The muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

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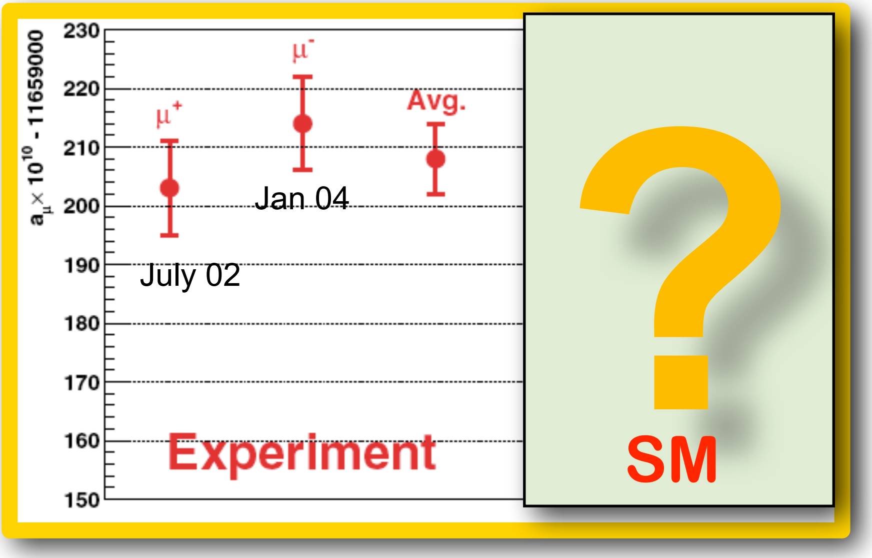
“Newton 1665” seminars
September 17 2020

- The muon g-2: recent theory progress
- Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection
- The MUonE project

The muon g-2: experimental status



- **BNL 821**: $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- **Fermilab E989**: new muon g-2 experiment aims at $\pm 16 \times 10^{-11}$ 0.14ppm. First 3 data taking completed. Analysis of run 1 (~1xBNL) in progress. First result expected very soon with BNL precision.
- **J-PARC**: Muon g-2 proposal. Phase-1 with ~ BNL precision.

The muon g-2: recent theory progress

White Paper of the Muon g-2 Theory Initiative:

arXiv:2006.04822

The muon g-2: the QED contribution

a_μ^{QED} = (1/2)(α/π)   Schwinger 1948

+ 0.765857426 (16) $(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

+ 24.05050988 (28) $(\alpha/\pi)^3$

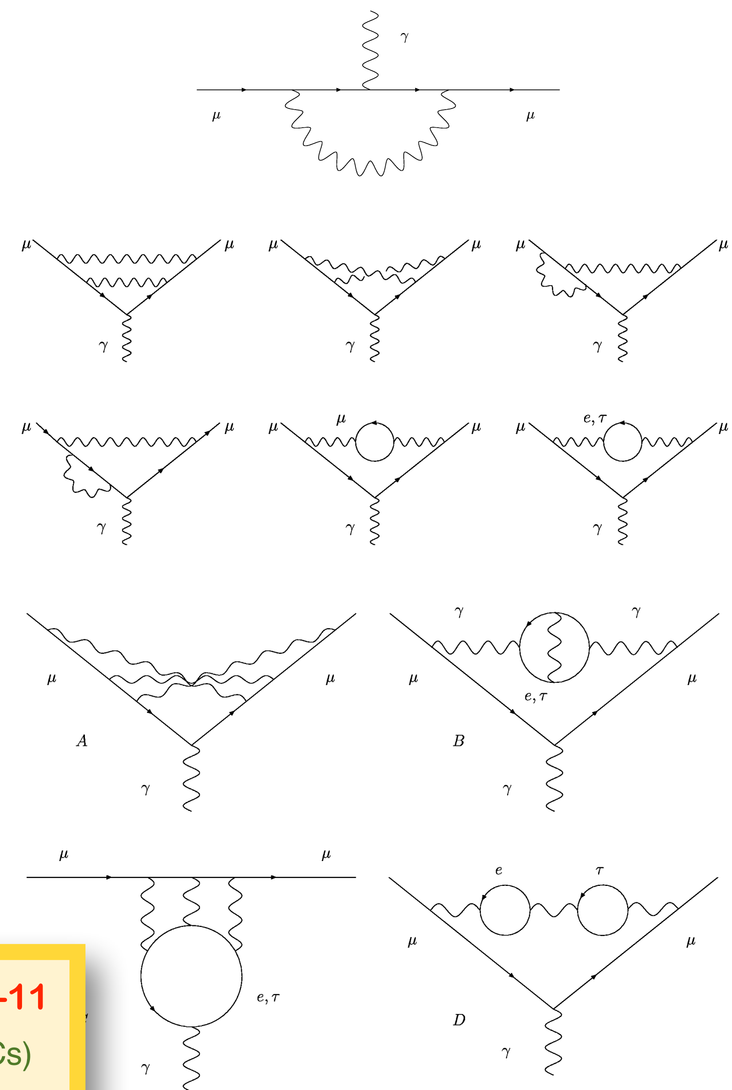
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

+ 130.8780 (60) $(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term). COMPLETED²!

+ 750.86 (88) $(\alpha/\pi)^5$ **COMPLETED!**

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.
Volkov 1909.08015: $A_1^{(10)}$[no lept loops] at variance, but negligible Δ.



…

Adding up, we get:

a_μ^{QED} = 116584718.931 (19)(100)(23) x 10^{-11}

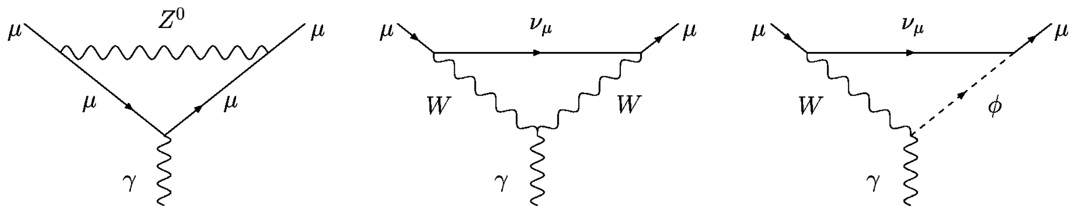
from 4-loop & 5-loop coeffs unc. ⮠   6-loop   ⮡ from α(Cs)

α=1/137.035999046(27) [0.2ppb] 2018

Muon g-2 TI WP: arXiv:2006.04822

The electroweak contribution

- ## One-loop term:



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M^2_{Z,W,H}}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

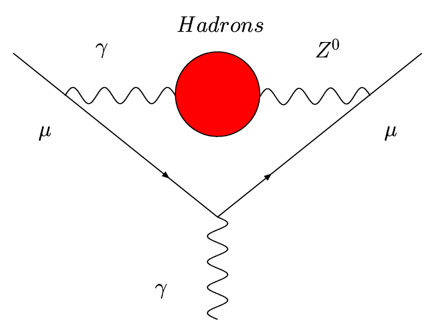
- ## One-loop plus higher-order terms:

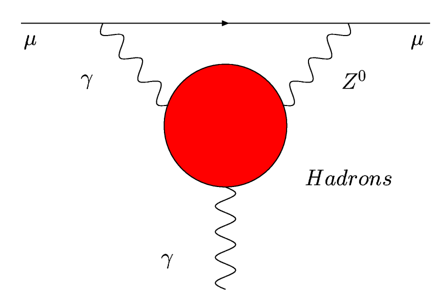
$a_\mu^{\rm EW} = 153.6\ (1.0) \times 10^{-11}$

Hadronic loop uncertainties (and 3-loop nonleading logs).

