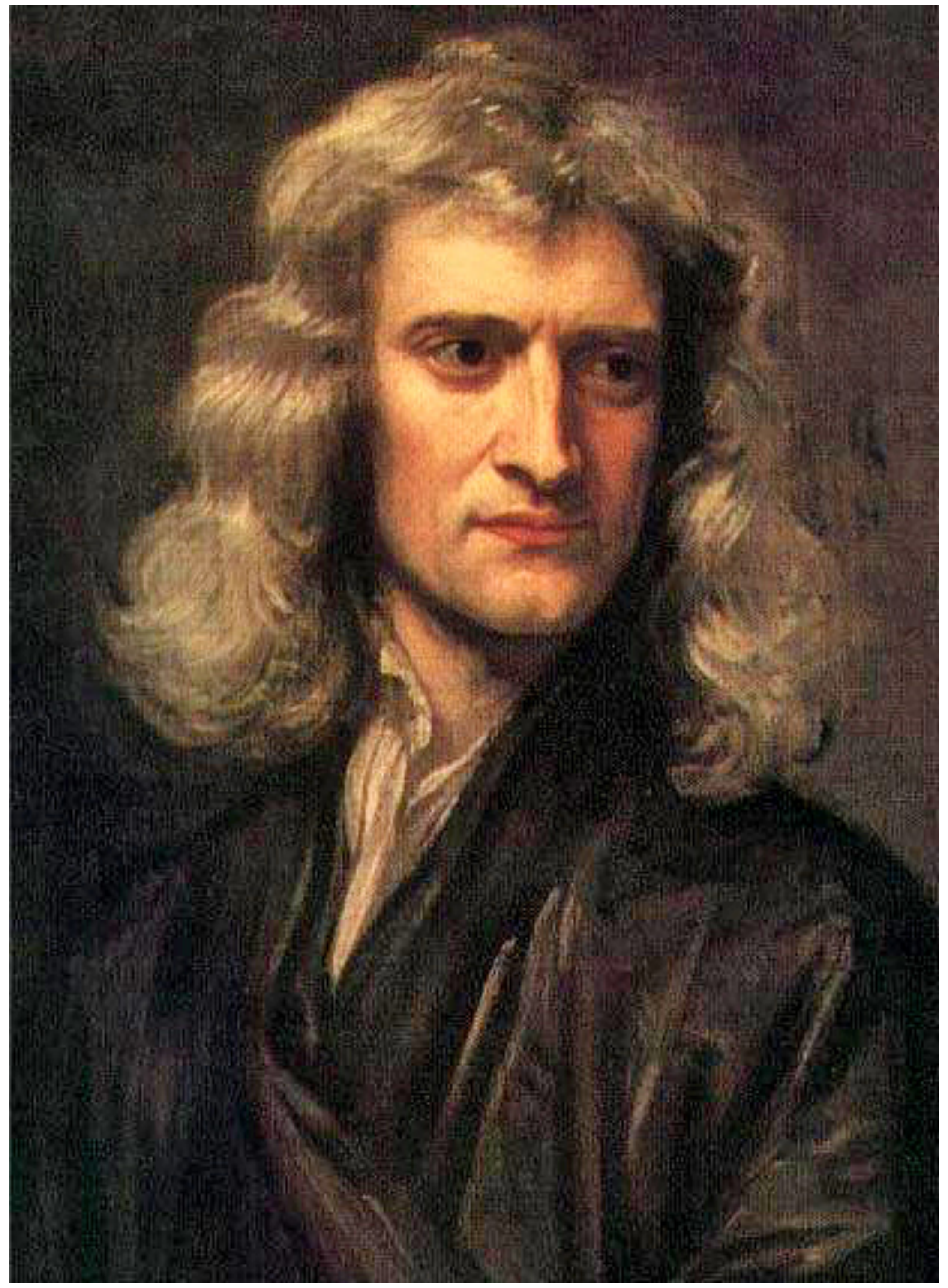
Gravity's Cheshire Cat !







Universal Laws of Gravitation



vs.

Microscopic Origin of Forces

Today's Science Fiction is Tomorrow's Reality

◉ Can we put a graviton in a quantum superposition in a lab? No !

◉ Can we study coalescing atoms, and study the loss of gravitons ? No!

◉ Can we witness Quantum Entanglement due to Quantum Nature of a Graviton ?

Yes!

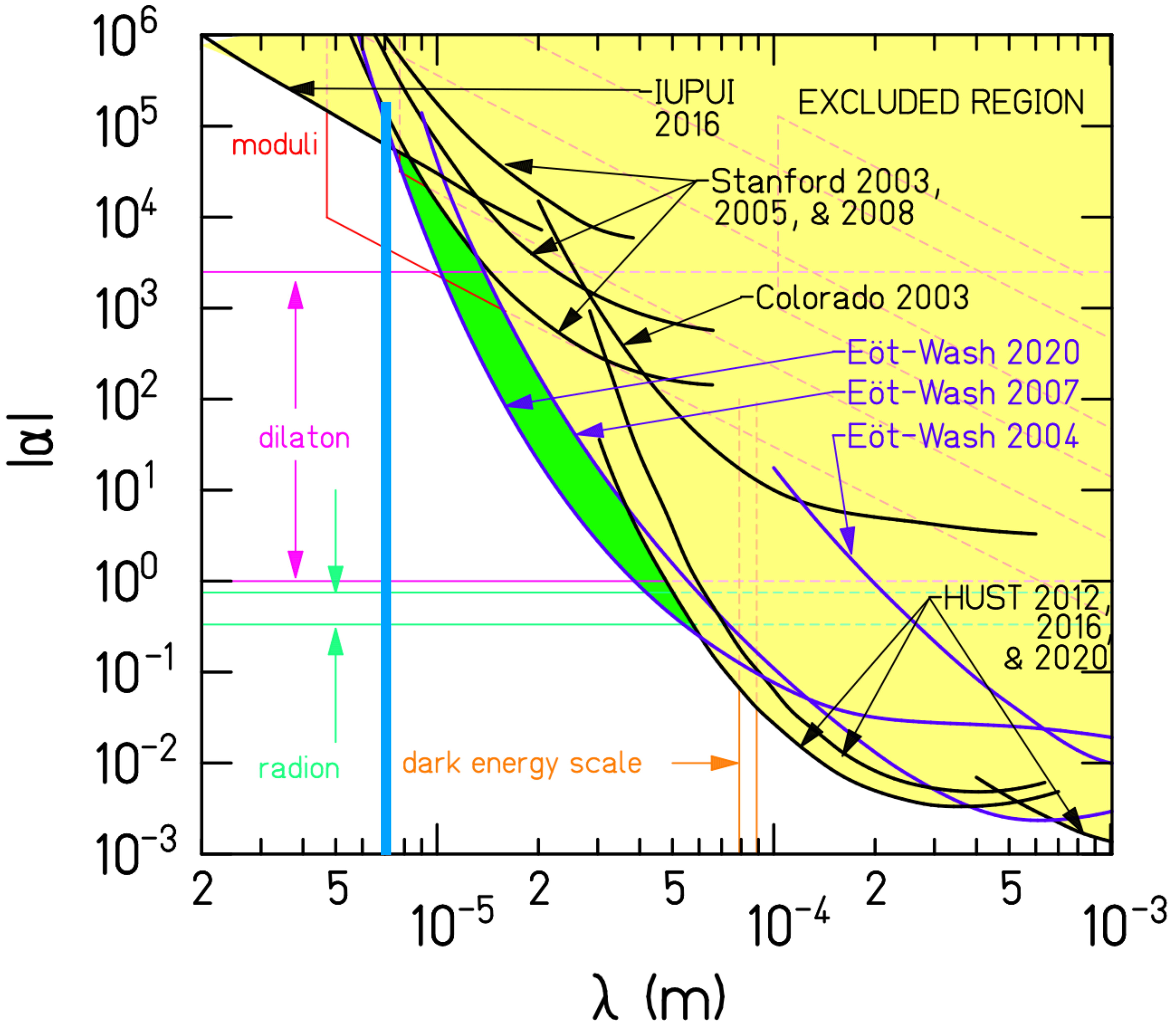
Lively discussions with

Sougato Bose, Jan Ambjorn, Abhay Ashtekar, Markus Aspelmeyers, Markus Arndt, Valeri Frolov, Gary Horowitz, Rob Myers, Don Page, Roger Penrose, Carlo Rovelli, Ashoke Sen, Gabriele Veneziano, Bob Wald, …

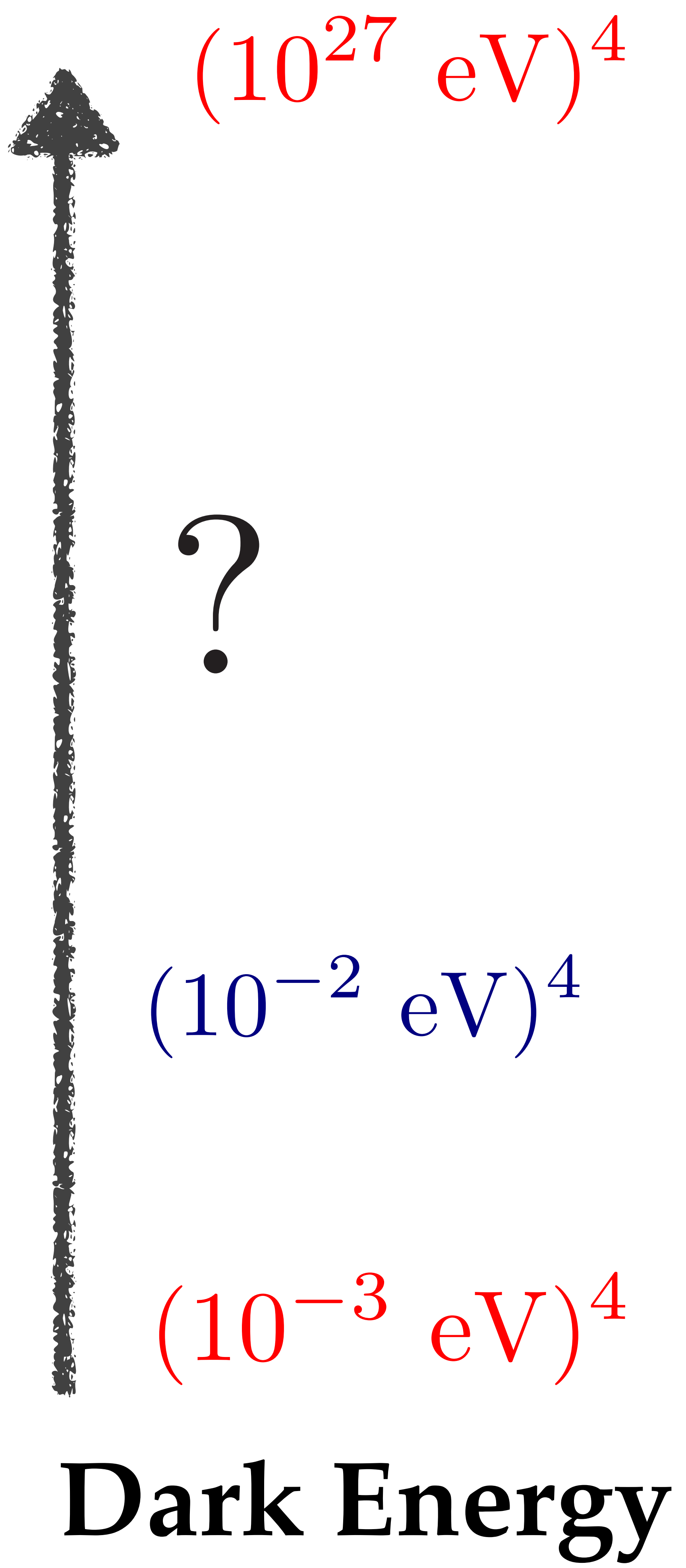
Things should be made as simple as possible, but no simpler! — Albert Einstein

- QG-Witness protocol for Quantum Gravity
- Reading the Witness
- Associated Challenges & Future Program
- Quantum Gravity

Constraints on Gravity, Newton's potential



$$V(r) = -\frac{Gm^2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$



J. G. Lee, E. G. Adelberger, T. S. Cook, S. M. Fleischer, B. R Heckel 2002.11761 [hep-ex]

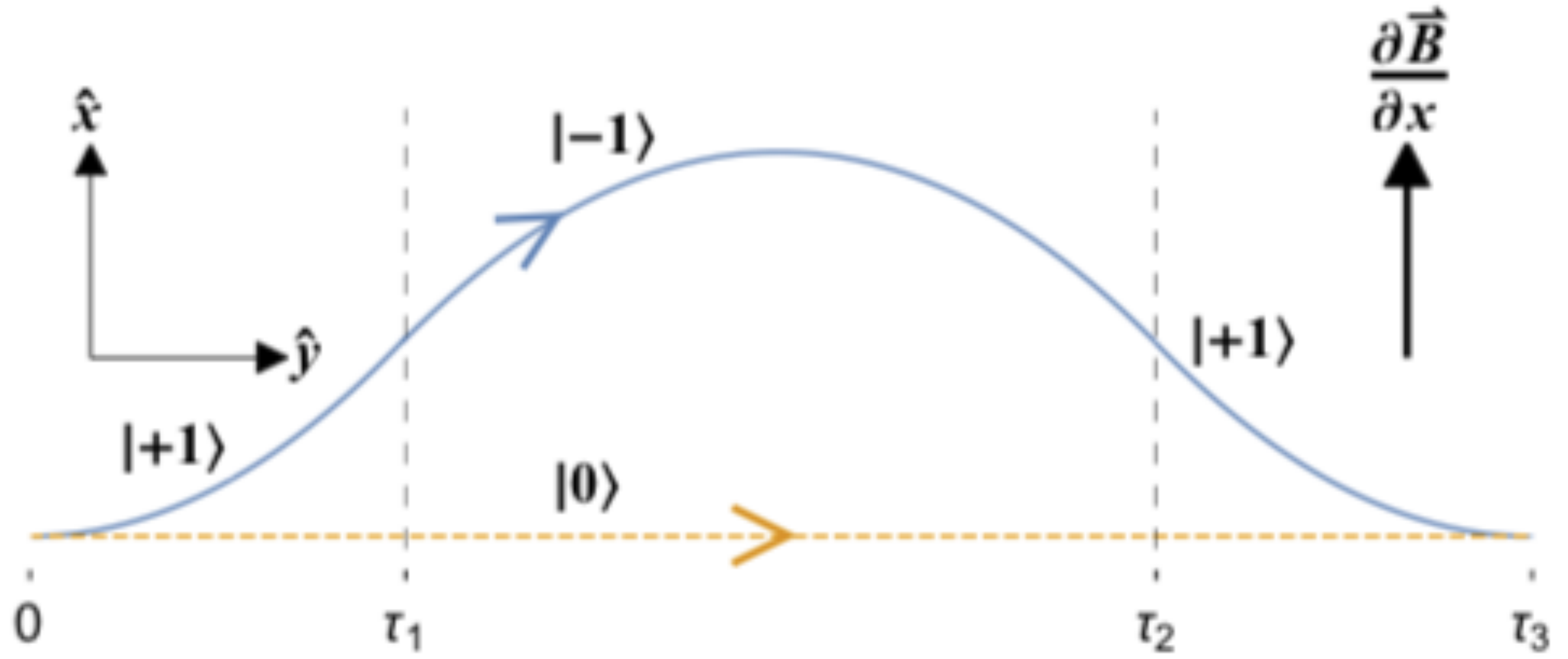
New effects in gravity could be around the corner !

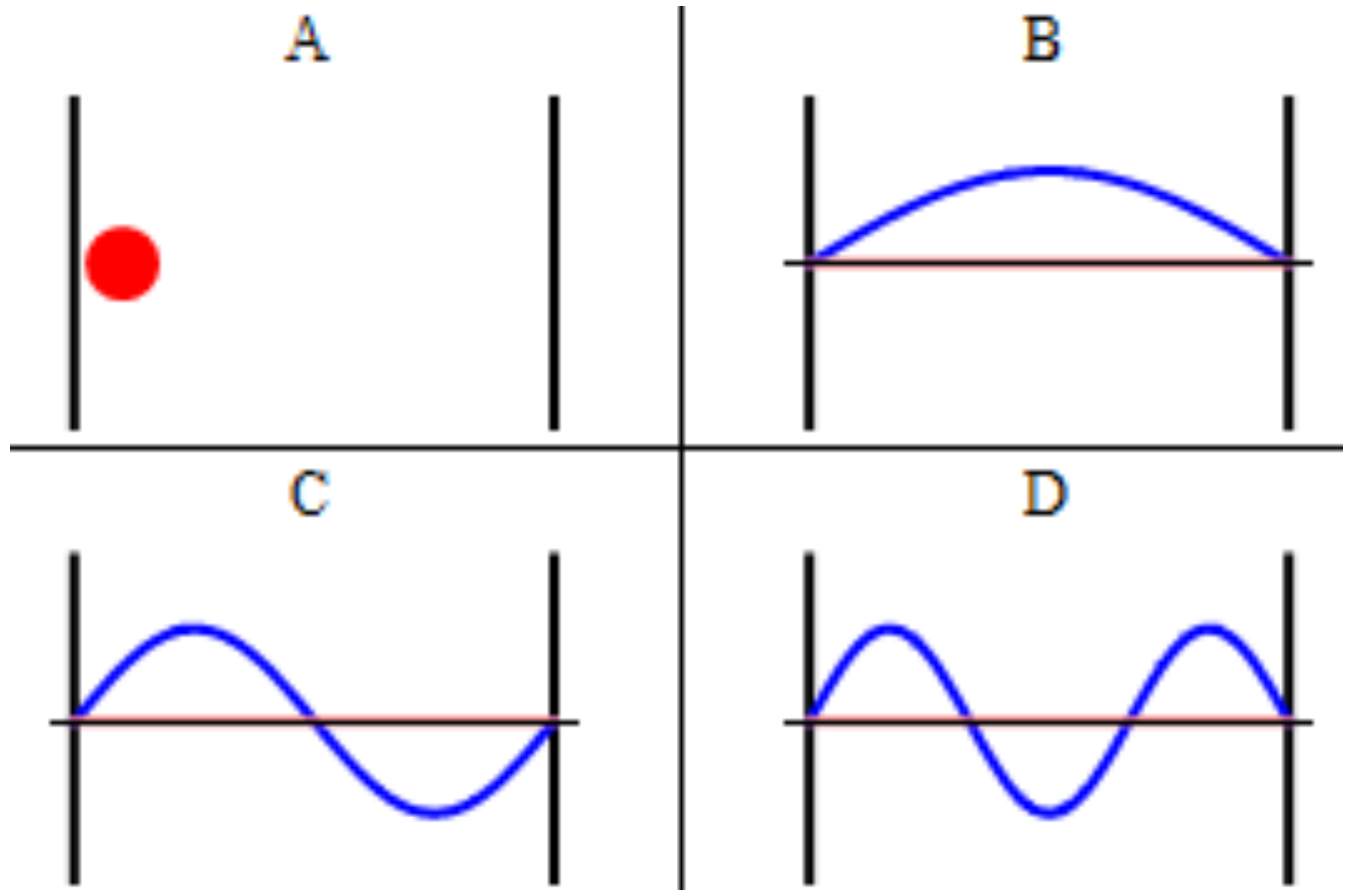
Classical Gravity Induced Quantum Phase

$$\Delta\phi \sim i\frac{S(G, G^2, G^2\hbar\cdots)}{\hbar}$$

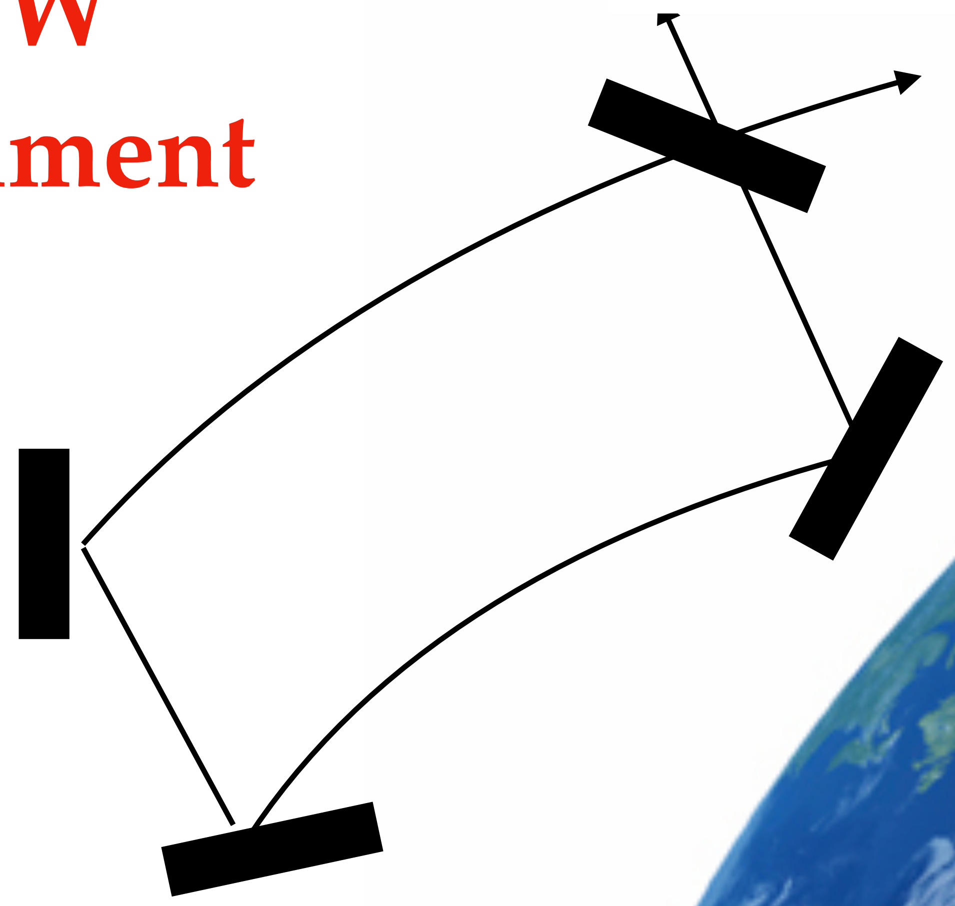
MIMAC. Mesoscopic Interferometer for Metric and Curvature

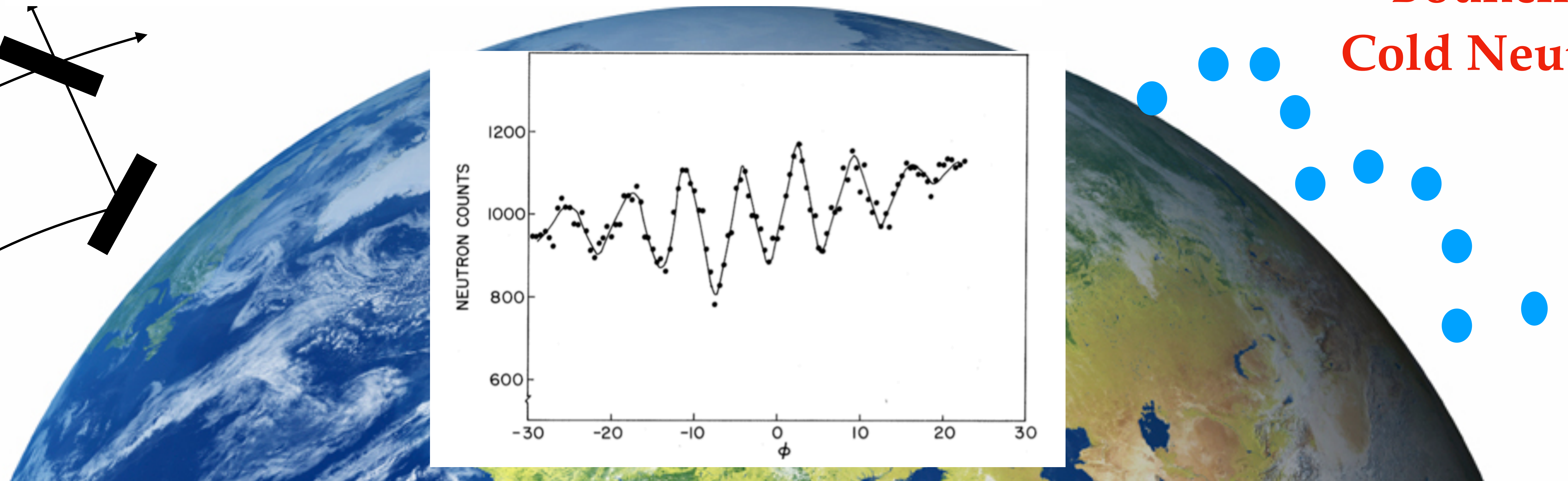
Marshman, Bose, Morley, Barker, Hoekstra





COW Experiment





Bouncing Cold Neutron

Initial Initiatives against Semi-Classical Gravity

VOLUME 47, NUMBER 14 PHYSICAL REVIEW LETTERS 5 OCTOBER 1981

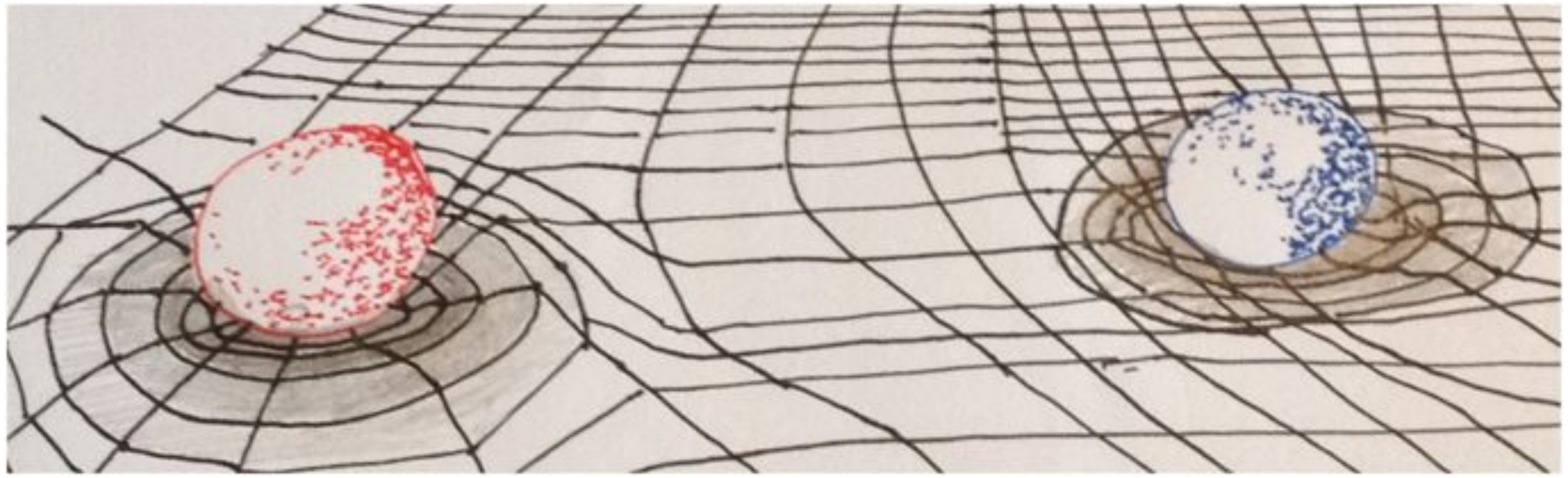
Indirect Evidence for Quantum Gravity

Don N. Page
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802

and

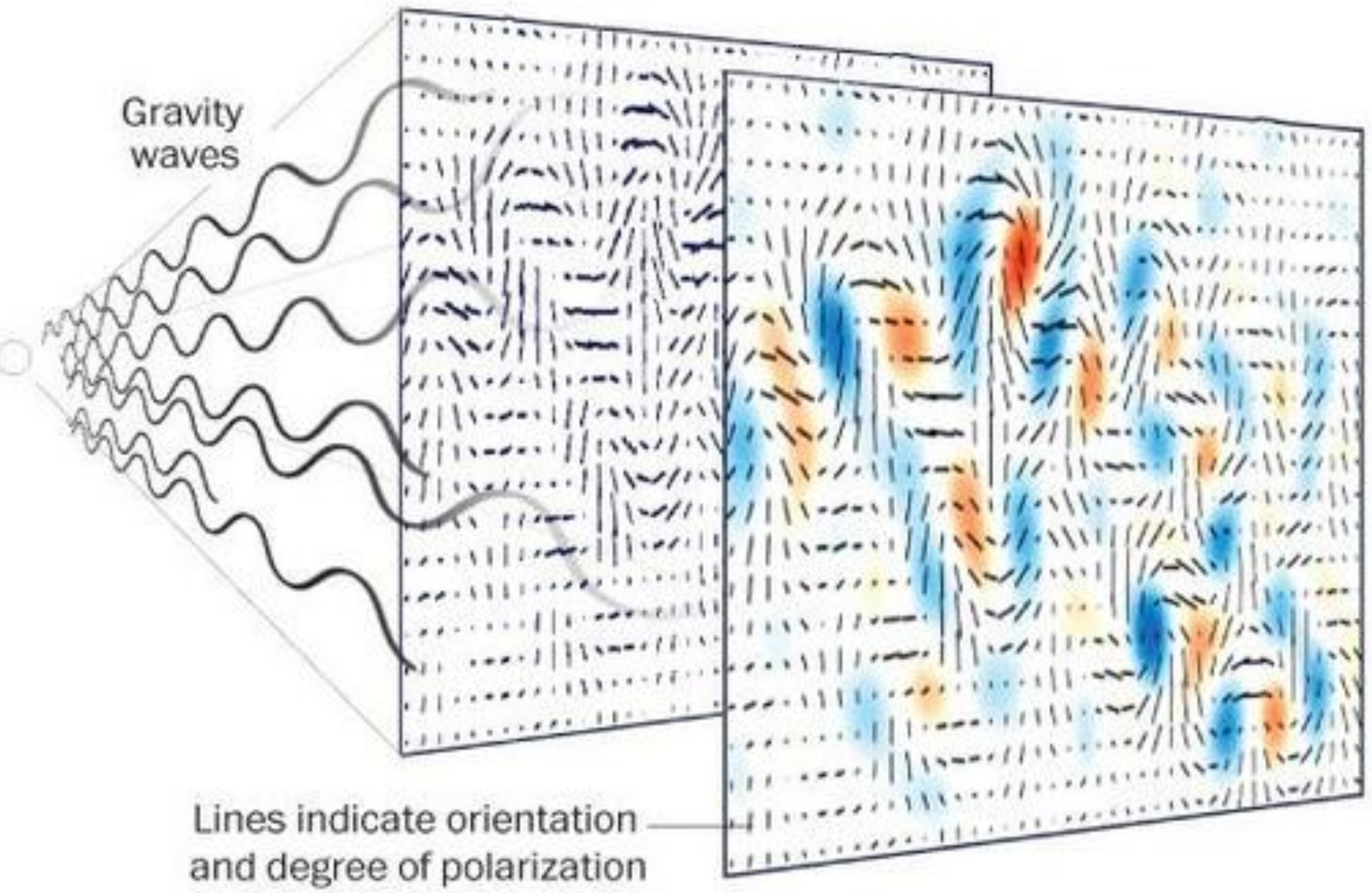
C. D. Geilker
Department of Physics, William Jewell College, Liberty, Missouri 64068
(Received 9 June 1981)

An experiment gave results inconsistent with the simplest alternative to quantum gravity, the semiclassical Einstein equations. This evidence supports (but does not prove) the hypothesis that a consistent theory of gravity coupled to quantized matter should also have the gravitational field quantized.



$$G_{\mu\nu} = \kappa^2 \langle T_{\mu\nu} \rangle$$

Primordial Gravitational Waves: is it a signature of QG?



