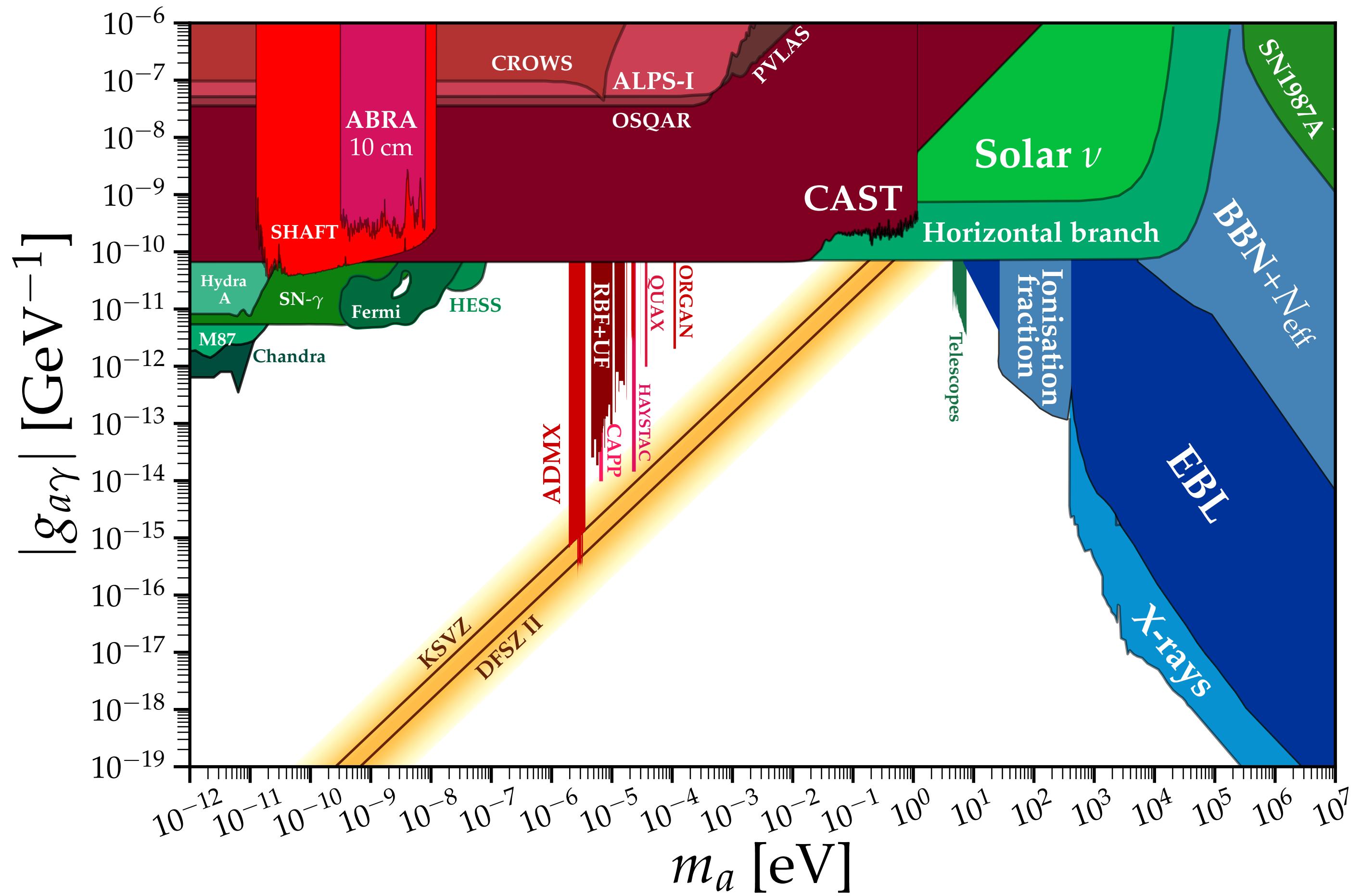


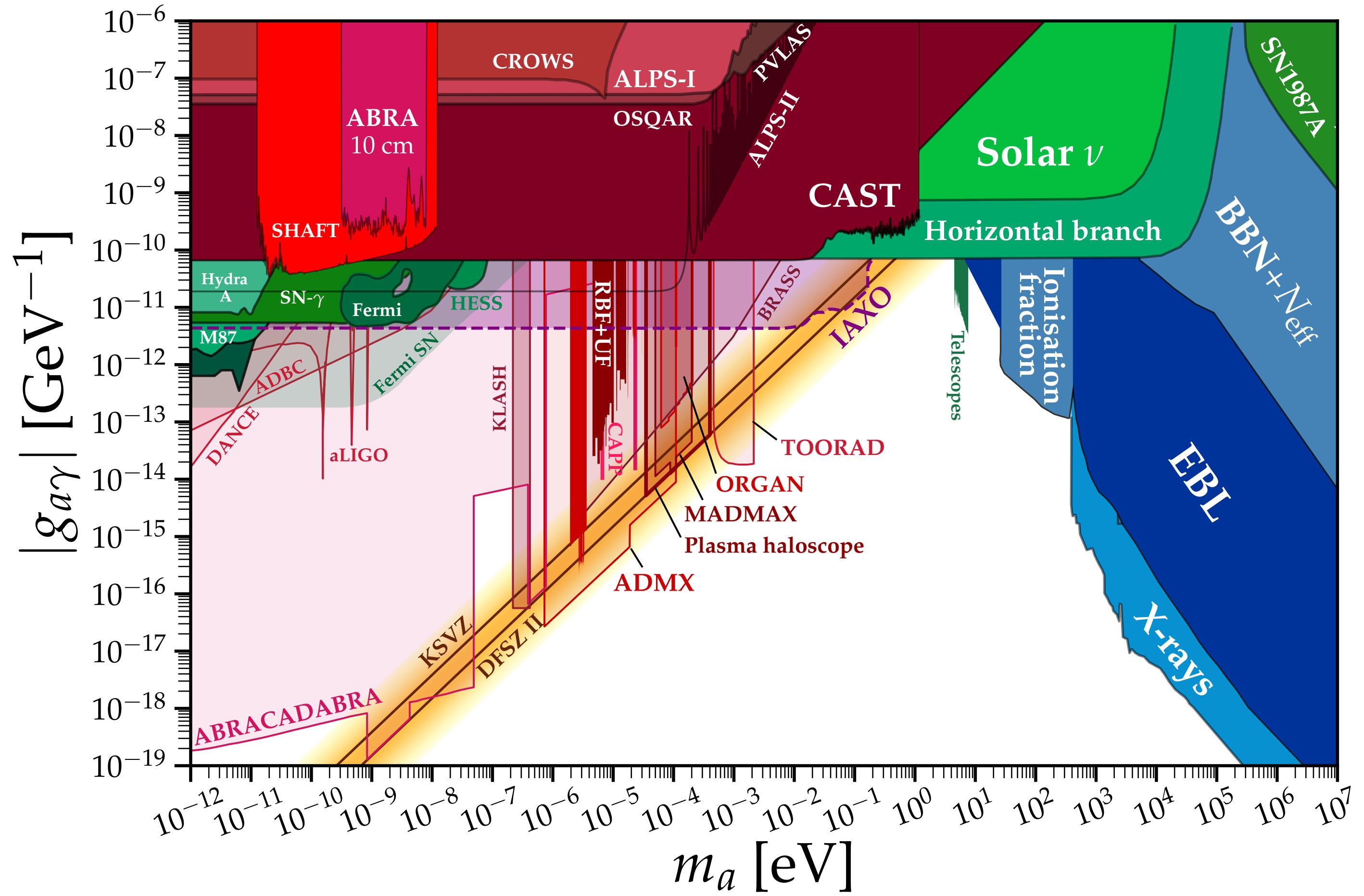


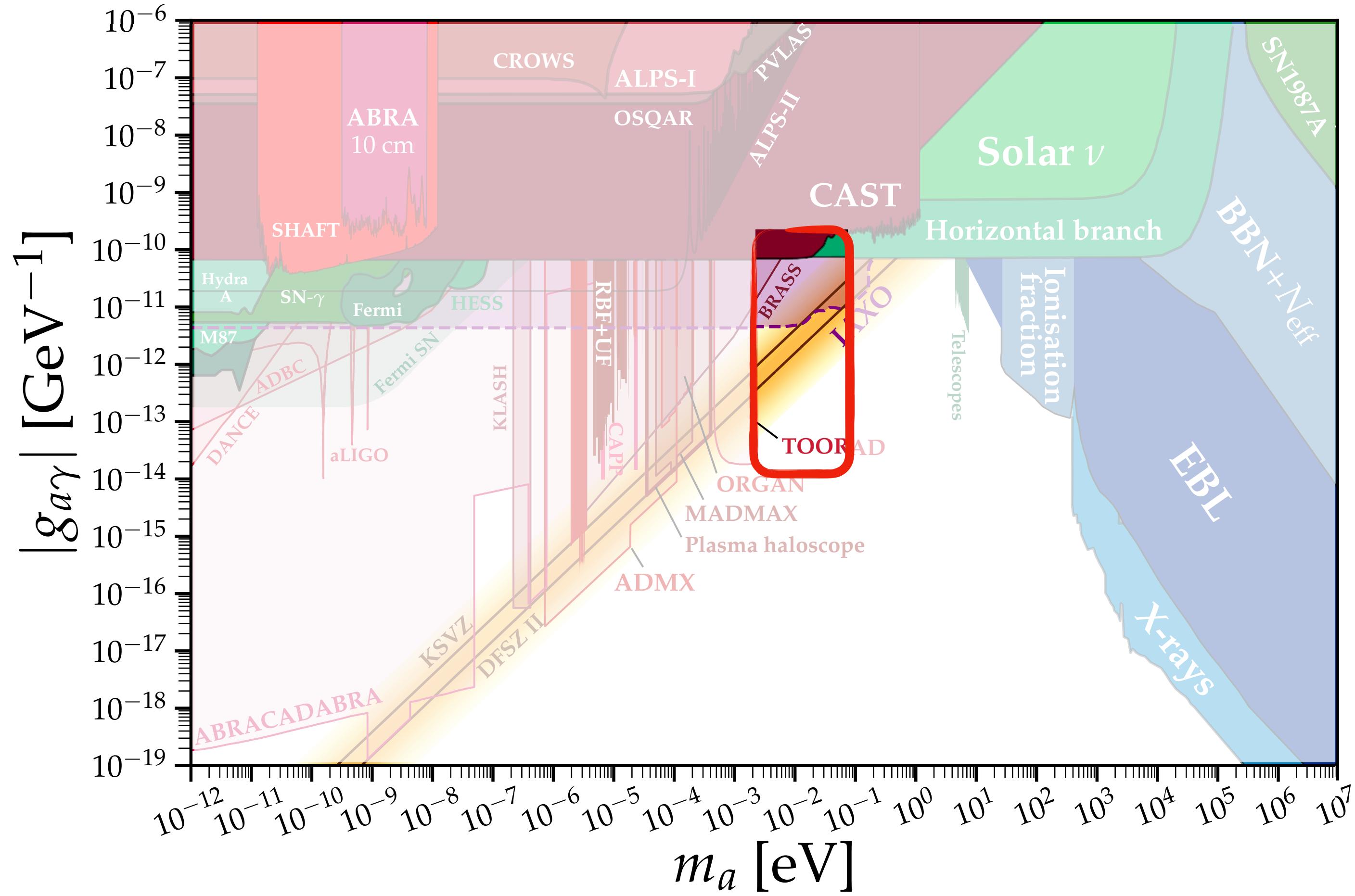
Axion detection with collective excitations

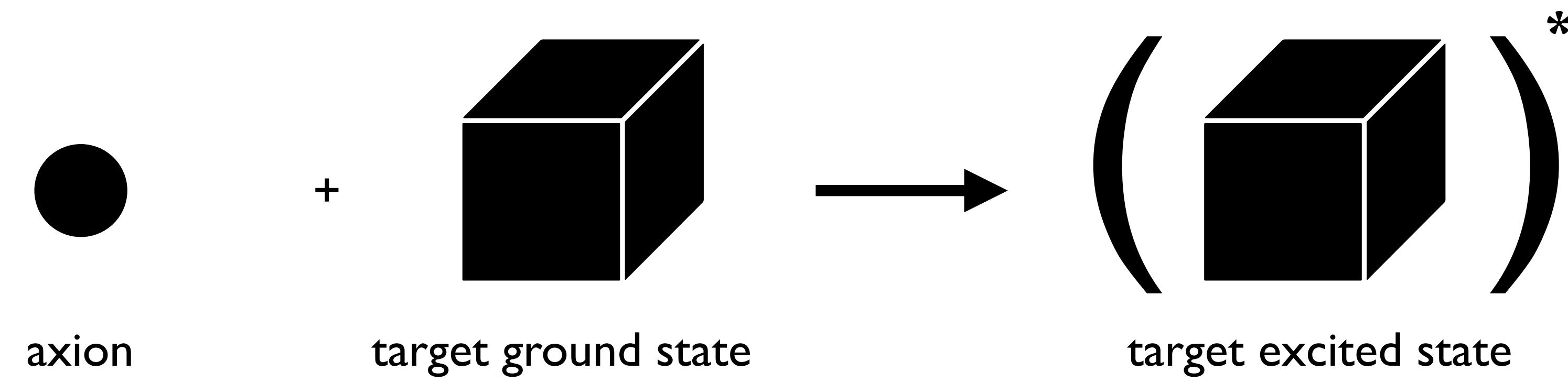
Andrea Mitridate

based on 2005.10256 with Tanner Trickle, Zhengkang Zhang and Kathryn M. Zurek





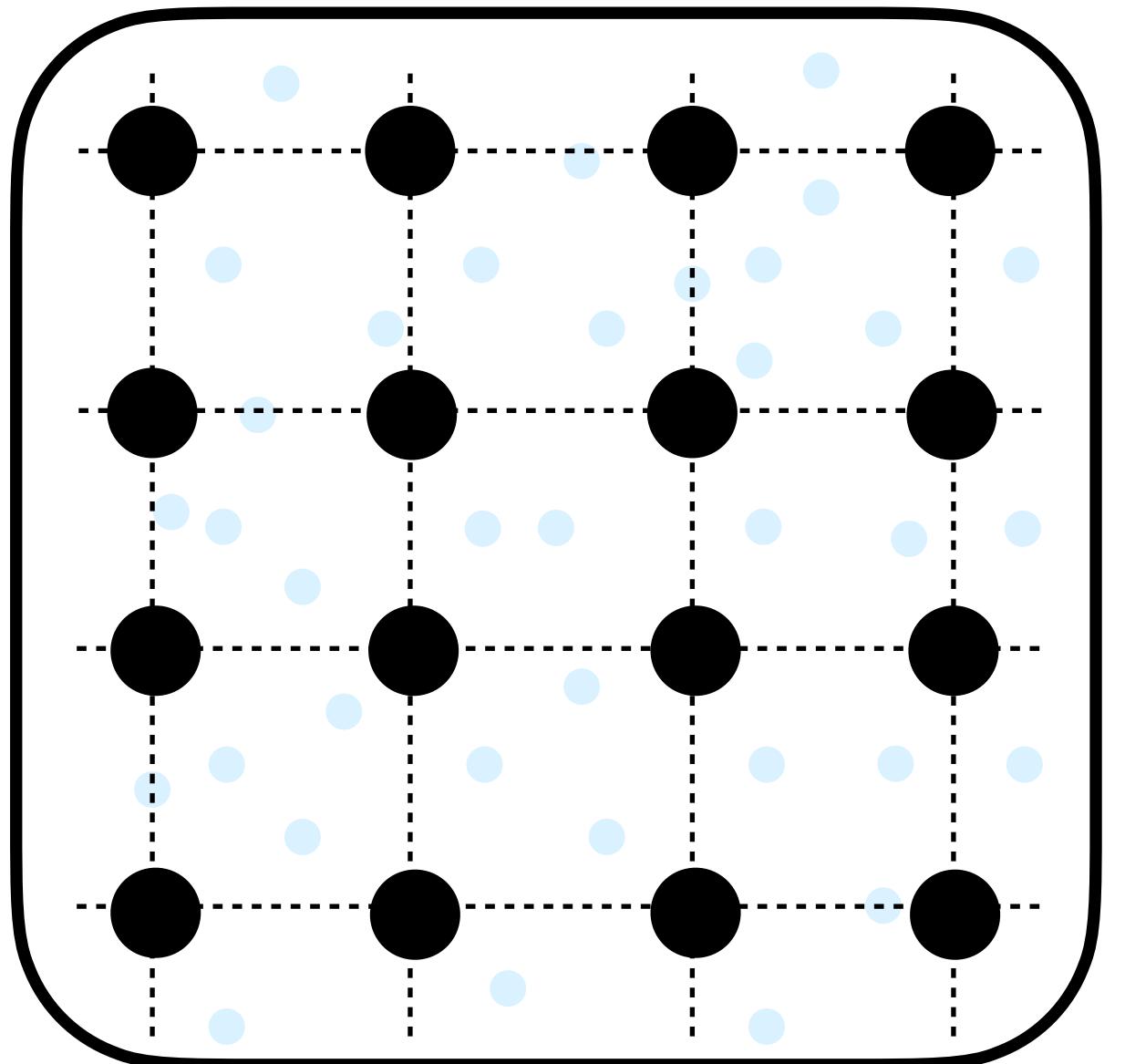
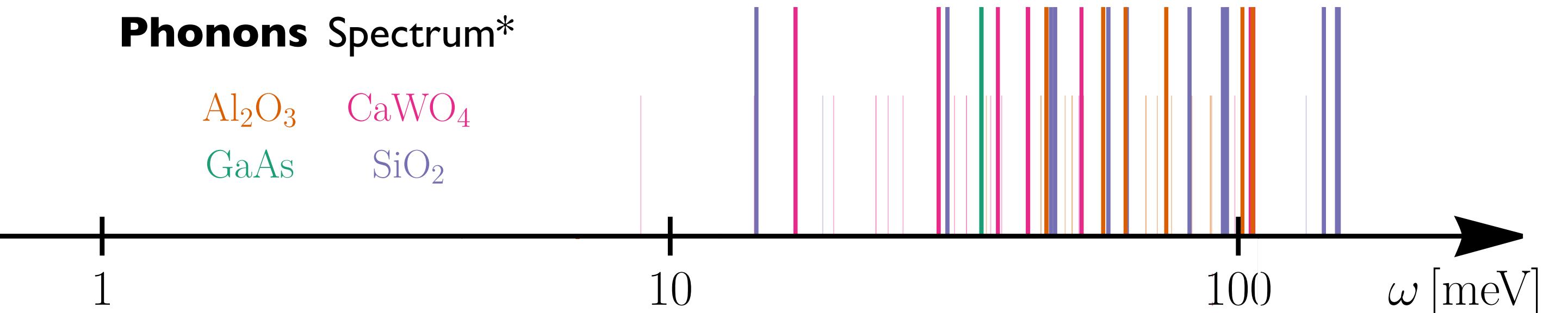
unconstrained window in the $(1 \div 10^2)$ meV range