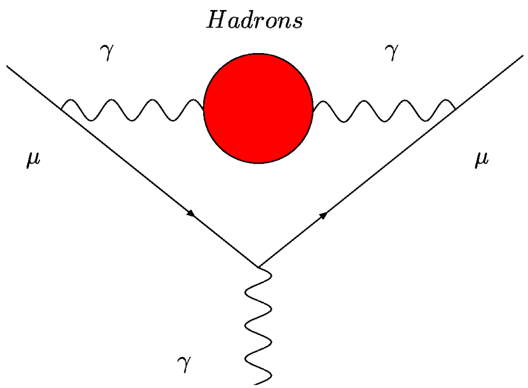
Muon g-2 TI WP: arXiv:2006.04822 .

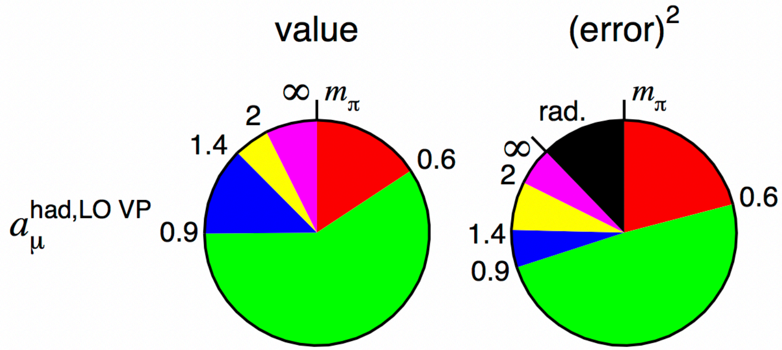
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.





The Hadronic LO contribution





Keshavarzi, Nomura, Teubner 2018

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \qquad a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds \, K(s) \, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

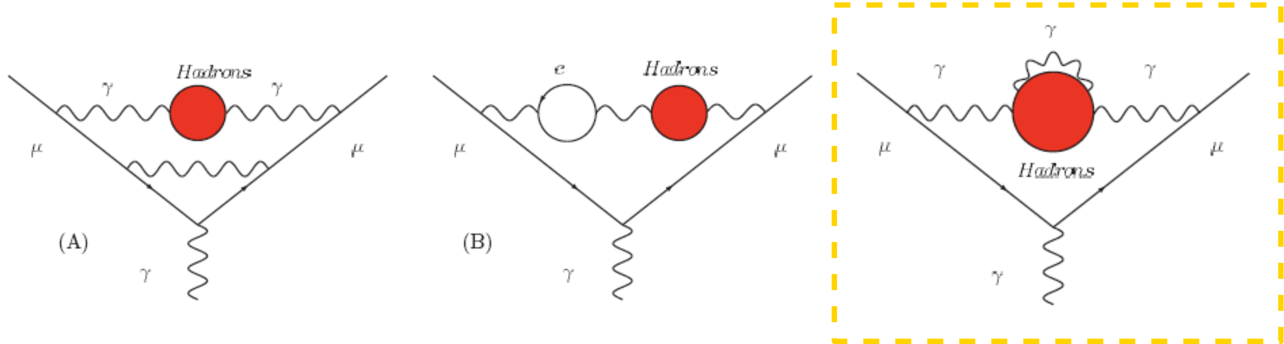
a_μ^{HLO} = 6895 (33) x 10^{-11}	F. Jegerlehner, arXiv:1711.06089
= 6939 (40) x 10^{-11}	Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921
= 6928 (24) x 10^{-11}	Keshavarzi, Nomura, Teubner, arXiv:1911.00367
= 6931 (40) x 10^{-11}	Muon g-2 TI WP: arXiv:2006.04822

- **Radiative Corrections to σ(s) are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585
- **Great progress in lattice QCD results. Recent BMW result with subpercent precision: a_μ^{HLO} = 7087(53)x10^{-11}. Tension with dispersive evaluations.** S. Borsanyi et al. 2002.12347.

The Hadronic HO VP contributions

- $O(\alpha^3)$ contributions of diagrams containing HVP insertions:

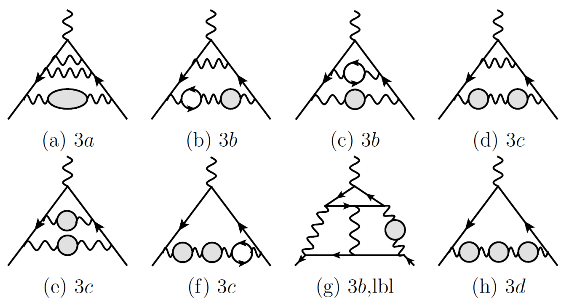
Already included



$$a_\mu^{HNLO}(vp) = -98.3\ (7) \times 10^{-11}$$

Krause '96; Keshavarzi, Nomura, Teubner 2019; Muon g-2 TI WP.

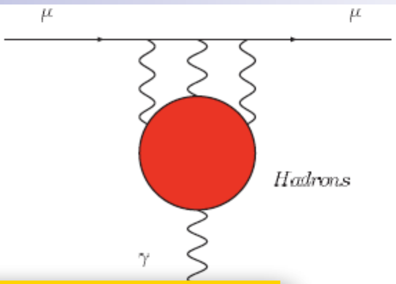
- $O(\alpha^4)$ contributions of diagrams containing HVP insertions:



$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

HNLO light-by-light



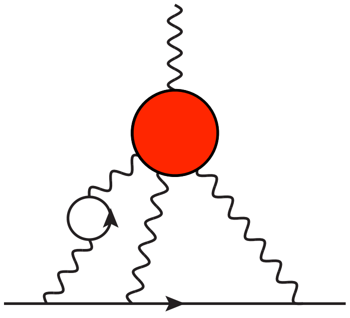
- This term had a troubled life! Nowadays:

$a_\mu^{HNLO}(lbl) =$	$+ 80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$=$	$+136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$=$	$+105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$=$	$+100\ (29) \times 10^{-11}$	Jegerlehner, arXiv:1705.00263
$=$	$+\ 92\ (19) \times 10^{-11}$	Muon g-2 TI WP, 2006.04822

- Significant improvements due to data-driven dispersive approach.
- Great progress on the lattice. Recent RBC result: $79(35)\times10^{-11}$ arXiv:1911.08123

HNNLO light-by-light

$$a_\mu^{HNNLO}(lbl) = 2\ (1) \times 10^{-11}$$



Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; Muon g-2 TI WP, 2006.04822

Comparing the SM prediction with the measured muon g-2 value:

$a_\mu^{EXP} = 116592089\ (63) \times 10^{-11}$ BNL E821

$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$ Muon g-2 TI

$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 279\ (76) \times 10^{-11}$ 3.7 σ

Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

Marciano, MP, Sirlin 2008 & 2010

Keshavarzi, Marciano, MP, Sirlin 2020

Is Δa_μ due to missed contributions in the hadronic cross section?