Caution!

Positive Detection of a B-mode polarization by BICEP (2013)

But the origin was not found to be primordial in nature.

$$\mathscr{P}_g = \frac{64[|\alpha_k|^2 + |\beta_k|^2]\hbar^2 H^2}{M_p^2}$$

Bunch-Davies quantum vacuum: $\alpha_k = -\frac{\sqrt{\pi}}{2}, \ \beta_k = 0$

Initial Conditions: Classical or Quantum?

Mere presence of $\hbar$ is not sufficient to say that gravity is quantum !

L. M. Krauss and F. Wilczek,
``Using Cosmology to Establish the Quantization of Gravity,''
[arXiv:1309.5343 [hep-th]].

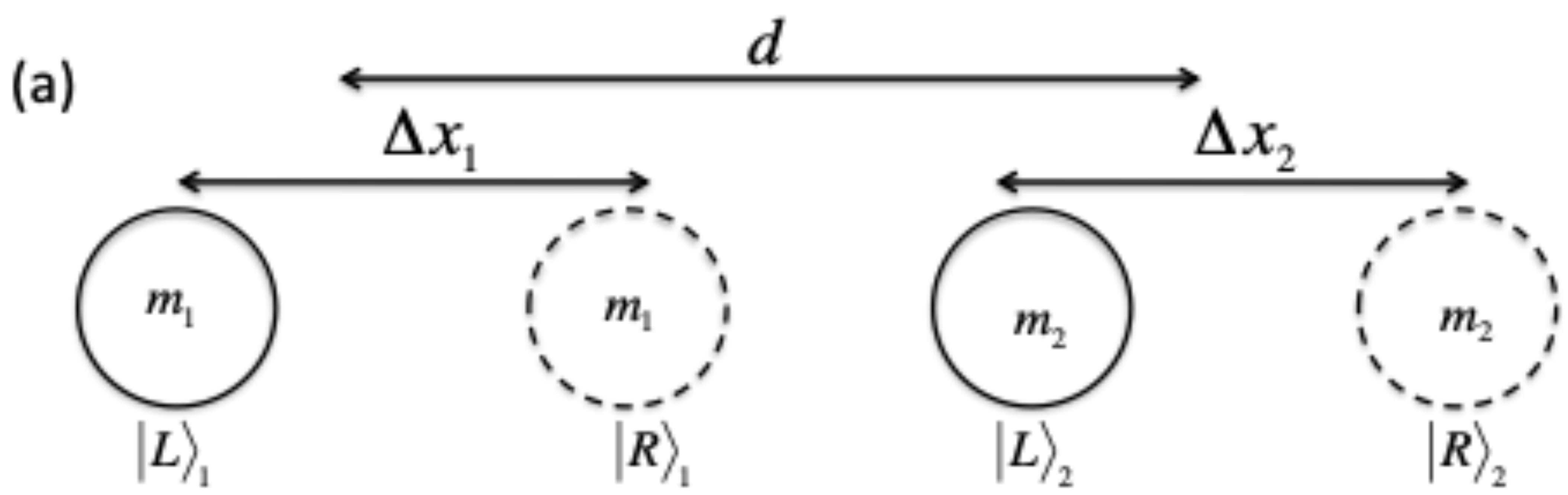
A. Ashoorioon, P. S. Bhupal Dev and AM,
``Implications of purely classical gravity for inflationary tensor modes,''
[arXiv:1211.4678 [hep-th]].

Witnessing Quantum Gravity

Razor Sharp Argument

- If the Witness is Positive then Gravity is Quantum
- If the Witness is Negative: Inconclusive !

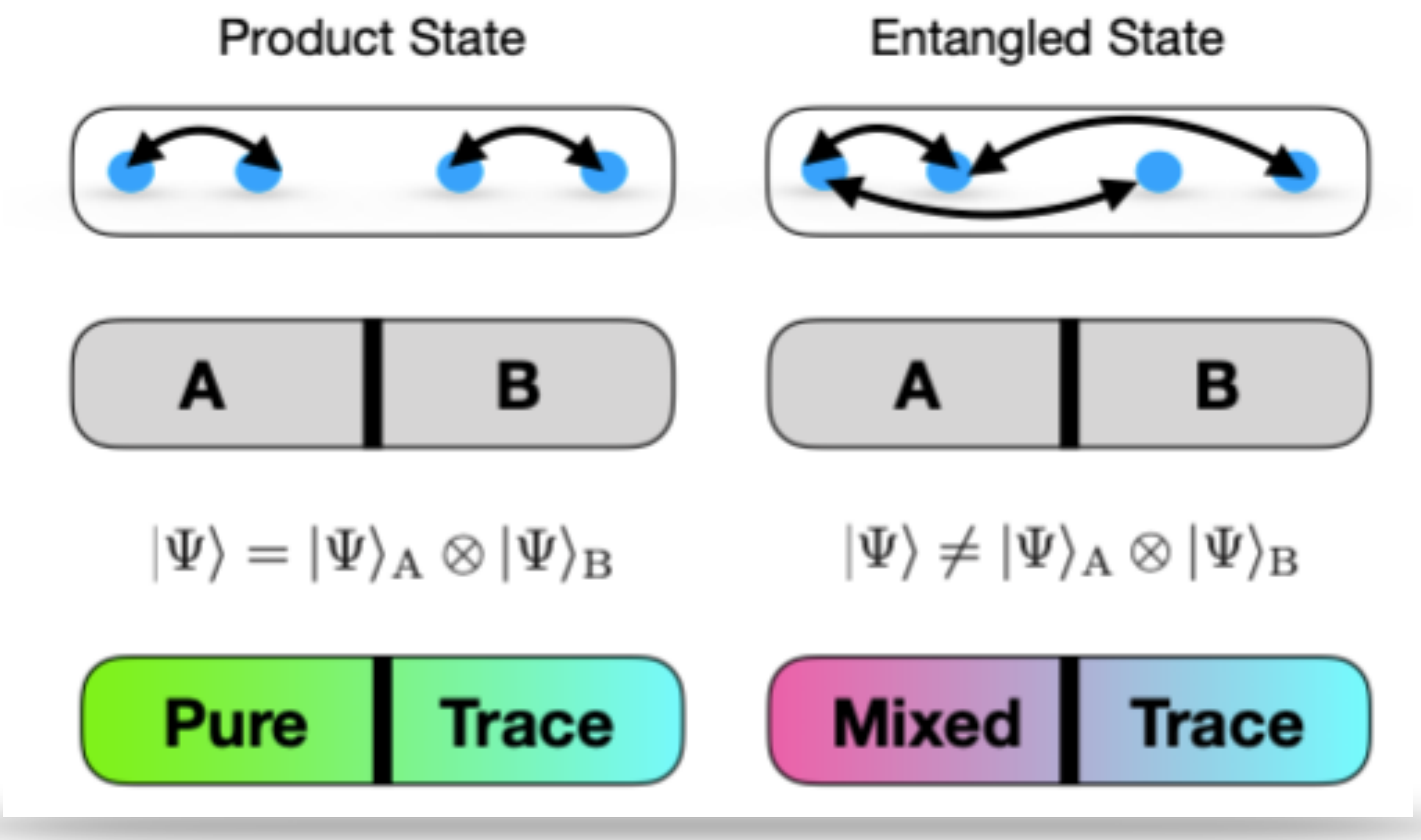
Key Ingredients



◉ **Macroscopic Quantum Superposition of Localised Objects**

Curvatures are Localised

10^{-21}Kg **[Markus Arndt's Lab, University of Vienna]**



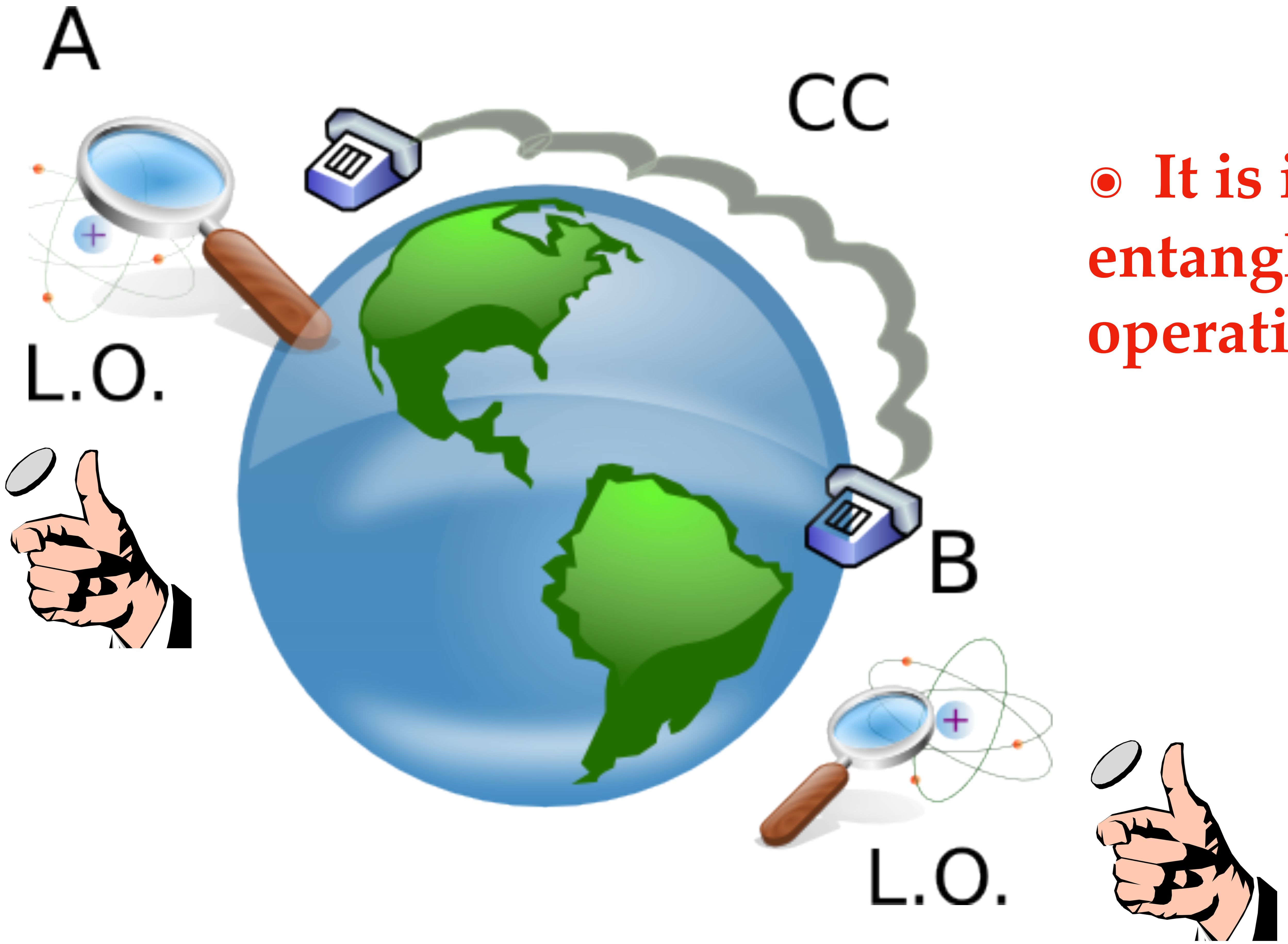
◉ **Quantum Entanglement**

$$e^+e^- + e^-e^+$$

$$\uparrow\downarrow - \downarrow\uparrow$$

Bose+AM+Morley+Ulbricht+Toros+Paternostro+Geraci+Barker+Kim+Milburn, PRL (2017) [1707.06050]

Local Operations & Classical Communication (LOCC)



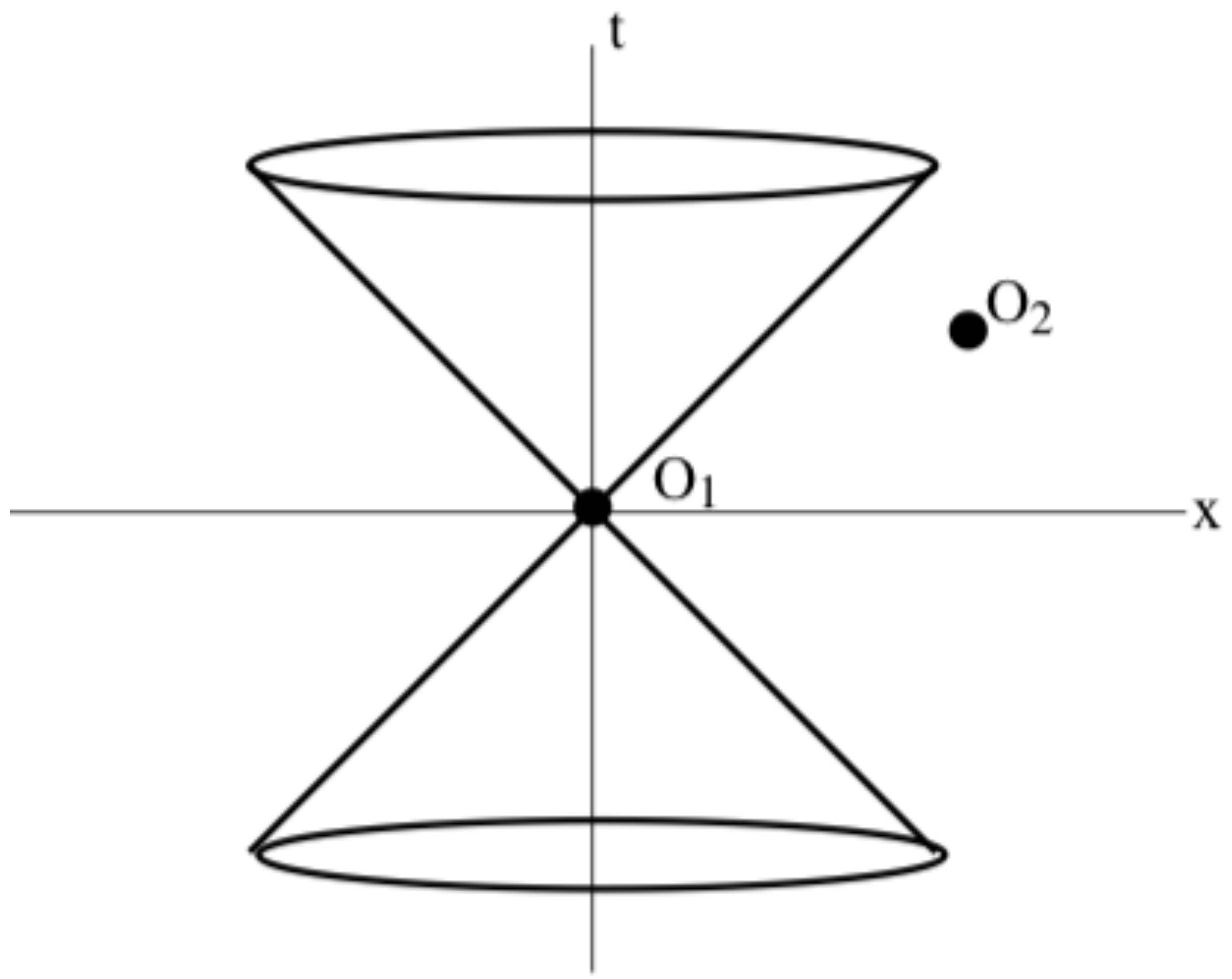
- It is impossible to generate/increase entanglement between A and B by local operations and classical communications

Bennett, et.al, (1996)

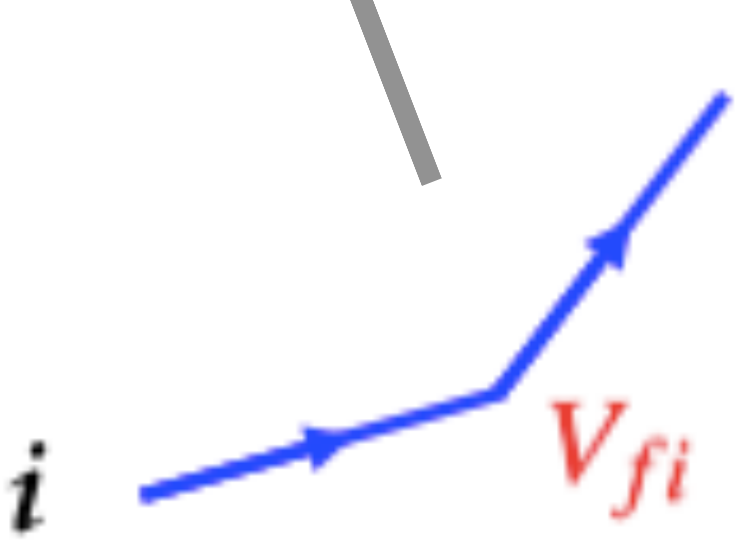
LOCC keeps Separable state remains Separable (Cannot Create Entanglement)

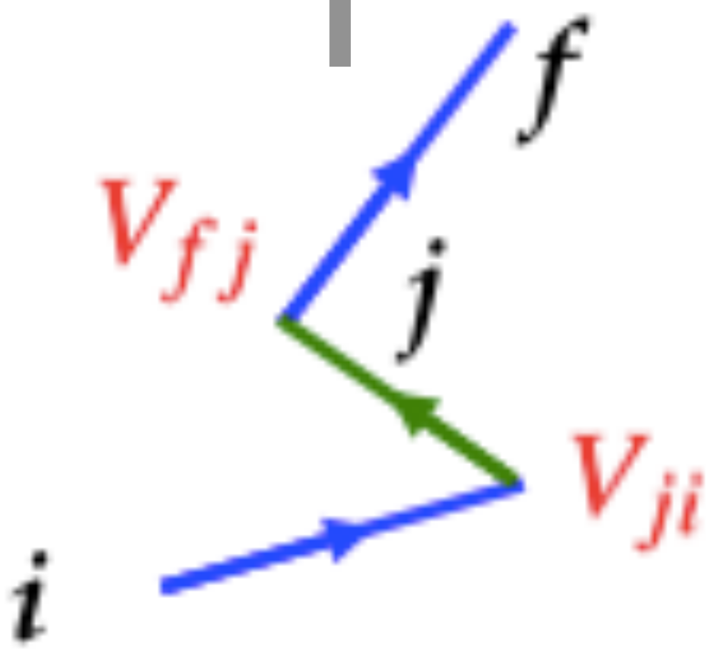
Quantum origin of V(r)

- "Classical picture" – particles act as sources for fields which give rise a potential in which other particles scatter – "action at a distance"
- "Quantum Field Theory picture" – forces arise due to the exchange of virtual particles. No action at a distance + forces between particles now due to particles



$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j} + \ldots$$





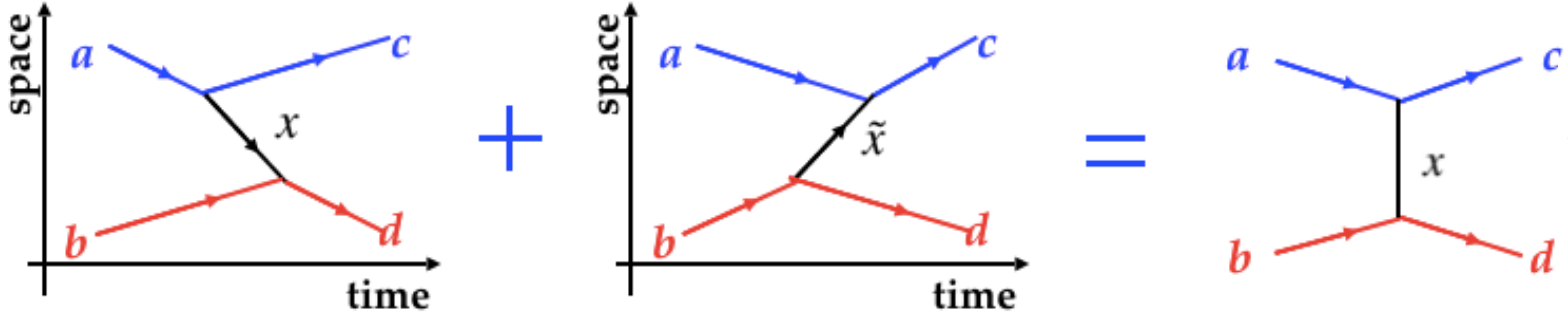
Violates Special Relativity

There are papers by Bei-lok Hu and Anastapoulos, who believe that tree level scattering is classical & not quantum!

These papers do not have the concept of an off-shell graviton mediator!

Quantum Mechanics to Quantum Field Theory

- The sum over all possible time-orderings is represented by a **FEYNMAN diagram**



$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$

- Momentum conserved at vertices
- Energy **not** conserved at vertices
- Exchanged particle **"on mass shell"**

$$E_x^2 - |\vec{p}_x|^2 = m_x^2$$

- Momentum **AND** energy conserved at interaction vertices
- Exchanged particle **"off mass shell"**

$$E_x^2 - |\vec{p}_x|^2 = q^2 \neq m_x^2$$

VIRTUAL PARTICLE

$$\Delta x \Delta p_x \geq \hbar$$

◉ On-shell (Follows Classical Equations of Motion): $E^2 = (pc)^2 + (mc^2)^2$

◉ Off-shell does not follow Classical Equations of Motion

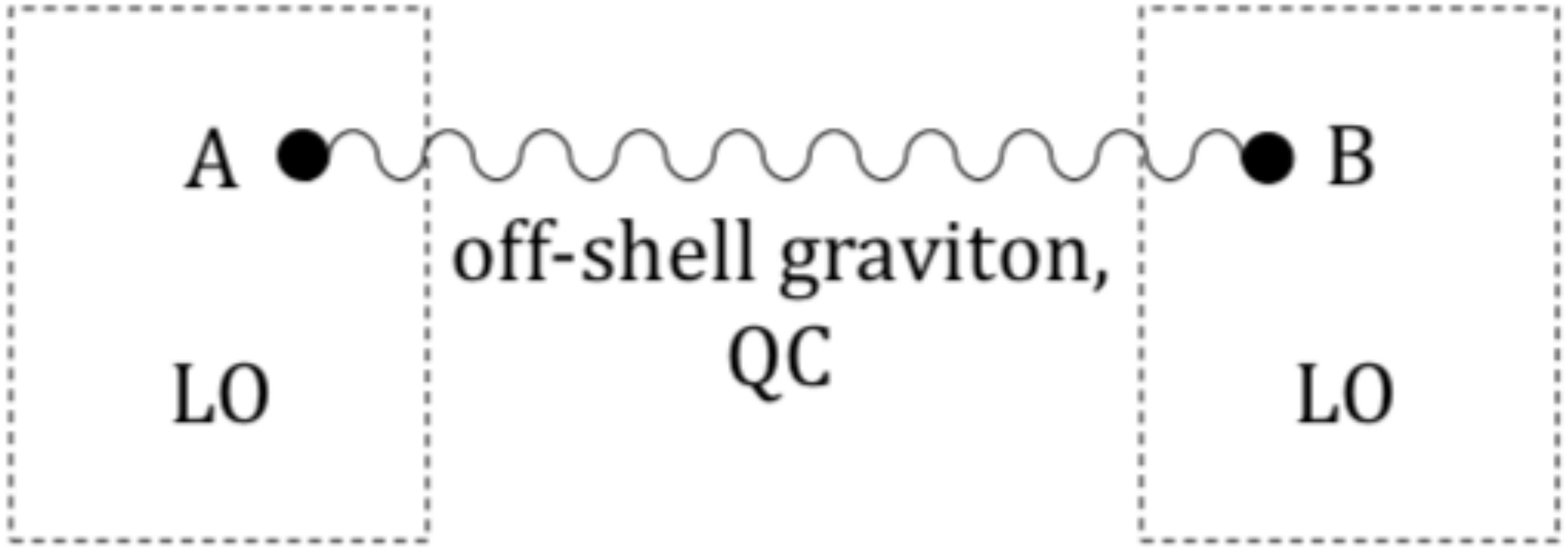
LOQC: Local operation & Quantum Communication

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad S_{EH} = \frac{1}{4}\int d^4x\, h_{\mu\nu}\, \mathcal{O}^{\mu\nu\rho\sigma}\, h_{\rho\sigma} + \mathcal{O}(\kappa h^3)$$

(1) Non-Classicality

Minimal Coupling : $\kappa h_{\mu\nu} T^{\mu\nu}$

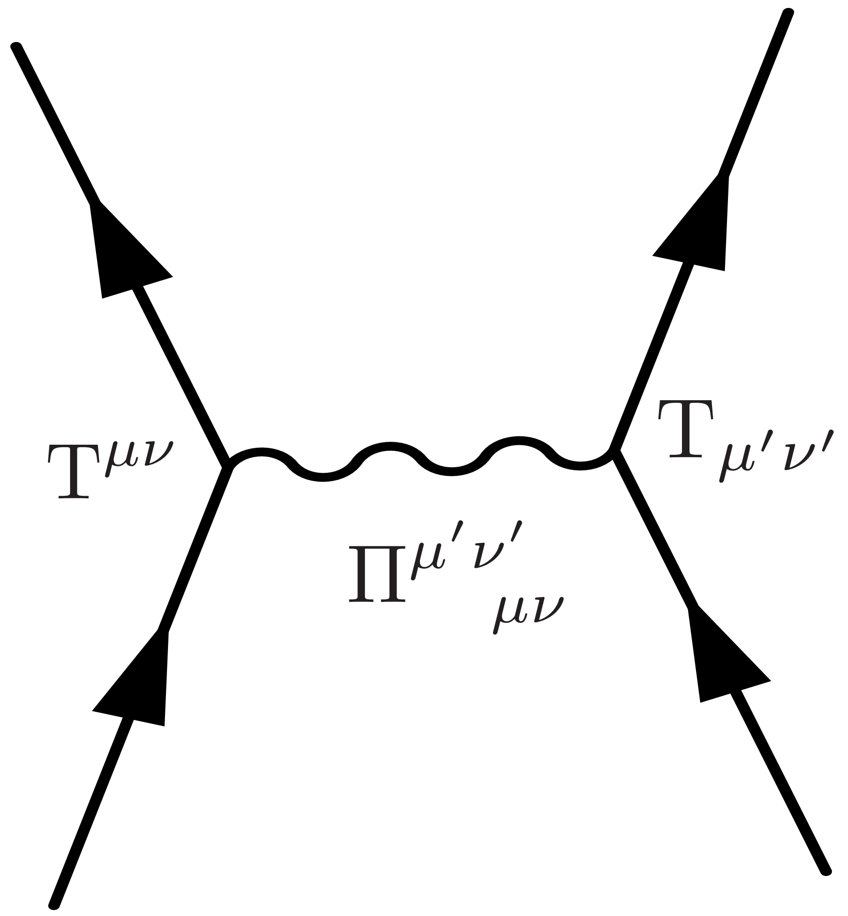
(2) Non-Commuting Operators



$$\mathcal{O}^{\mu\nu\rho\sigma} := \frac{1}{4}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right)\Box - \frac{1}{2}\eta^{\mu\nu}\eta^{\rho\sigma}\Box + \frac{1}{2}\left(\eta^{\mu\nu}\partial^\rho\partial^\sigma + \eta^{\rho\sigma}\partial^\mu\partial^\nu - \eta^{\mu\rho}\partial^\nu\partial^\sigma - \eta^{\mu\sigma}\partial^\nu\partial^\rho\right)$$

$$\Pi_{\mu\nu\rho\sigma}(k) = \left(\frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{k^2} - \frac{\mathcal{P}^0_{s,\,\mu\nu\rho\sigma}}{2k^2}\right)$$