absorption kinematics

deposited energy $\simeq m_a$

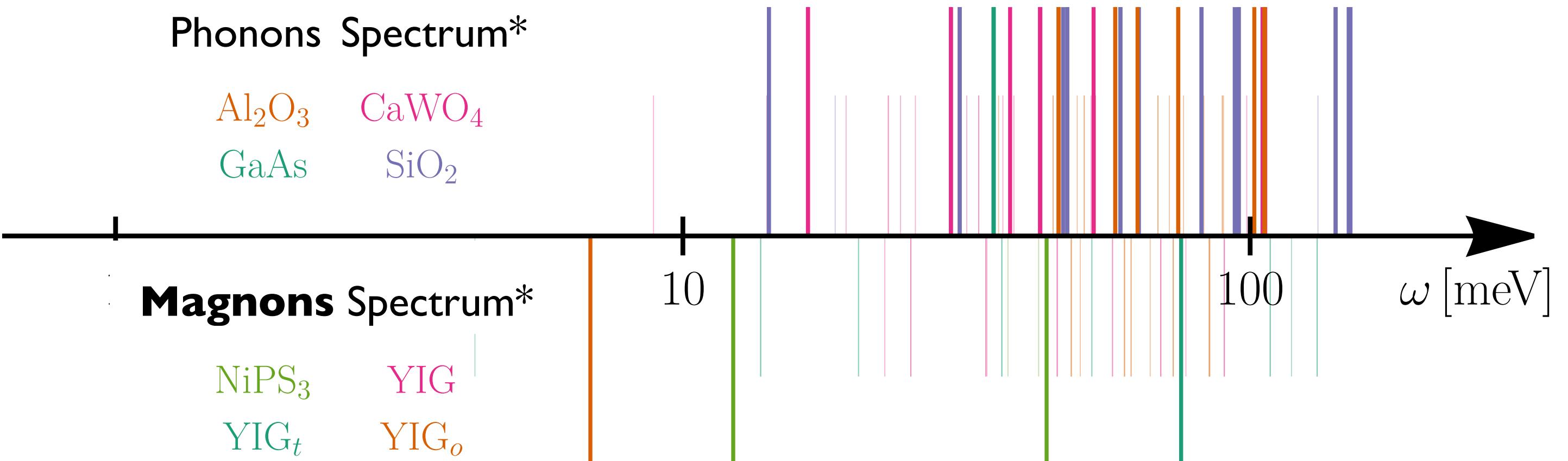
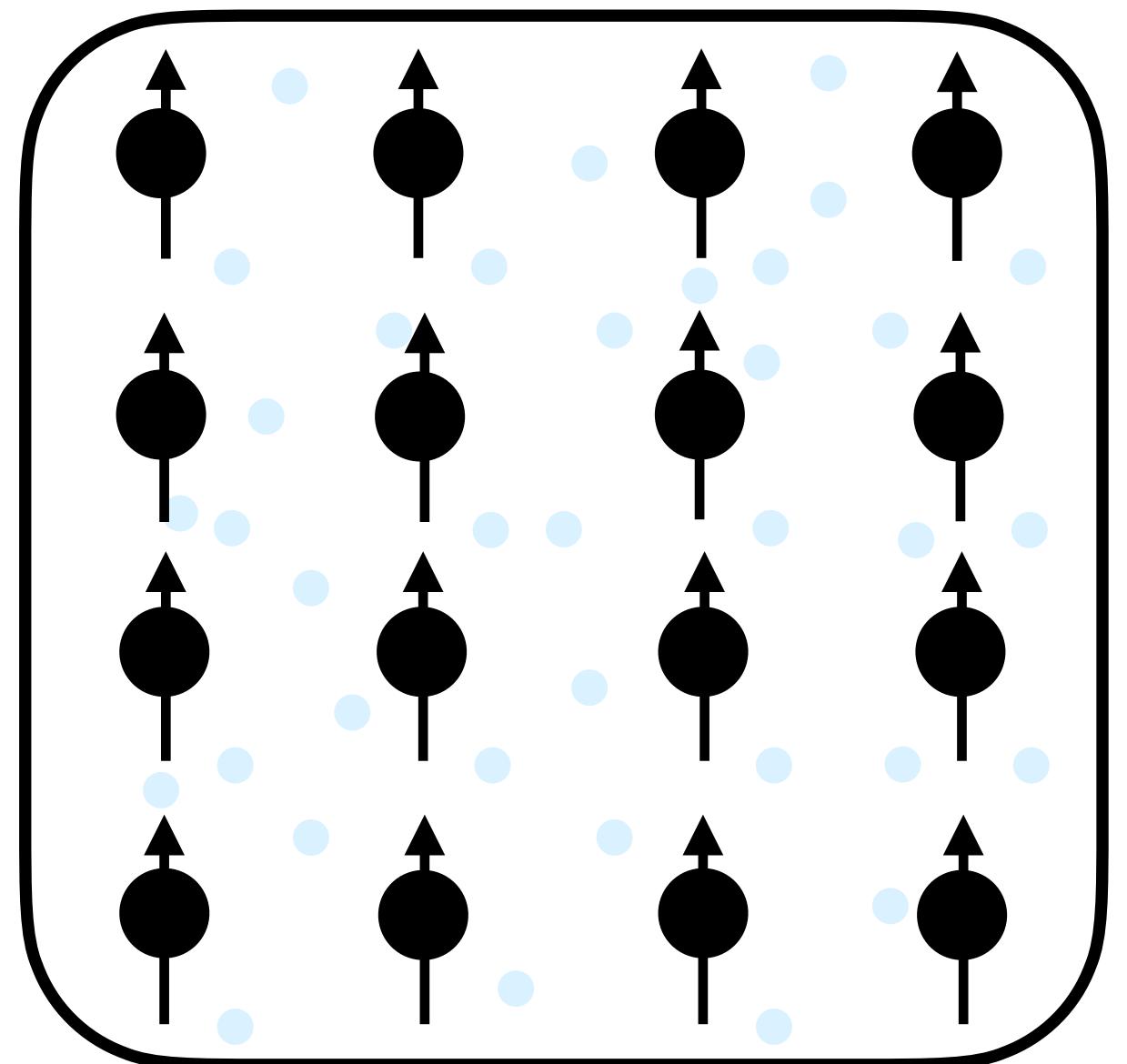
deposited momentum $\simeq 10^{-3}m_a$

