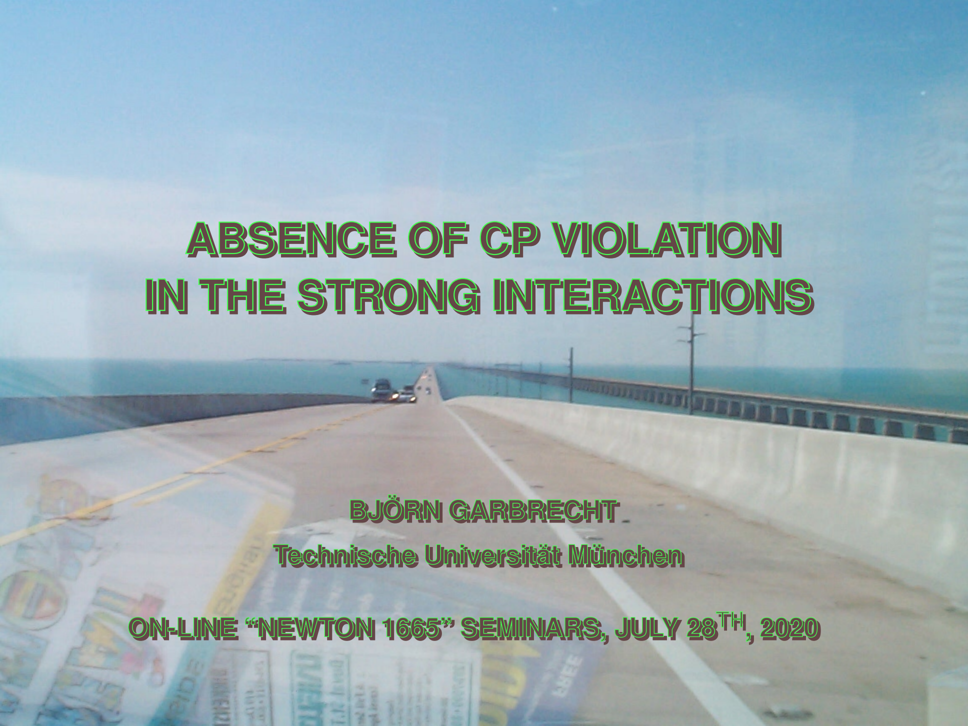

ABSENCE OF CP VIOLATION IN THE STRONG INTERACTIONS

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Outline

- CP-odd Lagrangians, correlations and effective operators
- Green's functions for fermions
- Interferences within the topological sectors
- Interferences among different topological sectors (are immaterial)

CP-odd Lagrangians, correlations and effective operators

CP-odd Lagrangian terms:

$$\mathcal{L} \supset -\sum_{j=1}^{N_f} \bar{\psi}_j m_j \mathrm{e}^{\mathrm{i}\alpha_j \gamma^5} \psi_j + \frac{1}{16\pi^2} \theta \, \mathrm{tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Rephasing invariant: $\bar{\theta} = \theta + \bar{\alpha}$, where $\bar{\alpha} = \sum_{j=1}^{N_f} \alpha_j$ [Fujikawa (1979,80)]

Instanton effects described by effective 't Hooft vertex: ['t Hooft (1976,86)]:

$$\mathcal{L} \to \mathcal{L} - \Gamma_{N_f} \mathrm{e}^{\mathrm{i}\xi} \prod_{j=1}^{N_f} (\bar{\psi}_j P_{\mathrm{L}} \psi_j) - \Gamma_{N_f} \mathrm{e}^{-\mathrm{i}\xi} \prod_{j=1}^{N_f} (\bar{\psi}_j P_{\mathrm{R}} \psi_j)$$

(Γ_{N_f} some coefficient)

$\xi = \theta$ (in general misaligned with masses) $\rightarrow$ CP violation
$\xi = -\bar{\alpha}$ (present claim, aligned with mass terms) $\rightarrow$ no CP violation

Note: Both comply with $\bar{\theta}$ being the only rephasing invariant CP phase

The effective vertex is chosen such that it generates the following **correlation functions** at tree level:

$$\langle\prod_{j=1}^{N_f}\psi_j(x_j)\bar{\psi}_j(x'_j)\rangle_{\rm conn} = \left(\mathrm{e}^{-\mathrm{i}\xi}\prod_{j=1}^{N_f}P_{\mathrm{L}j} + \mathrm{e}^{\mathrm{i}\xi}\prod_{j=1}^{N_f}P_{\mathrm{R}j}\right)\bar{H}(x_1,\ldots,x'_1,\ldots)$$