- Can Δa_μ be due to missing contributions in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3}, \;\; s_u < M_Z^2,$$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

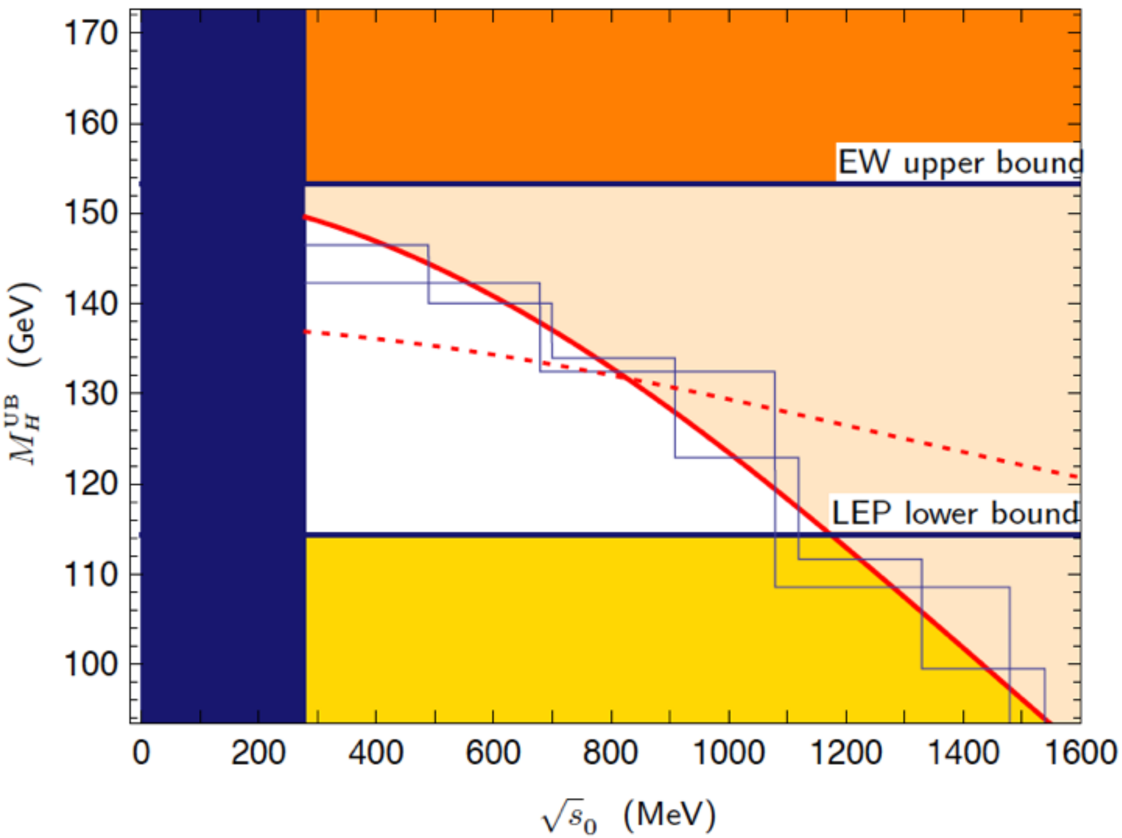
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

$\varepsilon>0$, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \longrightarrow$$

How much does the M_H upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?



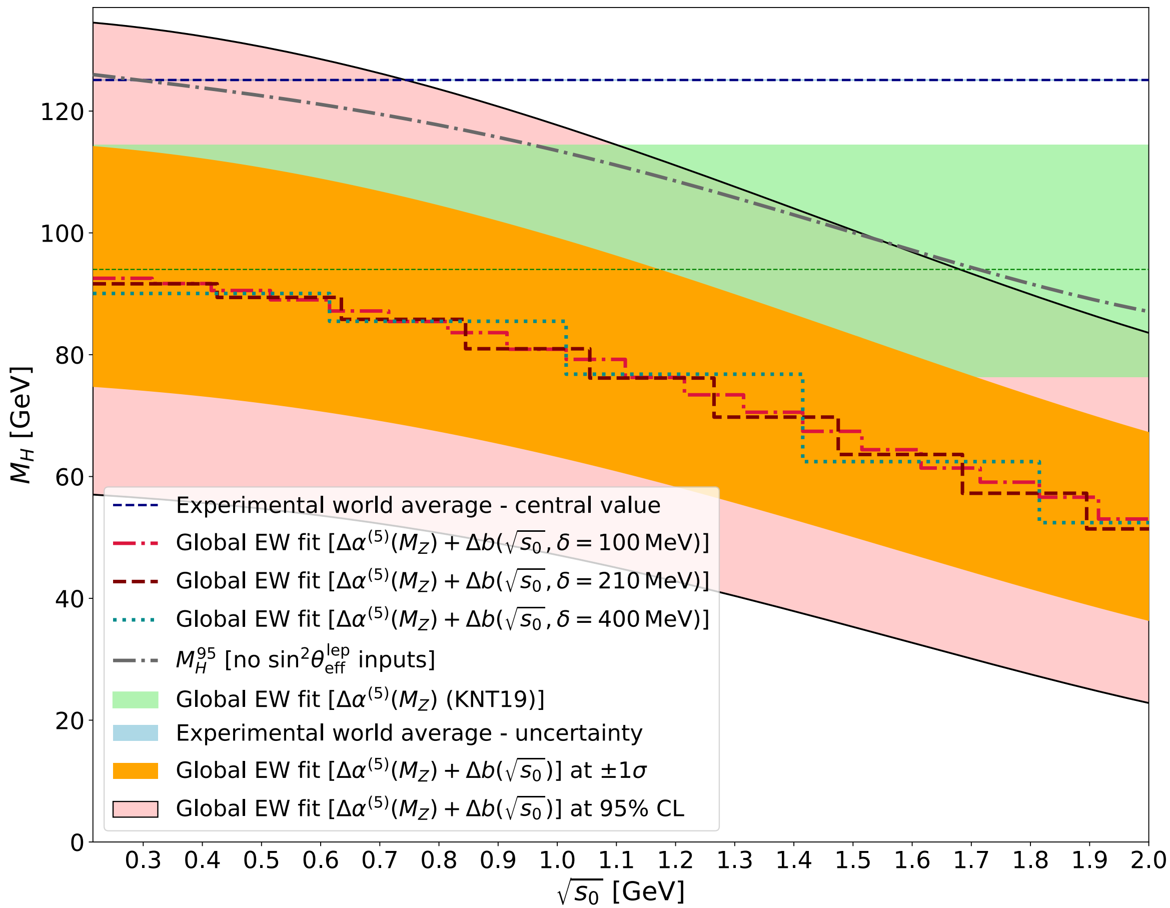
Marciano, MP, Sirlin, 2008 & 2010

Major update: Higgs discovered, improved EW observables (M_W, $\sin^2\theta$, M_{top}, …), updates to $\sigma(s)$, theory improvements, global fit, …