 : ions : electrons**Phonons Spectrum*** Al_2O_3 GaAs CaWO_4 SiO_2 

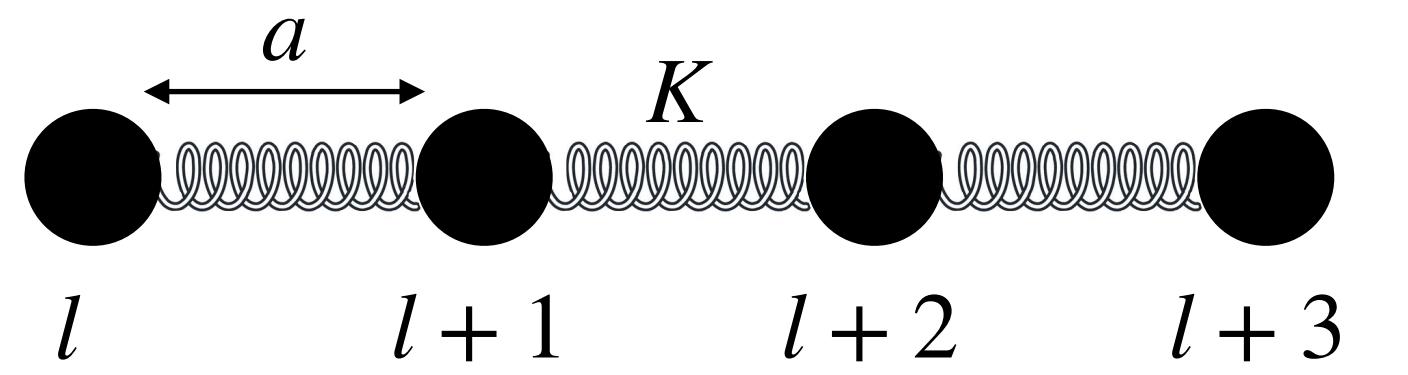
* at zero momentum

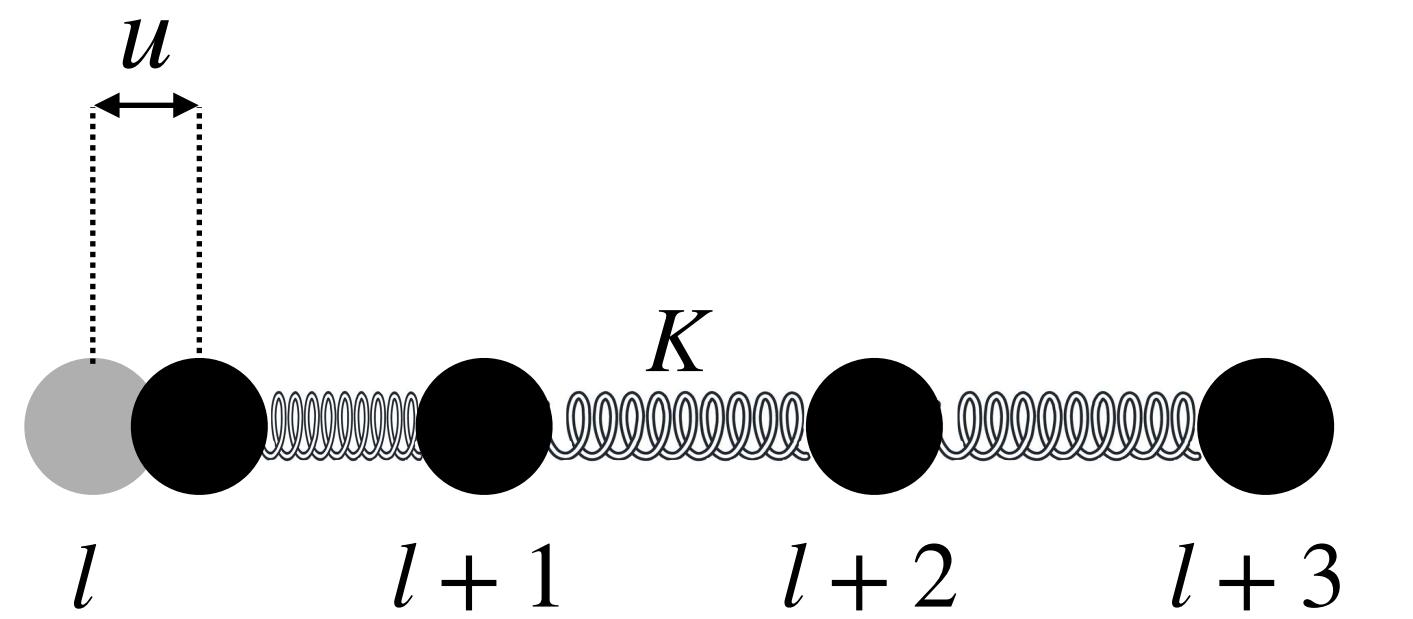
● : ions

● : electrons

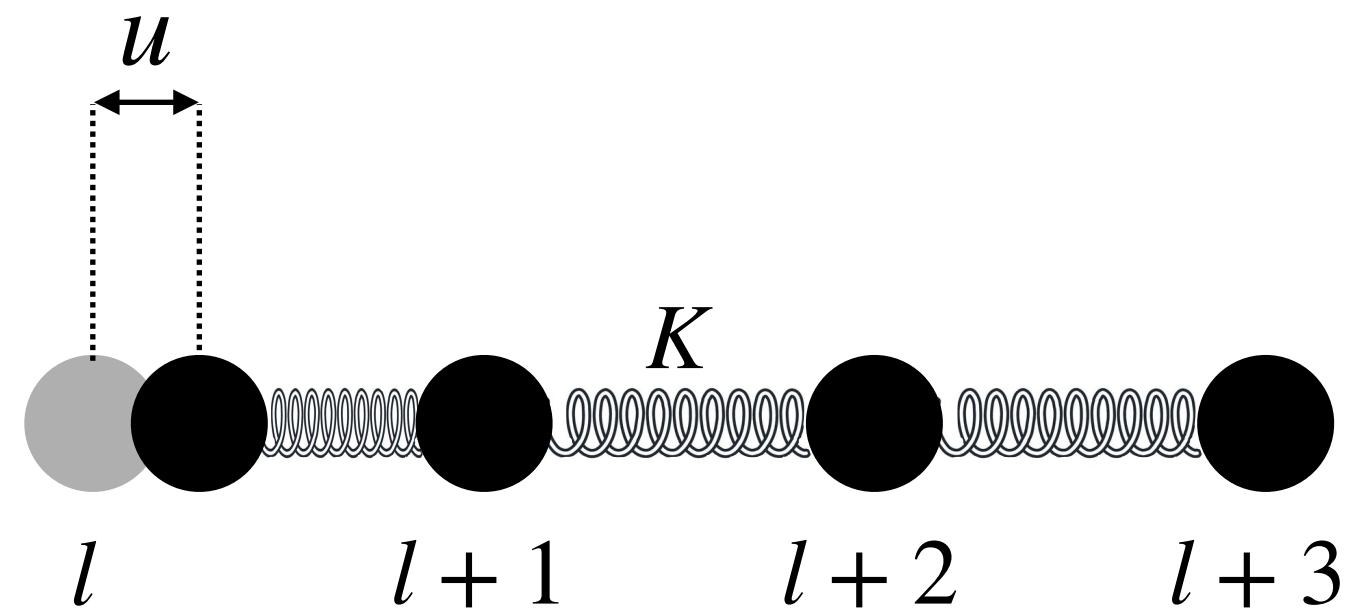


Phonons

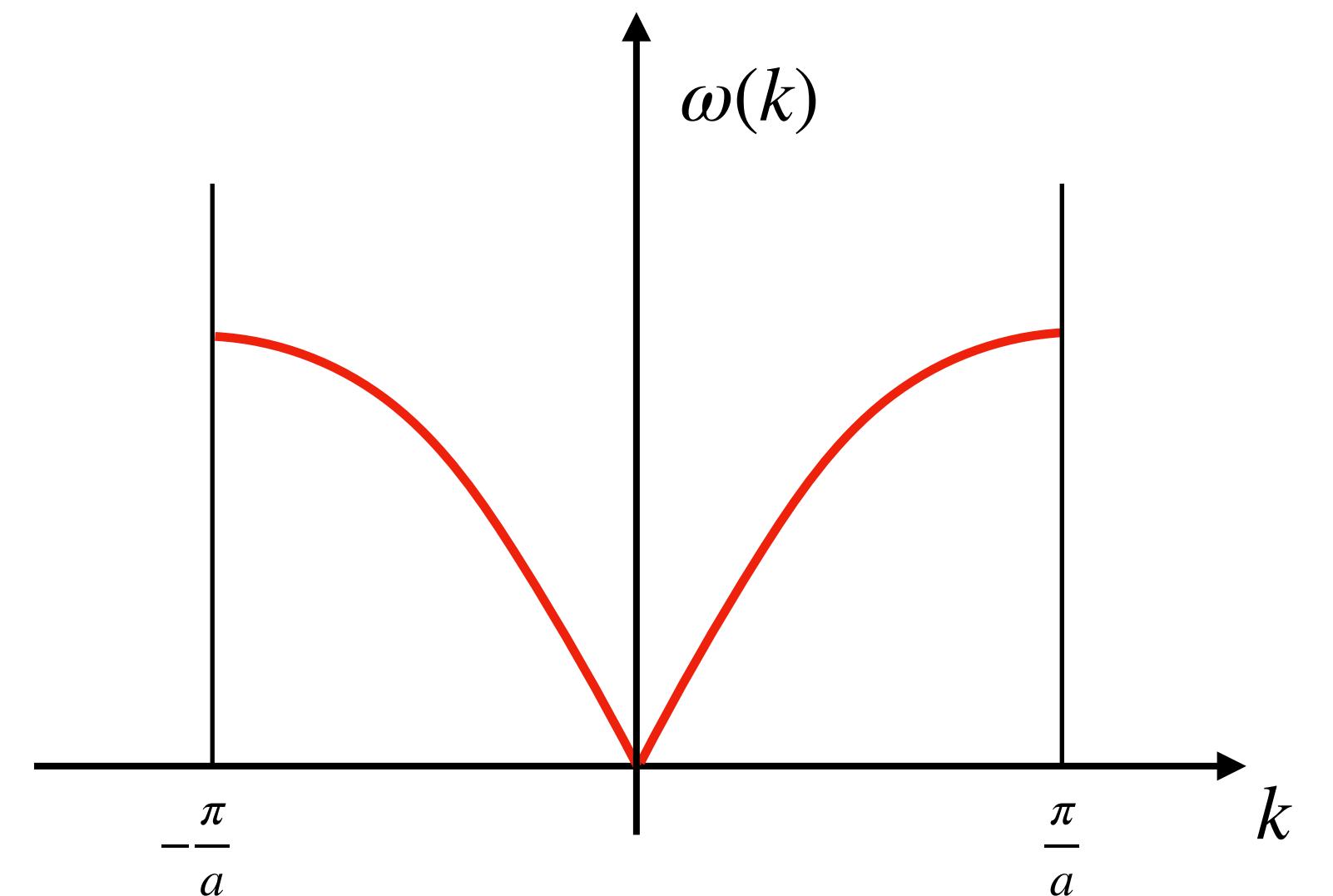




$$\mathcal{H} = \frac{K}{2} \sum_l [u_l - u_{l+1}]^2$$



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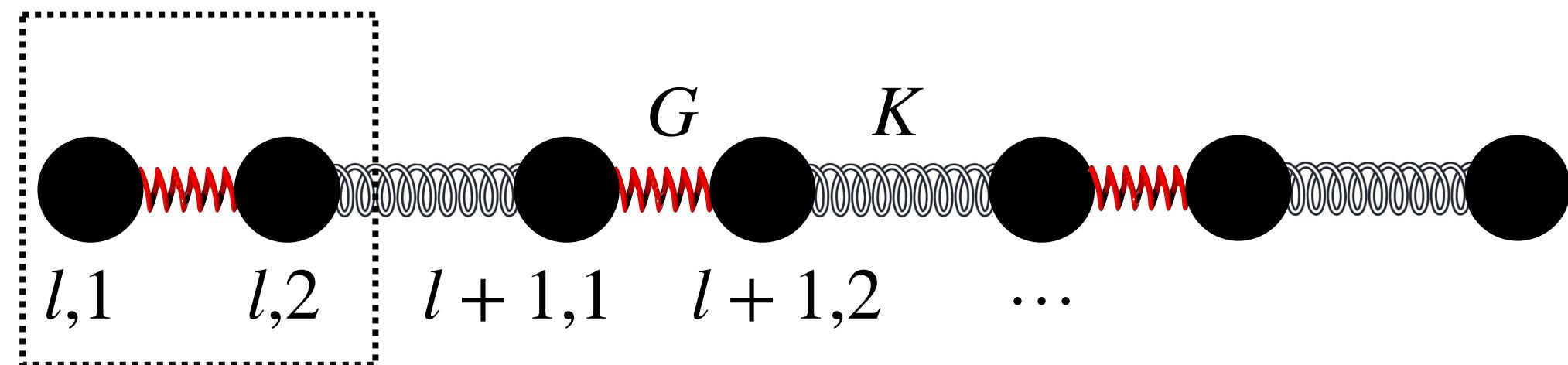


$$u_j \propto e^{i(k \cdot j a - \omega t)}$$

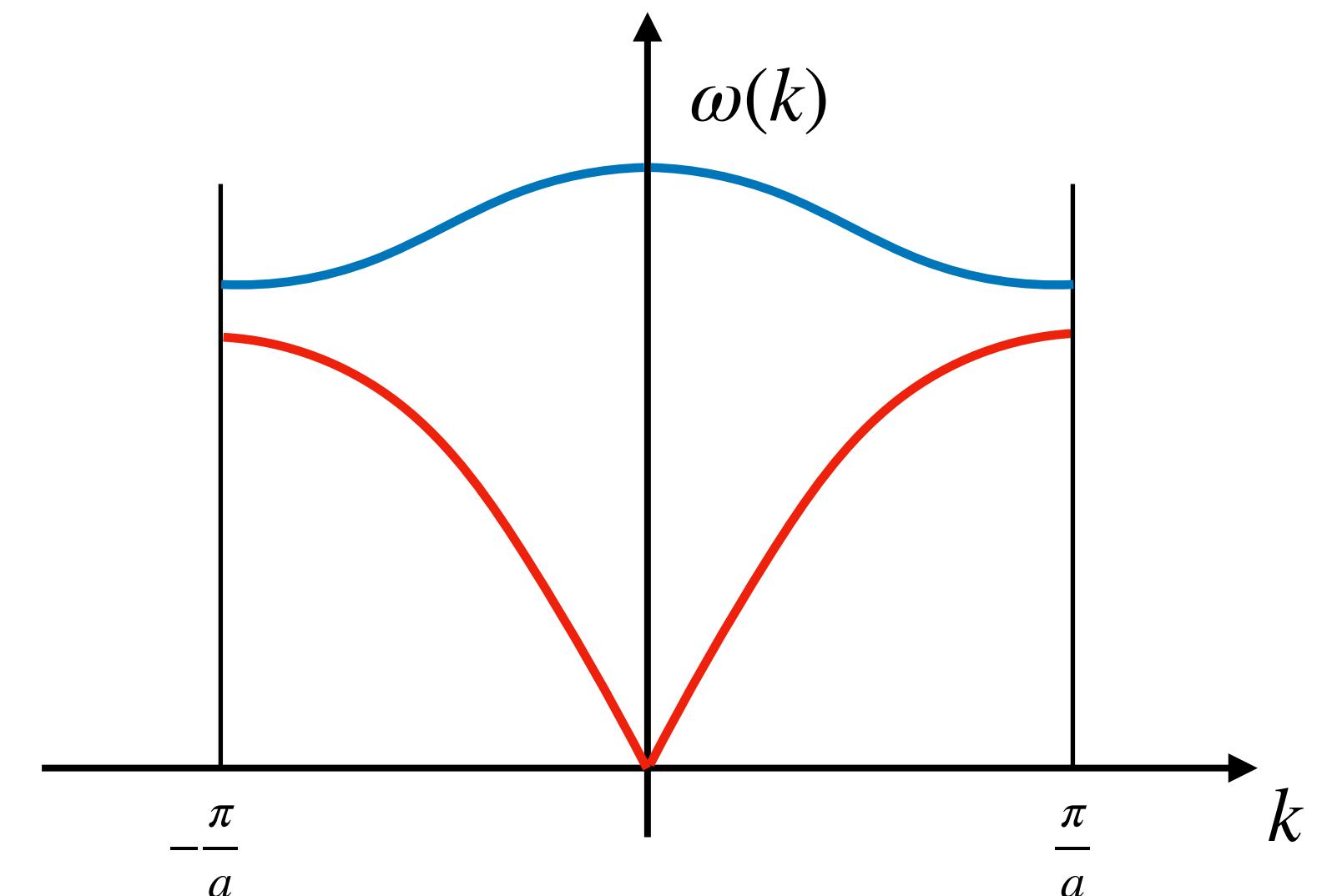
in-phase oscillations

gapless (acoustic) mode
(associated to broken translation symmetry)

primitive cell



$$\mathcal{H} = \frac{G}{2} \sum_l [u_{l,1} - u_{l,2}]^2 + \frac{K}{2} \sum_j [u_{l,2} - u_{l+1,1}]^2$$

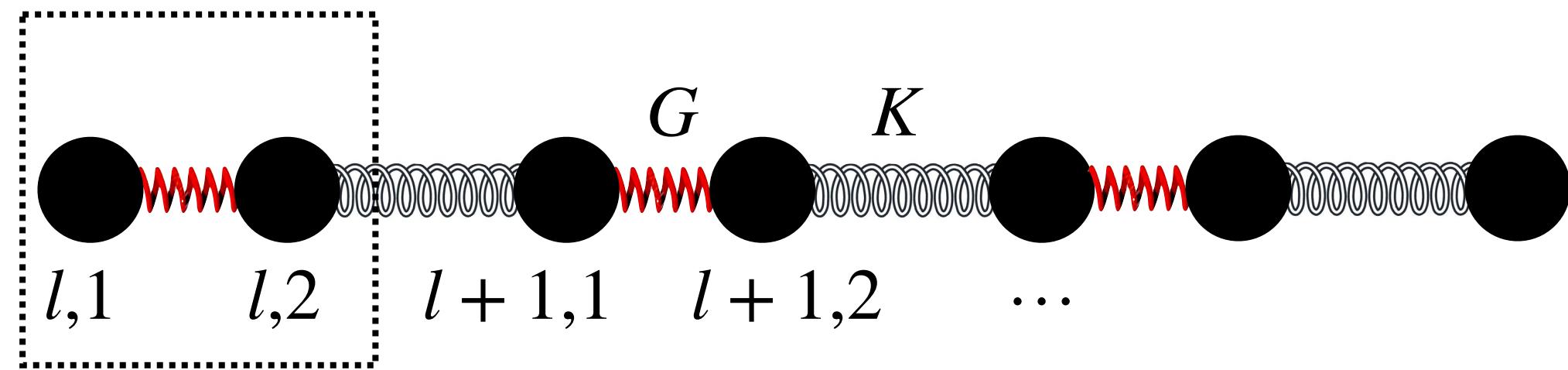


$$u_{lj} = \epsilon_j(k) e^{i(k \cdot la - \omega t)}$$

in and **out**-of-phase oscillations

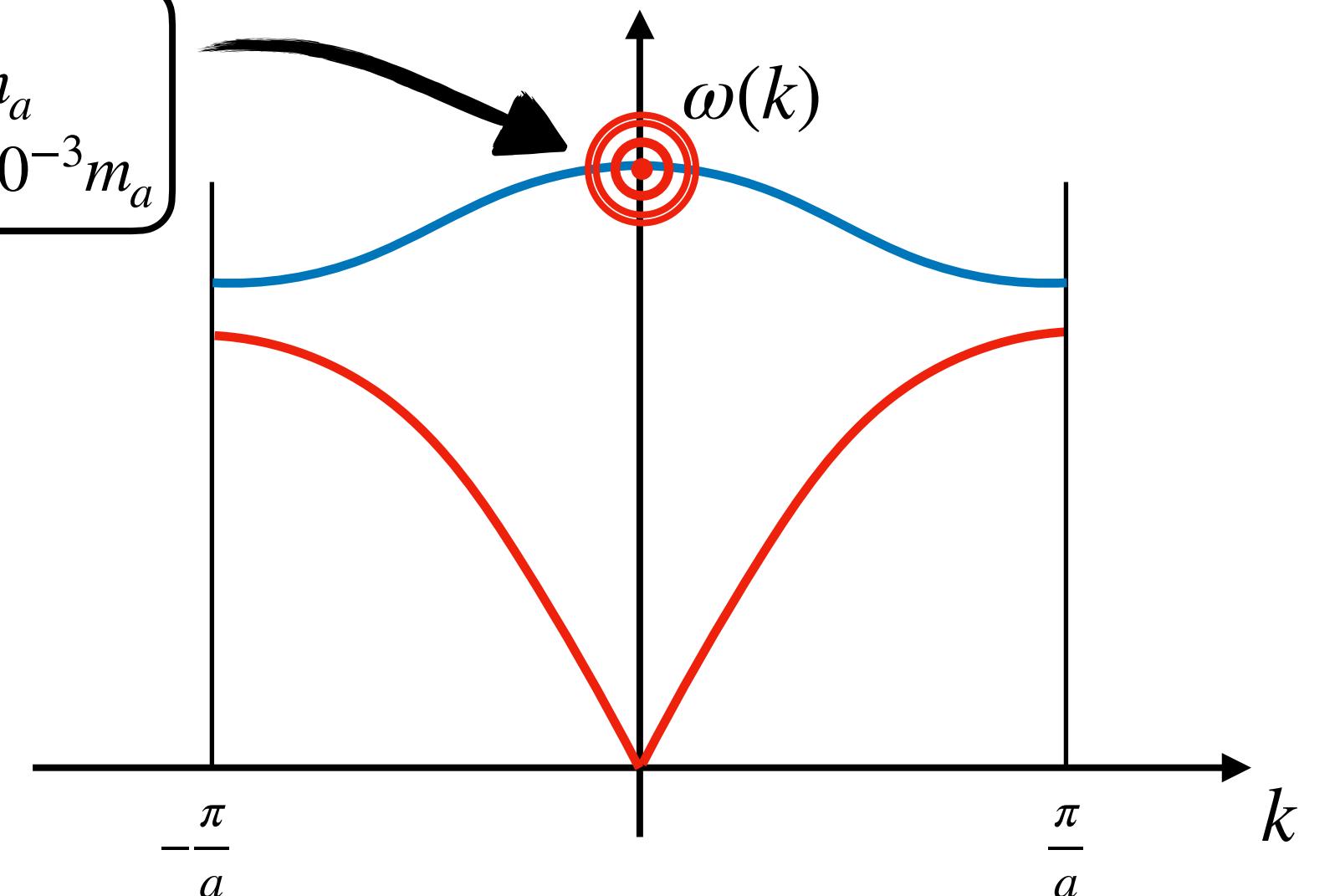
gapless (acoustic) and **gapped (optical)** modes

primitive cell



$$\mathcal{H} = \frac{G}{2} \sum_l [u_{l,1} - u_{l,2}]^2 + \frac{K}{2} \sum_j [u_{l,2} - u_{l+1,1}]^2$$