Unitary Local Interaction



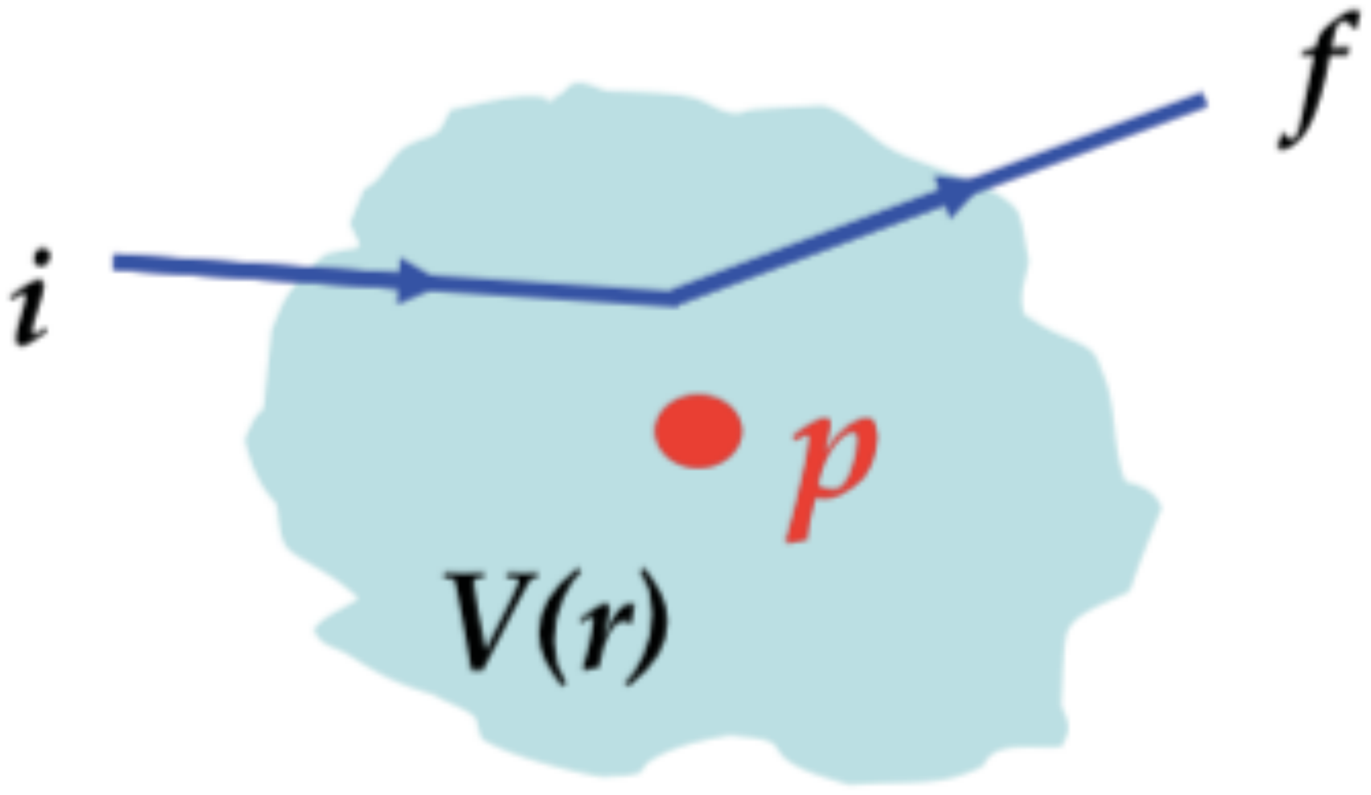
Unitary Local Interaction

$$\Phi(r) = -\kappa^2 \int \frac{d^3|\vec{k}|}{(2\pi)^3} T_1^{00}(k)\Pi_{0000}(k)T_2^{00}(-k)e^{i\vec{k}\cdot(\vec{r}}$$

$$= -\frac{\kappa^2 m}{2}\int \frac{d^3|\vec{k}|}{(2\pi)^3}\frac{1}{\vec{k}^2}e^{i\vec{k}\cdot(\vec{r})} = -\frac{Gm}{r},$$

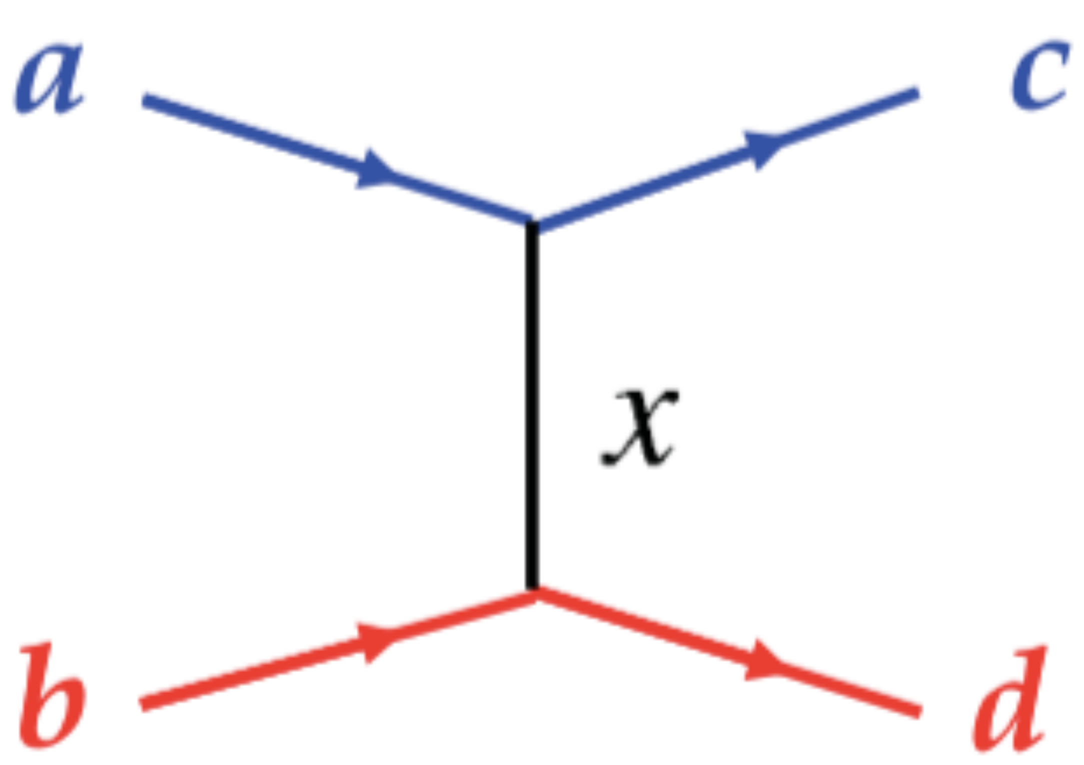
Marshman +Mazumdar+Bose, [ArXiv: 1907.01568] Phys. Rev. A. (2020)

What Aspects of Quantum Gravity Can We test in the Lab ?



$$\Delta\phi \sim G/\hbar$$

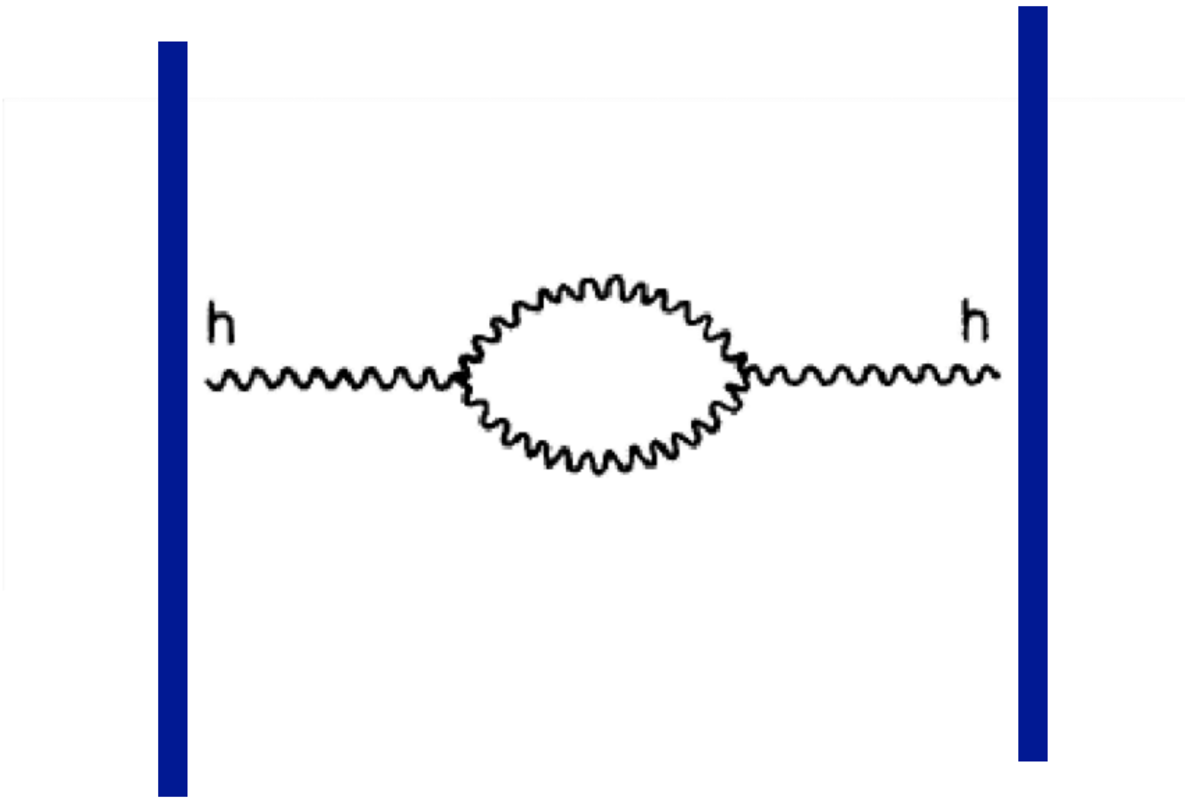
Classical



$$\Delta\phi \sim G/\hbar$$

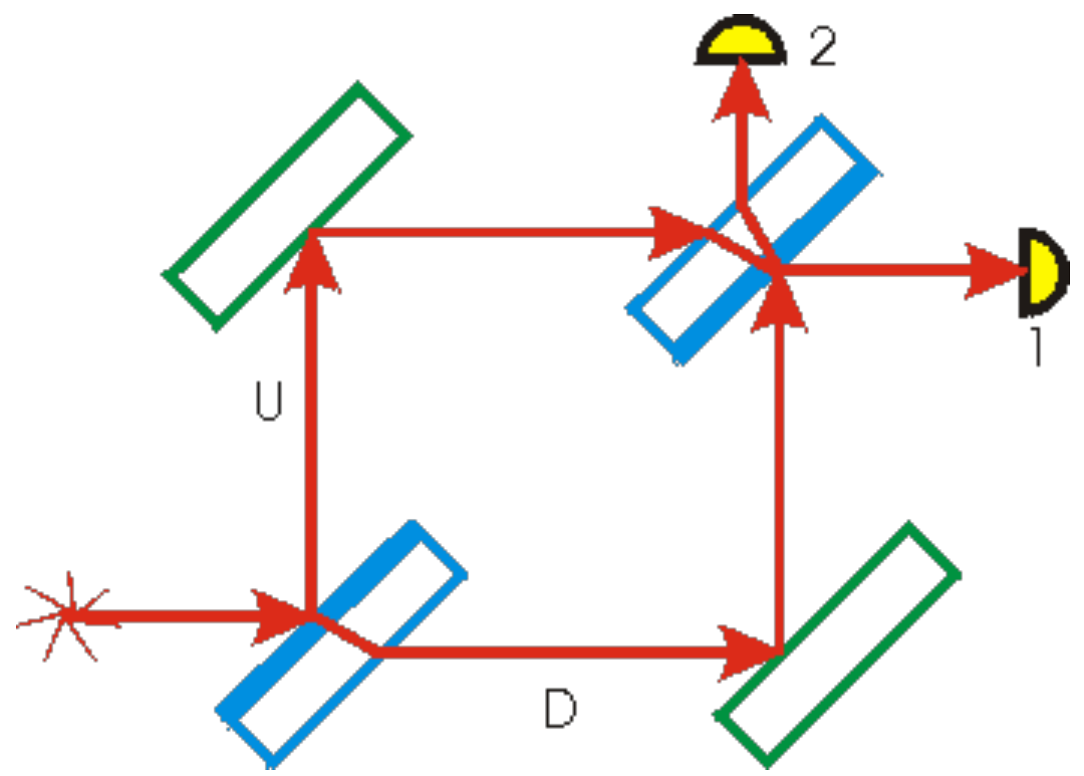
Quantum

2nd order in perturbation



$$\Delta\phi \sim G^2\hbar/\hbar$$

Quantum Loop Effect

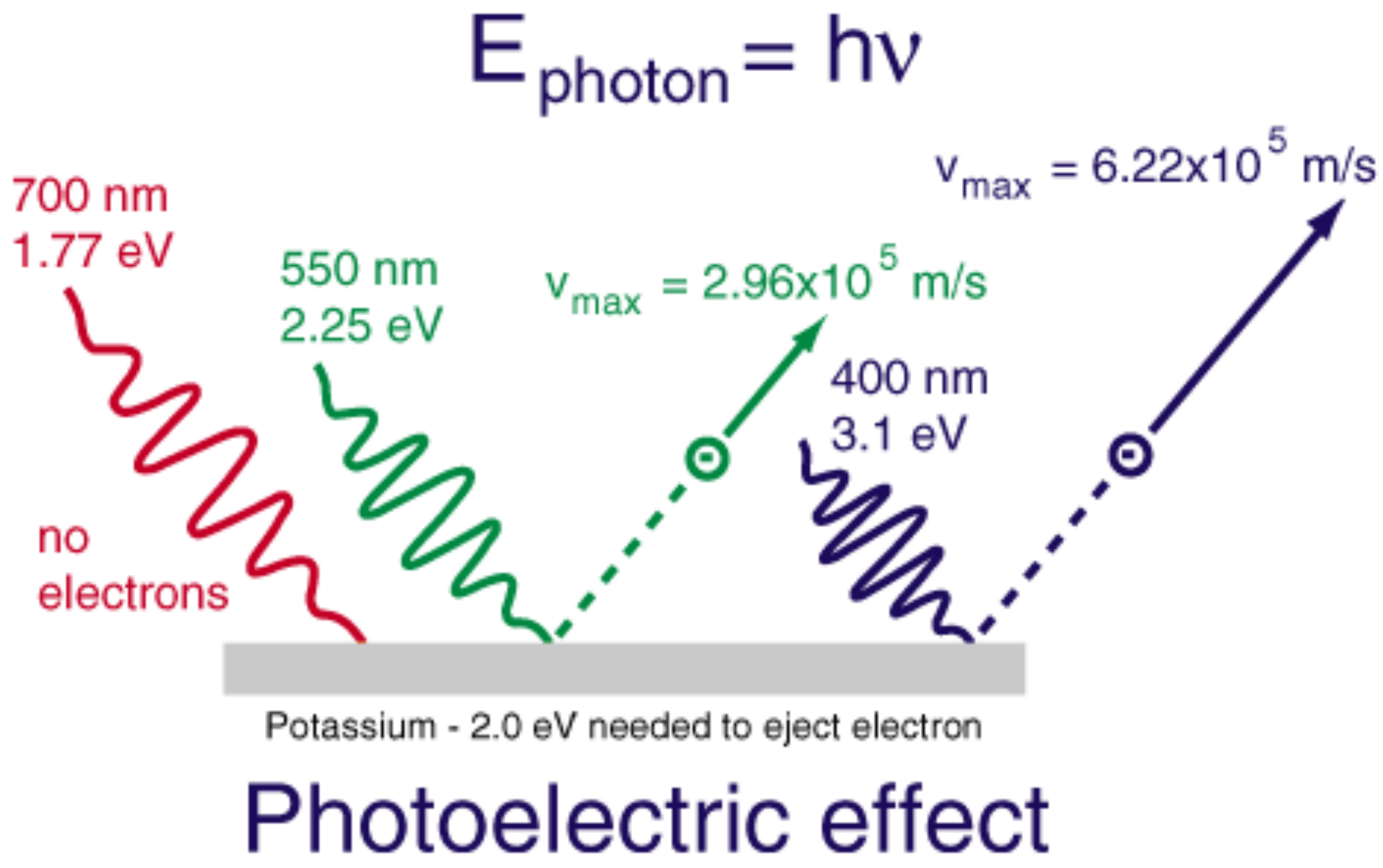


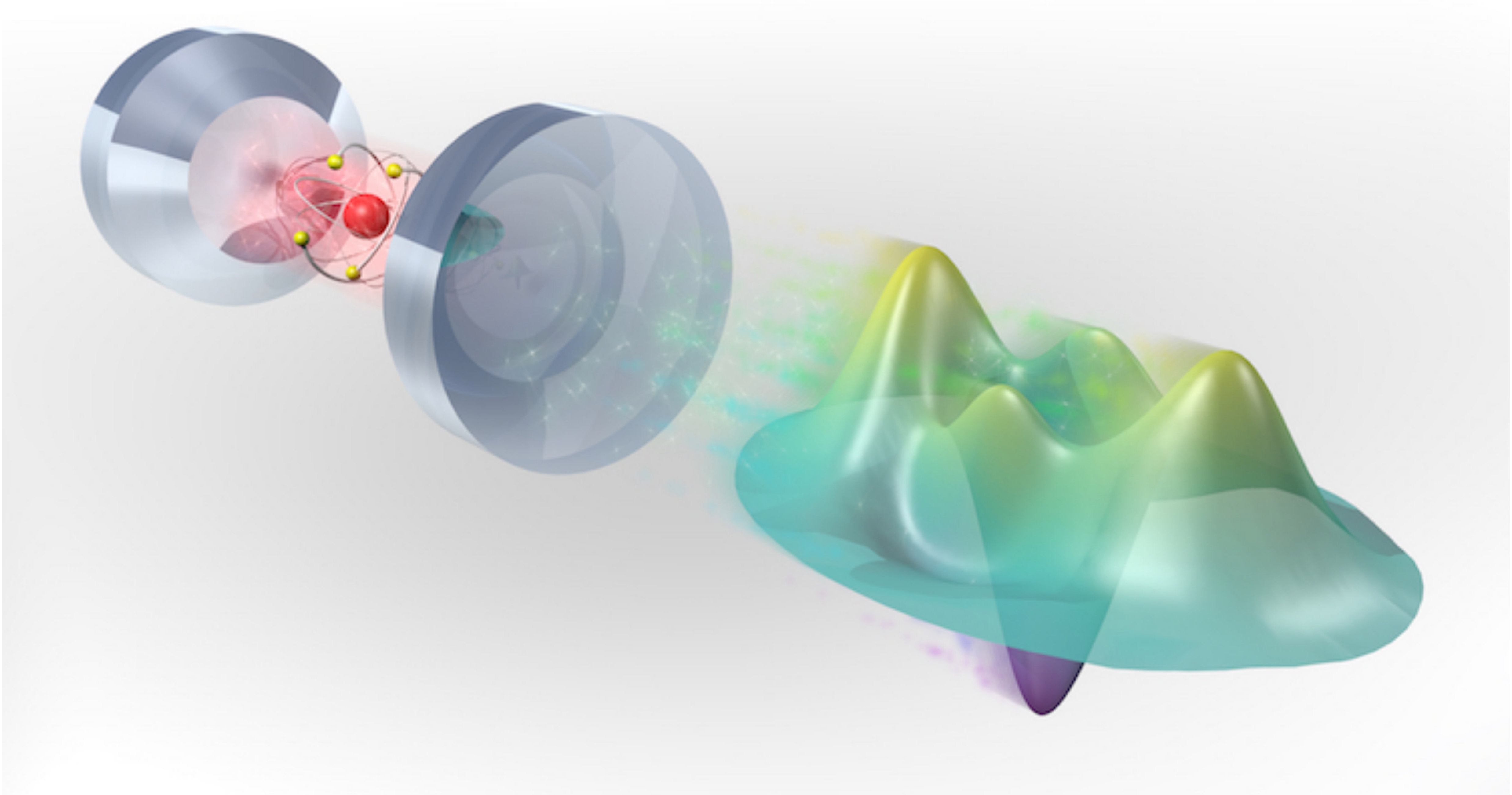
Fine Print

Quantum experiments are full of Off-shell processes

Beam splitter

From Photoelectric Effect to Schrödinger Cat States of Photon





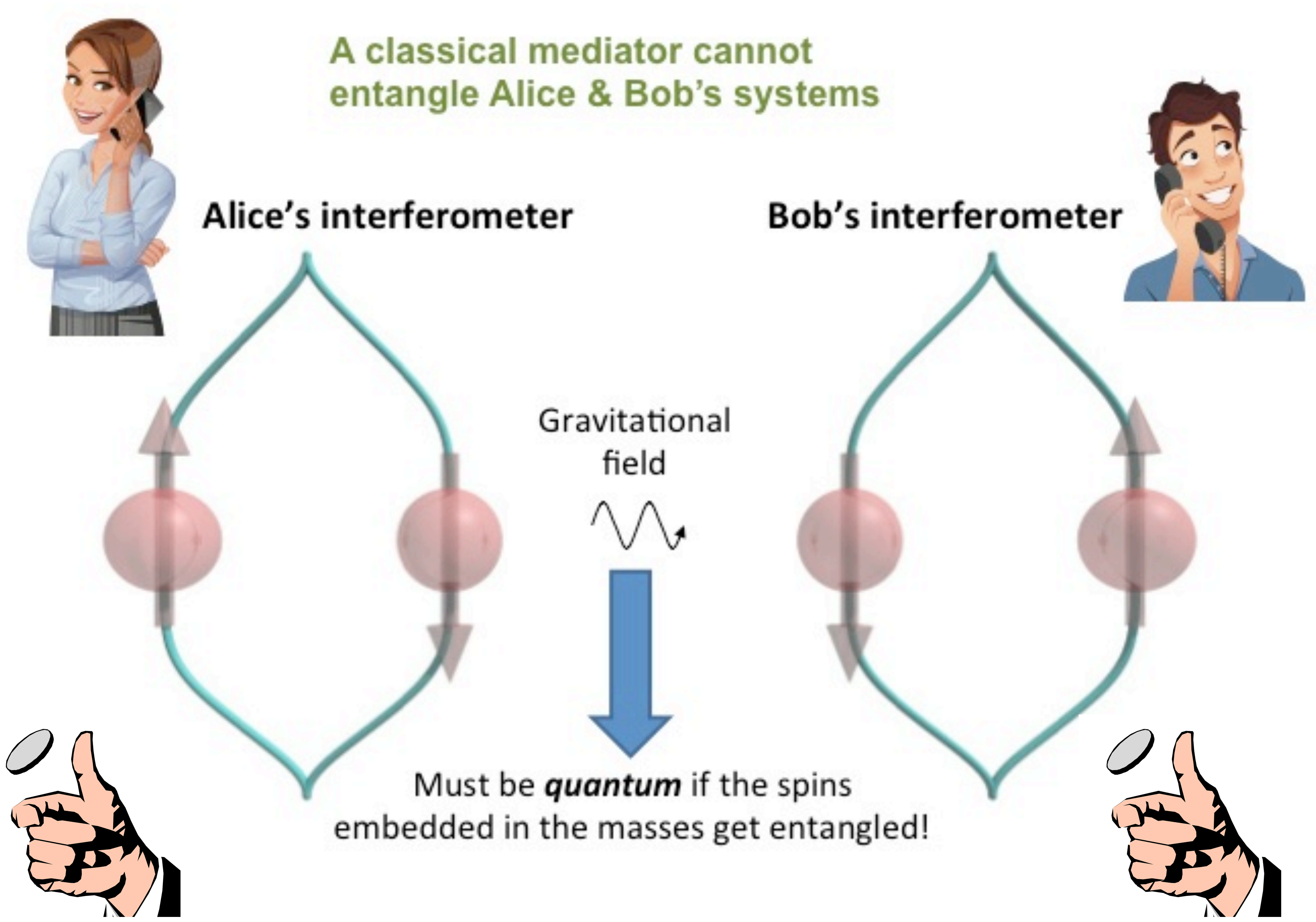
M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J.M. Raimond, S. Haroche,
Quantum Rabi Oscillation: A Direct Test of Field Quantization in a Cavity, Physical Review Letters, 76 (1996) 1800-1803.

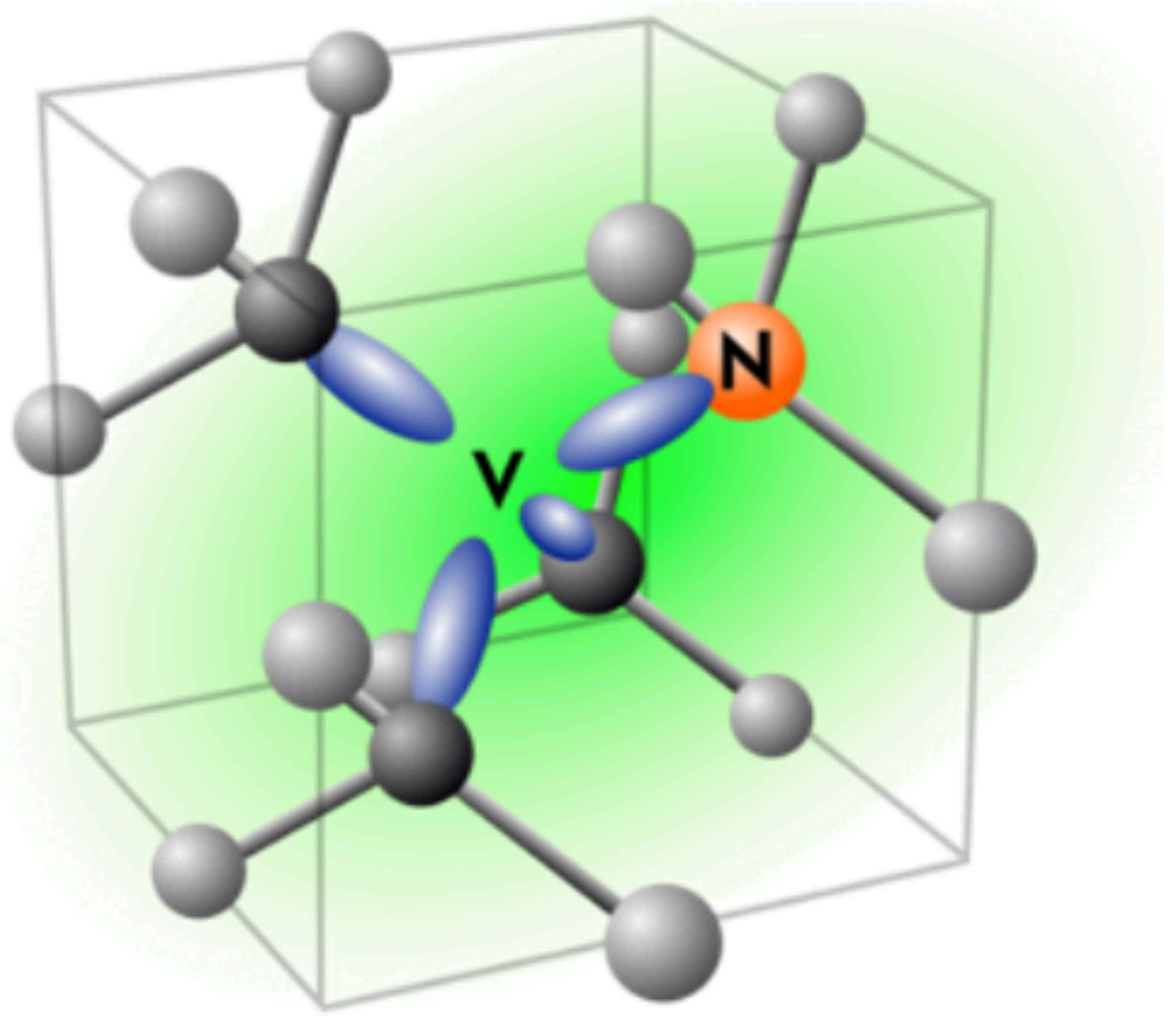
Creating Schrödinger cat (kitten) states with photons and observing their decoherence:
M. Brune, E. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J.M. Raimond, S. Haroche,
Observing the progressive decoherence of the "meter" in a quantum measurement, Physical Review Letters, 77 (1996) 4887-4890.