Parameter	Input value	Reference	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	[5]	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	[5]	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \times 10^4$	276.1(1.1)	[23]	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	[5]	173.0(4)	…
$\alpha_s(M_Z^2)$	0.1179(10)	[5]	0.1180(7)	…
M_Z (GeV)	91.1876(21)	[5]	91.1883(20)	…
Γ_Z (GeV)	2.4952(23)	[5]	2.4940(4)	…
Γ_W (GeV)	2.085(42)	[5]	2.0903(4)	…
$\sigma^0_{\rm had}$ (nb)	41.541(37)	[108]	41.490(4)	…
R^0_l	20.767(25)	[108]	20.732(4)	…
R^0_c	0.1721(30)	[108]	0.17222(8)	…
R^0_b	0.21629(66)	[108]	0.21581(8)	…
$\bar{m}_c$ (GeV)	1.27(2)	[5]	1.27(2)	…
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	[5]	$4.18^{+0.03}_{-0.02}$	…
$A^{0,l}_{\rm FB}$	0.0171(10)	[108]	0.01622(7)	…
$A^{0,c}_{\rm FB}$	0.0707(35)	[108]	0.0737(2)	…
$A^{0,b}_{\rm FB}$	0.0992(16)	[108]	0.1031(2)	…
A_ℓ	0.1499(18)	[75,108]	0.1471(3)	…
A_c	0.670(27)	[108]	0.6679(2)	…
A_b	0.923(20)	[108]	0.93462(7)	…
$\sin^2\theta^{\rm lep}_{\rm eff}(Q_{\rm FB})$	0.2324(12)	[108]	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\rm lep}_{\rm eff}$(Had Coll)	0.23140(23)	[100]	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

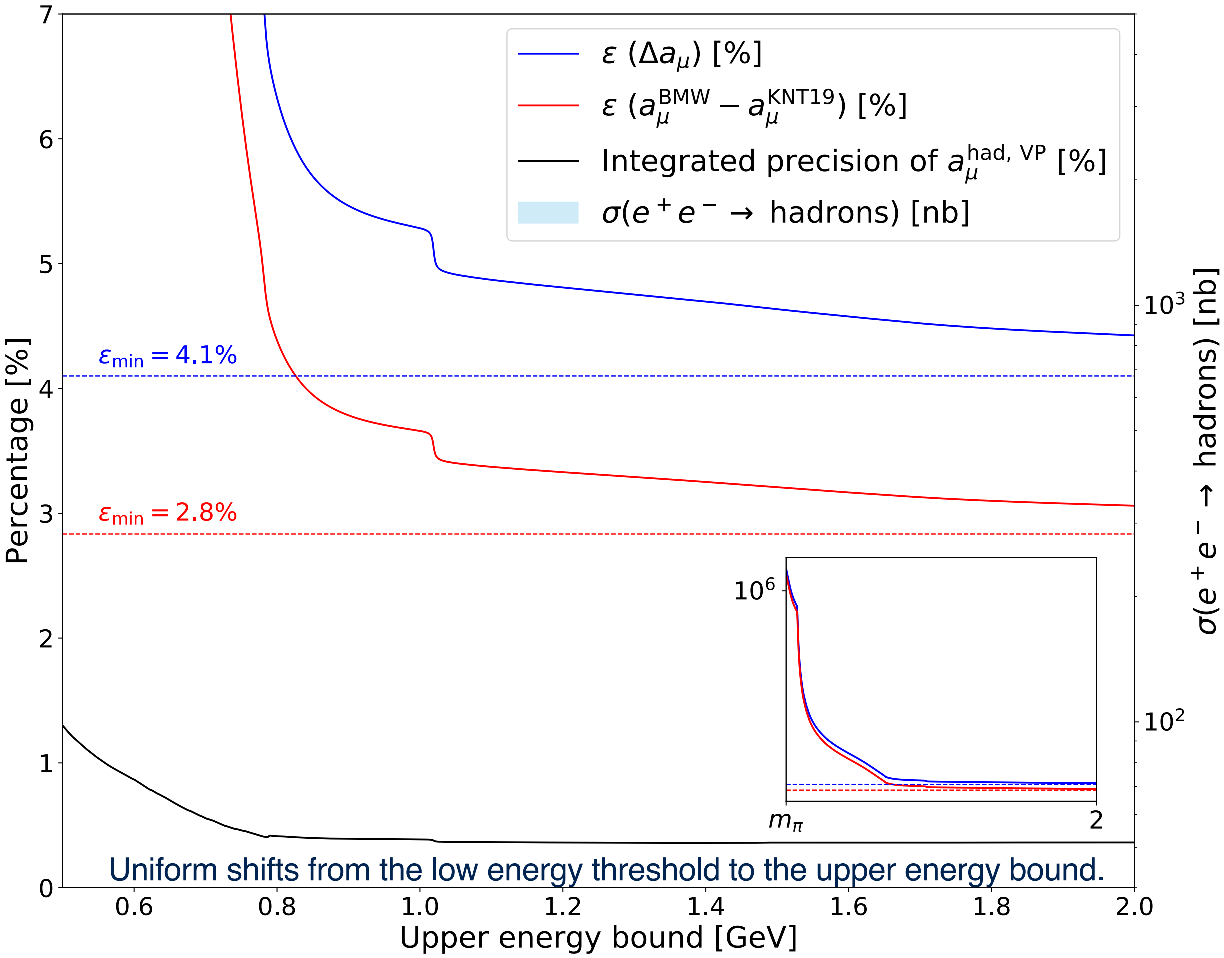
The muon g-2: connection with the SM Higgs mass (2020)



Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible, but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

How large are the required shifts $\Delta\sigma(s)$?



Shifts below 1 GeV conflict with the quoted exp. precision of $\sigma(s)$

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

e

- Using $\alpha = 1/137.036\,999\,046\,(27)$ [Cs 2018], the SM prediction for the electron g-2 is:

$$a_e^{SM} = 115\,965\,218\,16.2\,(0.1)\,(0.1)\,(2.3) \times 10^{-13}$$

δC_5^{qed} δa_e^{had} from $\delta\alpha$

- The (EXP — SM) difference is:

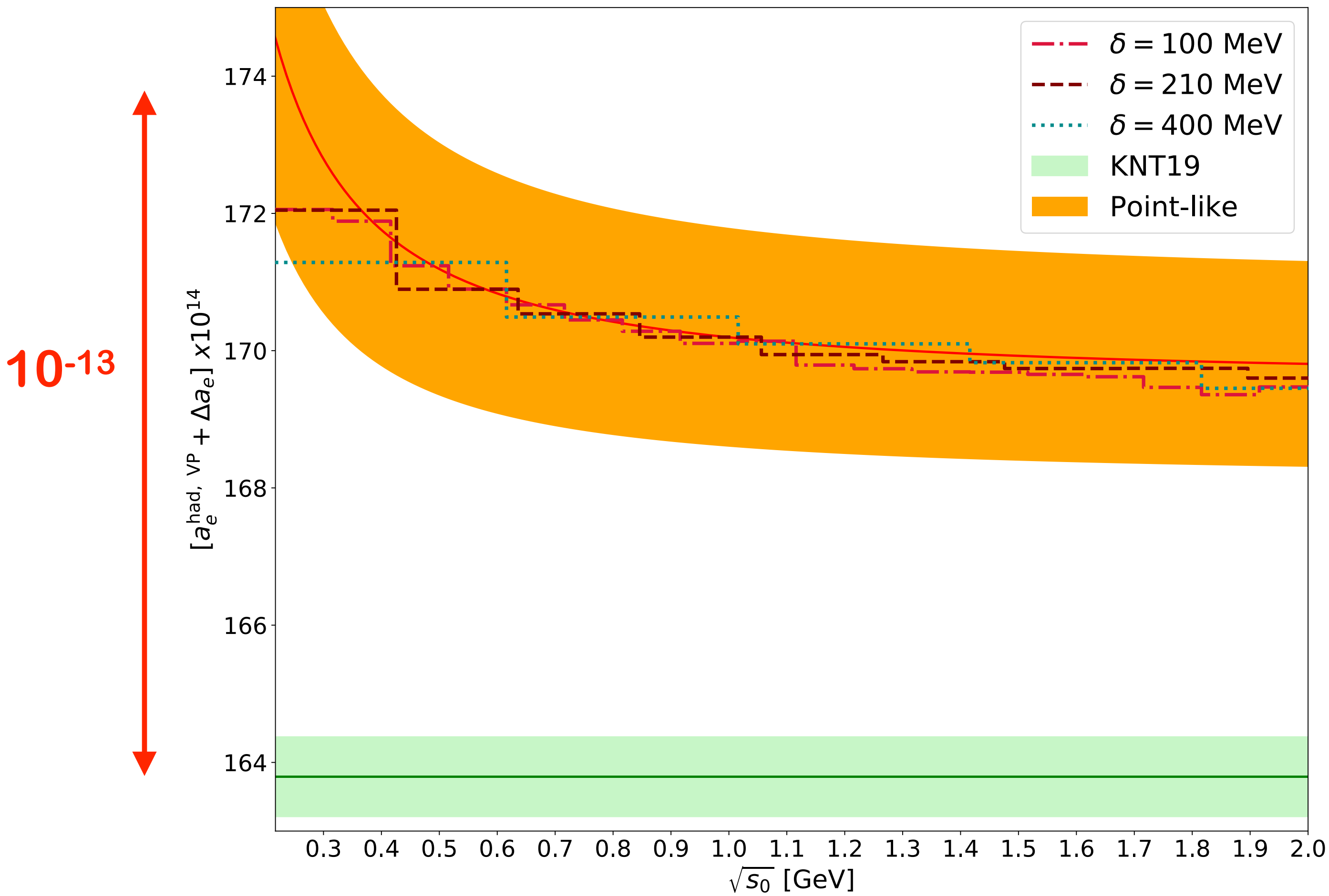
NB: negative!

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -\,8.9\,(3.6) \times 10^{-13}\ [2.5\sigma]$$

The hadronic VP contrib. is $16.66(6) \times 10^{-13}$ (up to NNLO) [QED 5-loop $a_e^{QED5} = 4.6 \times 10^{-13}$, $(m_e/m_\mu)^2\,\Delta a_\mu = 0.7 \times 10^{-13}$]

- NP sensitivity limited only by the experimental errors in α and a_e. May soon play a pivotal role in probing NP in the leptonic sector Giudice, Paradisi, MP 2012

Shift of the electron g-2

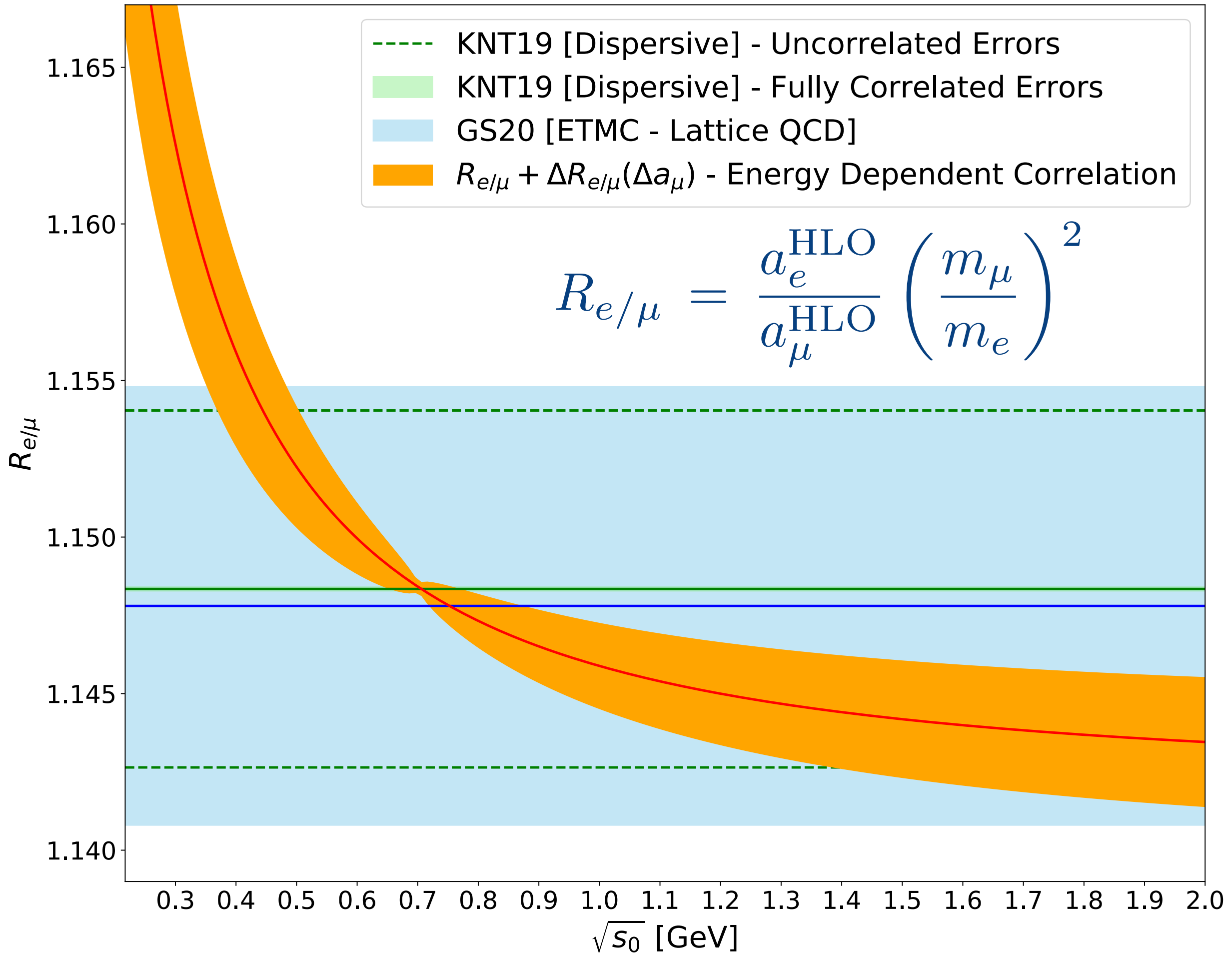


Shifts $\Delta\sigma(s)$ to fix Δa_μ only slightly increase the $|\Delta a_e| \sim 10^{-12}$ tension

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Shift of the e/μ g-2 scaled HLO ratio

e/μ



Good agreement between lattice [Giusti & Simula 2020] and KNT19.
Possible future bounds on very low energy shifts Δσ(s)?