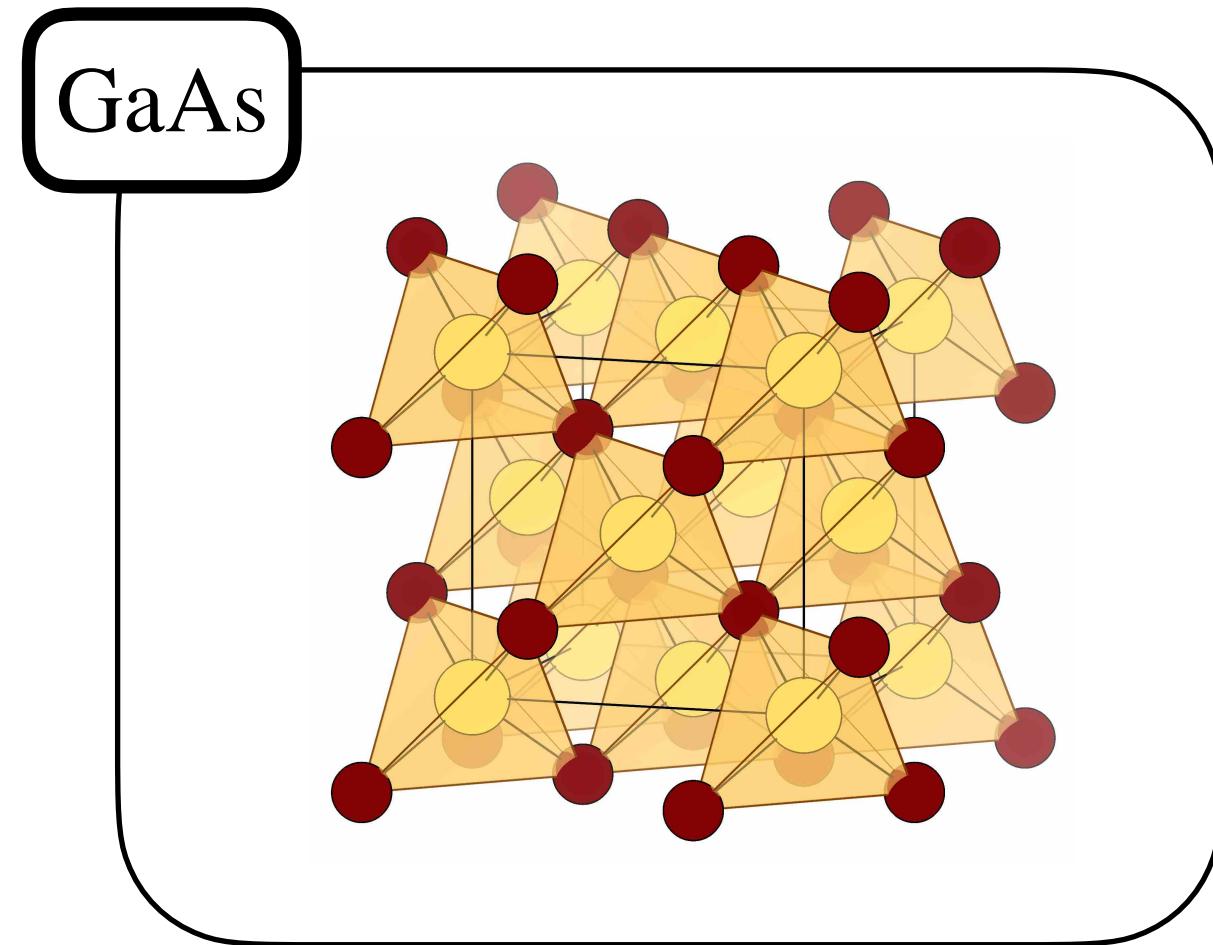
absorption kinematics
deposited energy $\simeq m_a$
deposited momentum $\simeq 10^{-3}m_a$



$$u_{lj} = \epsilon_j(k) e^{i(k \cdot la - \omega t)}$$

in and **out**-of-phase oscillations

gapless (acoustic) and **gapped (optical)** modes



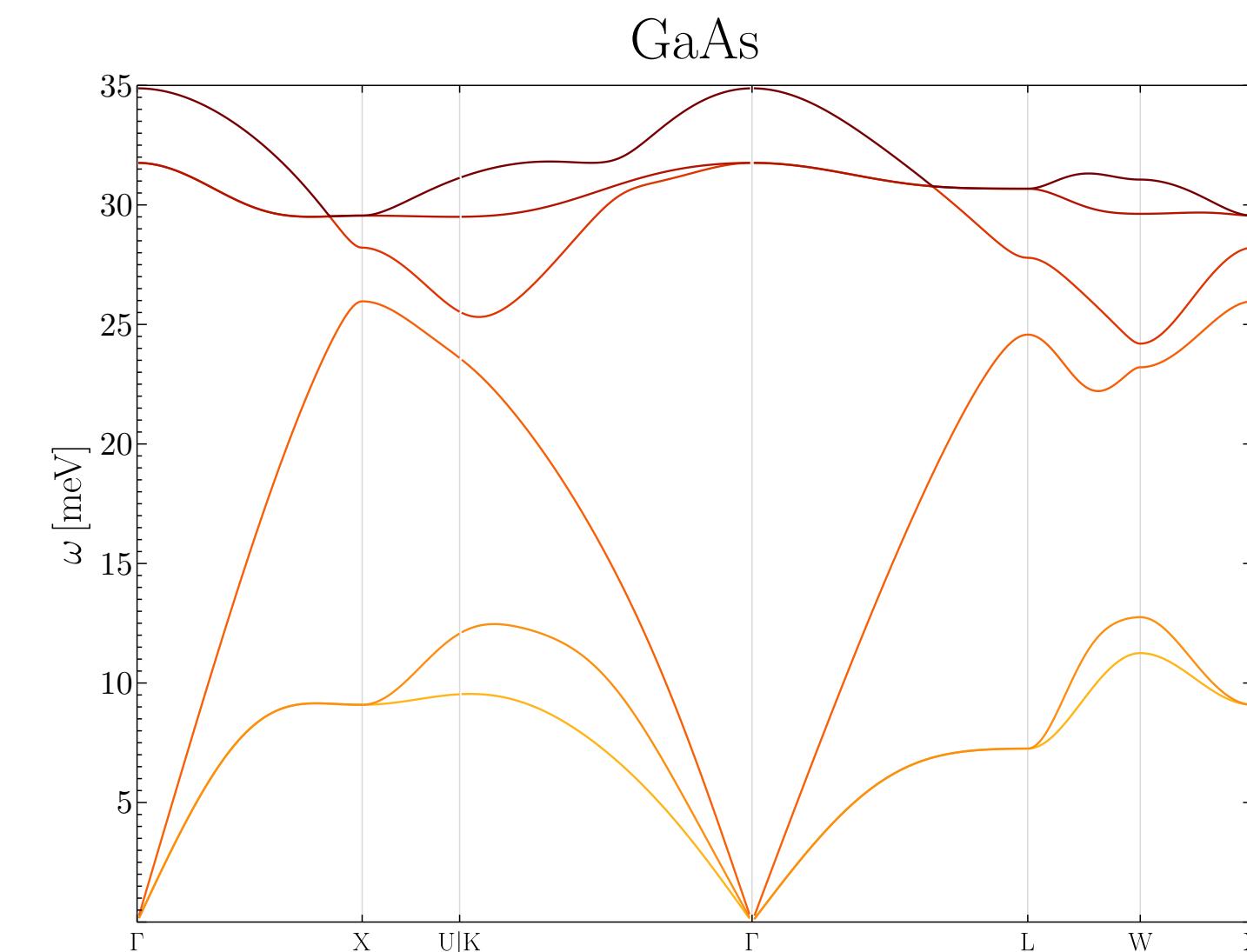
$$\mathcal{H} = \sum_{lj} \frac{\mathbf{p}_{lj}^2}{2m_j} + \frac{1}{2} \sum_{ll',jj'} \mathbf{u}_{lj} \cdot \mathbf{V}_{ll'jj'} \cdot \mathbf{u}_{l'j'}$$

↑

force constant matrix from DFT methods

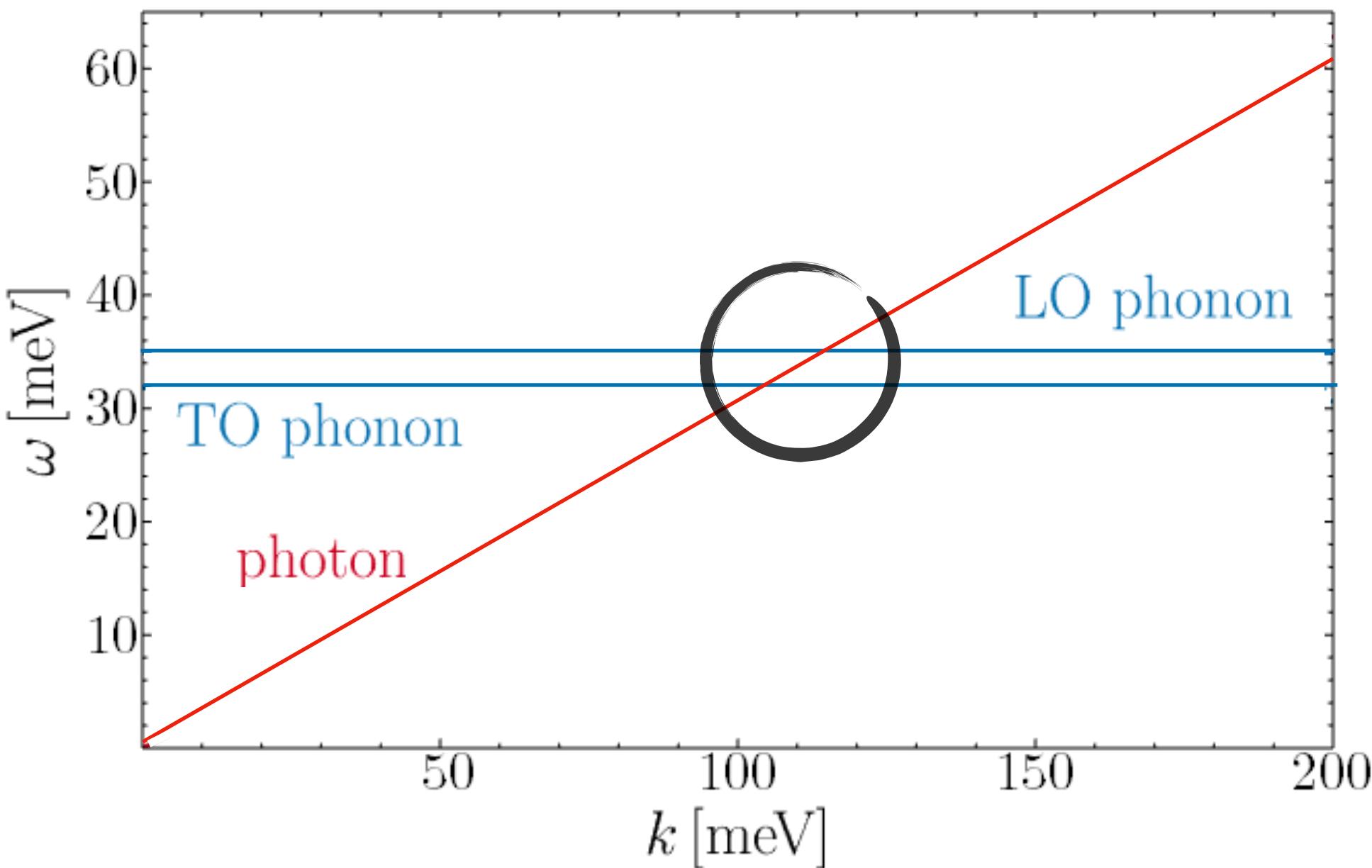
$$\mathbf{u}_{lj} = \sum_{\nu=1}^{3n} \sum_{\mathbf{k}} \frac{1}{\sqrt{2N m_j \omega_{\nu,\mathbf{k}}}} \left(\hat{a}_{\nu,\mathbf{k}} + \hat{a}_{\nu,-\mathbf{k}}^\dagger \right) e^{i \mathbf{k} \cdot \mathbf{x}_{lj}^0} \epsilon_{\nu,\mathbf{k},j}$$

a material with n ions in the primitive cell will have $3n$ phonon branches, 3 gapless and $3n - 3$ gapped



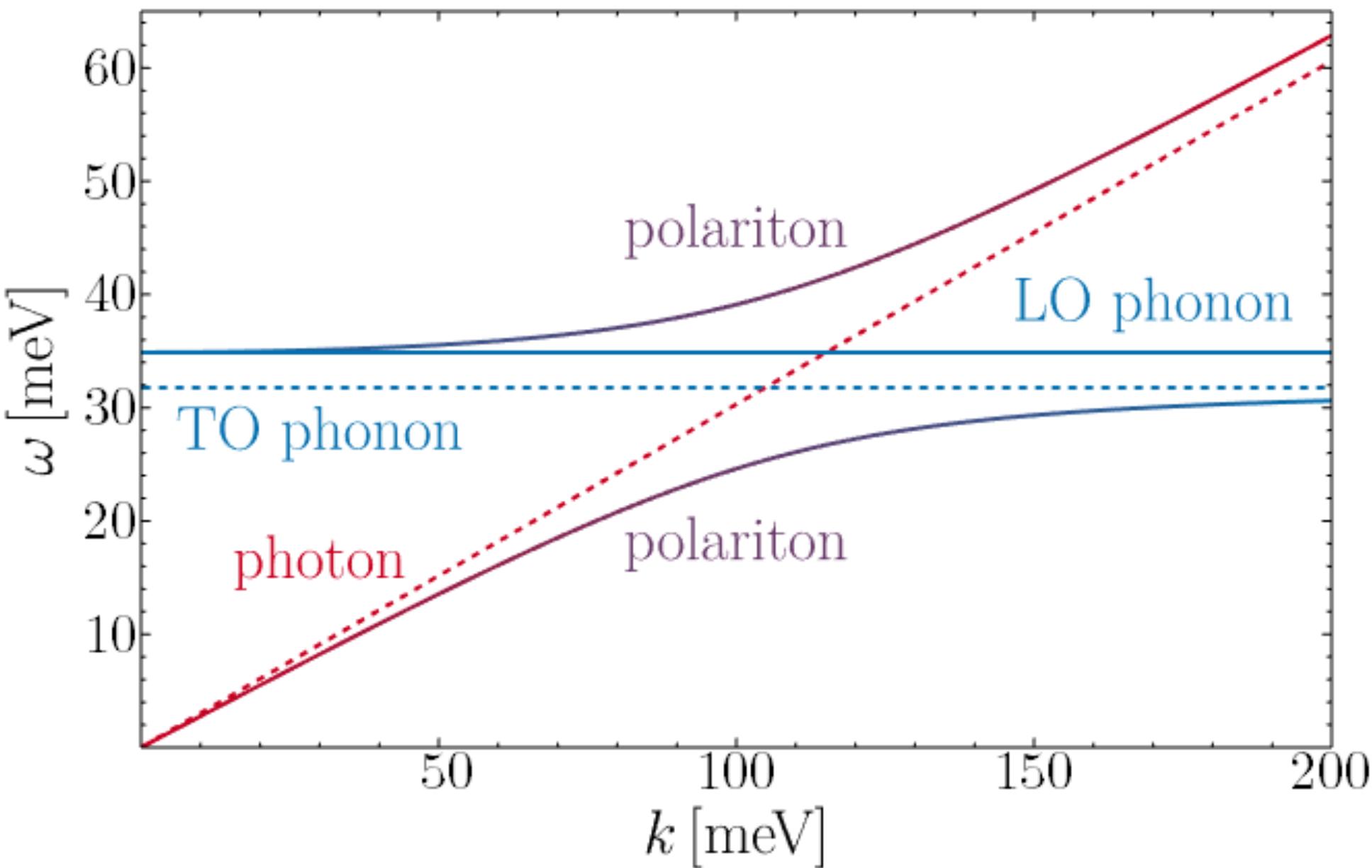
a technical but important note: photon-phonon mixing

at low momenta TO phonon and photon mixing is non-negligible



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at low momenta TO phonon and photon mixing is non-negligible



true physical states are **polaritons**, a mixing of photon and phonon states

axion coupling to phonons

in an **external magnetic field** the **axion** couples to **ions displacement**

Born charges \sim ion charge

$$\mathcal{L} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \xrightarrow{\text{NR limit}} \delta\mathcal{H} = -g_{a\gamma\gamma} \int d^3x a \mathbf{E} \cdot \mathbf{B} \xrightarrow{\text{in an external } \mathbf{B} \text{ field}} \delta\mathcal{H} = -e g_{a\gamma\gamma} \mathbf{a} \mathbf{B} \sum_{lj} \hat{\mathbf{B}} \cdot \varepsilon_{\infty}^{-1} \cdot \mathbf{Z}_j^* \cdot \mathbf{u}_{lj}$$

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this induces a phonon excitations rate (per unit time and detector mass)

$$R = \frac{2e^2 g_{a\gamma\gamma}^2 B^2}{m_{\text{cell}}} \frac{\rho_a}{m_a^2} \int d^3v_a f(\mathbf{v}_a) \sum_{\nu=6}^{3n+2} \frac{m_a \omega'_{\nu, \mathbf{p}} \gamma_{\nu, \mathbf{p}}}{(m_a^2 - \omega'^2_{\nu, \mathbf{p}})^2 + (m_a \gamma_{\nu, \mathbf{p}})^2} \left| \sum_j \sum_{\nu'=1}^{3n} \frac{1}{\sqrt{m_j \omega_{\nu', \mathbf{p}}}} f_j \cdot \epsilon_{\nu', \mathbf{p}, j}^* (\mathbb{U}_{\nu' \nu, \mathbf{p}}^* + \mathbb{V}_{\nu' \nu, -\mathbf{p}}) \right|^2$$

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axion phonon coupling strength

$$R = \frac{2e^2 g_{a\gamma\gamma}^2 B^2}{m_{\text{cell}}} \frac{\rho_a}{m_a^2} \int d^3v_a f(\mathbf{v}_a) \sum_{\nu=6}^{3n+2} \frac{m_a \omega'_{\nu, \mathbf{p}} \gamma_{\nu, \mathbf{p}}}{(m_a^2 - \omega'^2_{\nu, \mathbf{p}})^2 + (m_a \gamma_{\nu, \mathbf{p}})^2} \left| \sum_j \sum_{\nu'=1}^{3n} \frac{1}{\sqrt{m_j \omega_{\nu', \mathbf{p}}}} f_j \cdot \epsilon_{\nu', \mathbf{p}, j}^* (\mathbb{U}_{\nu' \nu, \mathbf{p}}^* + \mathbb{V}_{\nu' \nu, -\mathbf{p}}) \right|^2$$