Cf. leading contribution to two-point function

$$\begin{aligned}\langle\psi_i(x)\psi_j(x')\rangle =& \mathrm{i}S_{0\mathrm{inst}\,ij}(x,x')\\ \mathrm{i}S_{0\mathrm{inst}\,ij}(x,x') =& (-\gamma^\mu\partial_\mu + \mathrm{i}m_i\mathrm{e}^{-\mathrm{i}\alpha_i\gamma^5})\int\frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}p(x-x')}\frac{\delta_{ij}}{p^2-m_i^2+\mathrm{i}\epsilon}\end{aligned}$$

Again: $\xi=\theta/\xi=-\bar{\alpha}$ implies CP-violation/no CP-violation

Take $N_f=1$ from here onwards

Green's functions for fermions

Euclidean Green's function $S^{\mathrm{E}}(x^{\mathrm{E}}, x^{\mathrm{E}\prime})$ satisfies

$$(\not{D}^{\mathrm{E}} + m_{\mathrm{R}} + \mathrm{i}\gamma^5 m_{\mathrm{I}}) S^{\mathrm{E}}(x^{\mathrm{E}}, x^{\mathrm{E}\prime}) = \delta^4(x^{\mathrm{E}} - x^{\mathrm{E}\prime})$$

Spectral sum (first massless case):

$$\not{D}^{\mathrm{E}} \hat{\psi}^{\mathrm{E}}_{\lambda} = \left(\not{\partial}^{\mathrm{E}} + \gamma^{\mathrm{E}}_m A^{\mathrm{E}}_m\right) \hat{\psi}^{\mathrm{E}}_{\lambda} = \lambda^{\mathrm{E}} \hat{\psi}^{\mathrm{E}}_{\lambda}$$

$$\longrightarrow \quad S^{\mathrm{E}}(x^{\mathrm{E}}, x^{\mathrm{E}\prime}) = \sum\!\!\!\!\!\!\!\int_{\lambda^{\mathrm{E}}} \frac{\hat{\psi}^{\mathrm{E}}_{\lambda}(x^{\mathrm{E}}) \hat{\psi}^{\mathrm{E}\dagger}_{\lambda}(x^{\mathrm{E}\prime})}{\lambda^{\mathrm{E}}}$$

Spectral sum for $m = 0$ is ill-defined because of the fermionic zero mode $\lambda^{\rm E} = 0$ in the instanton background

Winding number: $\Delta n = \frac{1}{16\pi^2} \int {\rm d}^4 x F_{\mu\nu} \tilde{F}_{\mu\nu}$

Euclidean index theorem: Δn equals difference between number of right-handed and left-handed zero modes

$\rightarrow$ One left (right)-handed zero-mode for $\Delta n = -1$ ($\Delta n = 1$)

Left-handed zero mode ['t Hooft (1976)]

$$\hat{\psi}^{\rm E}_{0{\rm L}}(x^{\rm E}) = \left(\begin{array}{c} \chi^{\rm E}_0(x^{\rm E}) \\ \left(\begin{array}{c} 0 \\ 0 \end{array} \right) \end{array} \right), \quad \text{where} \quad \chi^{\rm E}_0(x^{\rm E}) = \frac{\varrho u}{\pi \left[\varrho^2 + (x^{\rm E})^2\right]^{\frac{3}{2}}}, \quad u^{\alpha b} = \varepsilon^{\alpha b}$$

Include mass @ first order in perturbation theory ($\Delta n = -1$ background) [Shifman, Vainshtein, Zakharov (1979)]

$$S^{\rm E}(x^{\rm E}, x^{\rm E\prime}) = \frac{\hat{\psi}^{\rm E}_0(x^{\rm E}) \hat{\psi}^{\rm E\dagger}_0(x^{\rm E\prime})}{m {\rm e}^{-{\rm i}\alpha}} + \sum\!\!\!\!\!\!\!\int_{\lambda^{\rm E} \neq 0} \frac{\hat{\psi}^{\rm E}_\lambda(x^{\rm E}) \hat{\psi}^{\rm E\dagger}_\lambda(x^{\rm E\prime})}{\lambda^{\rm E}}$$