Gerhard Rempe's group, Max Planck Institute of Quantum Optics

QGEM protocol: 2 Freely Falling Superposed masses

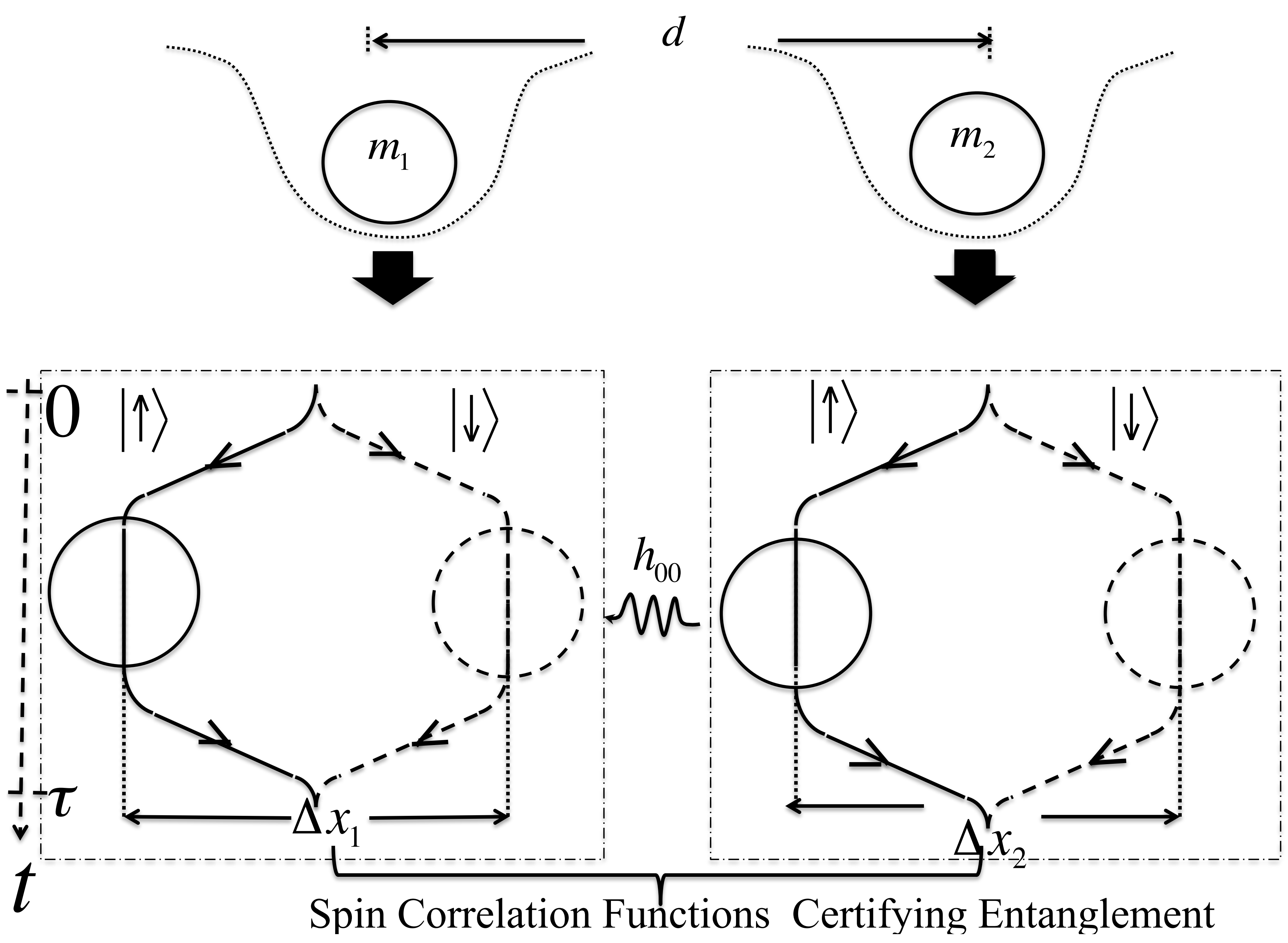
Macroscopic superposition with Spin Embedded Inside

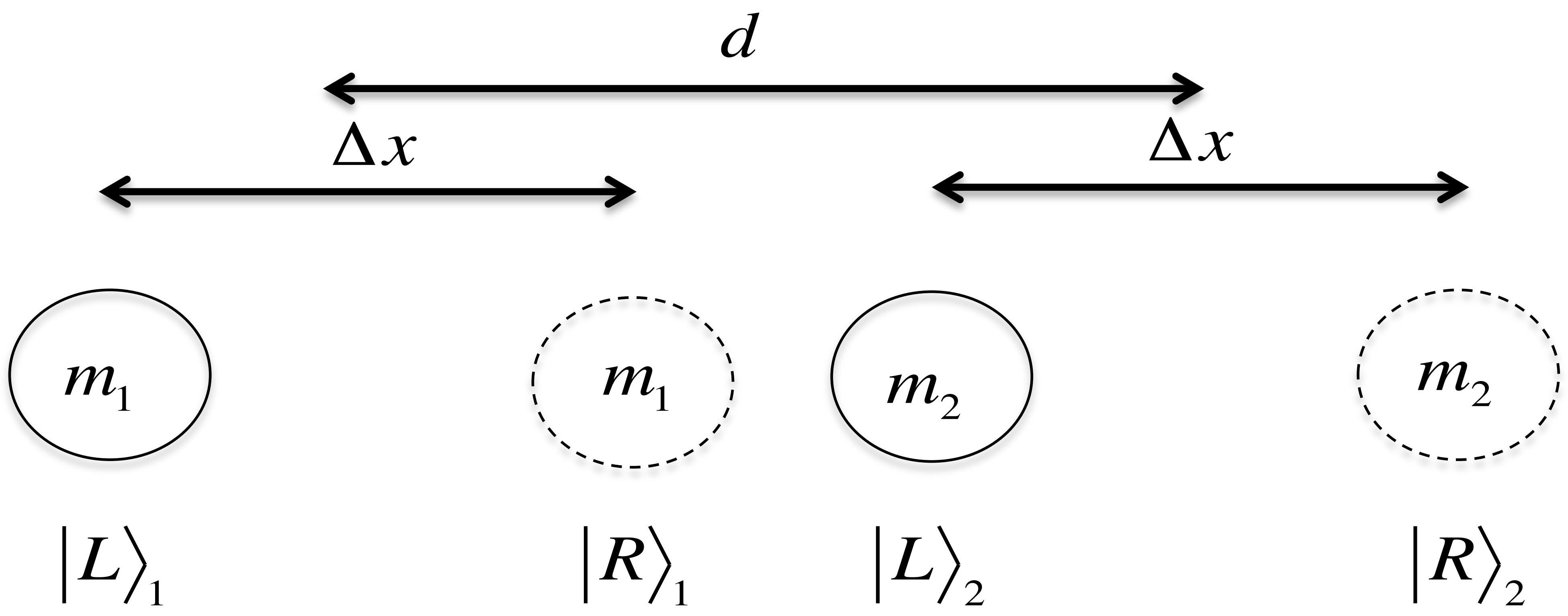




Use very strong atom chip gradients to push and split massive objects using just one spin!

QGEM protocol: 2 Freely Falling Superposed masses





If they interact *only* through the gravitational force

$$|\Psi(t=0)\rangle_{12} = \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1)\frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2)$$

$$= \frac{1}{2}(|L\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2 + |R\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2)$$

$$\rightarrow |\Psi(t=\tau)\rangle_{12} = \frac{1}{2}(e^{i\phi_{LL}}|L\rangle_1|L\rangle_2 + e^{i\phi_{LR}}|L\rangle_1|R\rangle_2 + e^{i\phi_{RL}}|R\rangle_1|L\rangle_2 + e^{i\phi_{RR}}|R\rangle_1|R\rangle_2),$$

where

$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}, \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)},$$

$$\phi_{LL} = \phi_{RR} \sim \frac{Gm_1m_2\tau}{\hbar d}$$

Maximum Entanglement Phase

Step 4: Witness spin entangled state:

$$|\Psi(t=t_{\text{End}})\rangle_{12} = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_1 \frac{1}{\sqrt{2}}(|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}}|\downarrow\rangle_2) + |\downarrow\rangle_1 \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|\uparrow\rangle_2 + |\downarrow\rangle_2)\}|C\rangle_1|C\rangle_2$$

through the correlations:

$$\mathcal{W} = |\langle\sigma_x^{(1)} \otimes \sigma_z^{(2)}\rangle - \langle\sigma_y^{(1)} \otimes \sigma_z^{(2)}\rangle|$$

$$\text{If } \mathcal{W} > 1 \implies \mathit{Graviton\ is\ quantum}$$

$$\Delta\phi_{LR} + \Delta\phi_{RL} \sim \mathcal{O}(1)$$

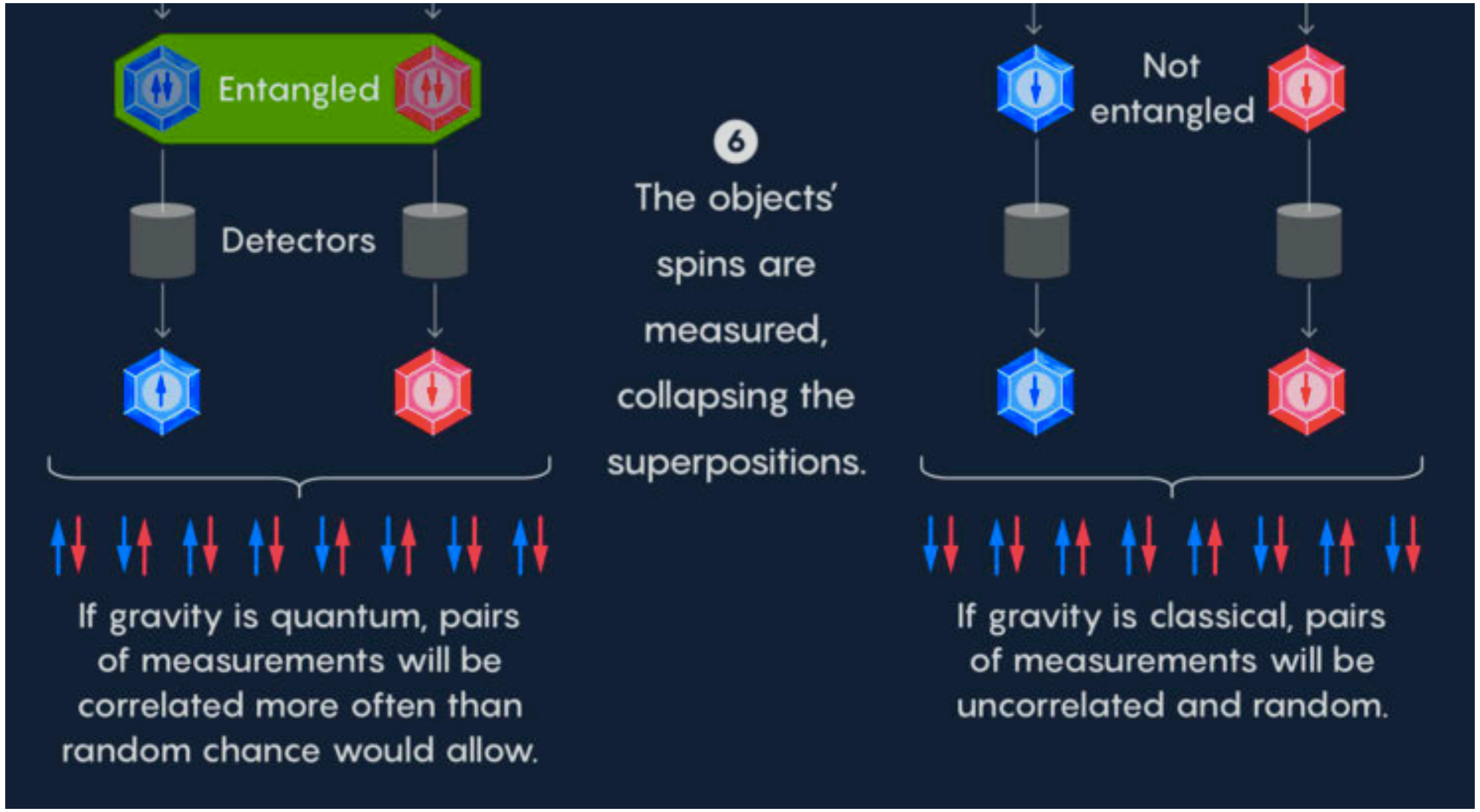
we have

$$\Delta\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)} >> \Delta\phi_{LR}, \Delta\phi_{LL}, \Delta\phi_{RR}$$

For mass ~ 10^(-14) kg (microspheres), separation at closest approach of the masses ~ 200 microns (to prevent Casimir interaction), **time ~ 1 seconds**, gives:
Scale of superposition ~ 100 microns, **Delta phi_{RL} ~ 1**

Planck's Constant fights Newton's Constant!

Entanglement Witness: Spin Correlation



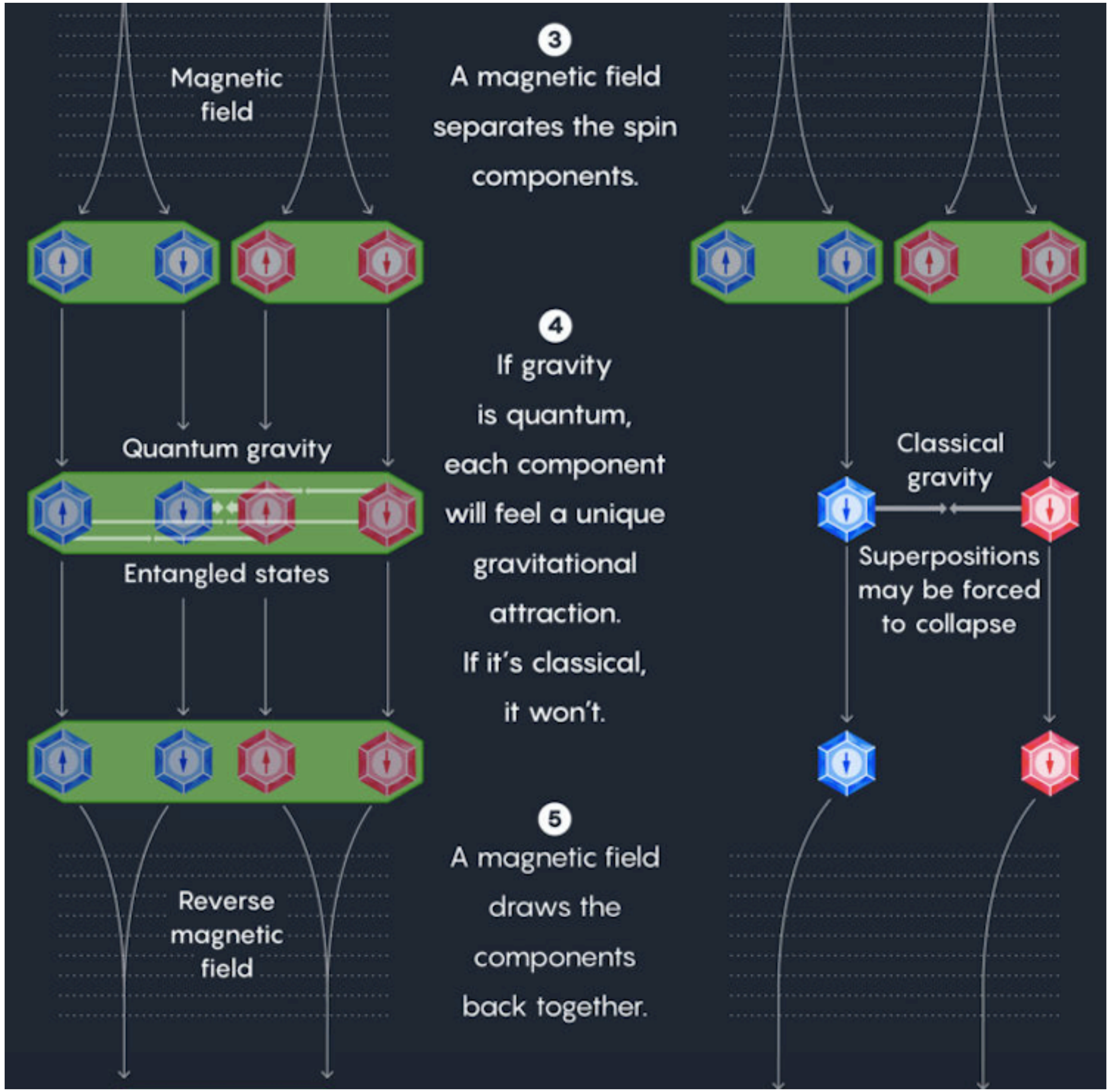
Basis Dependent Witness, similar to Bell's

$$\mathscr{W} = I^{(1)}I^{(2)} - \sigma_x^{(1)}\sigma_x^{(2)} - \sigma_y^{(1)}\sigma_z^{(2)} - \sigma_x^{(1)}\sigma_z^{(2)} \quad \mathrm{Tr}(\mathscr{W}\rho) < 0$$

H. Chevalier, A. Paige, and M. Kim arXiv:2005.13922 [quant-ph].

Basis Independent Witness: $S_A = -Tr_A \rho_A \log \rho_A = -S_B$ **Entanglement Entropy!**

Challenges for QGEM



M. Frimmer, K.Luszcz, S. Ferreiro, V. Jain, E.Hebestreit, and L.Novotny, Phys. Rev. A95, 061801 (2017).

$10^{-14} Kg$ Radius : 100nm

Neutralising e.m. charges

A magnetic field gradient of ~ 10^6 T/m and a time τacc ~ 500 m/s^2, Δx ~ 250μm, d-Δx ~200μm

Electronic spins coherent for 1s, which should be possible for macro-diamond below 77 K

To estimate collisional and thermal decoherence times of the orbital degree of freedom we consider the pressure P = $10^{-15} Pa$ and the temperature 0.15 K. the collisional decoherence time for a superposition size of Δx ~ 250μm is the same order of magnitude as the total microsphere's fall time τ + 2τacc ~ 3.5 s

T. Krisnanda, M. Zuppardo, M. Paternostro, T. Paterek, arXiv:1607.01140.

C. Wan, M. Scala, G. W. Morley, ATM. A. Rahman, H. Ulbricht, J. Bateman, P. F. Barker, S. Bose, and M. S. Kim, Phys. Rev. Lett. 117, 143003 (2016);

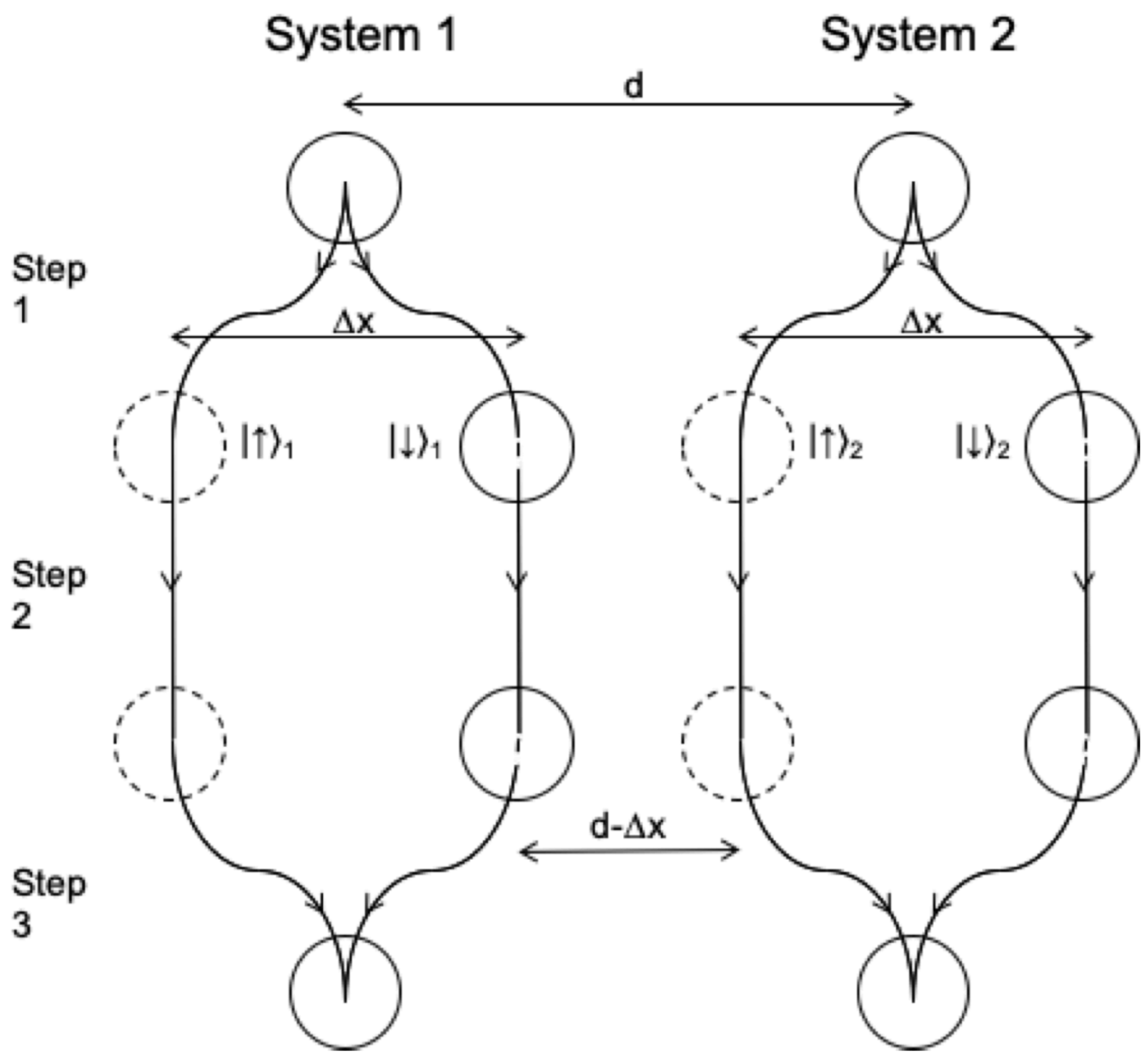
H. Pino, J. Prat-Camps, K. Sinha, B. P. Venkatesh, and O. Romero-Isart, arXiv:1603.01553v2

N. Bar-Gill, L.M. Pham, A. Jarmola, D. Budker, R. L. Walsworth, Nature Comm, 4, 1743 (2013),

S. Knowles, D. M. Kara and M. Atatu¨re, Nature Materials 13, 21 (2014),

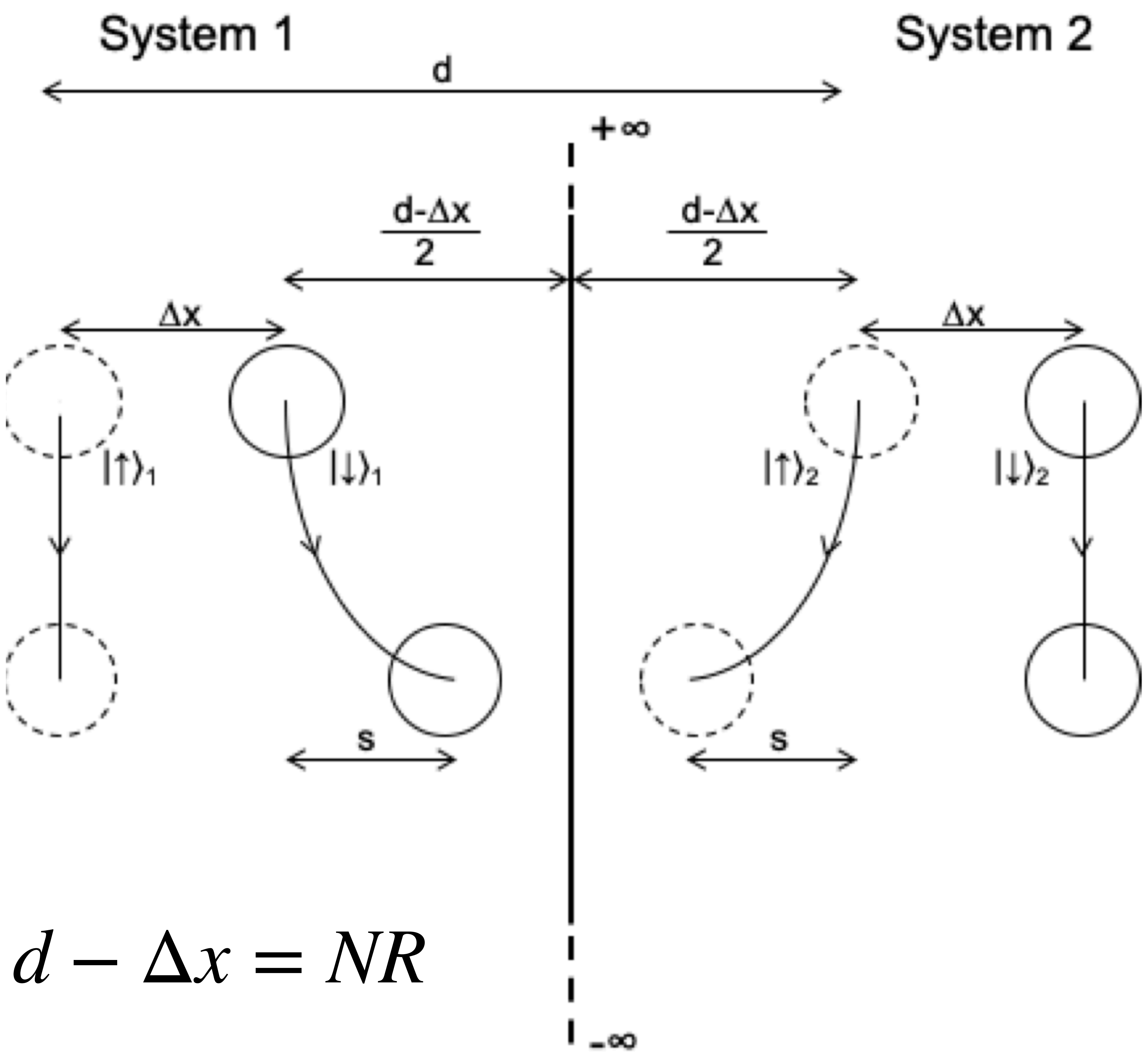
Kaltenbaek, Aspelmeyer, (2015)

QGEM Protocol : Improved Design



$$V_{CP} \sim -\frac{23\hbar c}{4\pi}\frac{R^6}{r^7}\left(\frac{\epsilon-1}{\epsilon+2}\right)^2$$

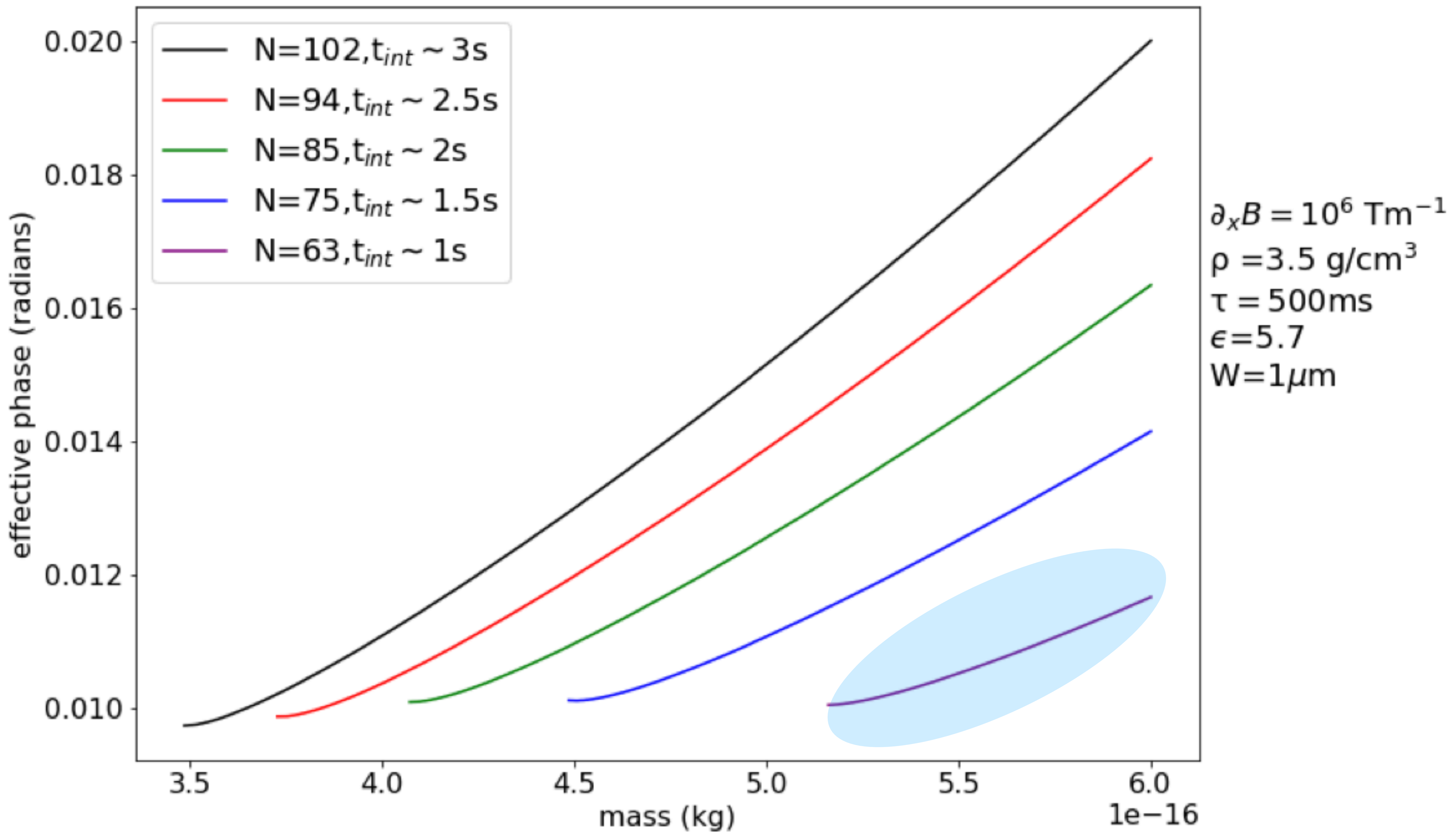
$$F_{\mathrm{ca}} = -\frac{3\hbar c}{2\pi}\left(\frac{\epsilon-1}{\epsilon+2}\right)\frac{R^3}{x^5}$$