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this induces a phonon excitations rate (per unit time and detector mass)

local axion field amplitude

$$R = \frac{2e^2 g_{a\gamma\gamma}^2 B^2}{m_{\text{cell}}} \left| \frac{\rho_a}{m_a^2} \right| \int d^3v_a f(\mathbf{v}_a) \sum_{\nu=6}^{3n+2} \frac{m_a \omega'_{\nu, \mathbf{p}} \gamma_{\nu, \mathbf{p}}}{(m_a^2 - \omega'^2_{\nu, \mathbf{p}})^2 + (m_a \gamma_{\nu, \mathbf{p}})^2} \left| \sum_j \sum_{\nu'=1}^{3n} \frac{1}{\sqrt{m_j \omega_{\nu', \mathbf{p}}}} f_j \cdot \epsilon_{\nu', \mathbf{p}, j}^* (\mathbb{U}_{\nu' \nu, \mathbf{p}}^* + \mathbb{V}_{\nu' \nu, -\mathbf{p}}) \right|^2$$

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this induces a phonon excitations rate (per unit time and detector mass)

DM velocity integral

$$R = \frac{2e^2 g_{a\gamma\gamma}^2 B^2}{m_{\text{cell}}} \frac{\rho_a}{m_a^2} \left[\int d^3v_a f(v_a) \sum_{\nu=6}^{3n+2} \frac{m_a \omega'_{\nu, \mathbf{p}} \gamma_{\nu, \mathbf{p}}}{(m_a^2 - \omega'^2_{\nu, \mathbf{p}})^2 + (m_a \gamma_{\nu, \mathbf{p}})^2} \left| \sum_j \sum_{\nu'=1}^{3n} \frac{1}{\sqrt{m_j \omega_{\nu', \mathbf{p}}}} f_j \cdot \epsilon_{\nu', \mathbf{p}, j}^* (\mathbb{U}_{\nu' \nu, \mathbf{p}}^* + \mathbb{V}_{\nu' \nu, -\mathbf{p}}) \right|^2 \right]$$

in an **external magnetic field** the **axion** couples to **ions displacement**

Born charges \sim ion charge

$$\mathcal{L} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \xrightarrow{\text{NR limit}} \delta\mathcal{H} = -g_{a\gamma\gamma} \int d^3x a \mathbf{E} \cdot \mathbf{B} \xrightarrow{\text{in an external } \mathbf{B} \text{ field}} \delta\mathcal{H} = -e g_{a\gamma\gamma} a \mathbf{B} \sum_{lj} \hat{\mathbf{B}} \cdot \varepsilon_{\infty}^{-1} \cdot \mathbf{Z}_j^* \cdot \mathbf{u}_{lj}$$

this induces a phonon excitations rate (per unit time and detector mass)

sum over final states

$$R = \frac{2e^2 g_{a\gamma\gamma}^2 B^2}{m_{\text{cell}}} \frac{\rho_a}{m_a^2} \int d^3v_a f(\mathbf{v}_a) \left(\sum_{\nu=6}^{3n+2} \frac{m_a \omega'_{\nu, \mathbf{p}} \gamma_{\nu, \mathbf{p}}}{(m_a^2 - \omega'^2_{\nu, \mathbf{p}})^2 + (m_a \gamma_{\nu, \mathbf{p}})^2} \left| \sum_j \sum_{\nu'=1}^{3n} \frac{1}{\sqrt{m_j \omega_{\nu', \mathbf{p}}}} f_j \cdot \epsilon_{\nu', \mathbf{p}, j}^* (\mathbb{U}_{\nu' \nu, \mathbf{p}}^* + \mathbb{V}_{\nu' \nu, -\mathbf{p}}) \right|^2 \right)$$

in an **external magnetic field** the **axion** couples to **ions displacement**

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Breit–Wigner

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this induces a phonon excitations rate (per unit time and detector mass)

$$R = \frac{2e^2 g_{a\gamma\gamma}^2 B^2}{m_{\text{cell}}} \frac{\rho_a}{m_a^2} \int d^3v_a f(\mathbf{v}_a) \sum_{\nu=6}^{3n+2} \frac{m_a \omega'_{\nu, \mathbf{p}} \gamma_{\nu, \mathbf{p}}}{(m_a^2 - \omega'^2_{\nu, \mathbf{p}})^2 + (m_a \gamma_{\nu, \mathbf{p}})^2} \left| \sum_j \sum_{\nu'=1}^{3n} \frac{1}{\sqrt{m_j \omega_{\nu', \mathbf{p}}}} f_j \cdot \epsilon_{\nu', \mathbf{p}, j}^* (\mathbb{U}_{\nu' \nu, \mathbf{p}}^* + \mathbb{V}_{\nu' \nu, -\mathbf{p}}) \right|^2$$

fixes selection rules

axion coupling to phonons

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photon - phonon mixing close to $\delta_{\nu\nu'}$

in an **external magnetic field** the **axion** couples to **ions displacement**

Born charges \sim ion charge

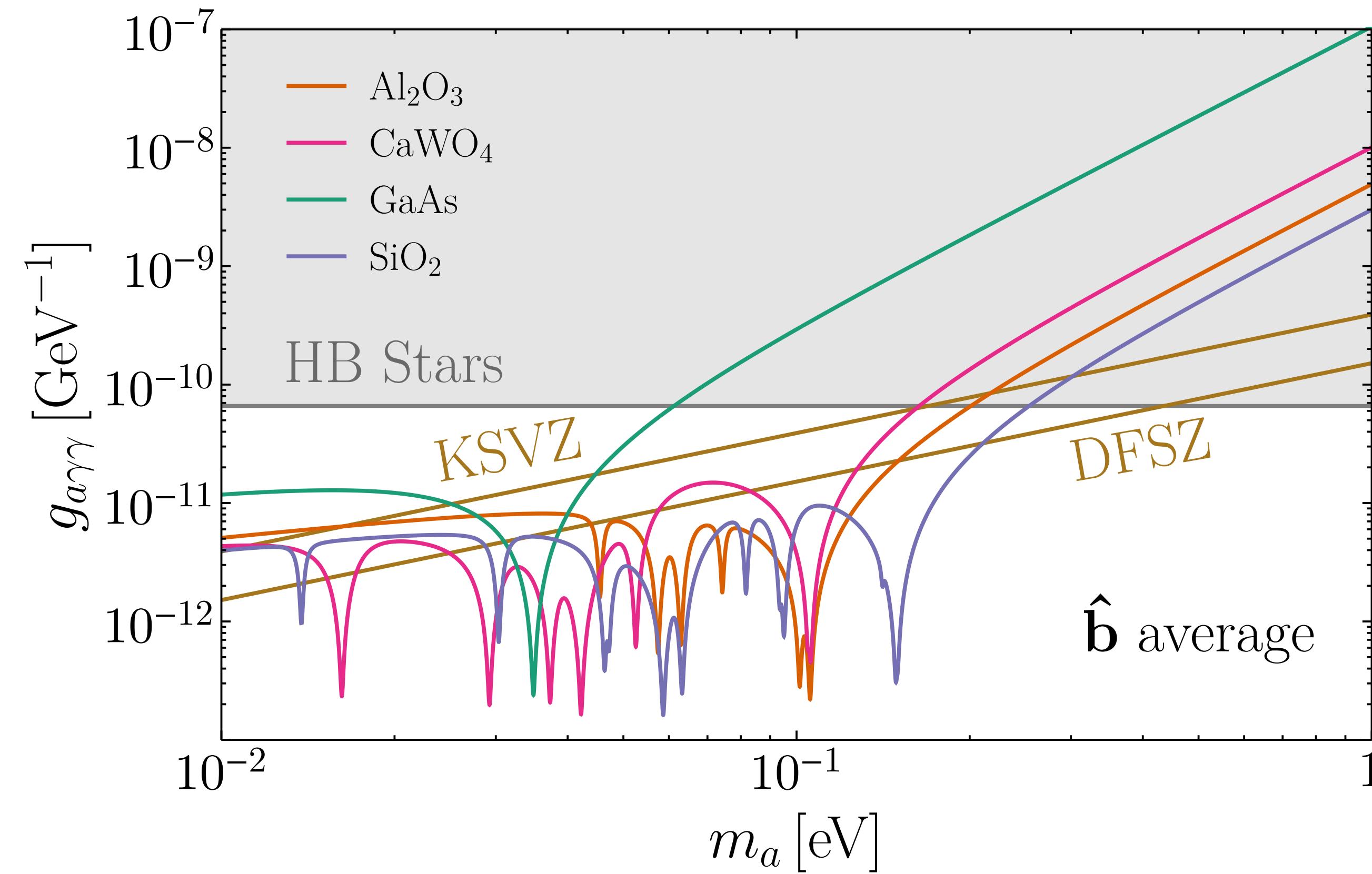
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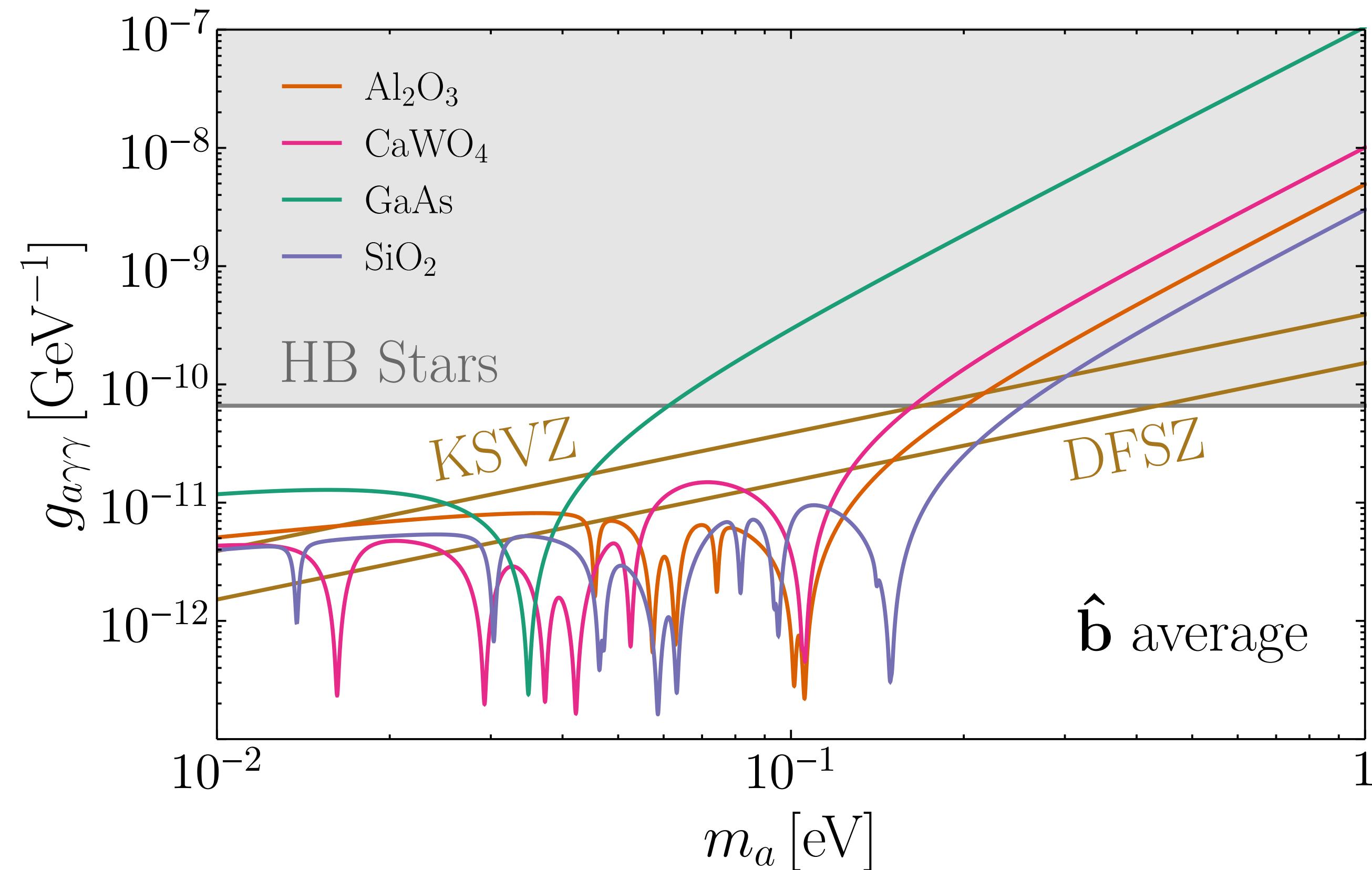
this induces a phonon excitations rate (per unit time and detector mass)

$$R = \frac{2e^2 g_{a\gamma\gamma}^2 B^2}{m_{\text{cell}}} \frac{\rho_a}{m_a^2} \int d^3v_a f(\mathbf{v}_a) \sum_{\nu=6}^{3n+2} \frac{m_a \omega'_{\nu, \mathbf{p}} \gamma_{\nu, \mathbf{p}}}{(m_a^2 - \omega'^2_{\nu, \mathbf{p}})^2 + (m_a \gamma_{\nu, \mathbf{p}})^2} \left| \sum_j \sum_{\nu'=1}^{3n} \frac{1}{\sqrt{m_j \omega_{\nu', \mathbf{p}}}} f_j \cdot \epsilon_{\nu', \mathbf{p}, j}^* (\mathbb{U}_{\nu' \nu, \mathbf{p}}^* + \mathbb{V}_{\nu' \nu, -\mathbf{p}}) \right|^2$$