Green's function in n-instanton, $\bar{n}$-anti-instanton background

$$\mathrm{i}S_{n,\bar{n}}(x,x') \approx \mathrm{i}S_{0\mathrm{inst}}(x,x') + \sum_{\bar{\nu}=1}^{\bar{n}} \frac{\varphi_{0\mathrm{L}}(x-x_{0,\bar{\nu}})\varphi_{0\mathrm{L}}^\dagger(x'-x_{0,\bar{\nu}})}{m\mathrm{e}^{-\mathrm{i}\alpha}} + \sum_{\nu=1}^{n} \frac{\varphi_{0\mathrm{R}}(x-x_{0,\nu})\varphi_{0\mathrm{R}}^\dagger(x'-x_{0,\nu})}{m\mathrm{e}^{\mathrm{i}\alpha}}$$

Comments:

- For small masses, zero-modes dominate close to the cores of the instantons, far away from the instantons the solution goes to the zero-instanton configuration [Diakonov, Petrov (1986)]
- Analytic continuation from Euclidean to Minkowski space verifies chiral block-structure [Ai, Cruz, BG, Tamarit (2020)]
- **Alignment** of phase α between Lagrangian mass and instanton-induced χSB $\longrightarrow$ No indication of CP-violation here
- Perhaps expected—θ-phase has not entered calculation thus far

Interferences within the topological sectors

Within a topological sector, interfere/sum/integrate over

- all instanton/anti-instanton numbers $n + \bar{n}$ with $\Delta n = n - \bar{n}$ fixed
- locations of all instantons/anti-instantons
- remaining collective coordinates

Choose θ-vacuum in Minkowski spacetime as $|\mathrm{vac}\rangle = \sum_{n_{\mathrm{CS}}} |n_{\mathrm{CS}}\rangle$

Absorb CP-odd phase in topological term/fermion mass

Evaluate correlation and partition function first for **fixed** Δn

$$
\begin{aligned}
&\langle \psi(x)\bar\psi(x')\rangle_{\Delta n} \\
=&\sum_m {}_{\mathrm{out}}\langle m+\Delta n|\psi(x)\bar\psi(x')|m\rangle_{\mathrm{in}} = \sum_{\substack{\bar n, n\geq 0\\ n-\bar n=\Delta n}} \int \mathcal{D}A_{\bar n,n}\mathcal{D}\bar\psi\mathcal{D}\psi\, \psi(x)\bar\psi(x')\mathrm{e}^{\mathrm{i}S_{\bar n,n}} \\
=& \sum_{\substack{\bar n, n\geq 0\\ n-\bar n=\Delta n}} \frac{1}{\bar n! n!}\left(\prod_{\bar\nu=1}^{\bar n}\int\limits_{VT} \mathrm{d}^4x_{0,\bar\nu}\mathrm{d}\Omega_{\bar\nu}J_{\bar\nu}\right)\left(\prod_{\nu=1}^{n}\int\limits_{VT} \mathrm{d}^4x_{0,\nu}\mathrm{d}\Omega_{\nu}J_{\nu}\right)\mathrm{i}S(x,x') \\
&\times\ \mathrm{e}^{-S_{\mathrm{E}}(\bar n+n)}\mathrm{e}^{-\mathrm{i}(\bar n-n)(\alpha+\theta)}(-\Theta\varpi)^{(\bar n+n)}
\end{aligned}
$$

$\mathrm{d}\Omega_\nu J_\nu$:	Zero modes & pertaining Jacobians
Θ,ϖ :	Reduced fermion & gauge/ghost determinants in instanton background
$\mathrm{i}S(x,x')$:	Fermion propagator in n instantons, $\bar n$ anti-instanton background
$S_{\bar n,n}$:	Action for saddle with n instantons, $\bar n$ anti-instantons
S_{E}:	Euclidean action for one (anti-)instanton

Likewise, partition function:

$$Z_{\Delta n} = \sum_m {}_{\mathrm{out}}\langle m + \Delta n | m \rangle_{\mathrm{in}} = \sum_{\substack{\bar{n}, n \geq 0 \\ n - \bar{n} = \Delta n}} \int \mathcal{D}A_{\bar{n},n} \mathcal{D}\bar{\psi} \mathcal{D}\psi \, \mathrm{e}^{\mathrm{i}S_{\bar{n},n}}$$

$$= \sum_{\substack{\bar{n}, n \geq 0 \\ n - \bar{n} = \Delta n}} \frac{1}{\bar{n}! n!} \left(-\textstyle\int \mathrm{d}\Omega \, J \, V T \, \Theta \, \varpi \, \mathrm{e}^{-S_{\mathrm{E}}} \right)^{(\bar{n}+n)} \mathrm{e}^{-\mathrm{i}(\bar{n}-n)(\alpha+\theta)}$$