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Recent related literature

- Crivellin, Hoferichter, Manzari and Montull, “Hadronic vacuum polarization: $(g\text{-}2)_\mu$ versus global electroweak fits,'' PRL125 (2020) 9, 091801 [arXiv:2003.04886].

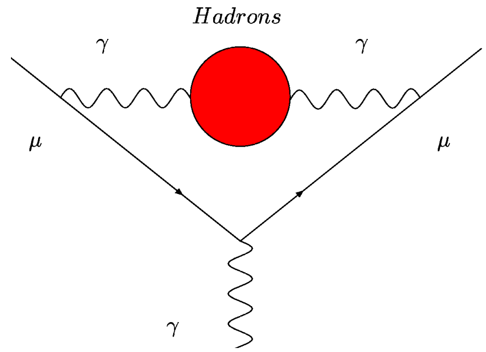
- Eduardo de Rafael, “On Constraints Between $\Delta\alpha_{had}(M_Z^2)$ and $(g_\mu\text{-}2)_{HVP}$,’’ arXiv:2006.13880.

- Malaescu and Schott, “Impact of correlations between a_μ and α_{QED} on the EW fit”, arXiv:2008.08107.

The MUonE project

Spacelike proposal for a_μ^{HLO}

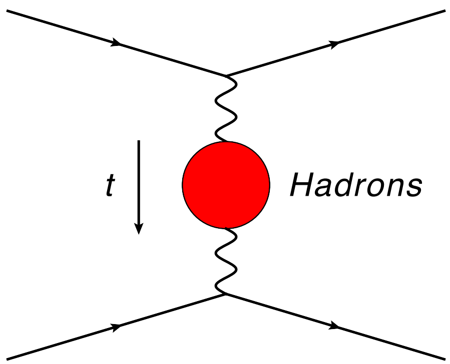
- **The leading hadronic contribution a_μ^{HLO} computed via the timelike formula:**



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\rm had}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

- **Alternatively, simply exchanging the x and s integrations:**



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

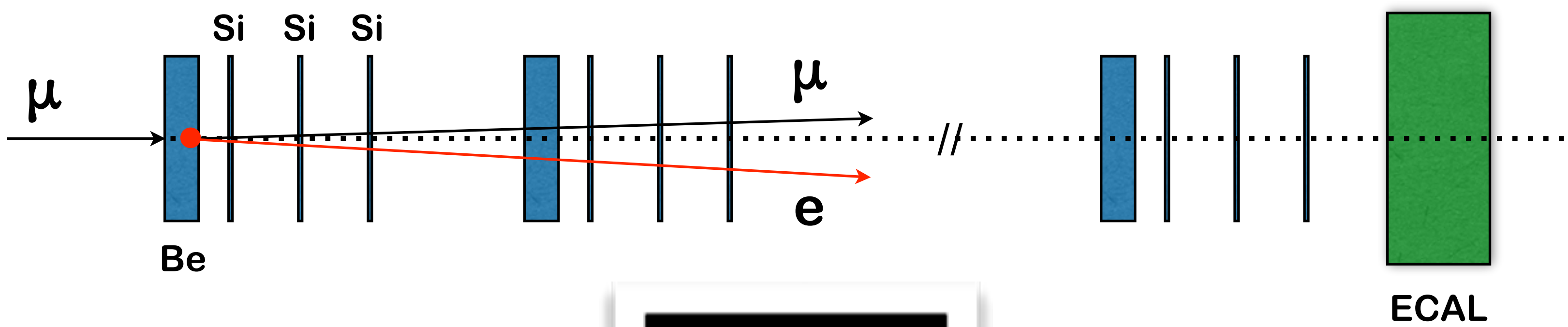
$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.

- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,
Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

- With 150 GeV muons, the high energy region inaccessible to MUonE contributes only 13% of the total a_μ^{HLO} integral. Recently it has been determined via lattice QCD Giusti&Simula and Marinkovic´&Cardoso 2019
- Statistics: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total thickness 60cm), 2 years of data taking (2×10^7 s/yr) → $\mathscr{L}_{int} \sim 1.5 \times 10^7\, nb^{-1}$.
- With this $\mathscr{L}_{int}$ we estimate that measuring the shape of $d\sigma/dt$ we can reach a statistical sensitivity of ~0.3% on a_μ^{HLO}, ie $\sim 20 \times 10^{-11}$.
- Systematic effects must be known at ≲ 10ppm!
- Theory: To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at ≲ 10ppm!
- Interplay and complementarity with lattice determination of a_μ^{HLO}
- LoI submitted to CERN SPSC in 2019. Test run in 2021 recently approved. Full-statistics run hopefully in 2022–24.

Conclusions

- **Is Δa_μ due to missed contributions in the hadronic $\sigma(s)$?**

  Shifts $\Delta\sigma(s)$ to fix Δa_μ conflict with the global EW fit above ~1 GeV