on resonance

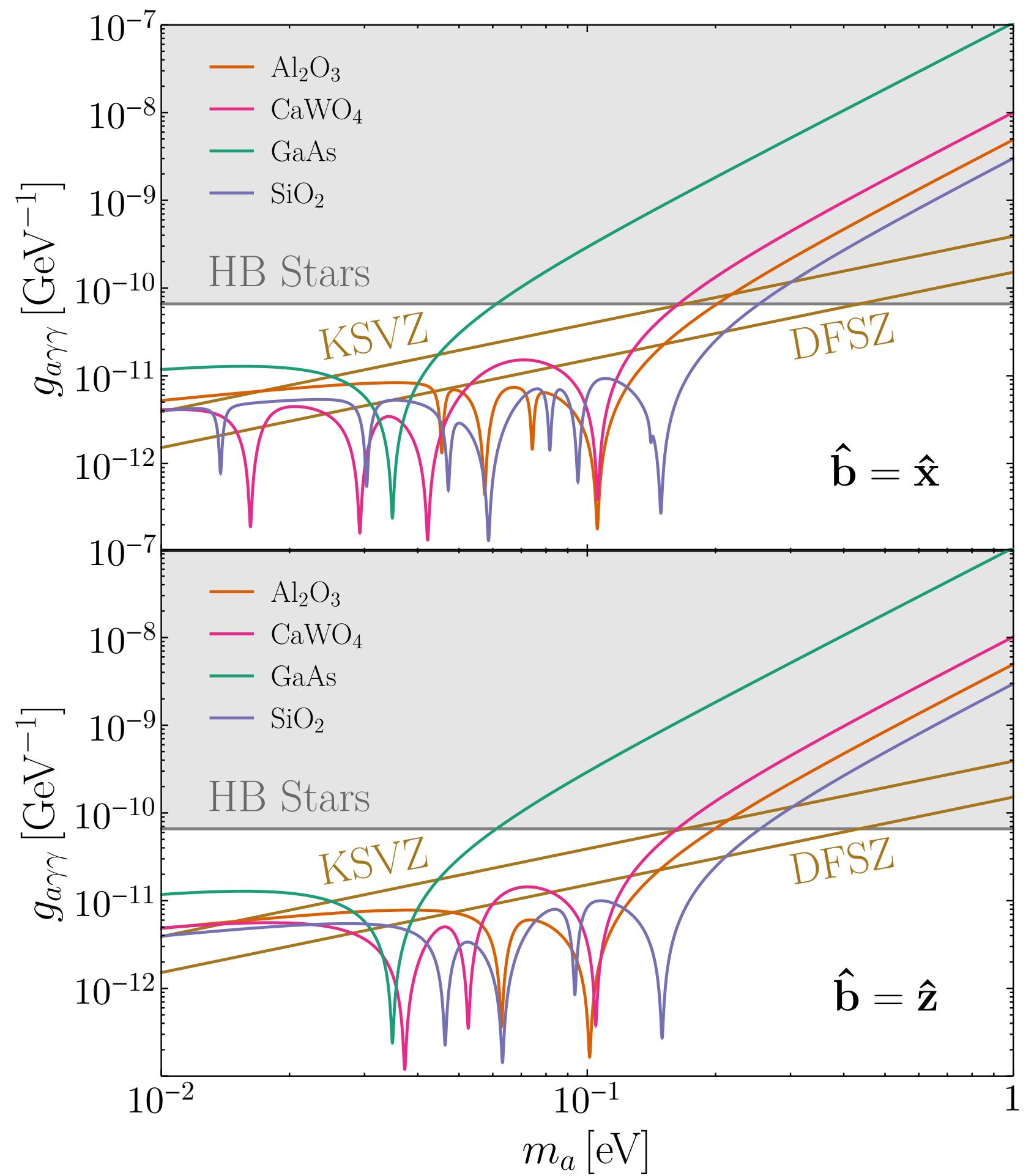
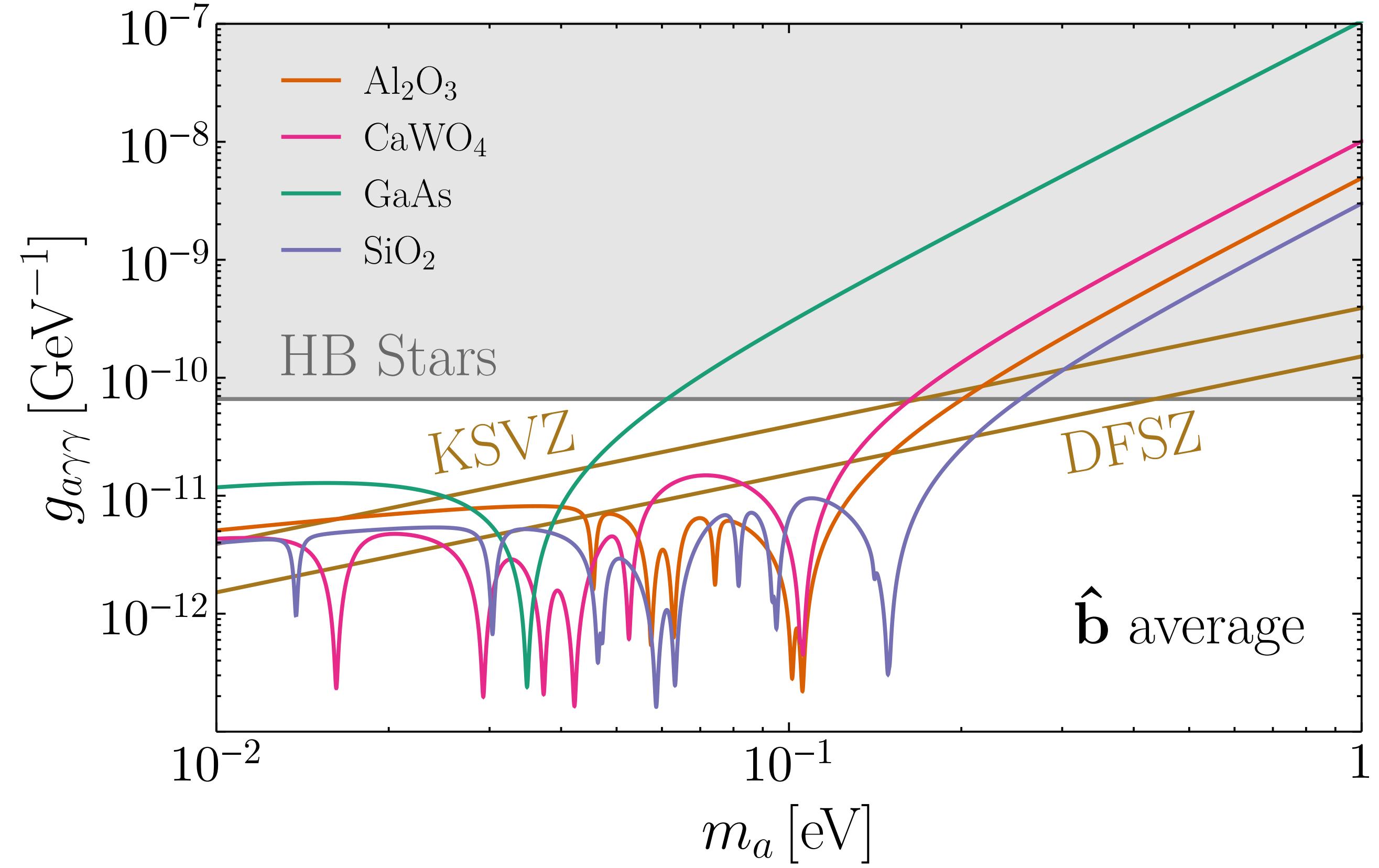
$$R \sim \frac{g_{a\gamma\gamma}^2 \rho_a}{m_a^3} \frac{Z^{*2} e^2 B^2}{\varepsilon_{\infty}^2 m_{\text{ion}}^2 \gamma} \sim (\text{kg}\cdot\text{yr})^{-1} \left(\frac{g_{a\gamma\gamma}}{10^{-13} \text{GeV}^{-1}} \right)^2 \left(\frac{100 \text{ meV}}{m_a} \right)^3 \left(\frac{B}{10 \text{T}} \right)^2 \left(\frac{\text{meV}}{\gamma} \right)$$



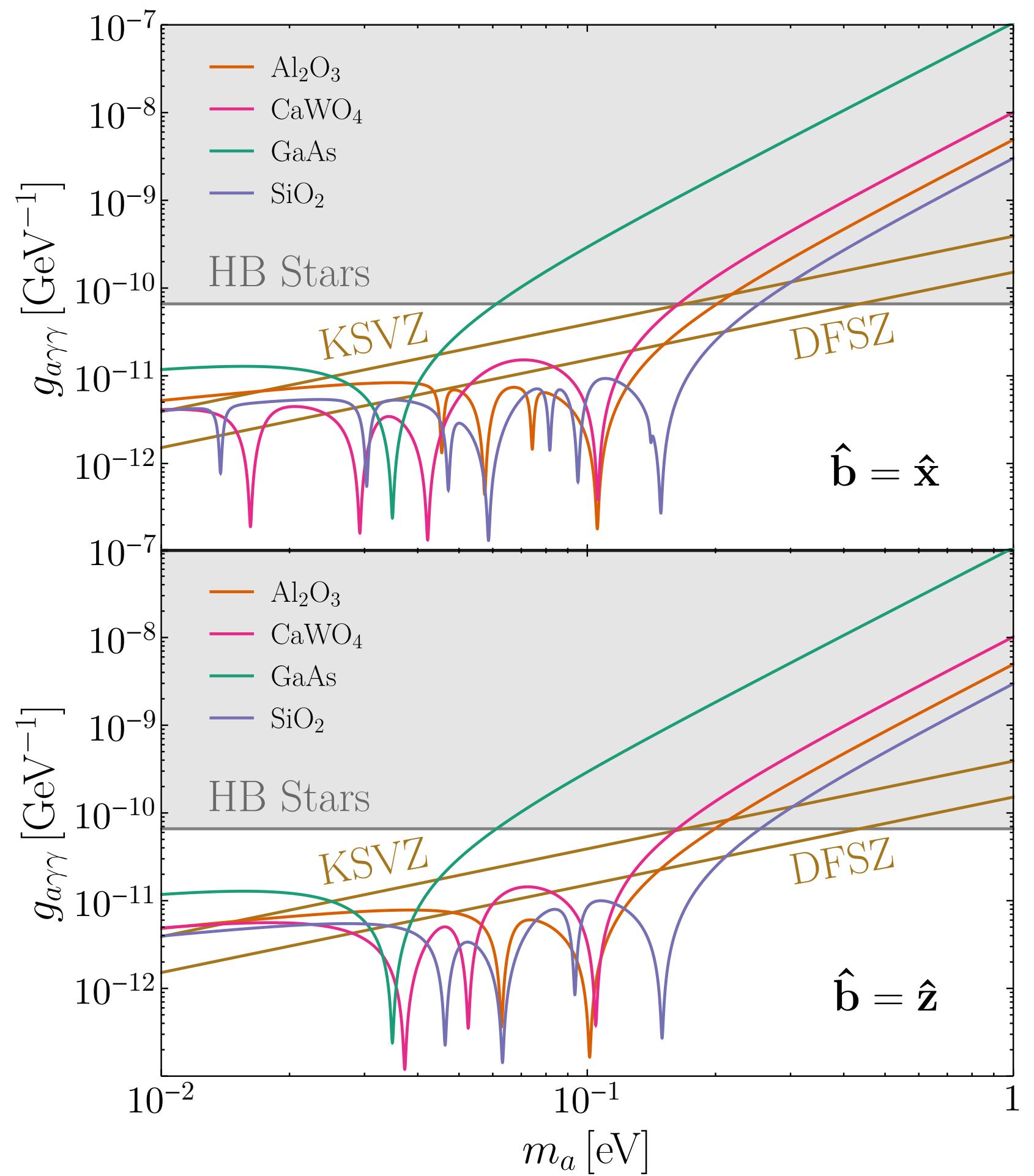
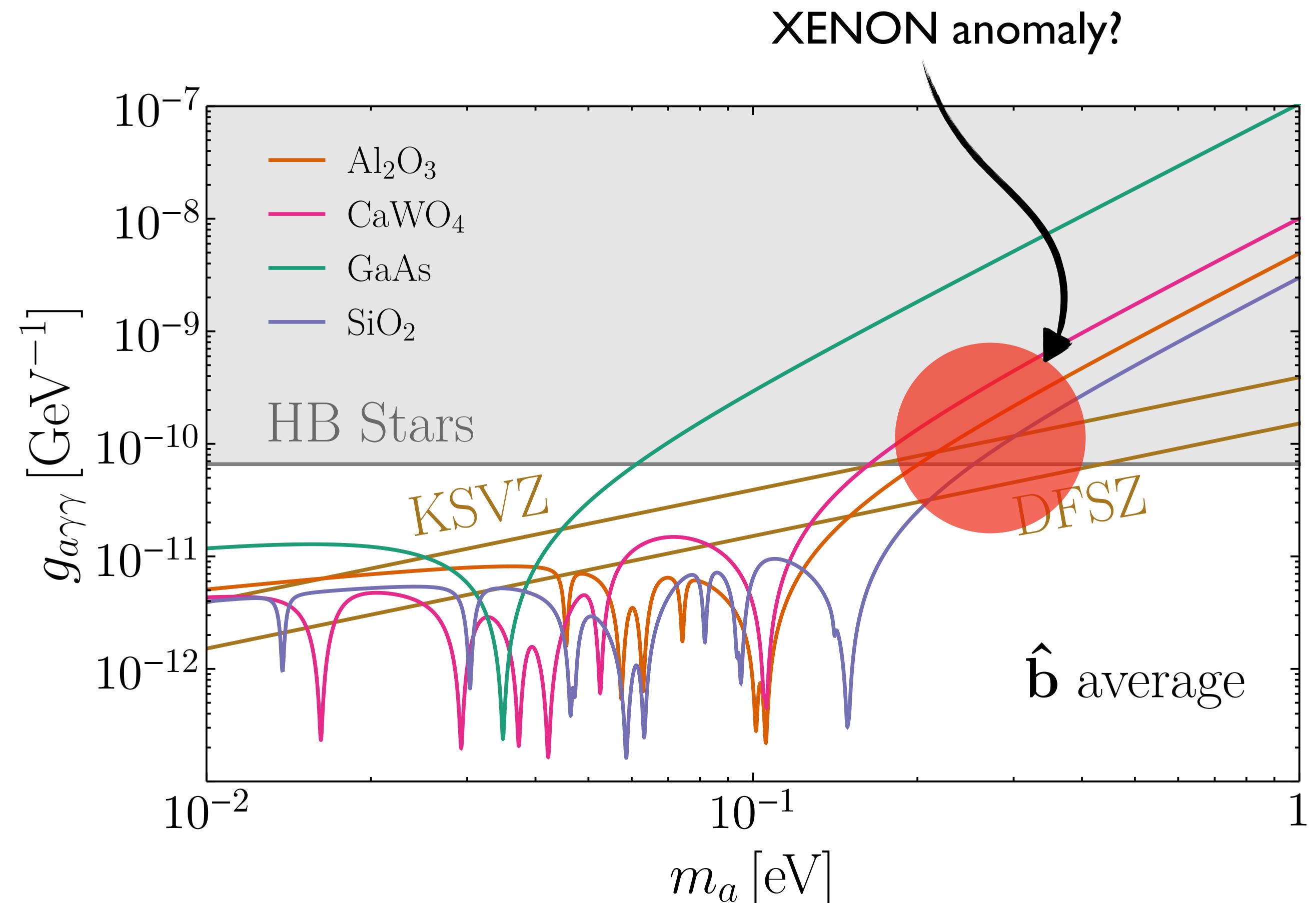


Optical branches	tot	Coupled
Al_2O_3	27	10
CaWO_4	33	11
GaAs	3 (deg)	3
SiO_2	24	20

coupled modes depend on the magnetic field orientation



coupled modes depend on the magnetic field orientation



read-out complicated by the strong external
magnetic field

standard TES read out does not work

possible solutions

helium evaporation

good efficiency

need very well
polished samples

...

photon read-out

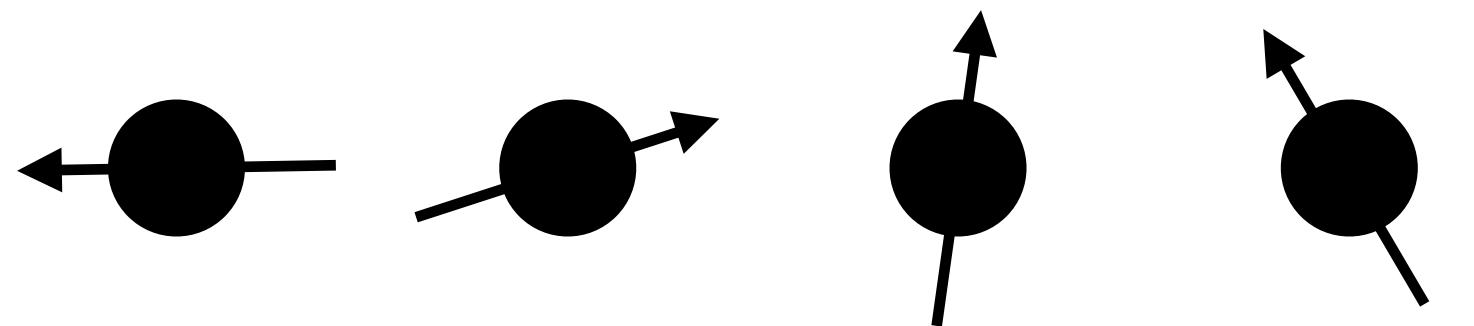
???

main open question:
TO-phonon BR into
photons

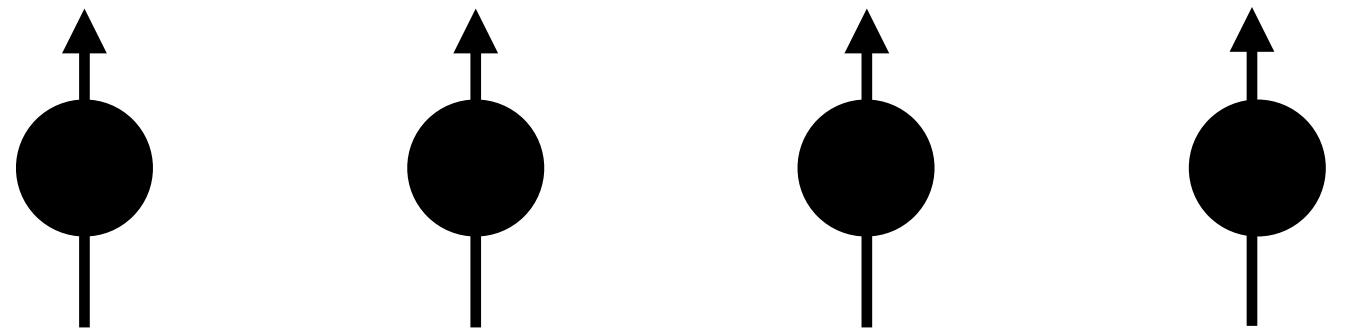
???