Integrating over all locations $\longrightarrow$ Correlation function for fixed Δn:

$$
\begin{aligned}
&\langle\psi(x)\bar\psi(x')\rangle_{\Delta n}\\
&=\sum_{\substack{\bar n,n\geq 0\\ n-\bar n=\Delta n}}\frac{1}{\bar n! n!}\left[\bar h(x,x')\left(\frac{\bar n}{m\mathrm{e}^{-\mathrm{i}\alpha}}P_{\mathrm{L}}+\frac{n}{m\mathrm{e}^{\mathrm{i}\alpha}}P_{\mathrm{R}}\right)(VT)^{\bar n+n-1}+\mathrm{i}S_{0\mathrm{inst}}(x,x')\,(VT)^{\bar n+n}\right]\\
&\qquad\qquad\times(\mathrm{i}\kappa)^{\bar n+n}(-1)^{n+\bar n}\mathrm{e}^{\mathrm{i}\Delta n(\alpha+\theta)}\\
&=\left[\left(\mathrm{e}^{\mathrm{i}\alpha}I_{\Delta n+1}(2\mathrm{i}\kappa VT)P_{\mathrm{L}}+\mathrm{e}^{-\mathrm{i}\alpha}I_{\Delta n-1}(2\mathrm{i}\kappa VT)P_{\mathrm{R}}\right)\frac{\mathrm{i}\kappa}{m}\,\bar h(x,x')+I_{\Delta n}(2\mathrm{i}\kappa VT)\mathrm{i}S_{0\mathrm{inst}}(x,x')\right]\\
&\qquad\qquad\times(-1)^{\Delta n}\mathrm{e}^{\mathrm{i}\Delta n(\alpha+\theta)}
\end{aligned}
$$

where $\mathrm{i}\kappa=\int\mathrm{d}\Omega\,J\,\Theta\,\varpi\,\mathrm{e}^{-S_{\mathrm{E}}}$, $I_\nu(x)$ is the modified Bessel function and $\bar h$ the spacetime-averaged correlation

Sum is dominated by particular value of $n\approx\bar n$: [Diakonov, Petrov (1986)]

$$
\langle n\rangle=\frac{\sum_{n=0}^{\infty}n\frac{(\alpha VT)^n}{n!}}{\sum_{n=0}^{\infty}\frac{(\alpha VT)^n}{n!}}=\alpha VT\,,\qquad\frac{\sqrt{\langle(n-\langle n\rangle)^2\rangle}}{\langle n\rangle}=\frac{1}{\sqrt{\alpha VT}}
$$

Cf. $\lim_{x\to\infty}I_{\Delta n}(\mathrm{i}x\,\mathrm{e}^{-\mathrm{i}0^+})/I_{\Delta n'}(\mathrm{i}x\,\mathrm{e}^{-\mathrm{i}0^+})=1$

$\longrightarrow$ No relative CP phase between mass and instanton induced breaking of χral symmetry—**alignment** in infinite-volume limit

Correspondingly, partition function for fixed Δn: [cf. Leutwyler, Smilga (1992)]

$$Z_{\Delta n} = I_{\Delta n}(2\mathrm{i}\kappa VT)\,(-1)^{\Delta n}\mathrm{e}^{\mathrm{i}\Delta n(\alpha+\theta)}$$

Note: The topological phase $\mathrm{e}^{\mathrm{i}\Delta n(\alpha+\theta)}$ multiplies $\langle\psi(x)\bar{\psi}(x')\rangle_{\Delta n}$ and $Z_{\Delta n}$ entirely—not just the contributions induced by instantons.

Other correlation functions (n point, stress-energy,...) are calculated from the Feynman diagram with the Green's function in the n instanton, $\bar{n}$ anti-instanton background.
Then it remains to average over n, $\bar{n}$, locations and remaining collective coordinates.