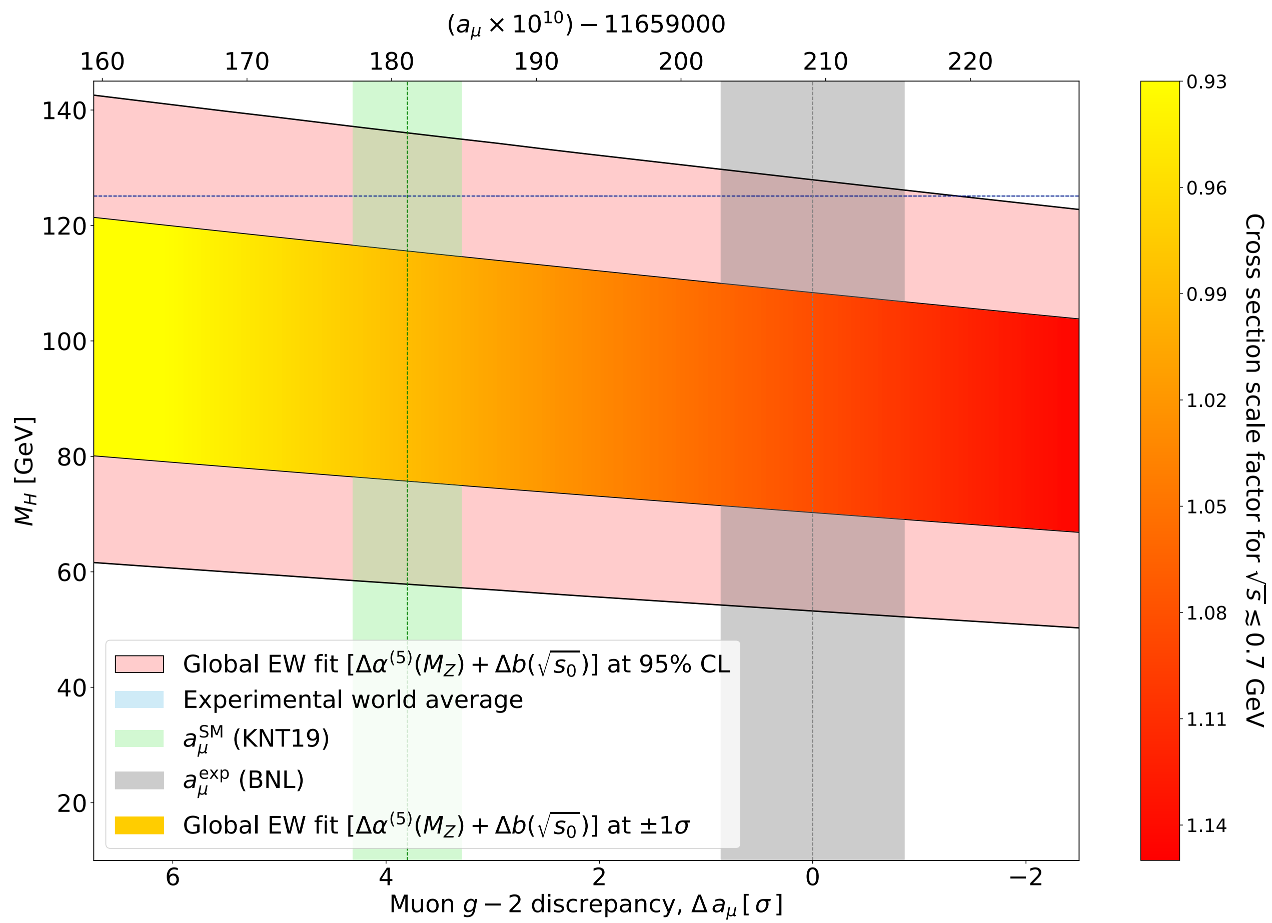
  Shifts below ~1 GeV conflict with the quoted exp. error of $\sigma(s)$.

- **Shifts $\Delta\sigma(s)$ to fix Δa_μ slightly increase the a_e tension ($R_{e/\mu}$ ok).**

- **MUonE will provide an independent (spacelike) determination of a_μ^{HLO} alternative to the dispersive and lattice ones.**

Backup

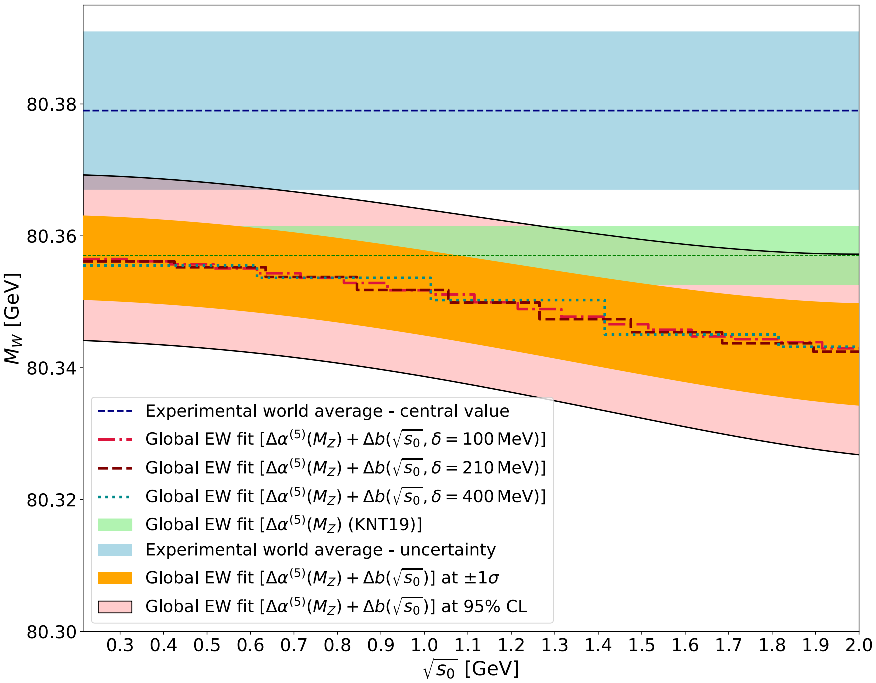
The muon g-2: connection with the SM Higgs mass (2020) - 2

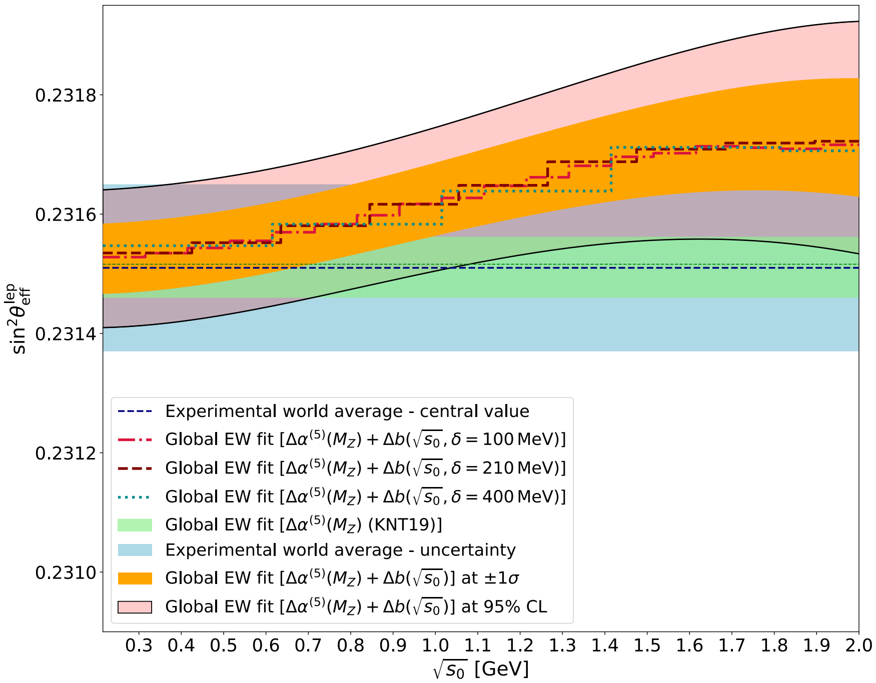


Uniform scaling of σ(s) below 0.7 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