Magnons

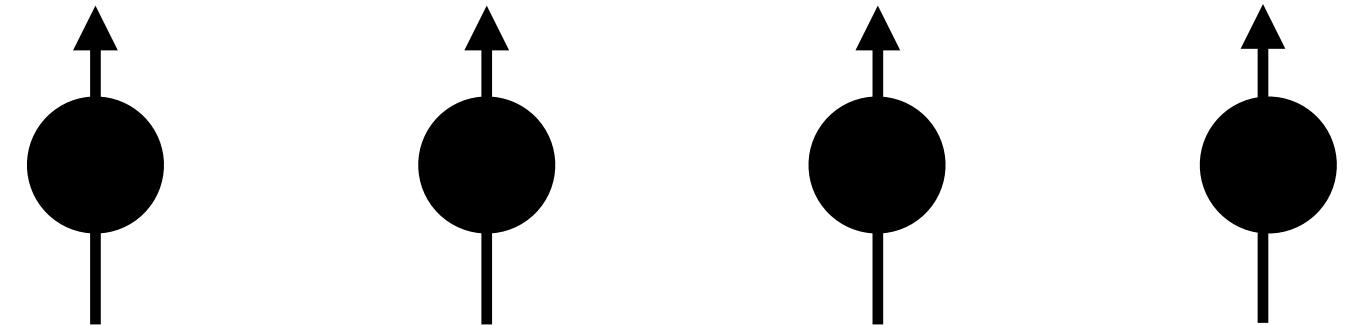
generic material



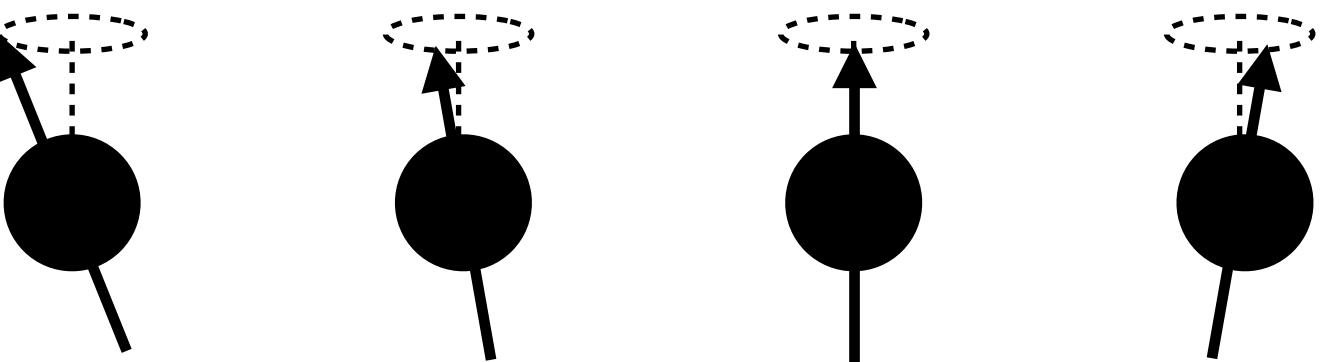
ferromagnet ($J > 0$)



$$\mathcal{H} = -J \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

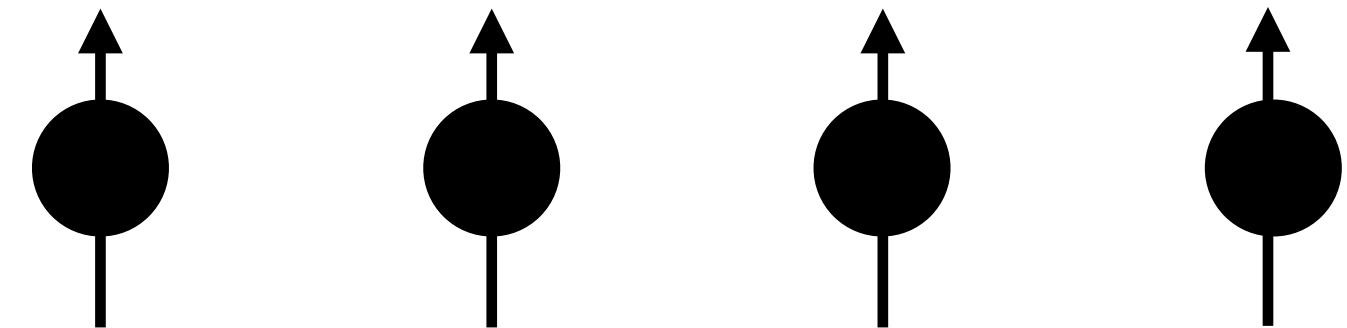
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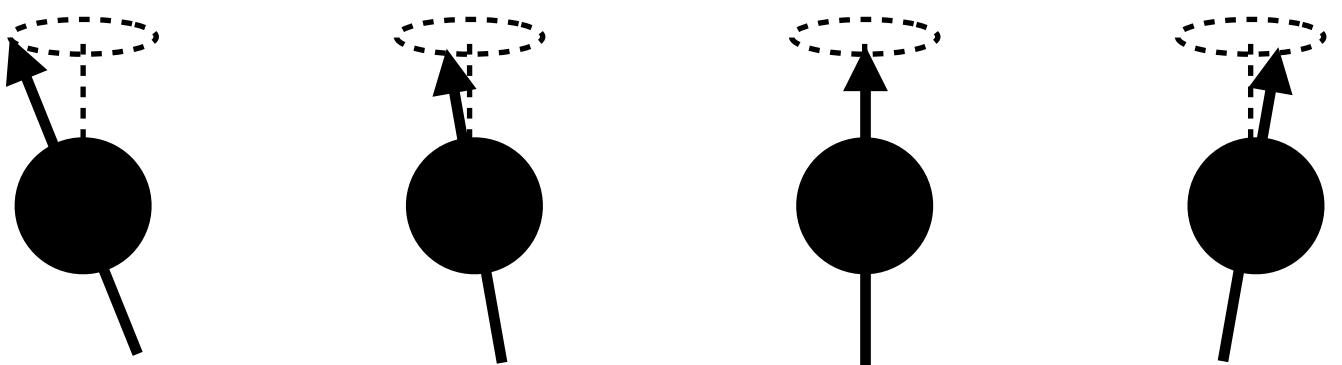


quanta of excitations about the magnetically order ground states are **magnons**

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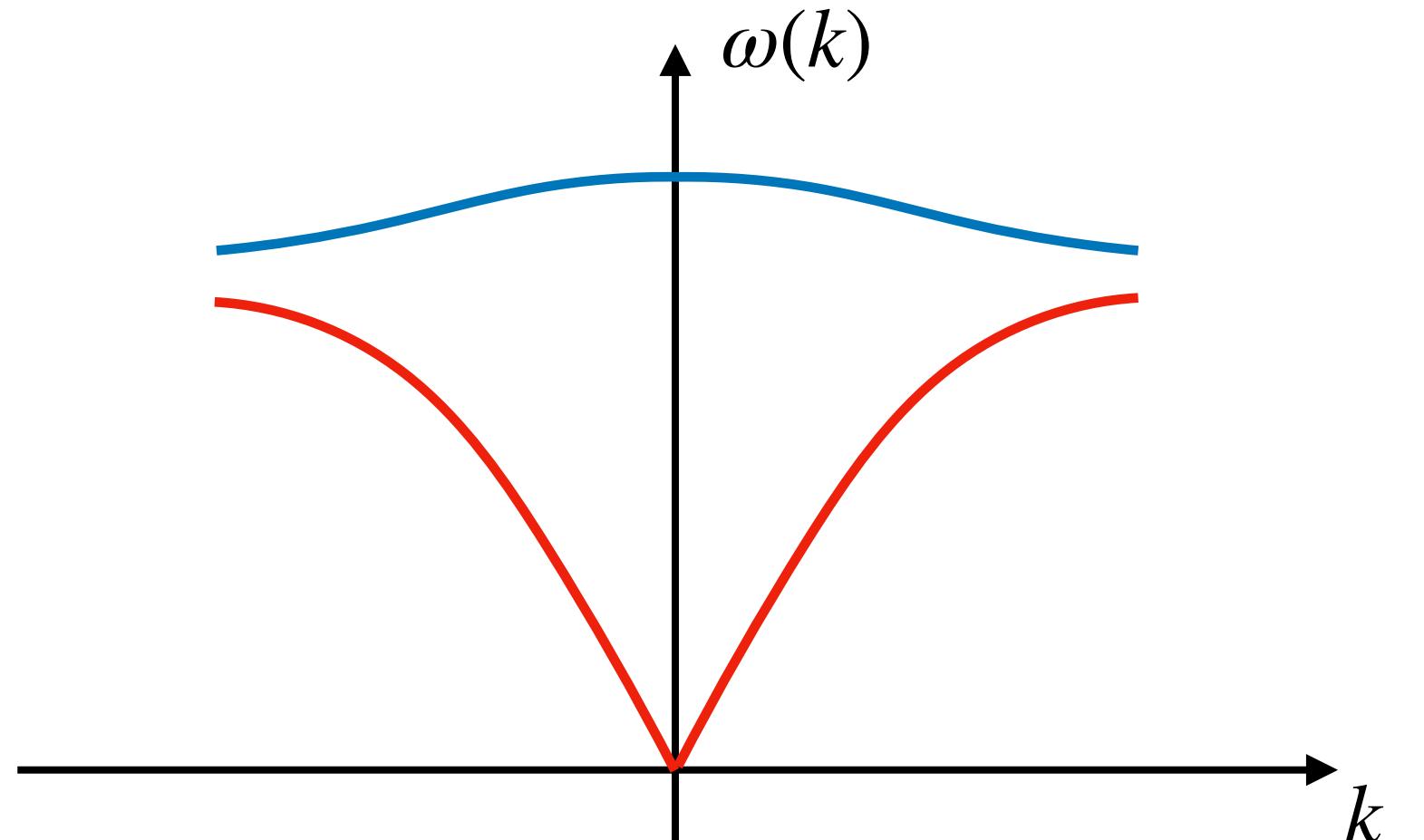
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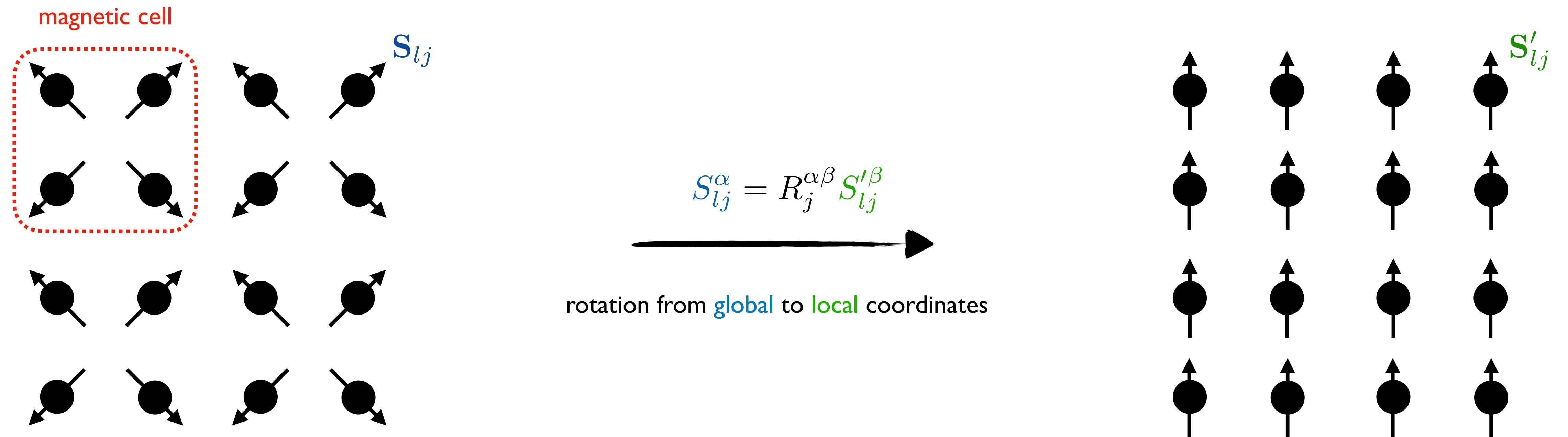