Interferences among topological sectors (are immaterial)

Partition function in θ-vacuum (recall phase resides in topological term):

$$Z = {}_{\rm out}\langle {\rm vac}|{\rm vac}\rangle_{\rm in} = \sum_{m,n} {}_{\rm out}\langle m|n\rangle_{\rm in} = \sum_{\Delta n=-\infty}^{\infty} \sum_{m} {}_{\rm out}\langle m+\Delta n|m\rangle_{\rm in} = \sum_{\Delta n=-\infty}^{\infty} Z_{\Delta n}$$

Fermion correlator

$$\langle \psi(x)\bar\psi(x')\rangle \equiv \frac{1}{Z}\,{}_{\rm out}\langle {\rm vac}|\psi(x)\bar\psi(x')|{\rm vac}\rangle_{\rm in}$$

$$= \frac{\sum_{\Delta n=-\infty}^{\infty} \sum_{n} {}_{\rm out}\langle n+\Delta n|\psi(x)\bar\psi(x')|n\rangle_{\rm in}}{\sum_{\Delta n=-\infty}^{\infty} Z_{\Delta n}} = \lim_{\substack{N\to\infty \\ N\in\mathbb{N}}} \lim_{VT\to\infty} \frac{\sum_{\Delta n=-N}^{N} \langle \psi(x)\bar\psi(x')\rangle_{\Delta n}}{\sum_{\Delta n=-N}^{N} Z_{\Delta n}}$$

$$= {\rm i}S_{0{\rm inst}}(x,x') + {\rm i}\kappa \bar h(x,x') m^{-1} {\rm e}^{-{\rm i}\alpha\gamma^5} \quad \text{(same as for fixed } \Delta n)$$

Recall: ${\rm i}S_{0{\rm inst}}(x,x') = (-\gamma^\mu\partial_\mu + {\rm i}m{\rm e}^{-{\rm i}\alpha\gamma^5}) \int \frac{{\rm d}^4p}{(2\pi)^4} {\rm e}^{-{\rm i}p(x-x')} \frac{1}{p^2-m^2+{\rm i}\epsilon}$

$\longrightarrow$ No relative CP-phase between mass and instanton term

Limits ordered the other way around

First sum over all Δn as well:

$$\sum_{\bar{n},n\geq 0}\frac{1}{\bar{n}!n!}\Big[\bar{h}(x,x')(\bar{n}\,m^{-1}\mathrm{e}^{\mathrm{i}\alpha}P_{\mathrm{L}}+n\,m^{-1}\mathrm{e}^{-\mathrm{i}\alpha}P_{\mathrm{R}})\,(VT)^{\bar{n}+n-1}+\mathrm{i}S_{0\mathrm{inst}}(x,x')\,(VT)^{\bar{n}+n}\Big] \times(-mi\kappa)^{\bar{n}+n}\mathrm{e}^{\mathrm{i}\Delta n(\alpha+\theta)}$$

$$=\left[-\left(\mathrm{e}^{-\mathrm{i}\theta}P_{\mathrm{L}}+\mathrm{e}^{\mathrm{i}\theta}P_{\mathrm{R}}\right)\frac{\mathrm{i}\kappa}{m}\bar{h}(x,x')+\mathrm{i}S_{0\mathrm{inst}}(x,x')\right]\mathrm{e}^{-2\mathrm{i}\kappa VT\cos(\alpha+\theta)}$$

$$Z\to\sum_{n,\bar{n}}\frac{1}{n!\bar{n}!}(-\mathrm{i}\kappa VT)^{\bar{n}+n}\mathrm{e}^{-\mathrm{i}(\bar{n}-n)(\alpha+\theta)}=\mathrm{e}^{-2\mathrm{i}\kappa VT\cos(\alpha+\theta)}$$

Then, $VT\to\infty$ trivial as VT-dependence cancels
$\longrightarrow$ Relative CP phase leading to CP-violating observables

However: The order of the limits is not a choice but dictated by the fact that boundary conditions for the theta-vacuum are imposed for $V=\infty$ and at $t=\pm\infty$.

Finite vs infinite spacetime volume—cluster decomposition

Consider expectation value of an operator $\mathcal{O}$ in spacetime volume Ω, interfere different topological sectors Δn:

$$\langle \mathcal{O} \rangle_\Omega = \frac{\sum_{\Delta n=-\infty}^{\infty} f(\Delta n) \int_{\Delta n} \mathcal{D}\phi\, \mathcal{O}\, \mathrm{e}^{-S_\Omega[\phi]}}{\sum_{\Delta n=-\infty}^{\infty} f(\Delta n) \int_{\Delta n} \mathcal{D}\phi\, \mathrm{e}^{-S_\Omega[\phi]}}$$