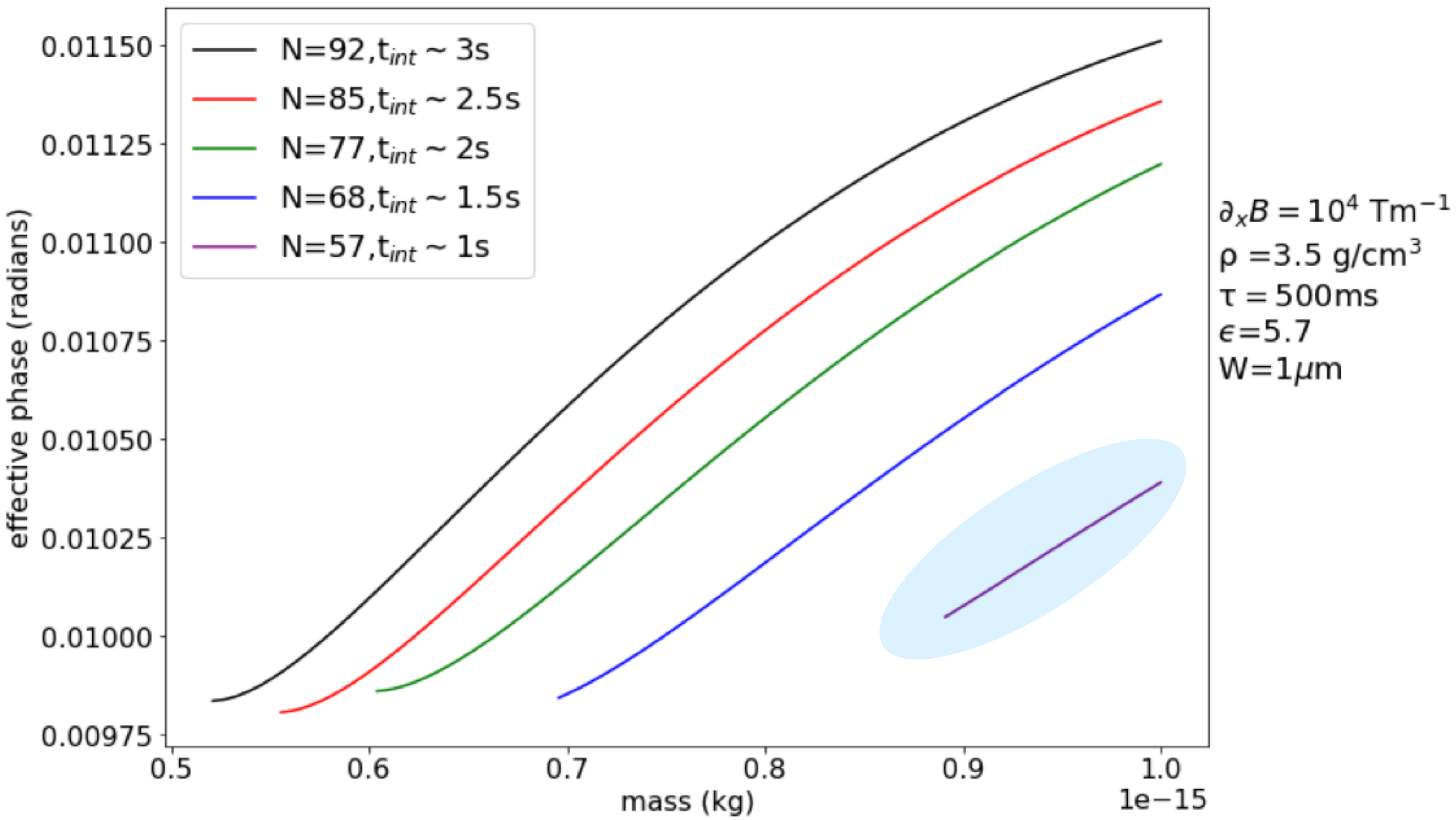


$$d - \Delta x = NR$$

Van de Kamp+Marshman +Bose+Mazumdar [2006.06931 [quant-ph]]

QGEM Protocol (Phase vs Mass)





$$10^{-14}\text{Kg} \Longrightarrow 10^{-16} - 10^{-15}\text{Kg}$$

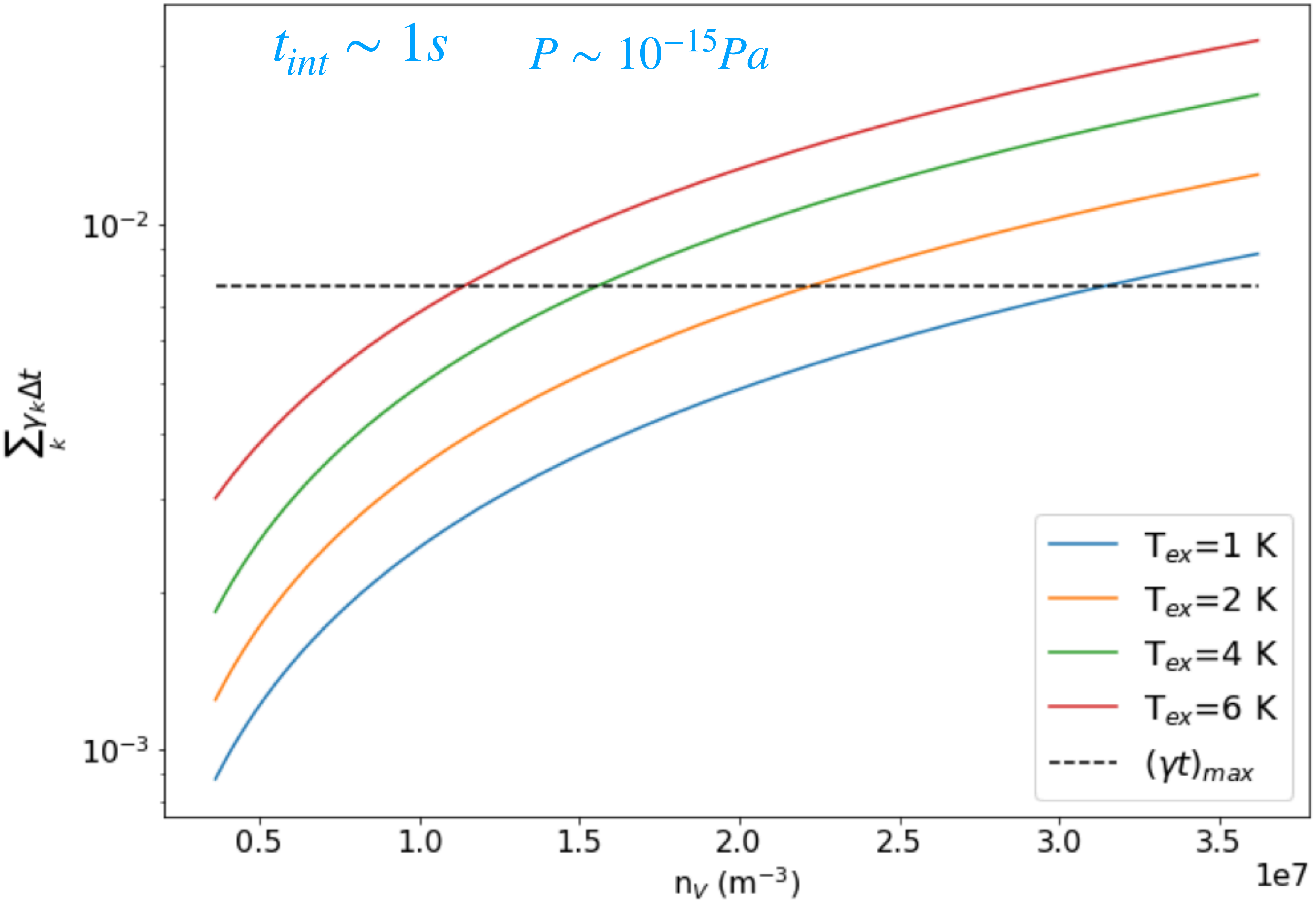
$$\Delta x \sim 200\mu m \Longrightarrow 20\mu m$$

$$P \sim 10^{-16} \Longrightarrow 10^{-15}Pa$$

Van de Kamp+Marshman +Bose+Mazumdar [2006.06931 [quant-ph]]

Sources of Dephasing/Decoherence

$$\mathscr{W} = I^{(1)}I^{(2)} - \sigma_x^{(1)}\sigma_x^{(2)} - \sigma_y^{(1)}\sigma_z^{(2)} - \sigma_x^{(1)}\sigma_z^{(2)} \qquad \mathrm{Tr}(\mathscr{W}\rho) < 0 \qquad \Rightarrow \gamma_{dec} t_{int} < \Phi/2 \sim 0.0075$$



ρ =3.5 g/cm^3
T_{int}=0.15K
m_{air}=28.97 amu
ϵ=5.68 + 1.1·10^{-4}i
$\partial_x B = 10^4$ Tm^{-1}
τ=500 ms
m=10^{-15} kg
N=57

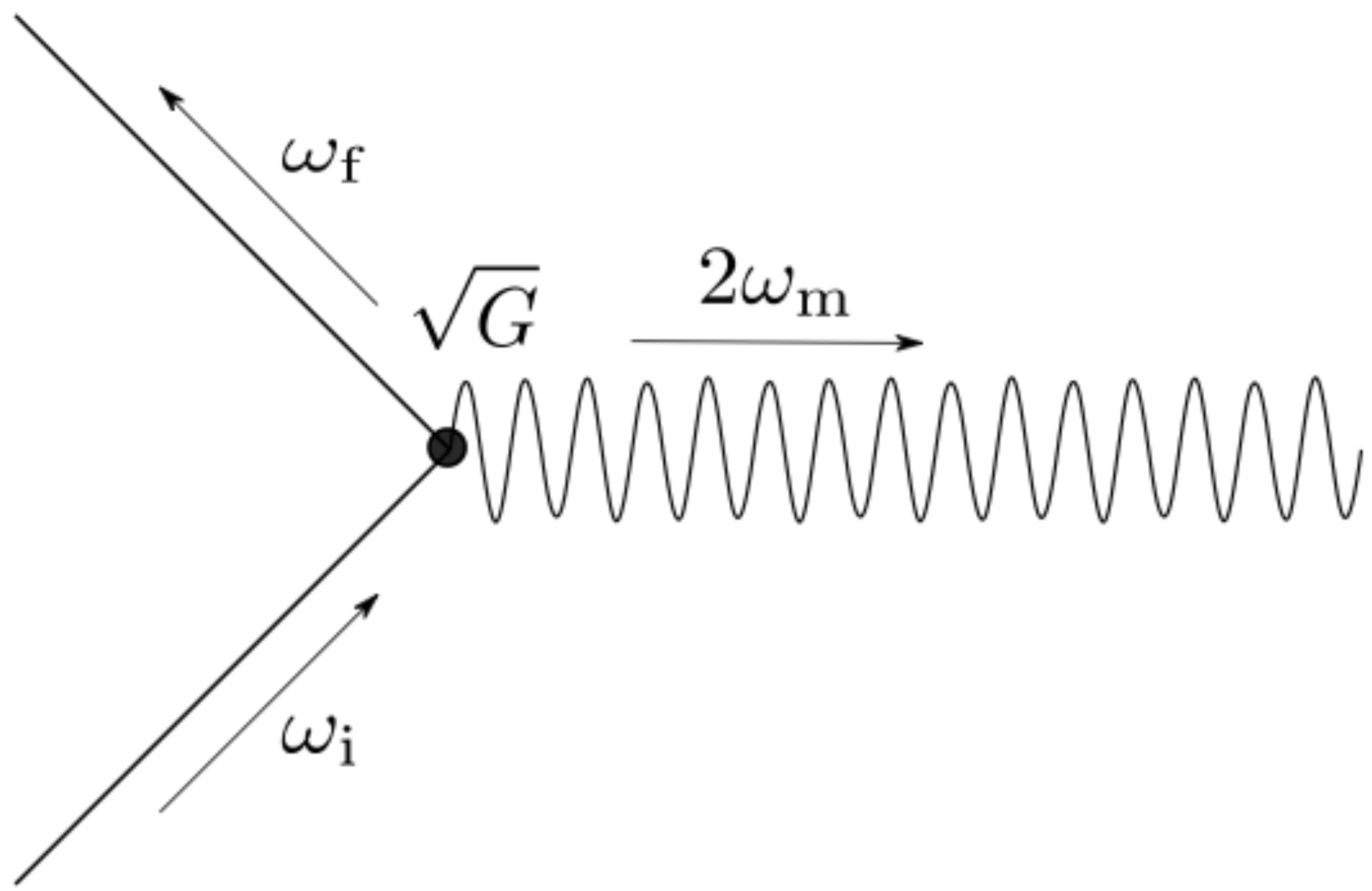
Main Sources of Decoherence

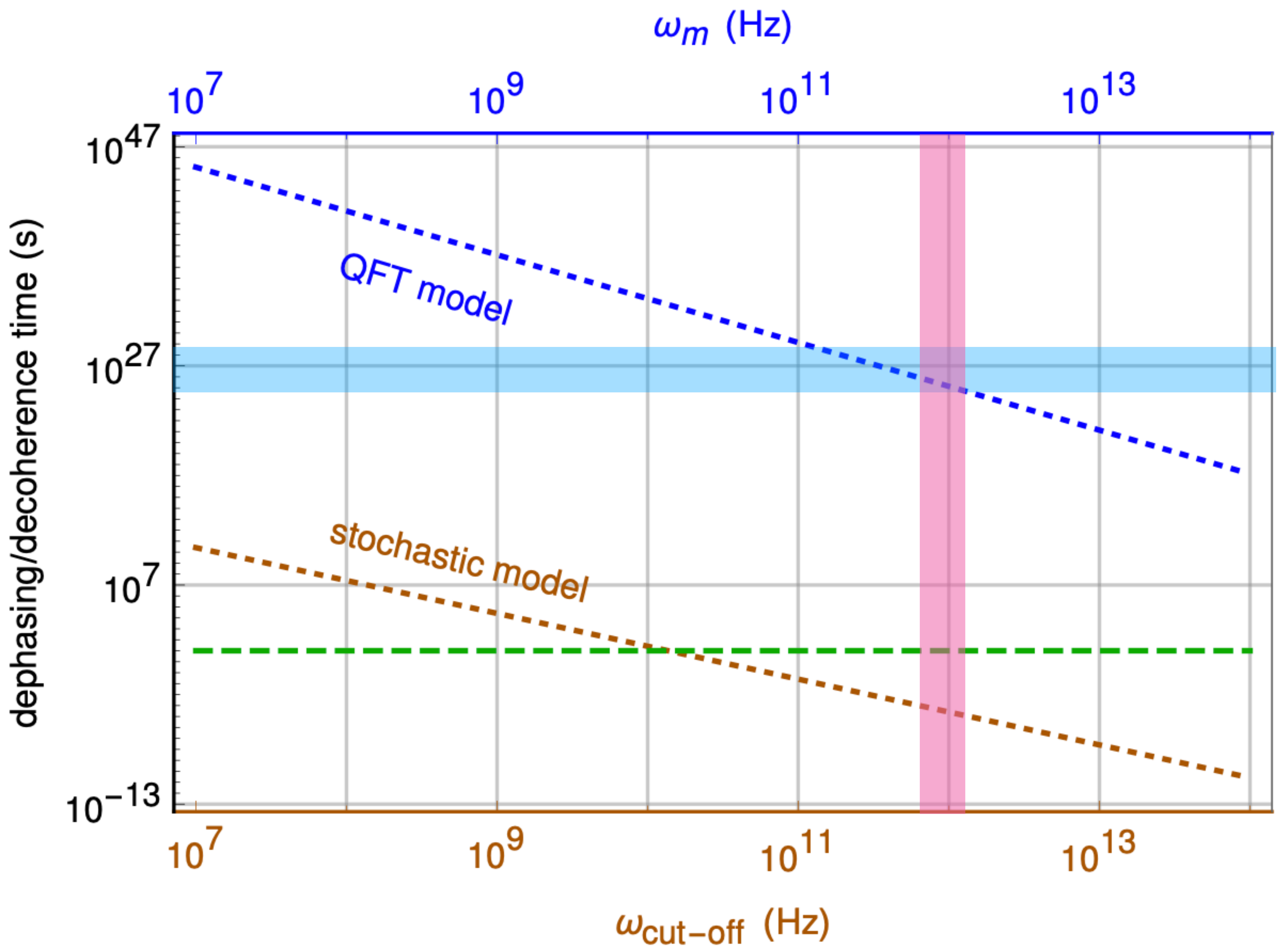
- **Collisional**
- **Black body emission, absorption**
- **Photon scattering**

- **At temperatures below 4k phonon vibration is not a problem for dephasing**

H. Van Kamp, R. Marshman, S. Bose, and A. Mazumdar. arXiv:2006.06931 [quant-ph].

Dephasing due to Graviton is extremely tiny

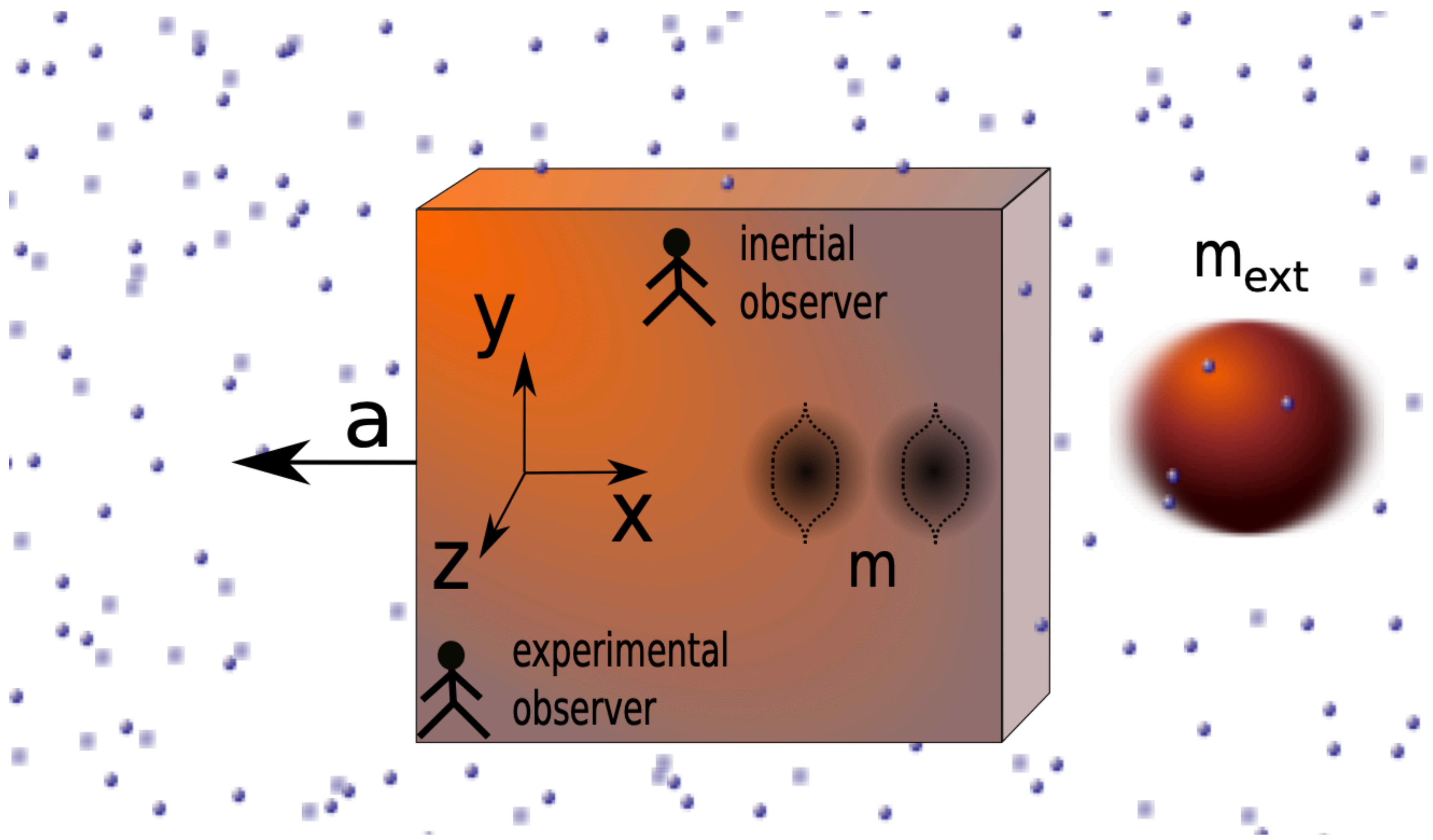




$$\gamma_{grav} = \frac{32G\hbar\omega_m^3}{15c^5} = \frac{32}{15}t_{pl}^2\omega_m^3$$

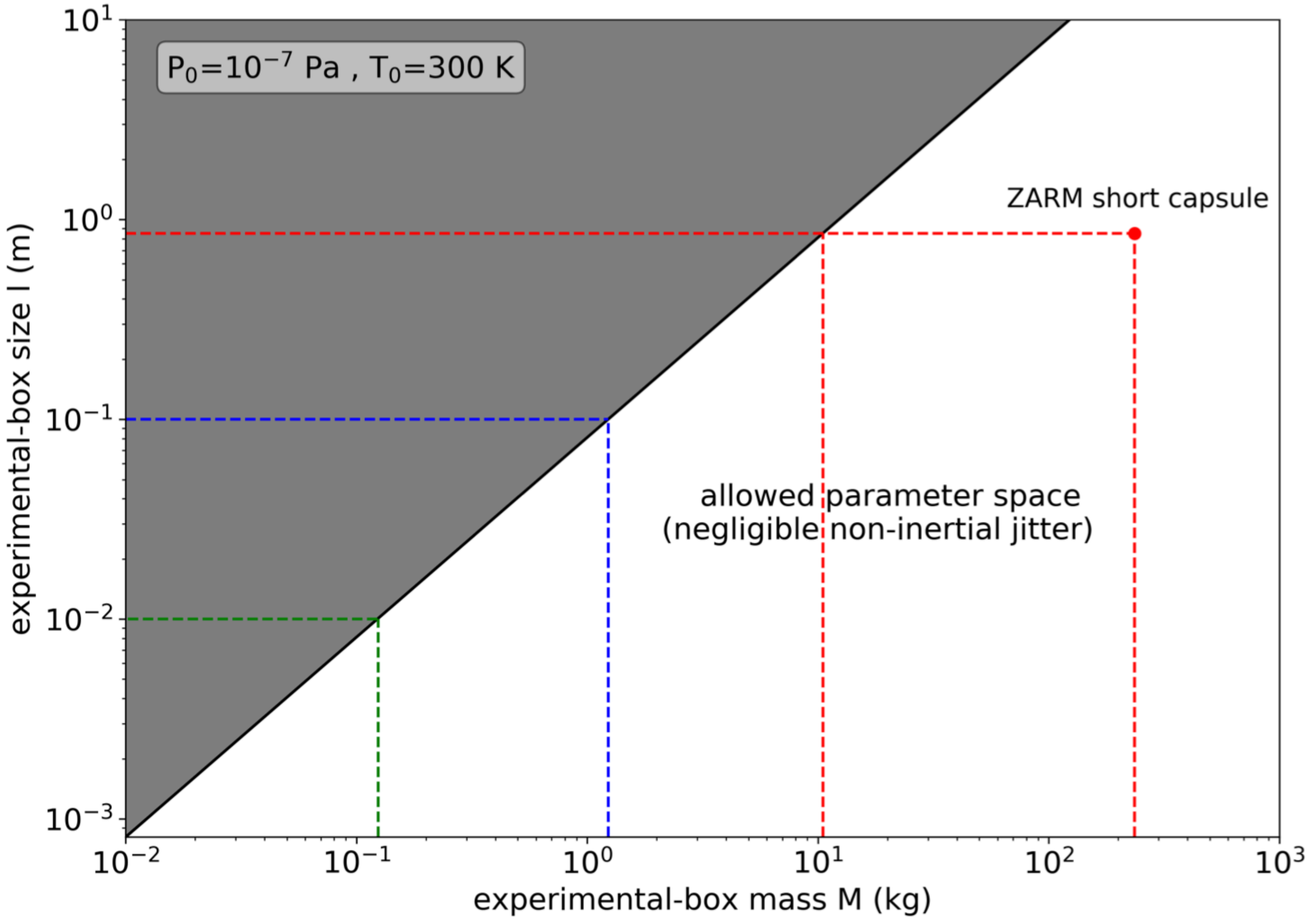
Toros + Bose + Mazumdar [2008.08609 gr-qc]

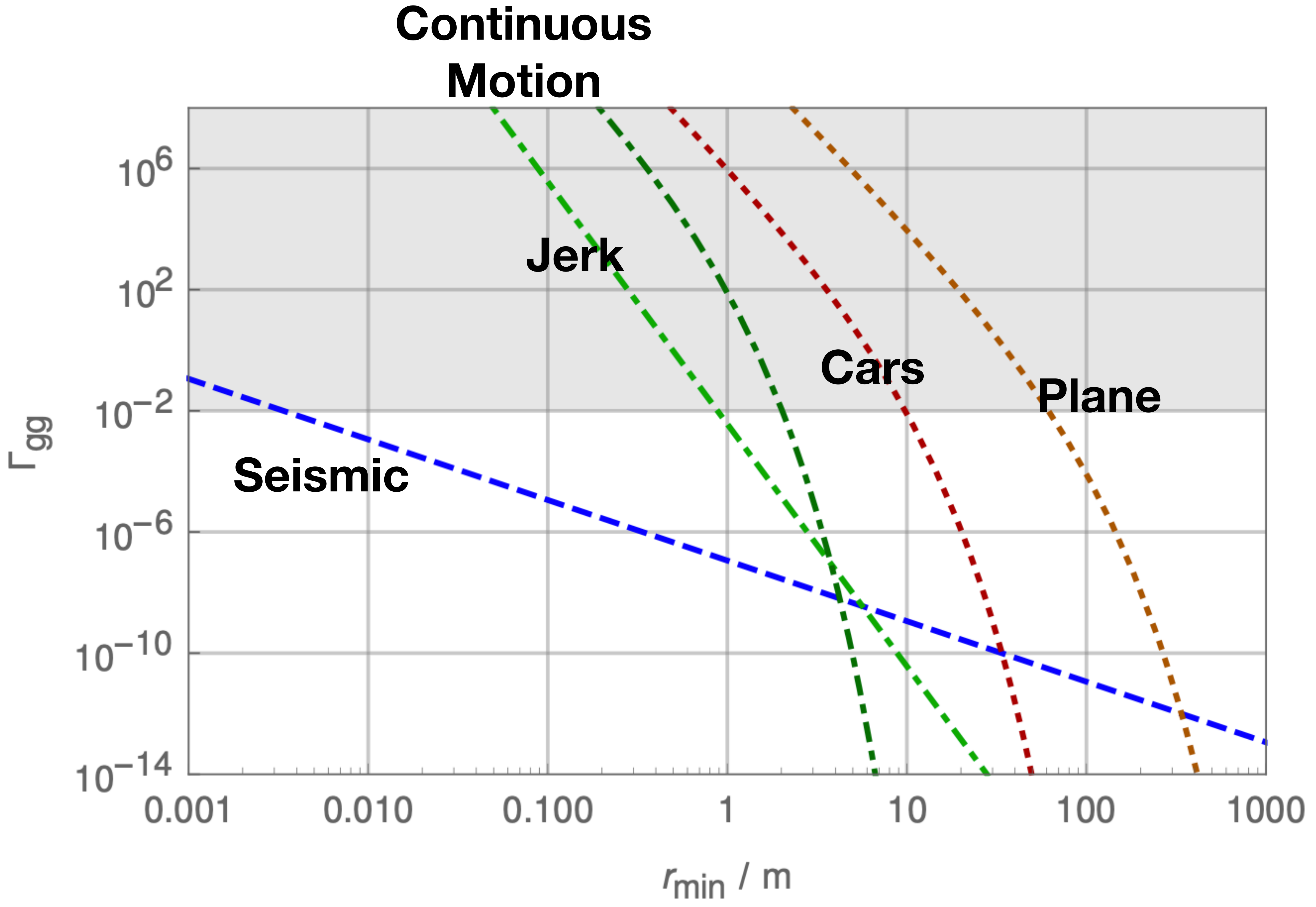
Relative Acceleration+Gravity Gradient Noise



$$\Phi_{eff} > \Gamma_d + \Gamma_{non-inertial} + \Gamma_{jitter} + \Gamma_{ggn}$$

Non-inertial, GGN (Atmospheric+Seismic+ Constant movement+ Jerks+Cars+Planes)





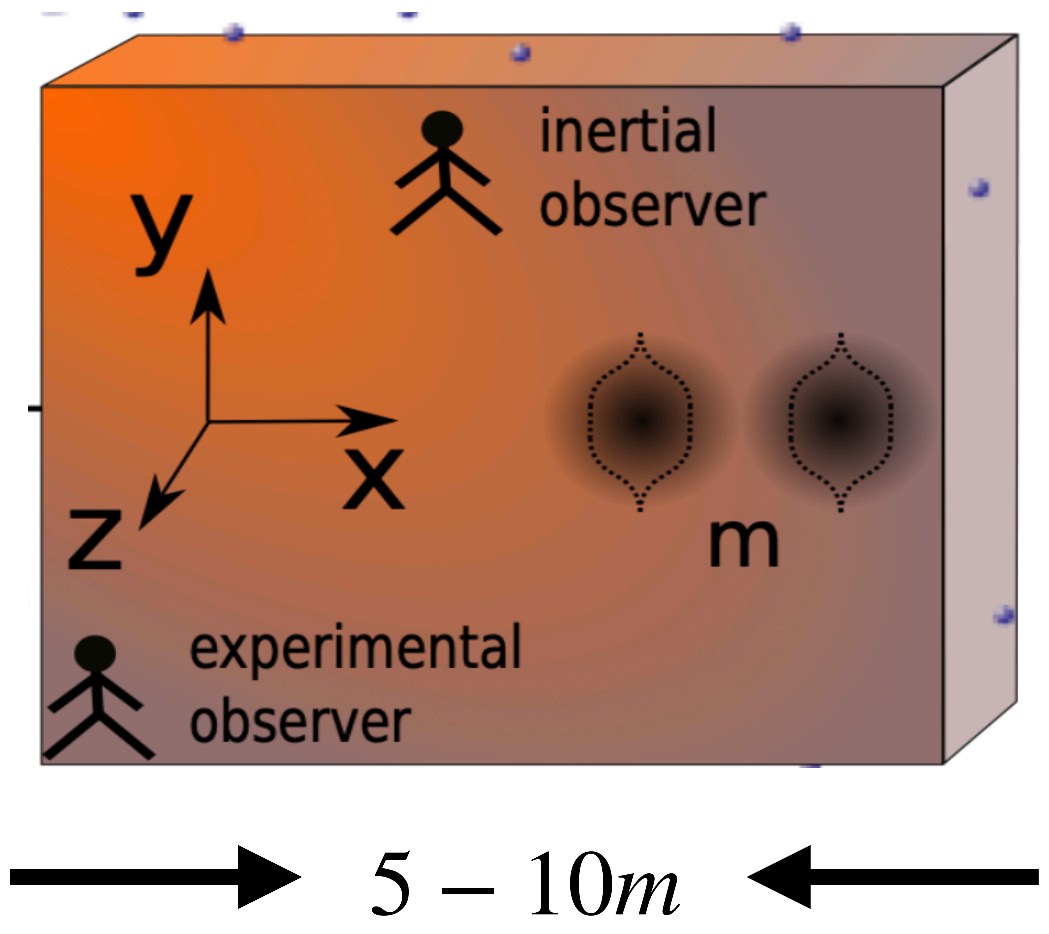
Toros+Van de Kamp+Kim+Mazumdar+ Bose, **2007.15029 [gr-qc]**