The muon g-2: connection with M_W and $\sin^2\theta$





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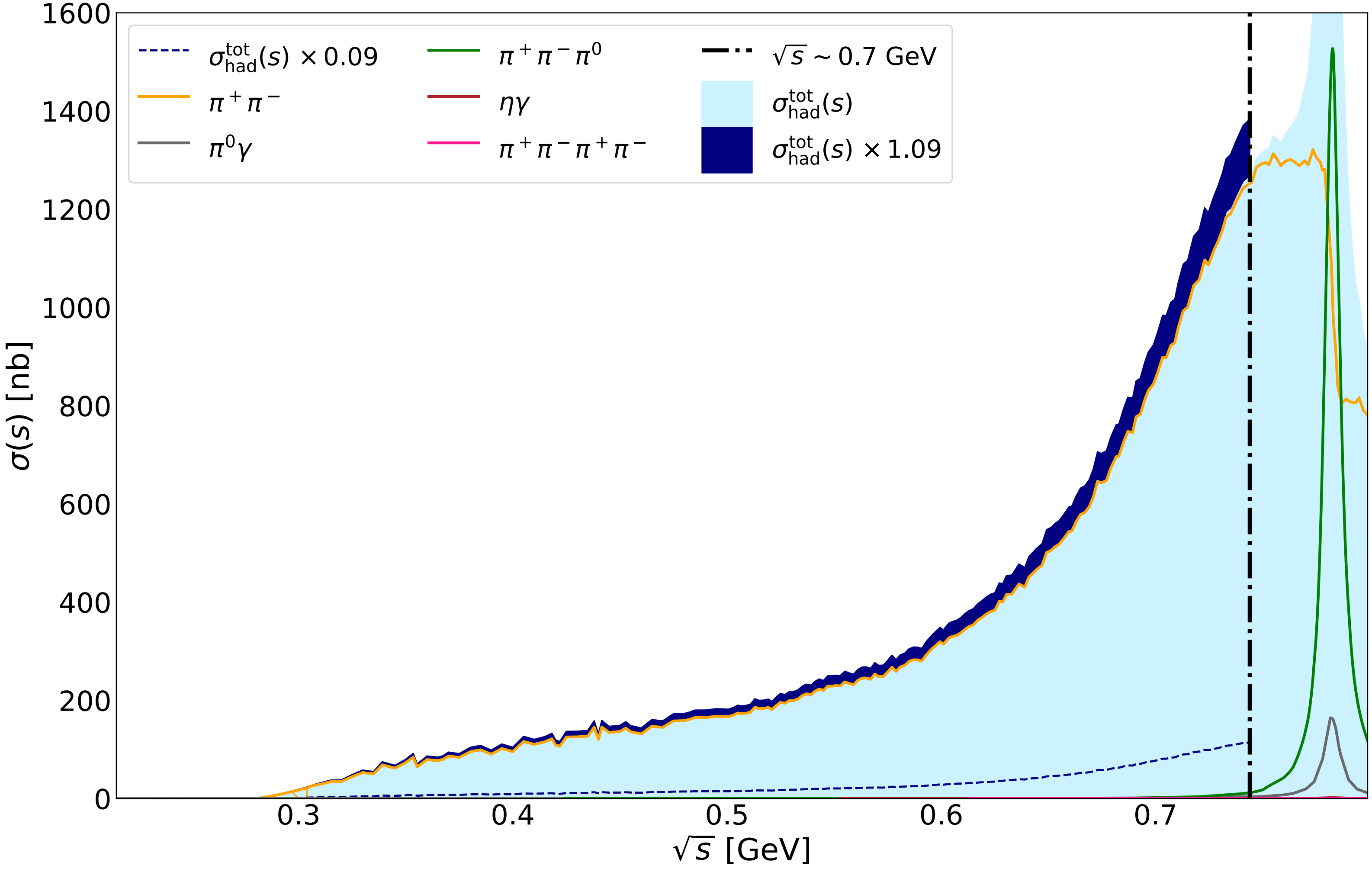

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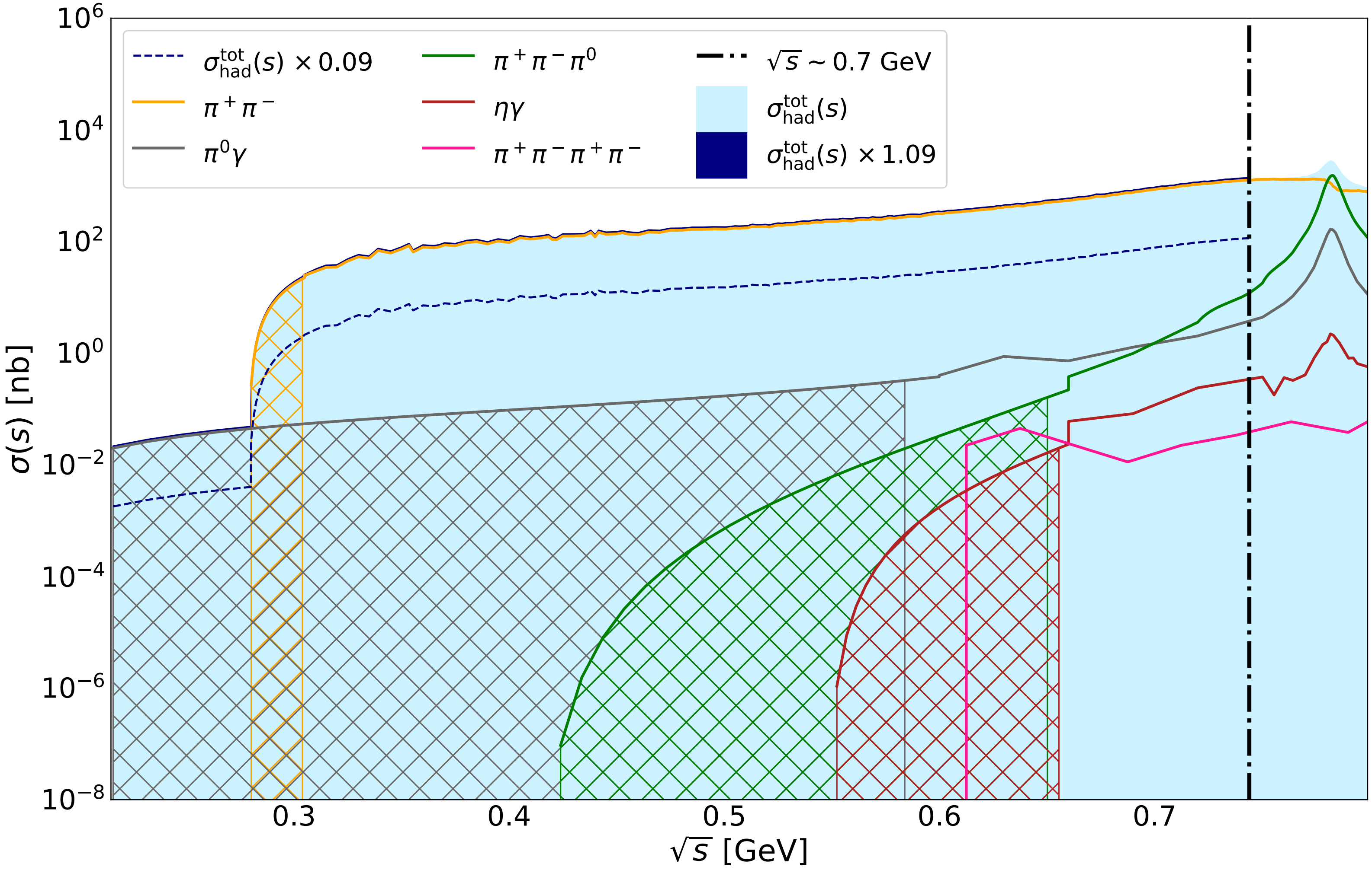

Keshavarzi, Marciano, MP, Sirlin, PRD 2020