Factorize path integral into volume contributions, $\Omega = \Omega_1 \cup \Omega_2$:
(Assume $\Delta n(\Omega) = \Delta n_1(\Omega_1) + \Delta n_2(\Omega_2)$)

$$\langle \mathcal{O}_1 \rangle_\Omega = \frac{\sum_{\Delta n_1=-\infty}^{\infty} \sum_{\Delta n_2=-\infty}^{\infty} f(\Delta n_1 + \Delta n_2) \int_{\Delta n_1} \mathcal{D}\phi\, \mathcal{O}_1\, \mathrm{e}^{-S_{\Omega_1}[\phi]} \int_{\Delta n_2} \mathcal{D}\phi\, \mathrm{e}^{-S_{\Omega_2}[\phi]}}{\sum_{\Delta n_1=-\infty}^{\infty} \sum_{\Delta n_2=-\infty}^{\infty} f(\Delta n_1 + \Delta n_2) \int_{\Delta n_1} \mathcal{D}\phi\, \mathrm{e}^{-S_{\Omega_1}[\phi]} \int_{\Delta n_2} \mathcal{D}\phi\, \mathrm{e}^{-S_{\Omega_2}[\phi]}}$$

Independence of $\langle \mathcal{O}_1 \rangle_\Omega$ from the fluctuations in Ω_2 is achieved if the contributions from Ω_2 cancel:

$$f(\Delta n_1 + \Delta n_2) = f(\Delta n_1) f(\Delta n_2) \Rightarrow f(\Delta n) = \mathrm{e}^{\mathrm{i}\Delta n(\alpha+\theta)}$$

Now keep Δn fixed:

$$\langle \mathcal{O}_1 \rangle_\Omega = \frac{\sum\limits_{\Delta n_1=-\infty}^{\infty} f(\Delta n) \int\limits_{\Delta n_1} \mathcal{D}\phi \, \mathcal{O}_1 \, \mathrm{e}^{-S_{\Omega_1}[\phi]} \int\limits_{\Delta n_2 = \Delta n - \Delta n_1} \mathcal{D}\phi \, \mathrm{e}^{-S_{\Omega_2}[\phi]}}{\sum\limits_{\Delta n_1=-\infty}^{\infty} f(\Delta n) \int\limits_{\Delta n_1} \mathcal{D}\phi \, \mathrm{e}^{-S_{\Omega_1}[\phi]} \int\limits_{\Delta n_2 = \Delta n - \Delta n_1} \mathcal{D}\phi \, \mathrm{e}^{-S_{\Omega_2}[\phi]}}$$

$$= \frac{\sum\limits_{\Delta n_1=-\infty}^{\infty} f(\Delta n) I_{\Delta n - \Delta n_1}(2\mathrm{i}\kappa\Omega_2) \int\limits_{\Delta n_1} \mathcal{D}\phi \, \mathcal{O}_1 \, \mathrm{e}^{-S_{\Omega_1}[\phi]}}{\sum\limits_{\Delta n_1=-\infty}^{\infty} f(\Delta n) I_{\Delta n - \Delta n_1}(2\mathrm{i}\kappa\Omega_2) \int\limits_{\Delta n_1} \mathcal{D}\phi \, \mathrm{e}^{-S_{\Omega_1}[\phi]}}$$

$$\underset{\Omega_2 \gg \Omega_1}{\approx} \frac{\sum\limits_{\Delta n_1=-\infty}^{\infty} \int\limits_{\Delta n_1} \mathcal{D}\phi \, \mathcal{O}_1 \, \mathrm{e}^{-S_{\Omega_1}[\phi]}}{\sum\limits_{\Delta n_1=-\infty}^{\infty} \int\limits_{\Delta n_1} \mathcal{D}\phi \, \mathrm{e}^{-S_{\Omega_1}[\phi]}}$$

The theory with fixed Δn (gauge invariant) in large but finite volumes complies with cluster decomposition [cf. Leutwyler, Smilga (1992)].

An observer made up of local quantum fields can *anyway* not observe interference between different Δn.

Conclusions

In infinite spacetime volume, the interferences between topological sectors that may lead to misaligned CP-odd phases between correlations from mass and instanton contributions disappear.

In finite volumes, the interference remains. However, it should not be observable within local QFT.

There is agreement between the theories in infinite volumes (with Δn either free or fixed) and in large, finite volumes for fixed Δn.

In a subvolume Ω_1, we can work with free Δn_1 when accounting for the topological conservation of the total Δn.