Underground Laboratory

Drop Tower

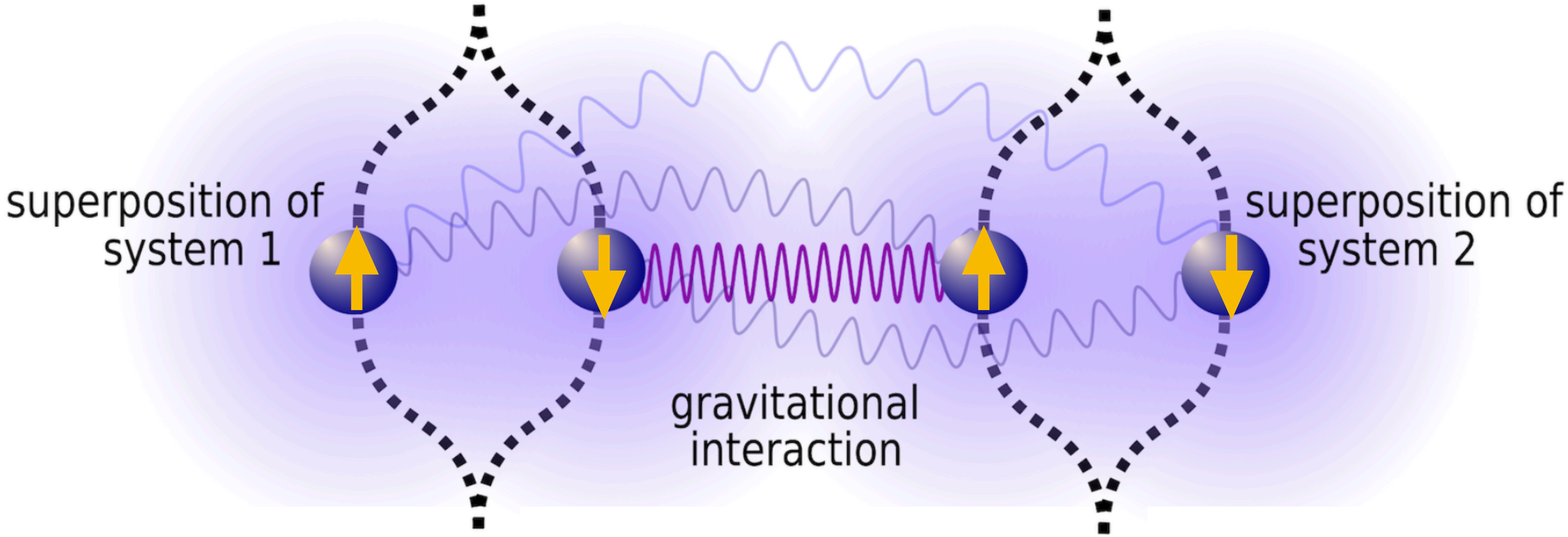
$50 - 100m$

Seismic Noise is Low



Toros+Van de Kamp+Kim+Mazumdar+ Bose, 2007.15029 [gr-qc]

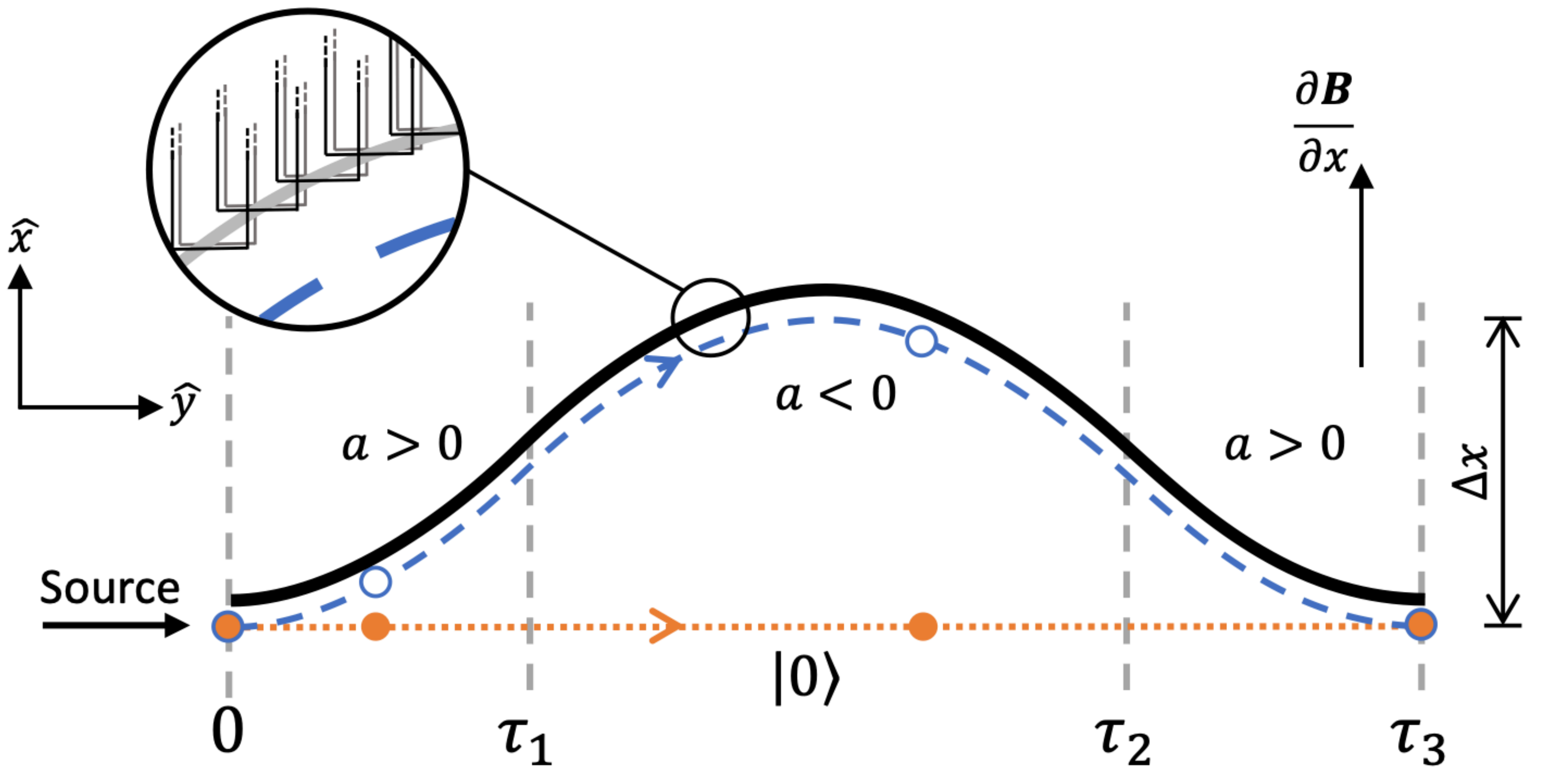
Formidable Challenges



$$H = \frac{1}{2m}\vec{p}^{\,2} + mg\hat{z} + \hbar D\hat{S}_z^2 - \frac{\chi_v V}{2\mu_0}\vec{B}^2 - \hbar\gamma_e\vec{S}\cdot\vec{B}$$

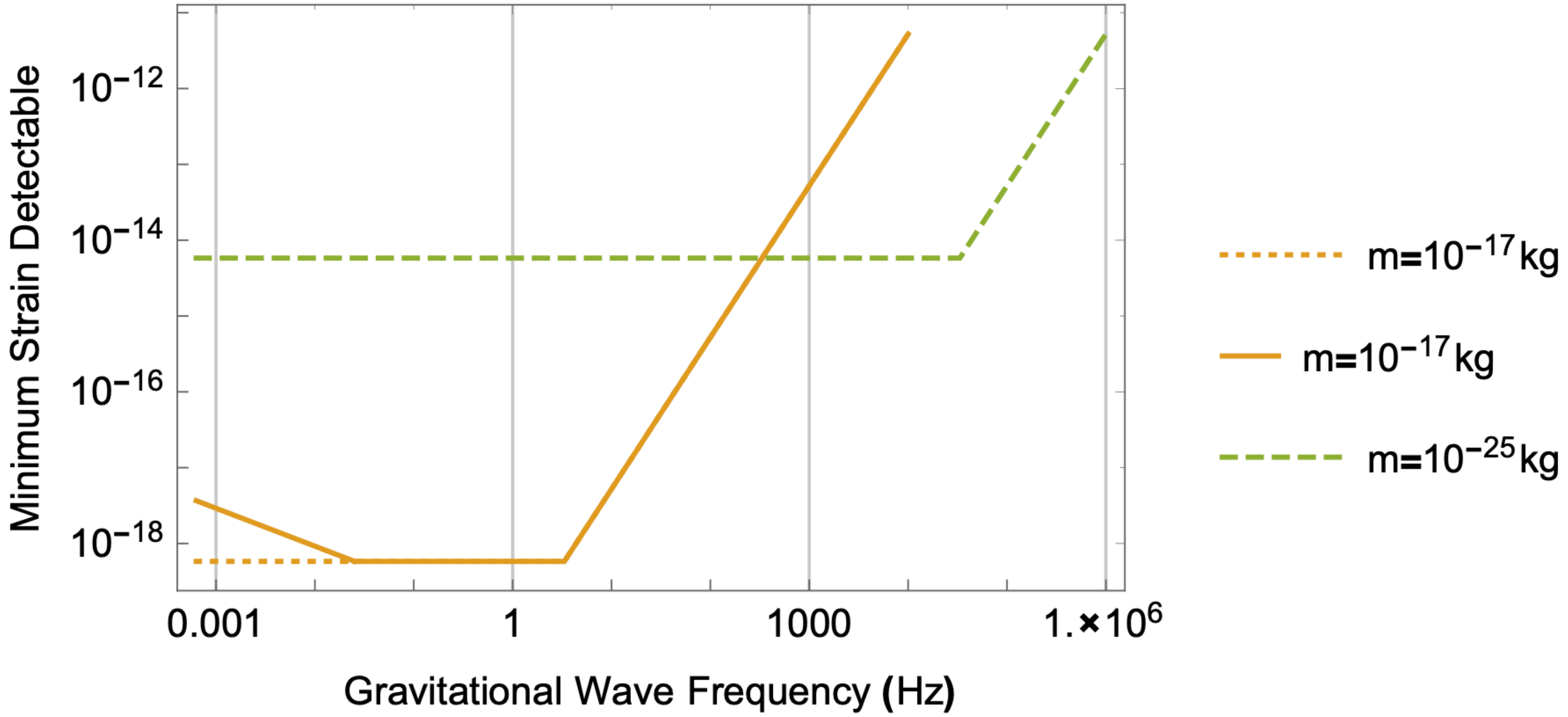
Zero-field Splitting **Diamagnetic part** **NV-part**

MIMAC: Mesoscopic Interferometer for Metric and Curvature



Sub-Hz Gravitational Wave Detector

We need to Improve upon the Strain Sensitivity



Theoretical Aspects

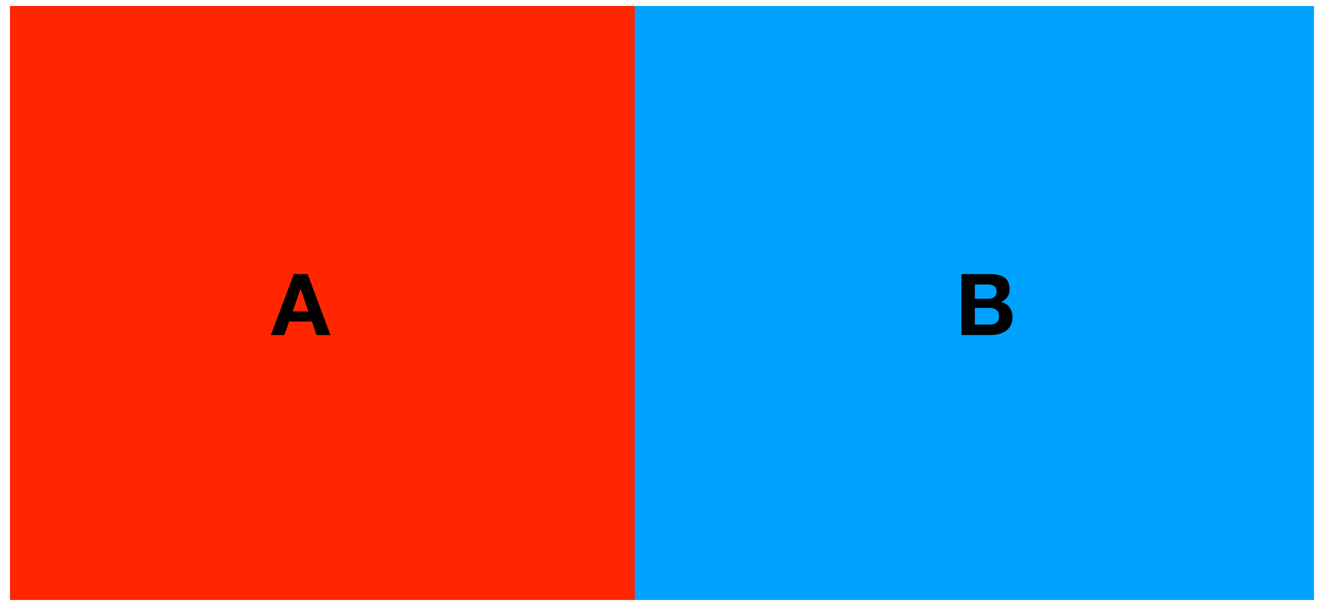
Alice, Bob and Eve

We are
all
Entangled
:
If Gravity is
QUANTUM !

We can test it !

Only Entropy is Entanglement Entropy

$$|\Psi\rangle = \sum_{i=1}^{N} C_i \psi_i^A \otimes \psi_i^B$$



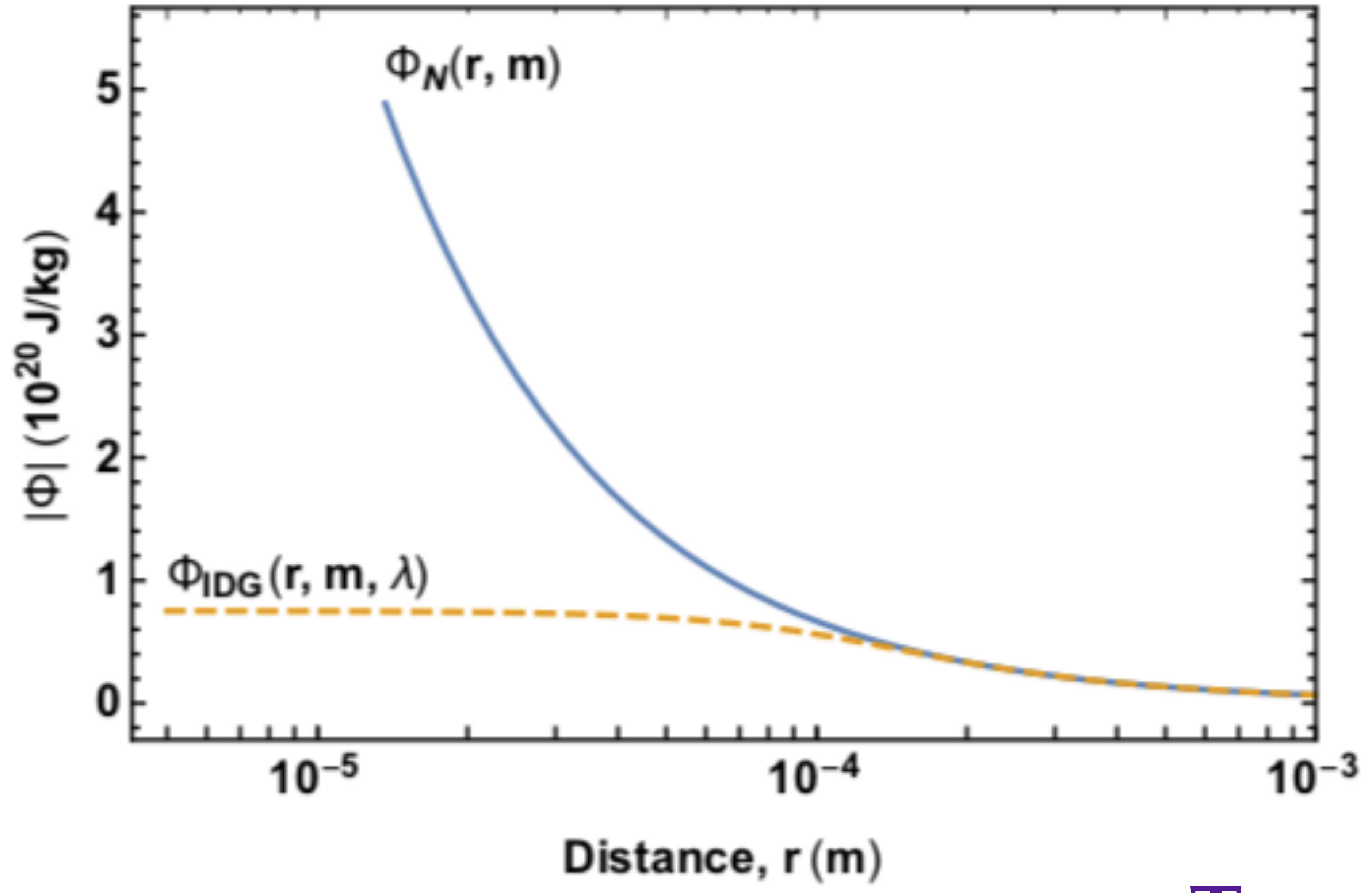
We can trace out the degrees of freedom of 'B' to get a reduced density matrix describing 'A'

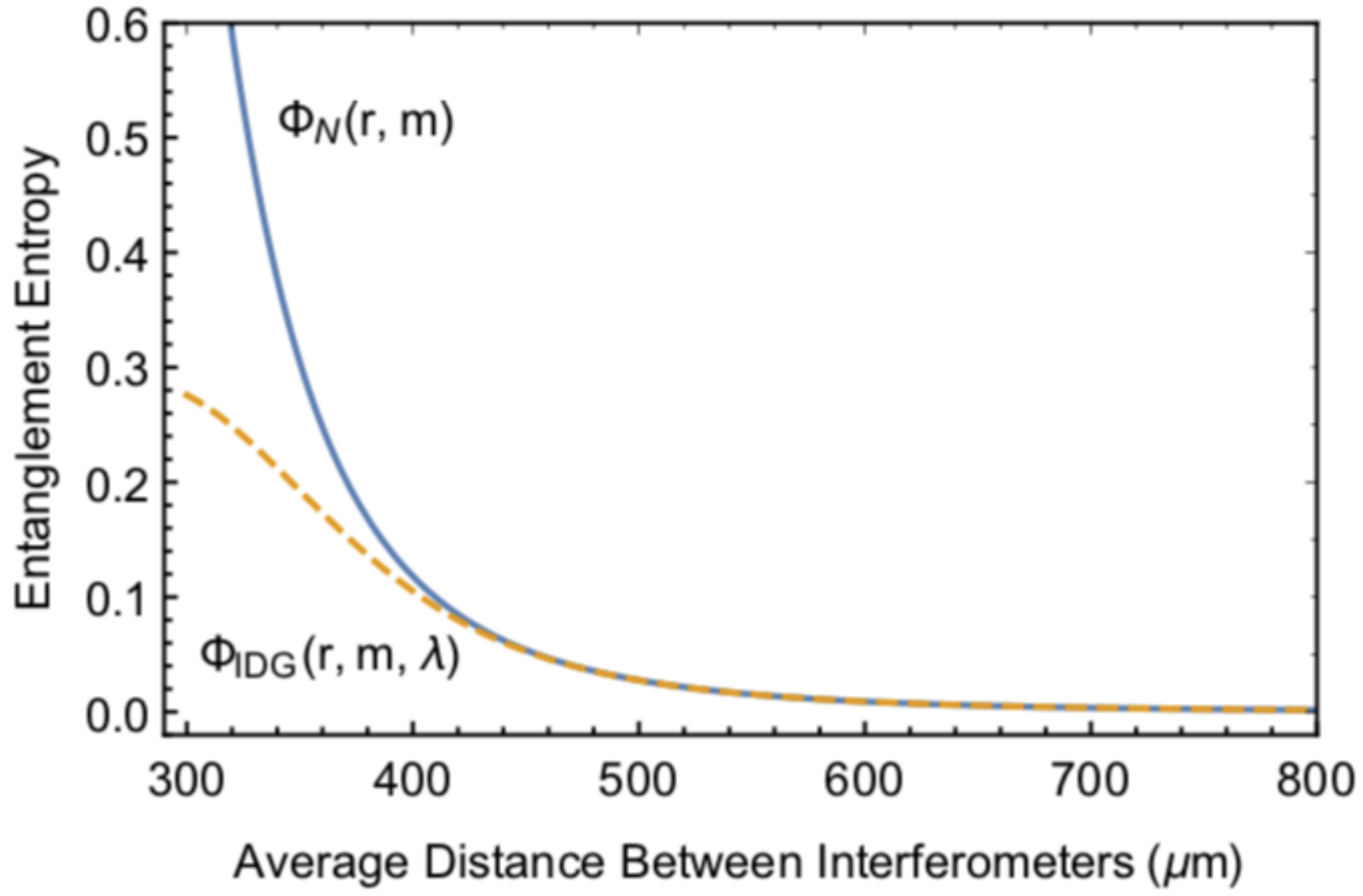
$$\rho_A = \sum_{i=1}^{N} |C_i|^2 \psi_i^A\rangle\langle\psi_i^A|$$

$$S(A) = -Tr[\rho_A \ln \rho_A] \; = \sum_{i=1}^{N} |C_i|^2 \ln |C_i|^2$$

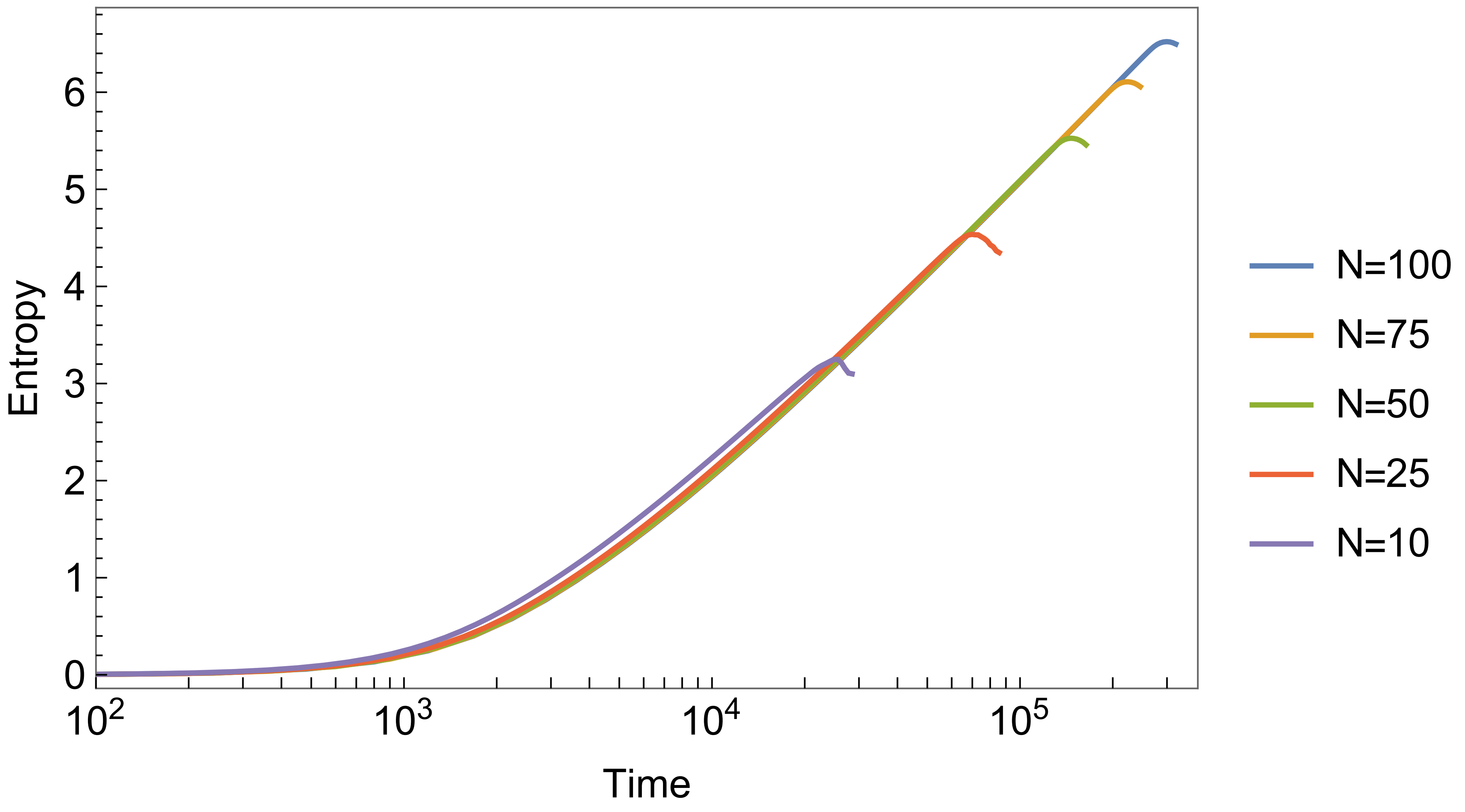
$$S(A) + S(B) = 0$$

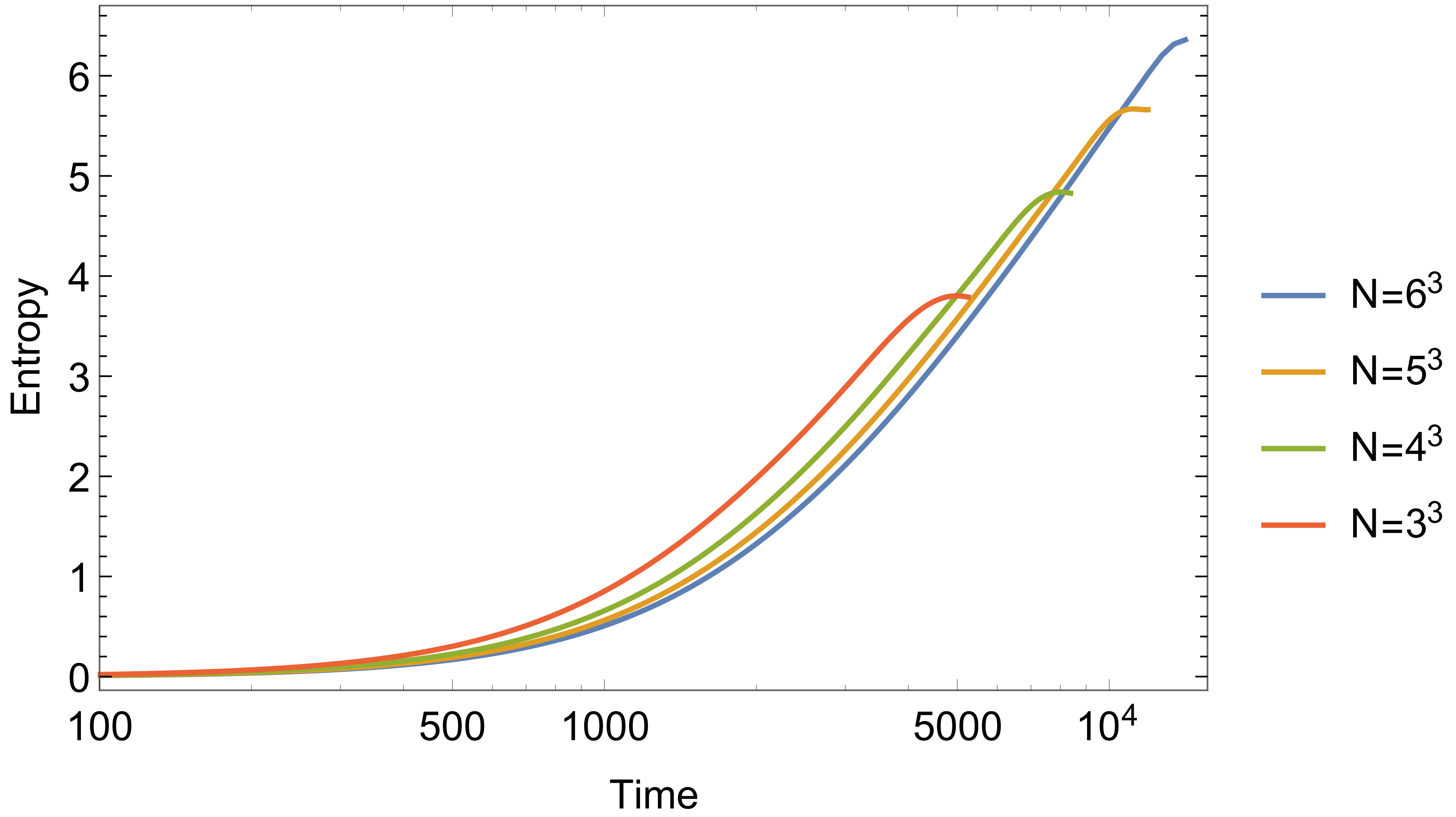
Entanglement Entropy & it's Universal Growth



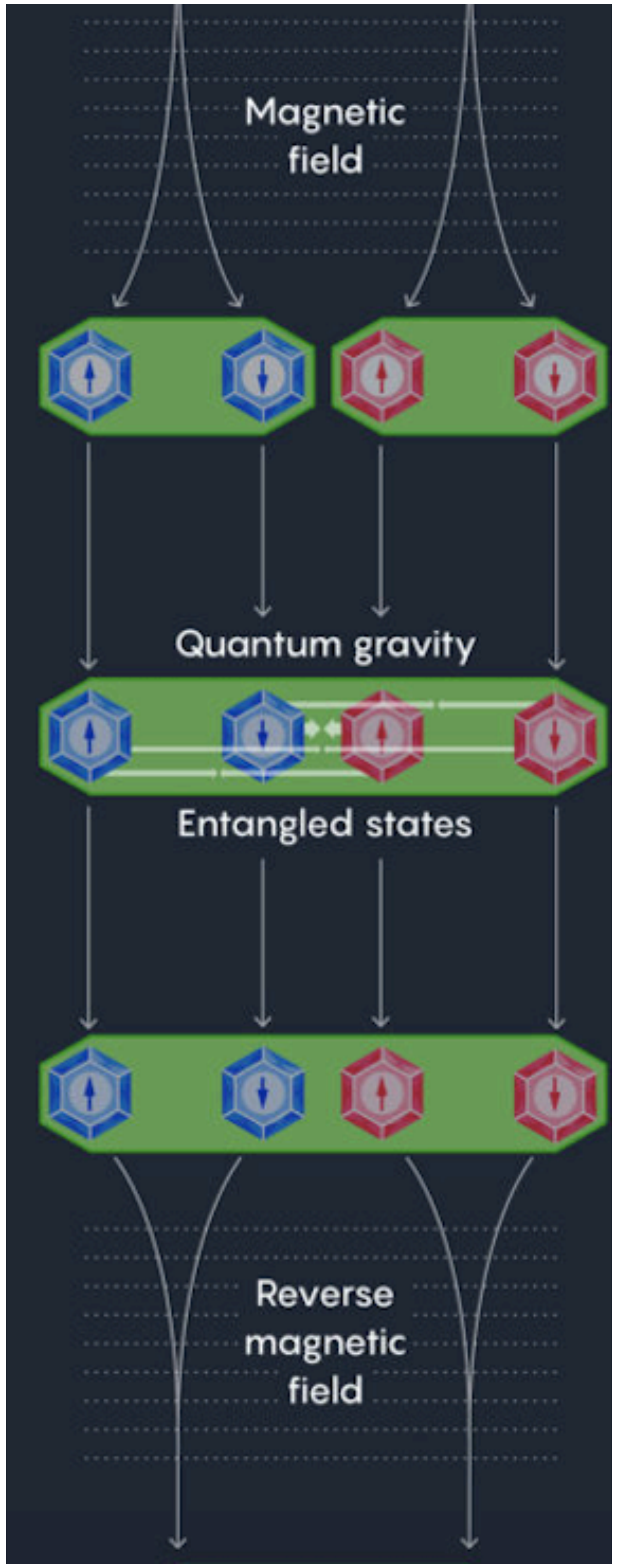


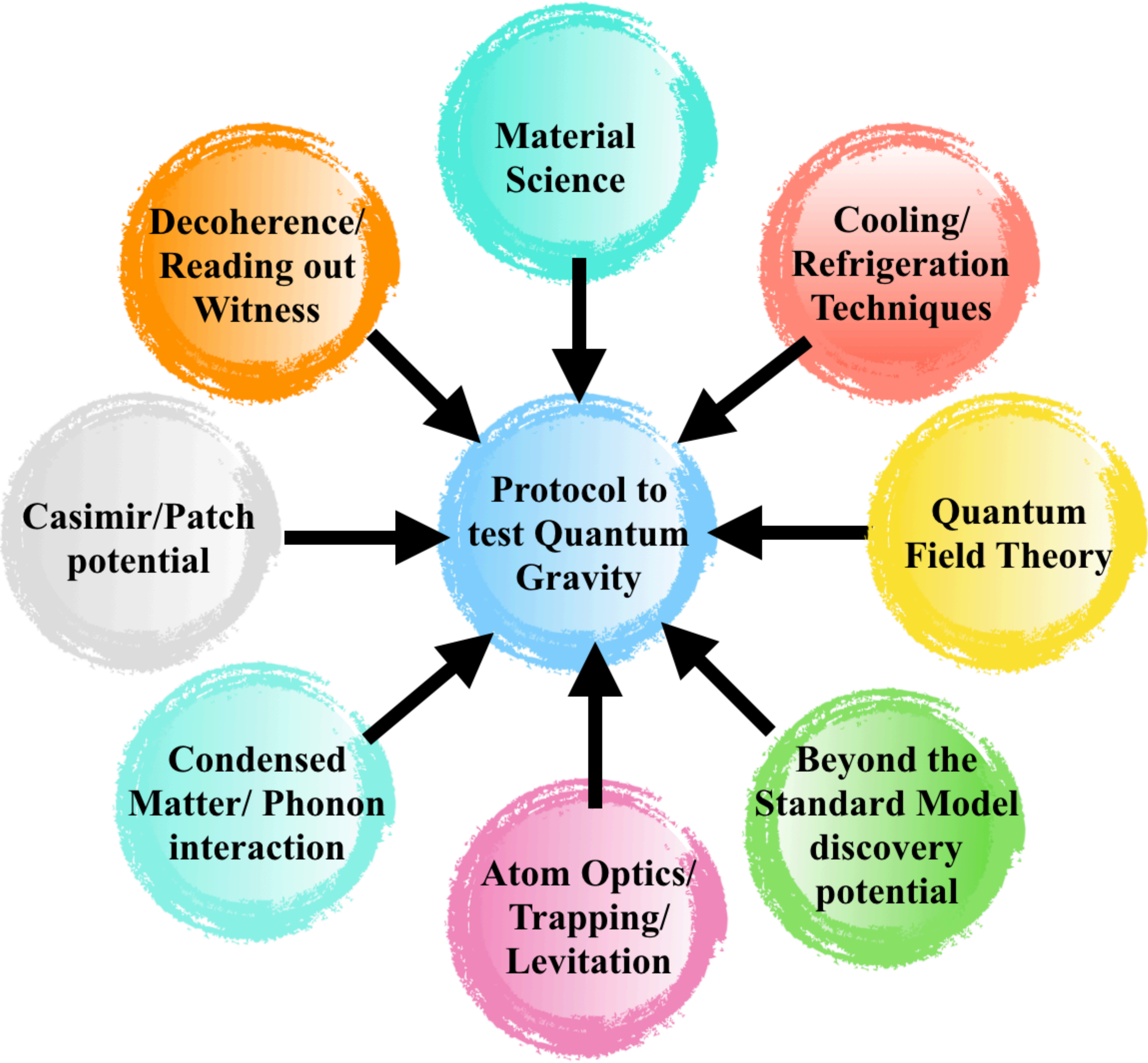
Two Self-Gravitating systems





QGEM Protocol Unites All



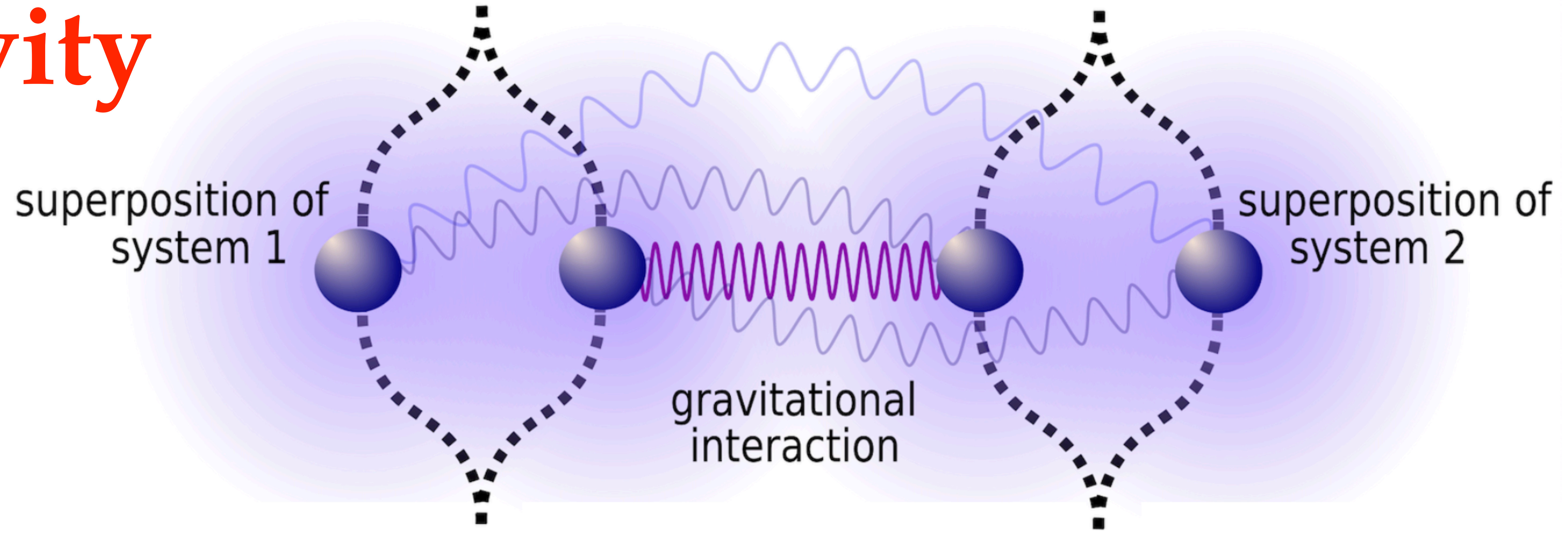


- Foundations Of QM
- Foundations of QFT
- Foundations of Gravity
- Penrose's Collapse Model
- Gravitational Waves
- Axion/Hidden sector

Conclusion & Future Outlook

Testing Quantum Gravity

The Final Frontier !



One Protocol to rule them all,
One Protocol to find them,
One Protocol to bring them all,
and Quantum Gravity which binds them.

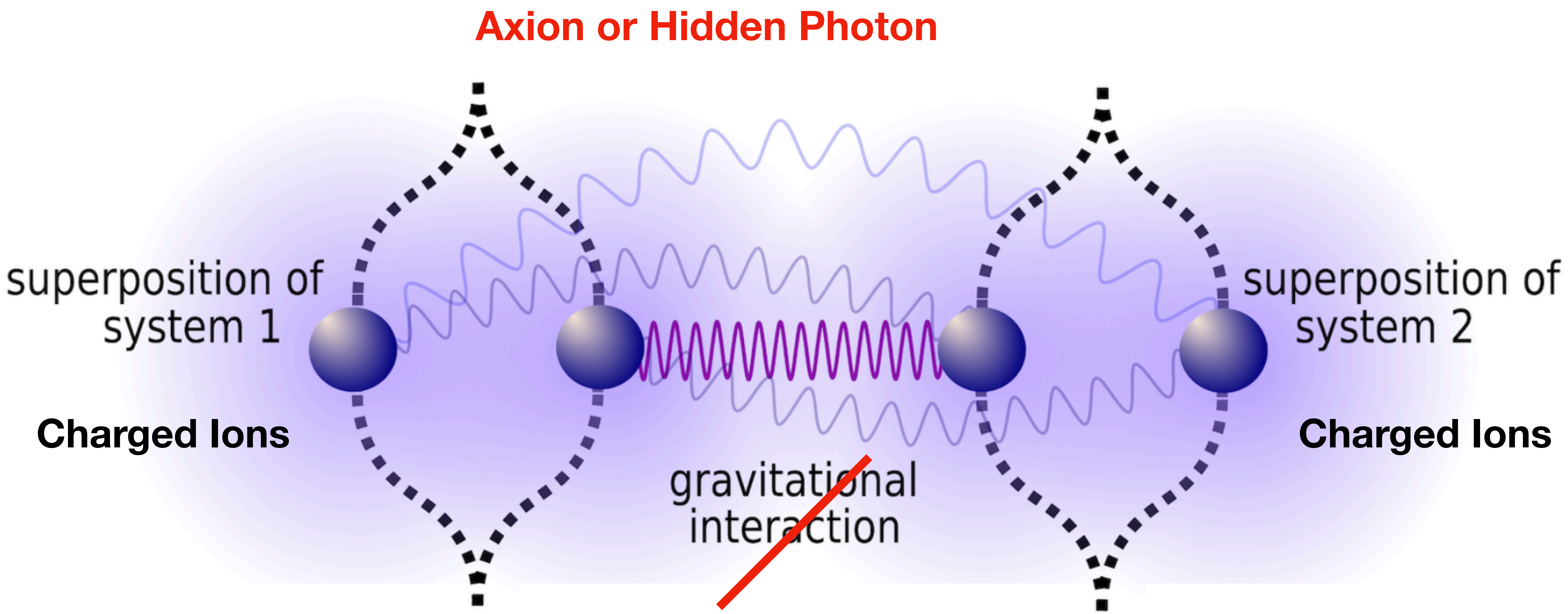
We can test QGEM Protocol

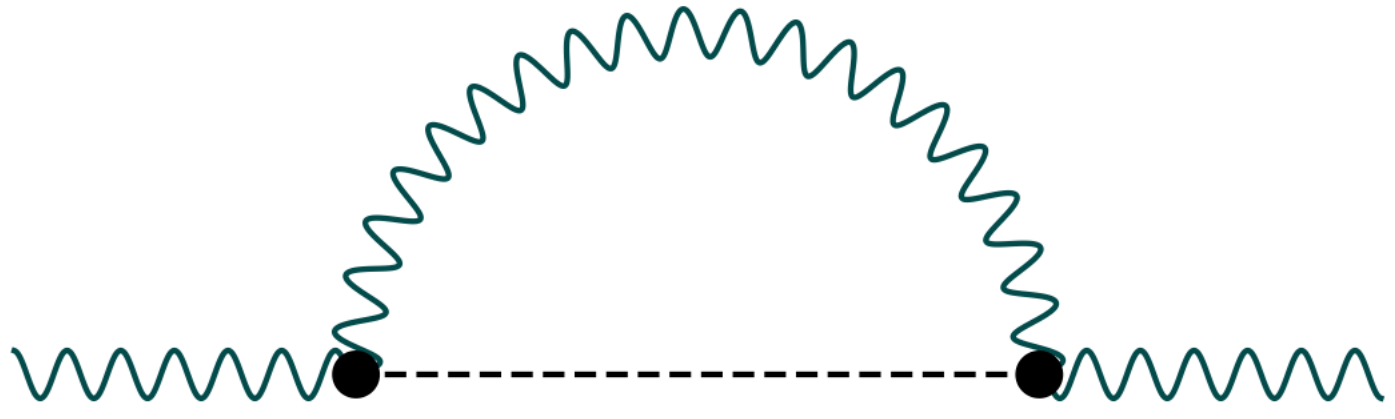
Bose+AM+Morley+Ulbricht+Toros+Paternostro+Geraci+Barker+Kim+Milburn, PRL (2017) [1707.06050]

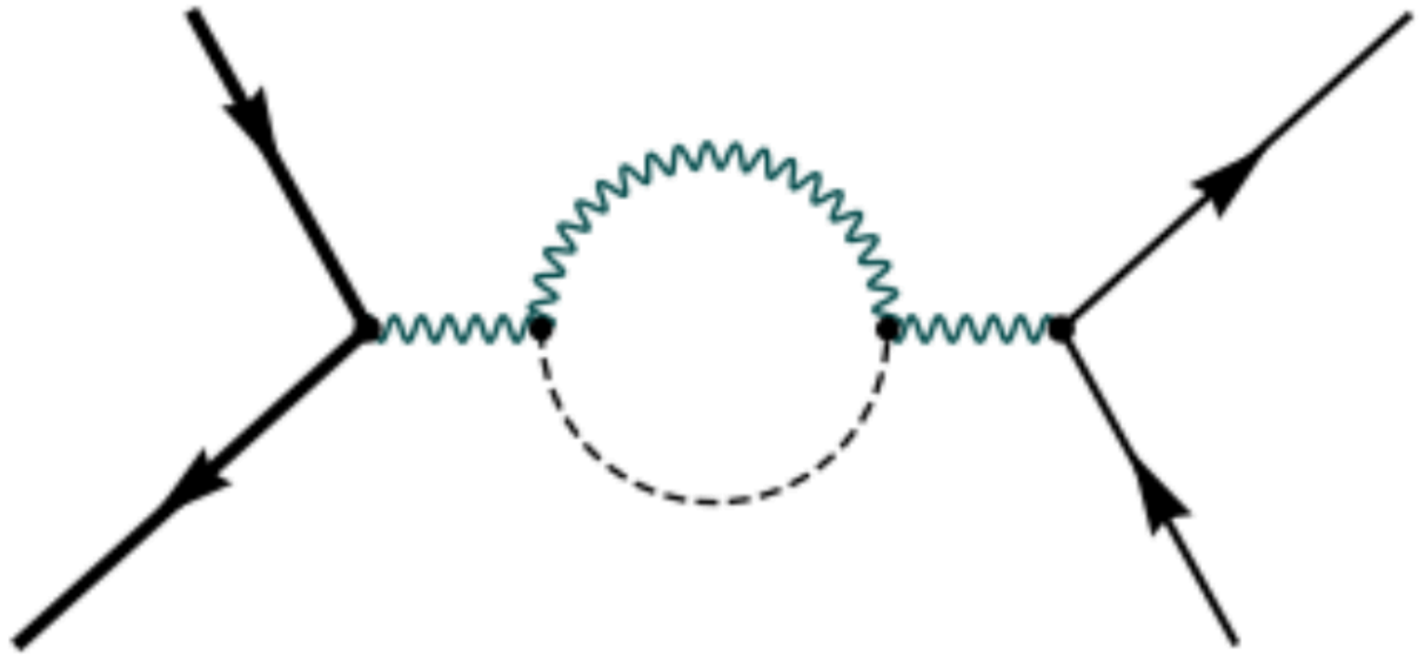
Marshman+AM+Bose, PRA (2020) [1907.01568]

Extra slides

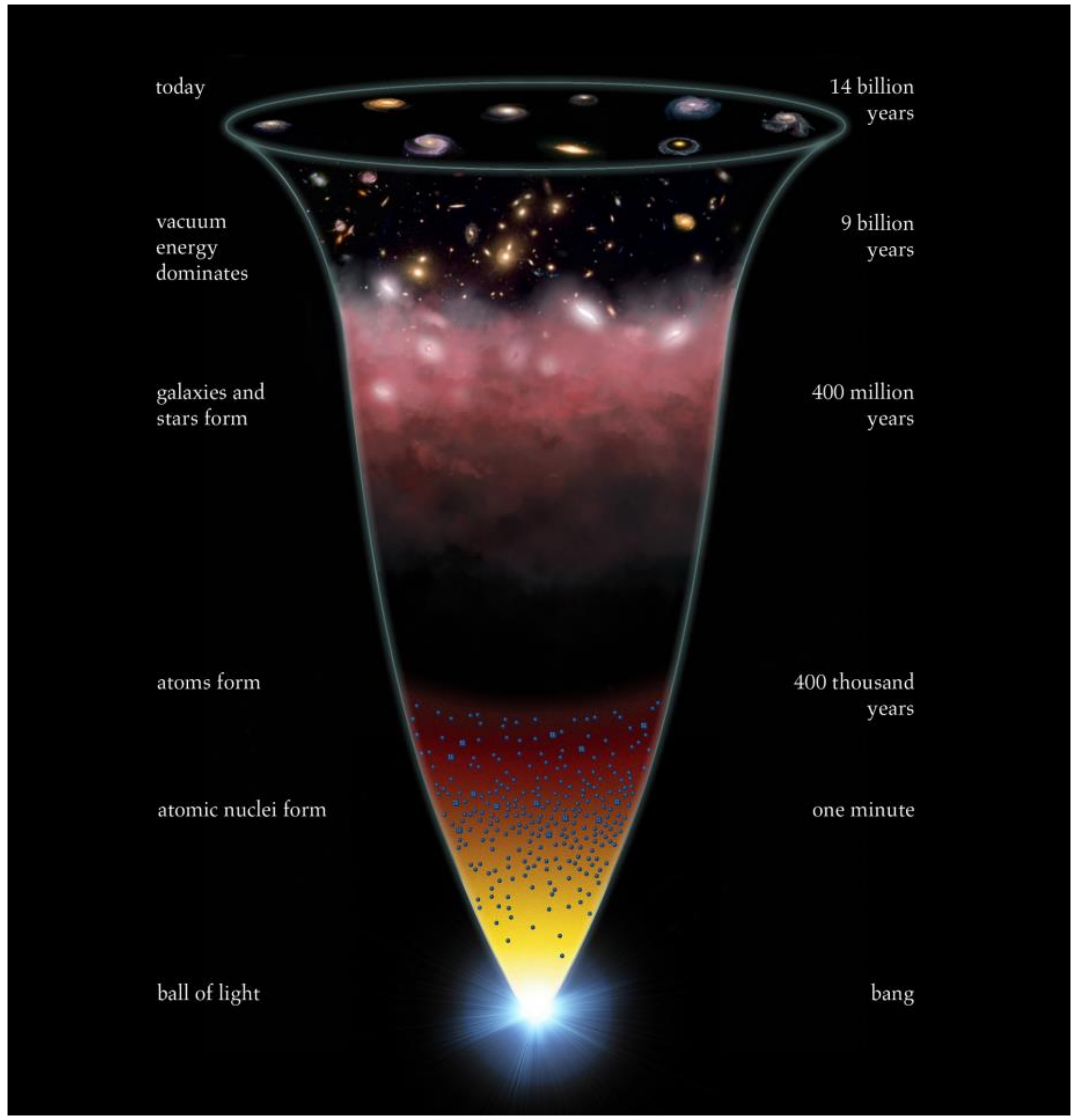
Probing Beyond the Standard Model Physics

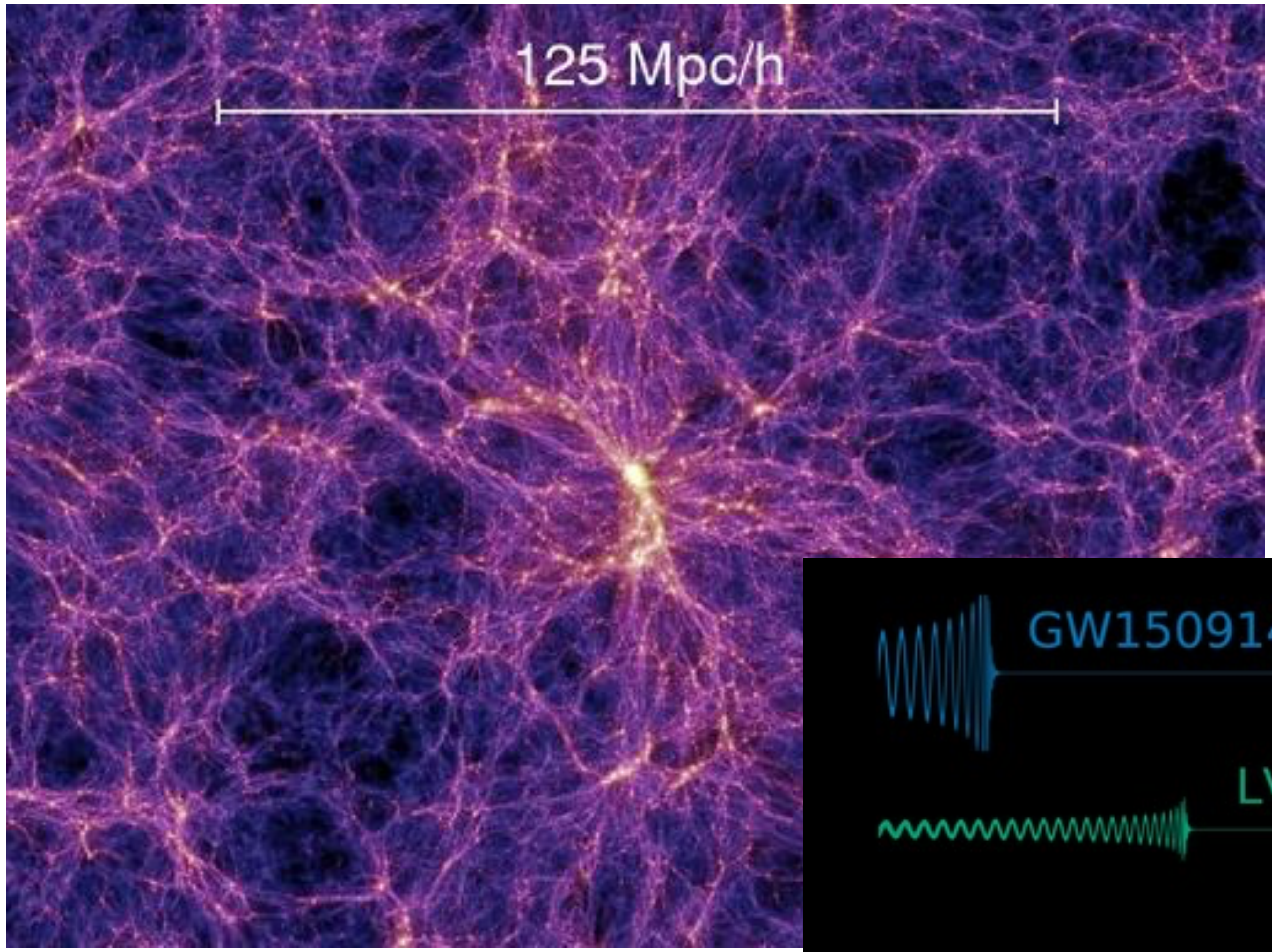


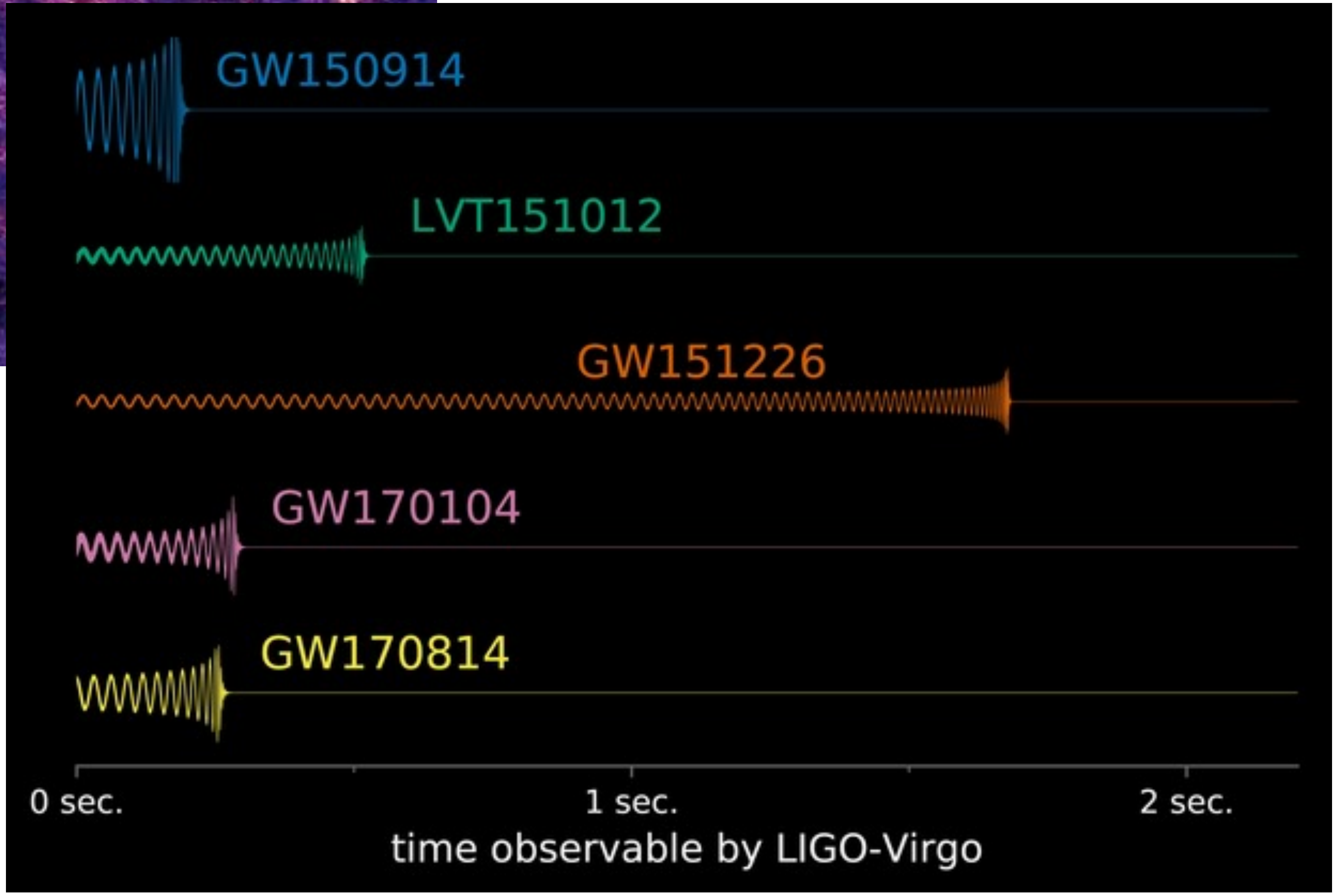




Gravity is Universal & Successful



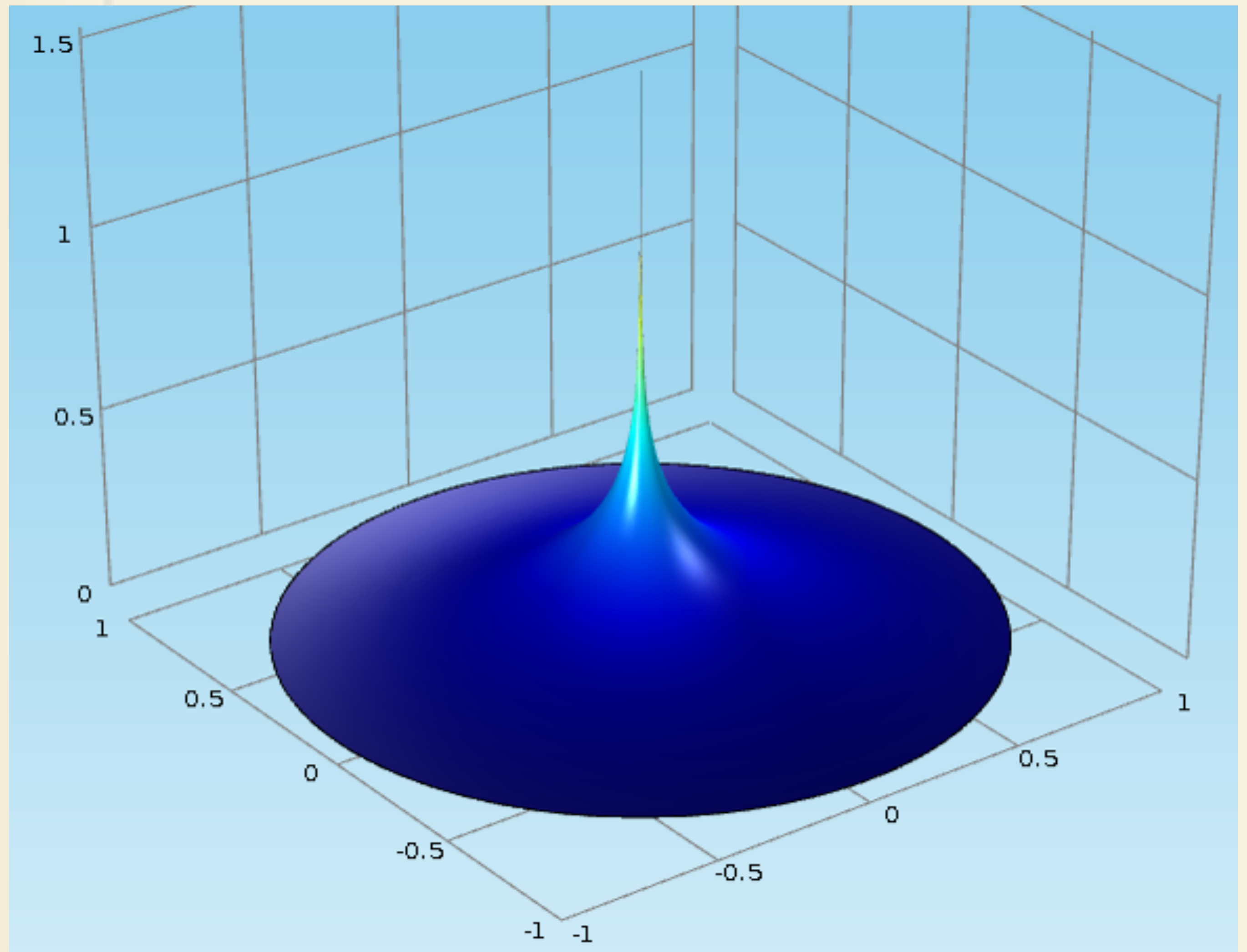


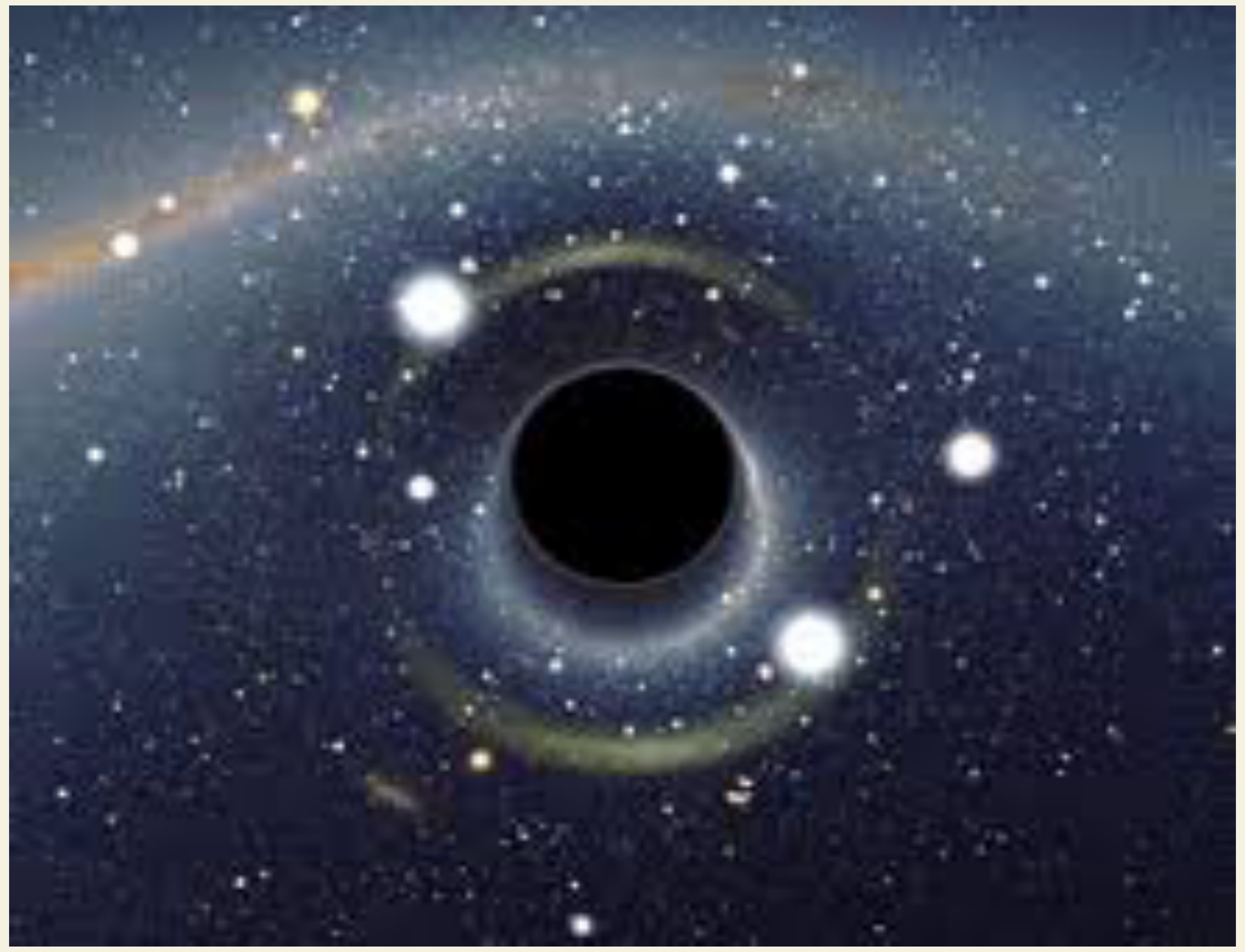


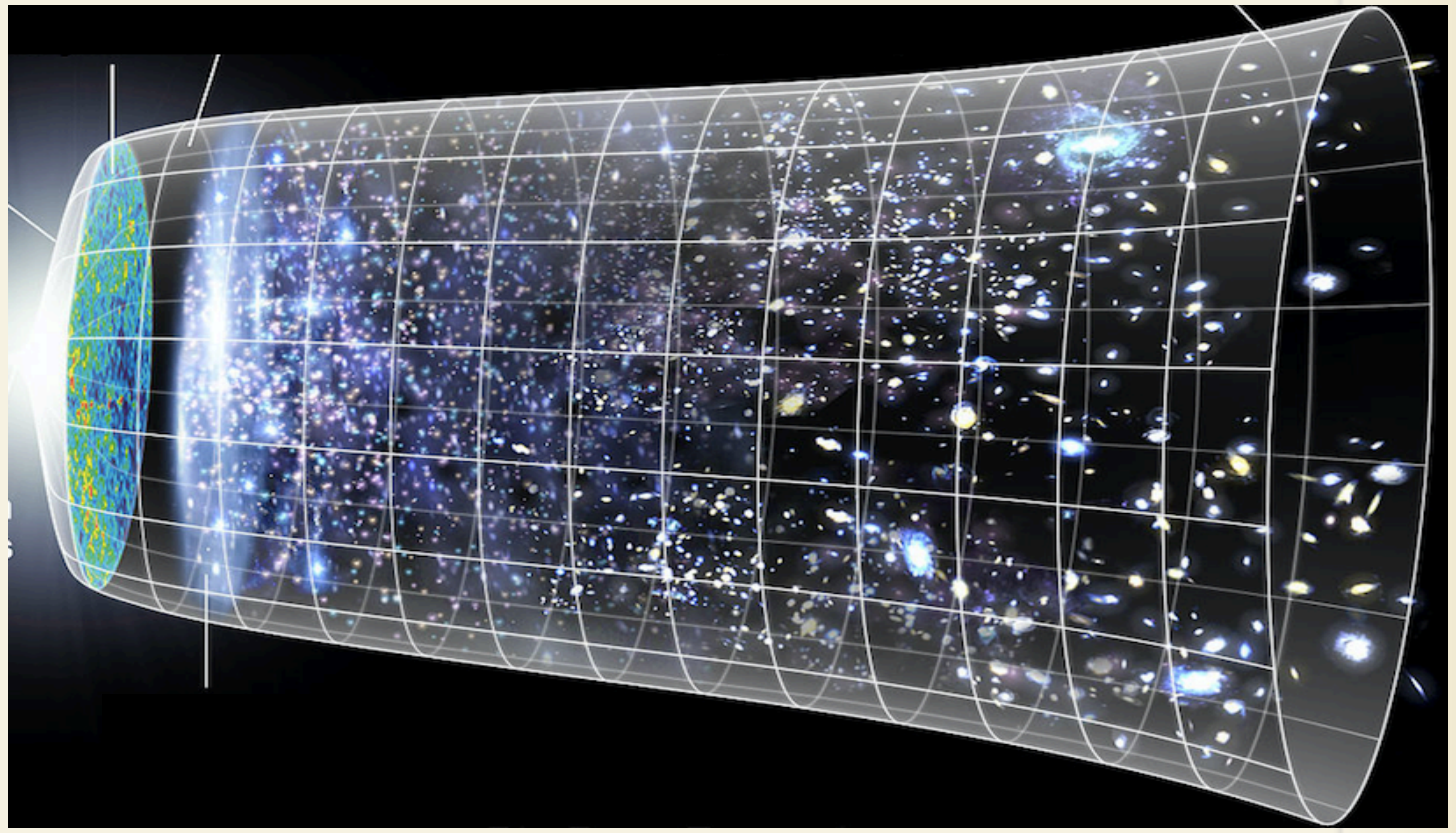
In the simplest case: Minimal Coupling with matter

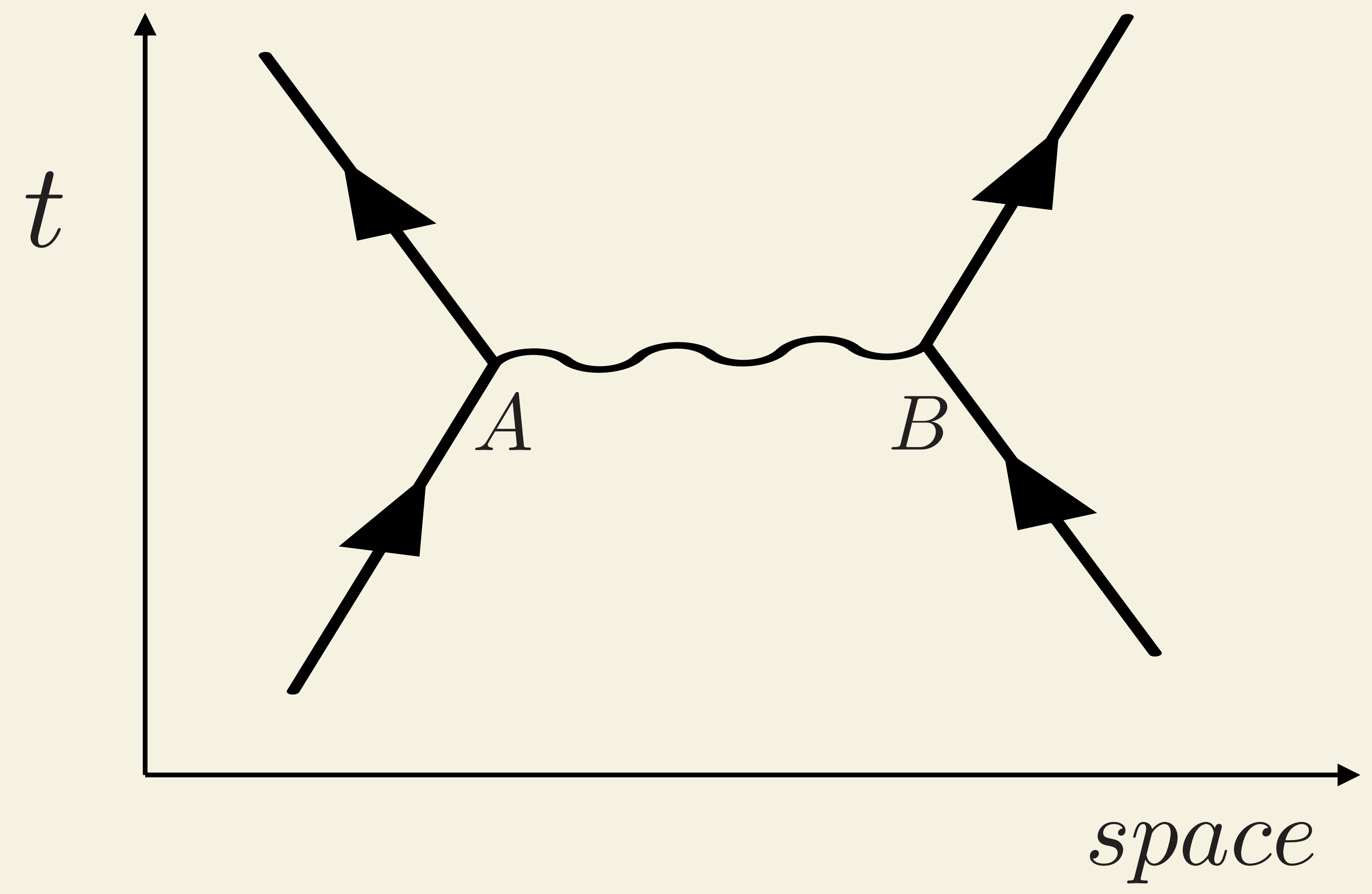
$$\kappa h^{\mu\nu} T_{\mu\nu}$$

Cosmological & Black hole Singularities









$$V \sim \frac{1}{r}$$

Graviton or Photon
(mediator is massless)

Conservation of Probability will not allow singularities to develop, Non-local effects will forbid singularities to form

Macroscopic Spatial Superposition

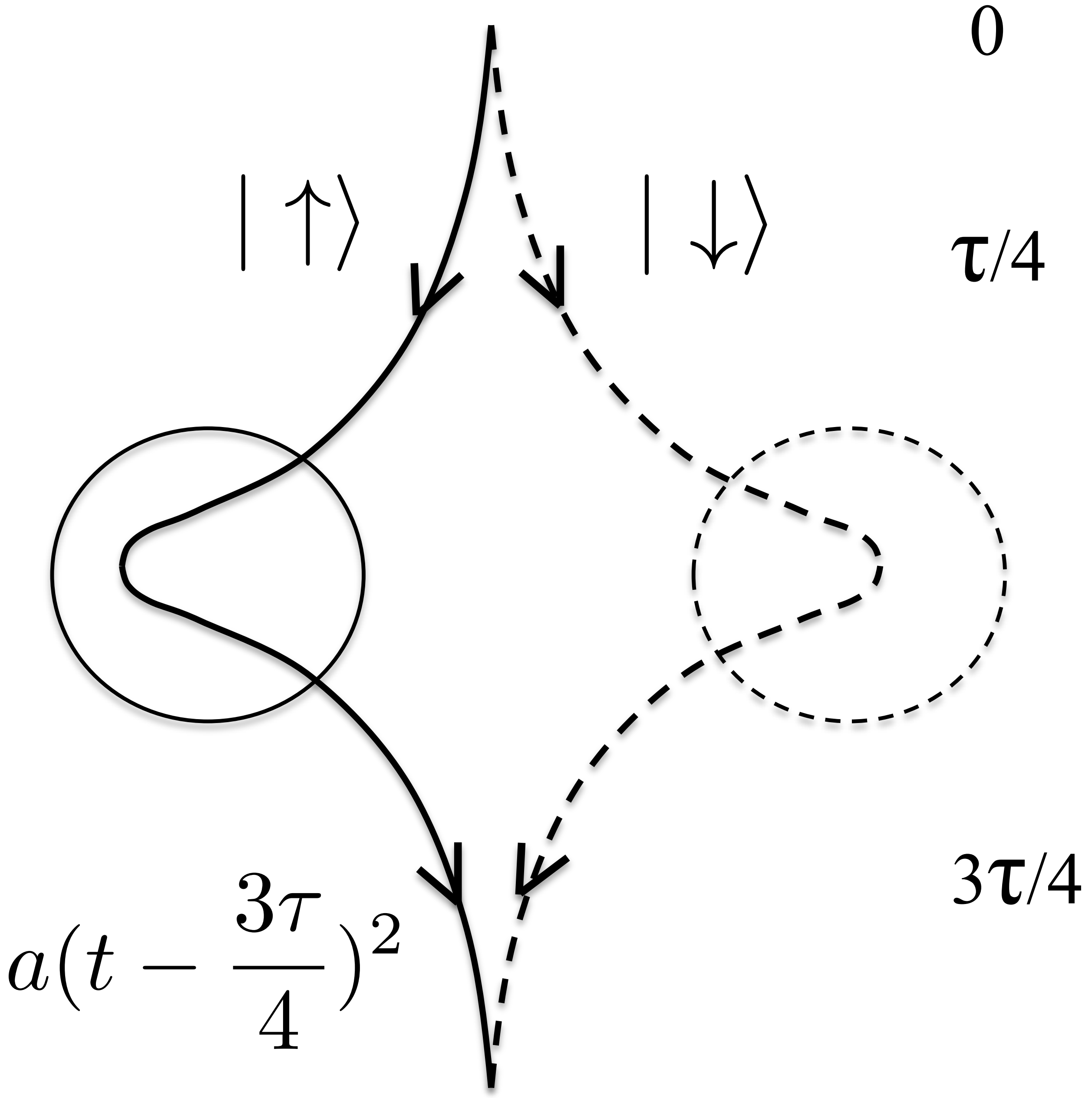
How can we increase the scale of the superposition?

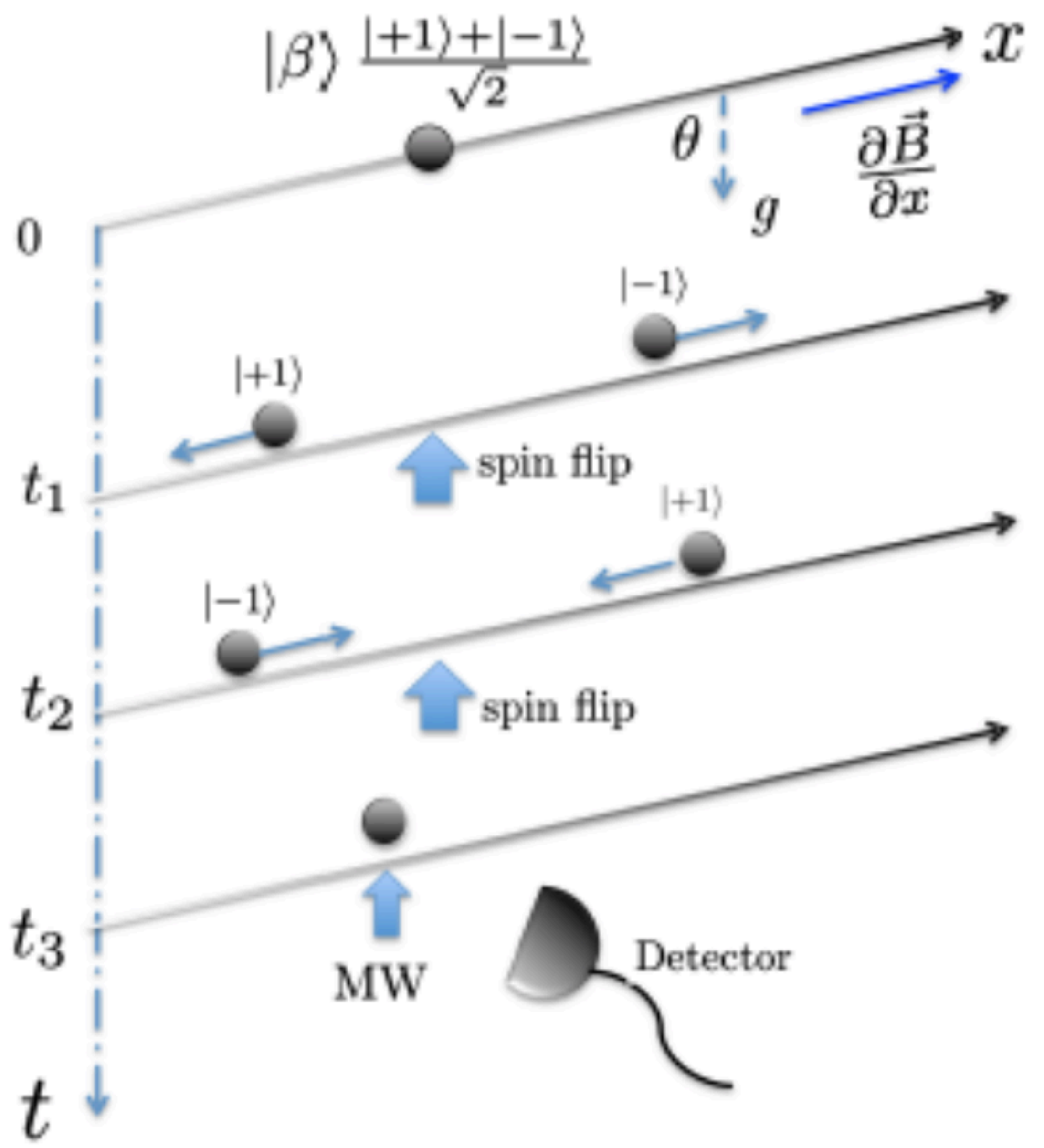
Free particle in an inhomogeneous magnetic field (acceleration +*a* or –*a*)

$$x_\sigma(t,j) = x_j(0) \pm \frac{1}{2}at^2$$

$$= \frac{a\tau}{4}(t-\frac{\tau}{4}) \mp \frac{1}{2}a(t-\frac{\tau}{4})^2$$

$$= \frac{1}{2}a(\frac{\tau}{4})^2 \mp \frac{a\tau}{4}(t-\frac{3\tau}{4}) \pm \frac{1}{2}a(t-\frac{3\tau}{4})^2$$





$$\Delta x = 2s = 2\frac{g\mu_B}{m\hbar}S_x\partial_x B(\tau/2)^2 = \frac{g\mu_B\partial_x B}{2m}\tau^2$$

100 micron separation for 1 sec

M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, S. Bose, PRL. **111**, 180403 (2013)

J. S. Pedernales, G. W. Morley, and M. B. Plenio:1906.00835 (2019)

Van de Kamp+Marshman +Bose+Mazumdar [2006.06931 [quant-ph]]

Probing UV Gravity

(1) GR: $$\lim_{k^2\to 0} \Pi = (\mathcal{P}^2/k^2) - (\mathcal{P}_s^0/2k^2) \equiv \Pi_{GR}$$

(2) F(R) Gravity:

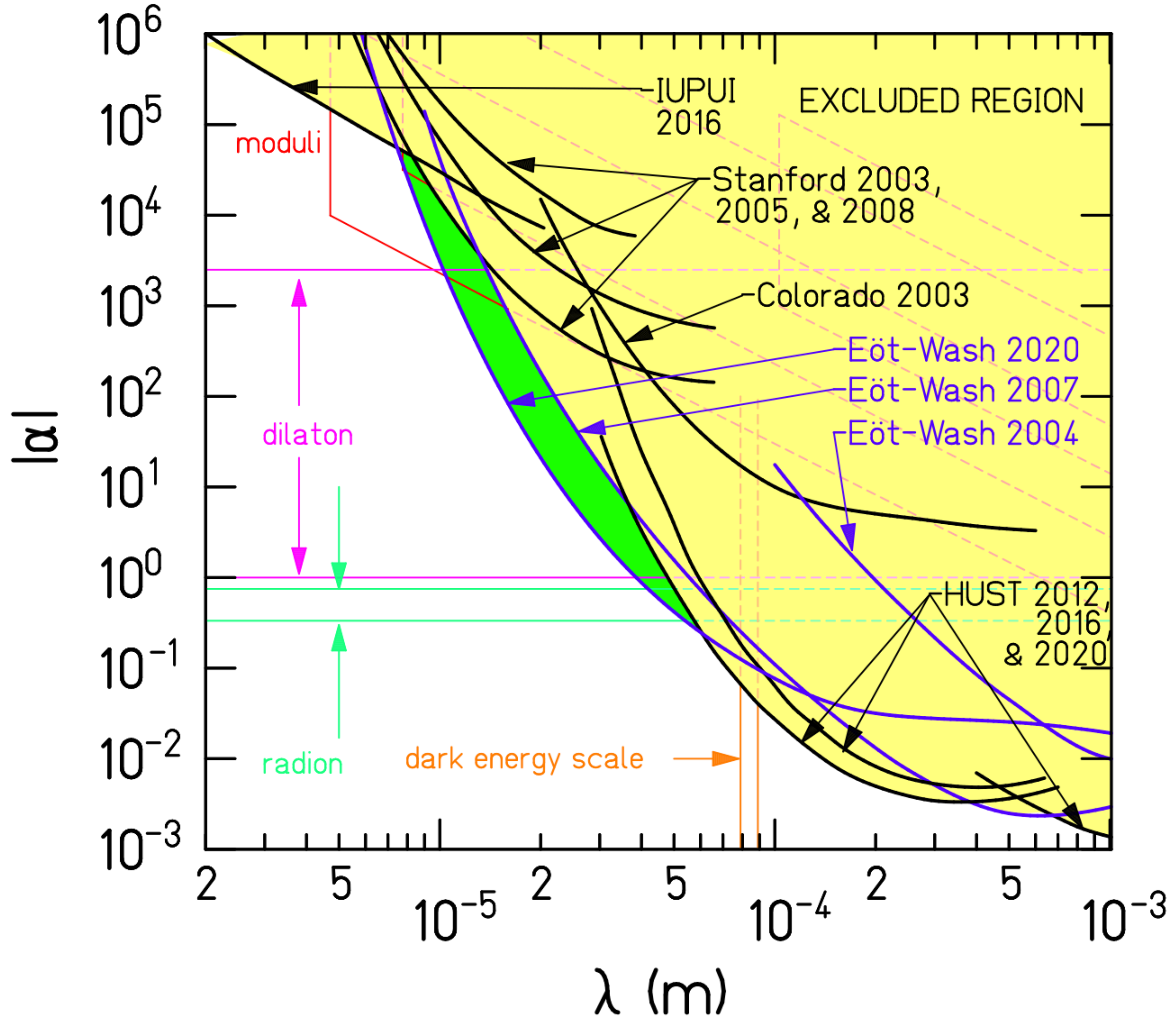
$$\mathcal{L}(R) = \mathcal{L}(0) + \mathcal{L}'(0)R + \frac{1}{2}\mathcal{L}''(0)R^2 + \cdots$$

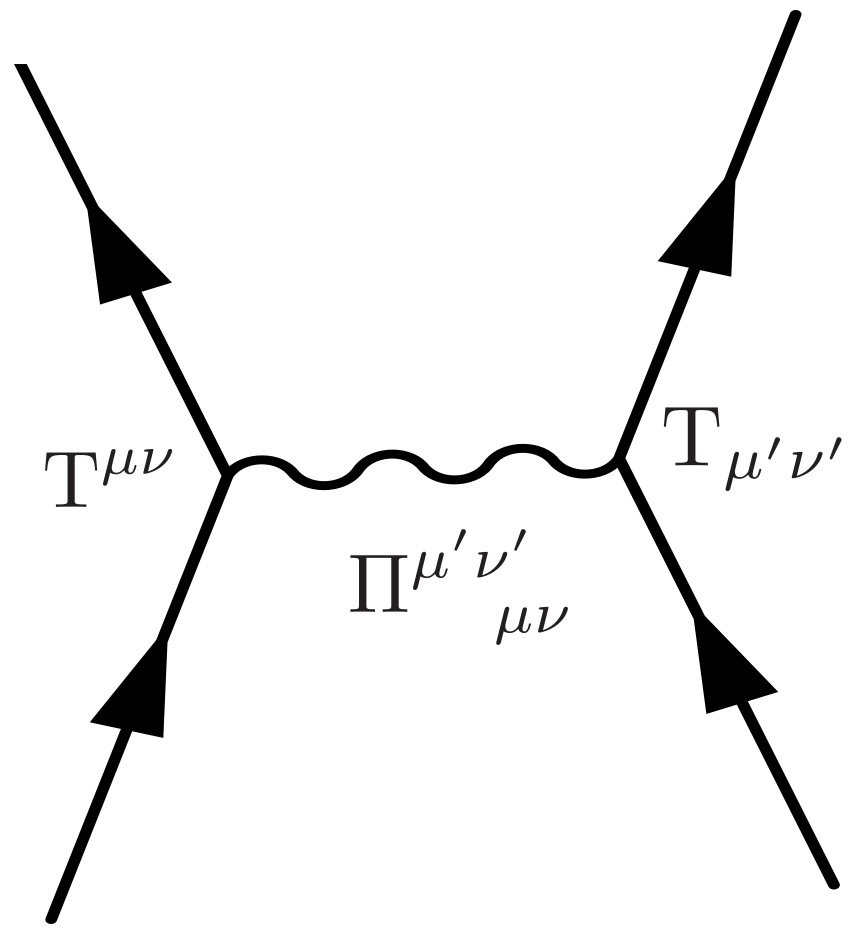
$$\Pi = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}_s^0}{k^2+m^2}, \quad m^2 = \frac{1}{3\mathcal{L}''(0)}$$

(3) Weyl Gravity: $$\mathcal{L} = R - \frac{1}{m^2}C^2$$

$$C^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$$\Pi = \frac{\mathcal{P}^2}{k^2\left(1-(k/m)^2\right)} - \frac{\mathcal{P}_s^0}{2k^2} = \Pi_{GR} - \frac{\mathcal{P}^2}{k^2+m^2}$$





$$V \sim \frac{1}{r}e^{-mr} + \cdots$$

P. Van Nieuwenhuizen, Nucl. Phys. B60, 478-492 (1973)
T. Biswas, T. Koivisto and A. Mazumdar, ``Nonlocal theories of gravity: the flat space propagator,'' arXiv:1302.0532 [gr-qc]