quanta of excitations about the magnetically order ground states are **magnons**

gapless mode associated to broken rotation symmetry

gapped modes arise when inequivalent ions are present in the primitive cell

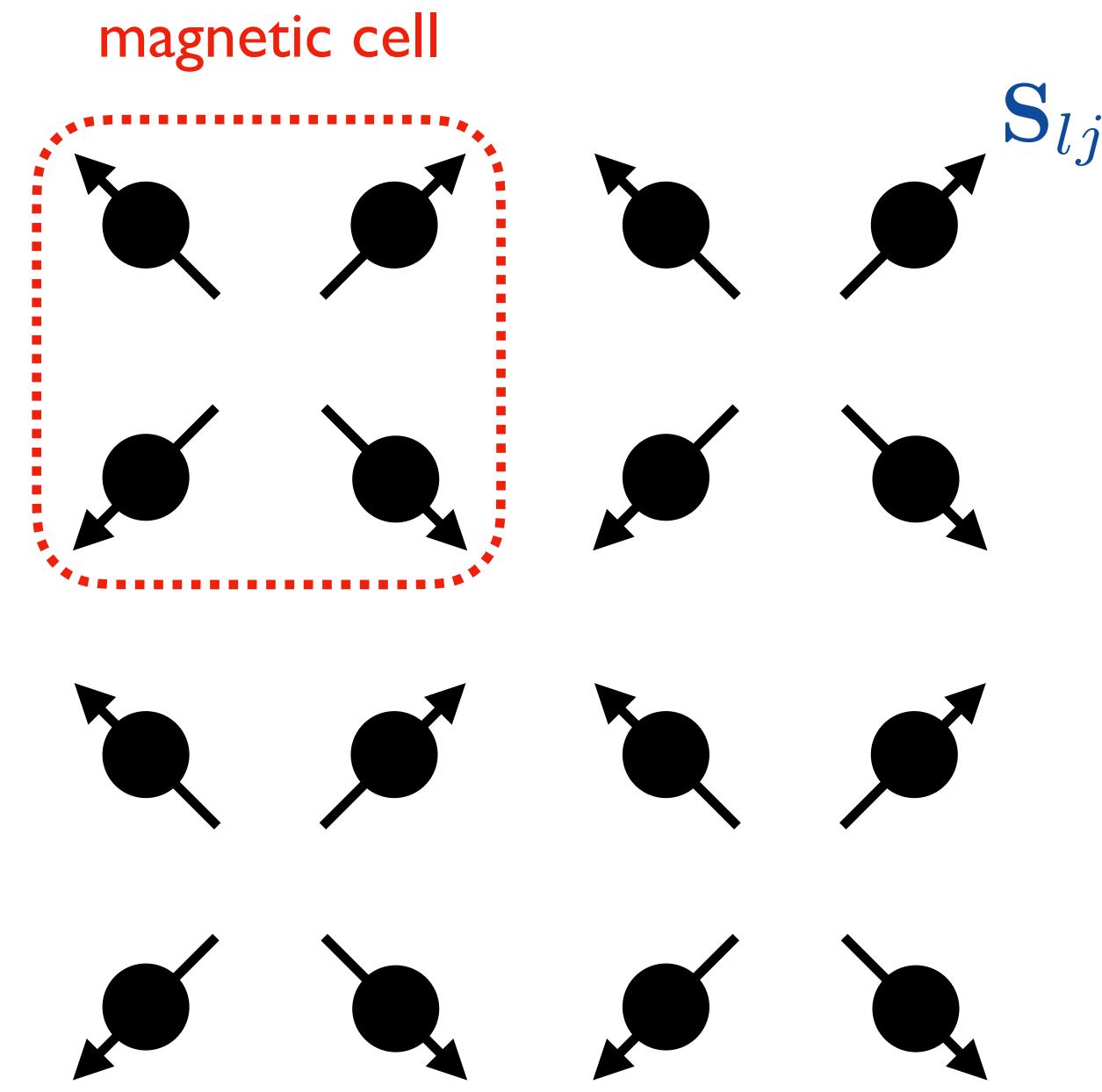




$$S'^+_j = (2S_j - \hat{a}_{lj}^\dagger \hat{a}_{lj})^{1/2} \hat{a}_{lj}$$

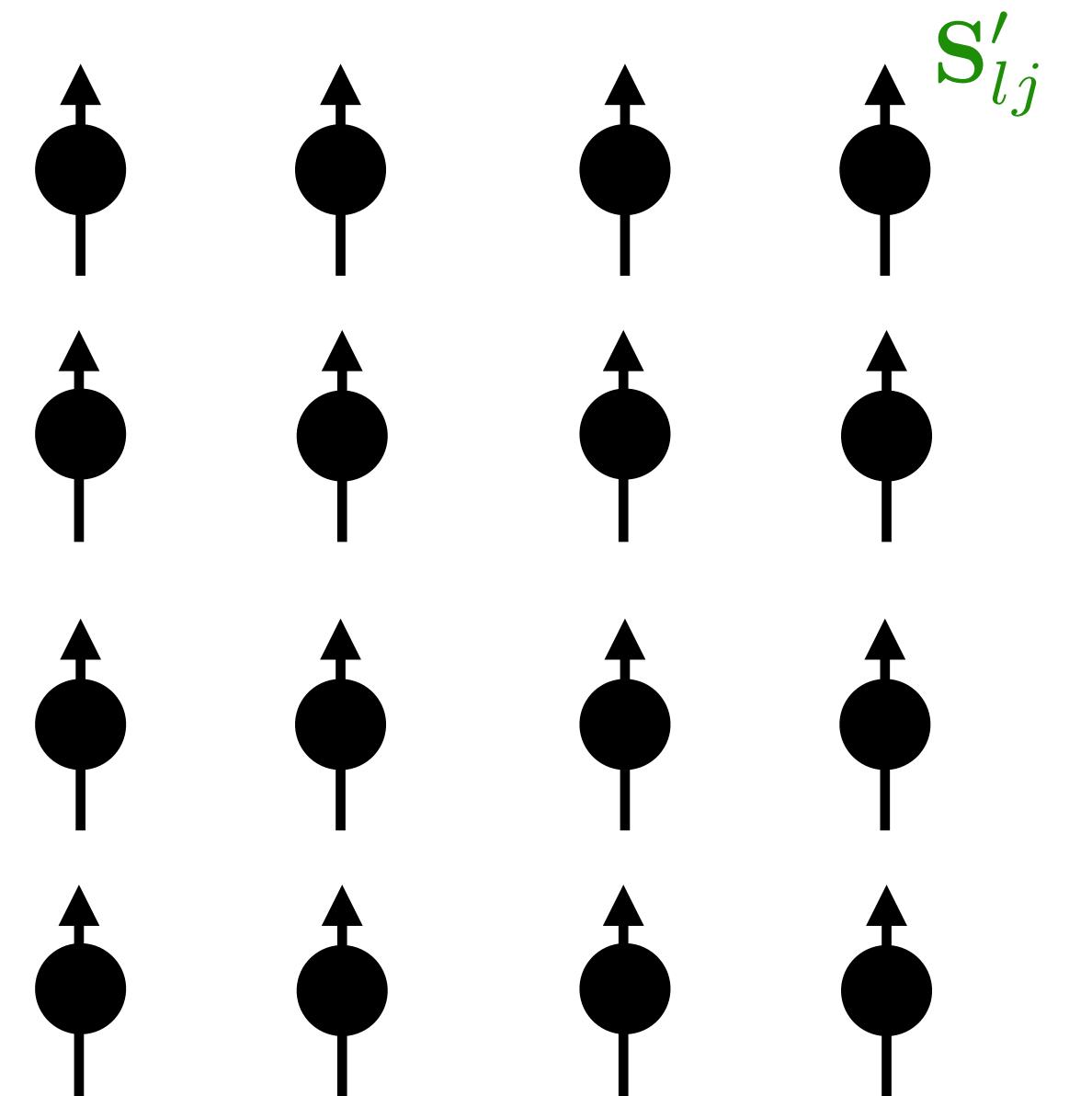
$$S'^-_j = \hat{a}_{lj}^\dagger (2S_j - \hat{a}_{lj}^\dagger \hat{a}_{lj})^{1/2}$$

$$S'^z_j = S_j - \hat{a}_{lj}^\dagger \hat{a}_{lj}$$



$$S_{lj}^{\alpha} = R_j^{\alpha\beta} S'^{\beta}_{lj}$$

rotation from **global** to **local** coordinates



$$S'^+_j = (2S_j - \hat{a}_{lj}^\dagger \hat{a}_{lj})^{1/2} \hat{a}_{lj}$$

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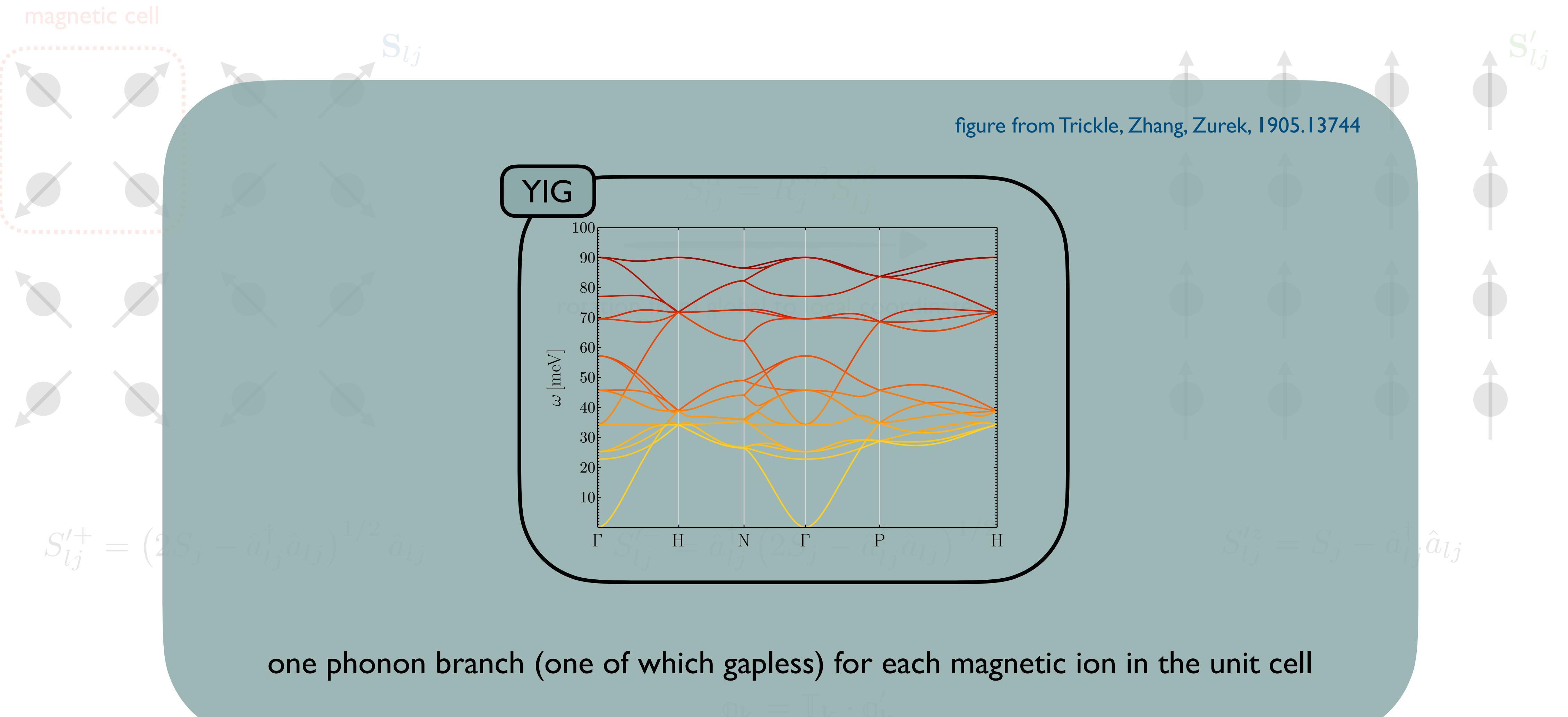
$$S'^z_j = S_j - \hat{a}_{lj}^\dagger \hat{a}_{lj}$$

$$\mathcal{H} = \sum_{ll'jj'} \mathbf{S}_{lj} \cdot \mathbf{J}_{ll'jj'} \cdot \mathbf{S}_{l'j'}$$

$$\mathbb{O}_{\mathbf{k}} = \mathbb{T}_{\mathbf{k}} \cdot \mathbb{O}'_{\mathbf{k}}$$

$\mathbb{O}_{\mathbf{k}} \equiv \left[\hat{a}'_{1,\mathbf{k}}, \dots, \hat{a}'_{n,\mathbf{k}}, \hat{a}'_{1,-\mathbf{k}}^\dagger, \dots, \hat{a}'_{n,-\mathbf{k}}^\dagger \right]^T$

$$\mathcal{H} = \sum_{\nu=1}^n \sum_{\mathbf{k}} \omega_{\nu,\mathbf{k}} \hat{a}'_{\nu,\mathbf{k}}^\dagger \hat{a}'_{\nu,\mathbf{k}}$$



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the **axion wind** couples to **ions spins**

g_j : Landé g-factor

$$\mathcal{L} \supset \frac{g_{aee}}{2m_e} (\partial_\mu a) (\bar{\psi} \gamma^\mu \gamma^5 \psi) \xrightarrow{\text{NR limit}} \delta\mathcal{H} = \frac{g_{aee}}{m_e} \nabla a \cdot \mathbf{s}_e \xrightarrow{} \delta\mathcal{H} = \sum_j \frac{g_{aee}(g_j - 1)}{m_e} \nabla a \cdot \mathbf{S}_j$$

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this induces a magnon excitations rate (per unit time and detector mass)

$$R = \frac{g_{aee}^2}{m_e^2 m_{\text{cell}}} \frac{\rho_a}{m_a^2} \int d^3 v_a f(\mathbf{v}_a) \sum_{\nu=6}^{3n+2} \frac{m_a \omega'_{\nu, \mathbf{p}} \gamma_{\nu, \mathbf{p}}}{(m_a^2 - \omega'^2_{\nu, \mathbf{p}})^2 + (m_a \gamma_{\nu, \mathbf{p}})^2} \left| \sum_j \sum_{\nu'=1}^{3n} \frac{(g_j - 1)}{\sqrt{m_j \omega_{\nu', \mathbf{p}}}} \mathbf{p}_a \cdot \epsilon_{\nu', \mathbf{p}, j}^* (\mathbb{U}_{\nu' \nu, \mathbf{p}}^* + \mathbb{V}_{\nu' \nu, -\mathbf{p}}) \right|^2$$

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sets the signal intensity

sets selection rules

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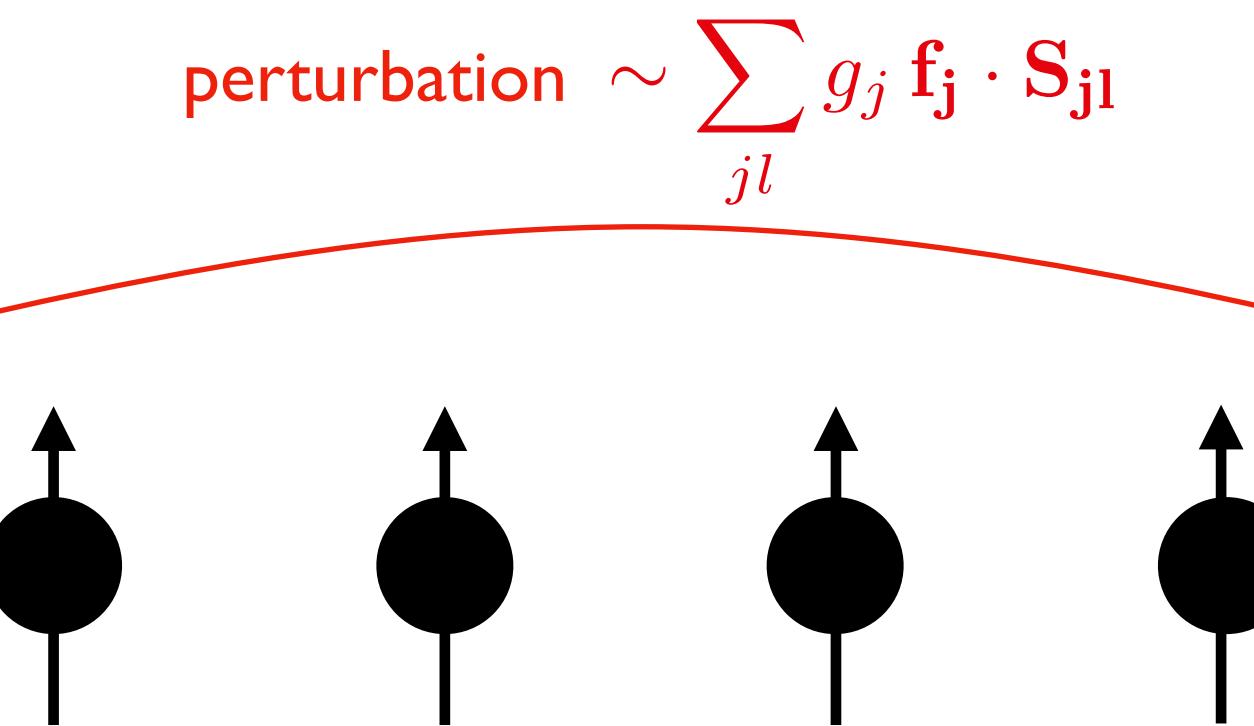
sets the signal intensity

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on resonance

$$R \sim \frac{g_{aee}^2 \rho_a v_a^2}{m_e^2} \frac{n_s}{\rho_T \gamma} \sim (\text{kg}\cdot\text{yr})^{-1} \left(\frac{g_{aee}}{10^{-15}} \right)^2 \left(\frac{\mu\text{eV}}{\gamma} \right)$$

for $\mathbf{k} \rightarrow 0$ only gapless modes can be excited



$$\frac{d}{dt}(\mathbf{S}_{lj} \cdot \mathbf{S}_{l'j'}) = (g_j \mathbf{f}_j - g_{j'} \mathbf{f}_{j'}) \cdot (\mathbf{S}_{lj} \times \mathbf{S}_{l'j'}) = 0$$

$$\begin{aligned} \mathbf{f}_j &= \mathbf{f}_{j'} \text{ for } \mathbf{k} \rightarrow 0 \\ g_j &= g_{j'} \end{aligned}$$

$$\Rightarrow \delta E = 0$$

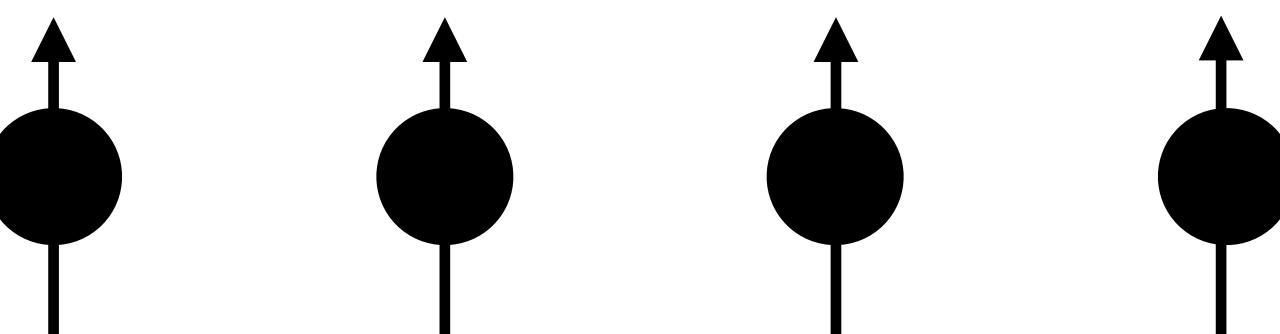
$$\mathcal{H} \propto \sum \mathbf{S}_{lj} \cdot \mathbf{S}_{l'j'}$$

i.e. only gapless mode can be excited

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Non-degenerate g factors

$$\text{perturbation} \sim \sum_{jl} g_j \mathbf{f}_j \cdot \mathbf{S}_{jl}$$



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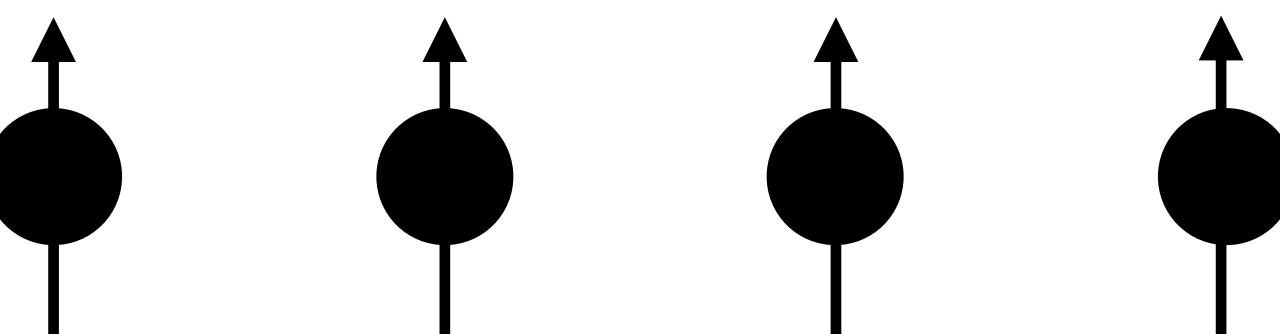
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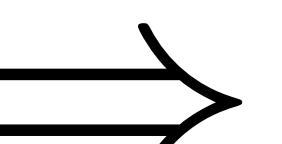
External magnetic field

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$$\mathcal{H} \propto \sum \mathbf{S}_{lj} \cdot \mathbf{S}_{l'j'}$$

$$\mathcal{H} \propto \sum \mathbf{S}_{lj} \cdot \mathbf{S}_{l'j'} + \mu_B \mathbf{B} \cdot \sum g_j \mathbf{S}_{lj}$$

idea exploited by QUAX exp.

Barbieri, Cerdonio, Fiorentini, Vitale ('89)
Barbieri et al 1606.02201

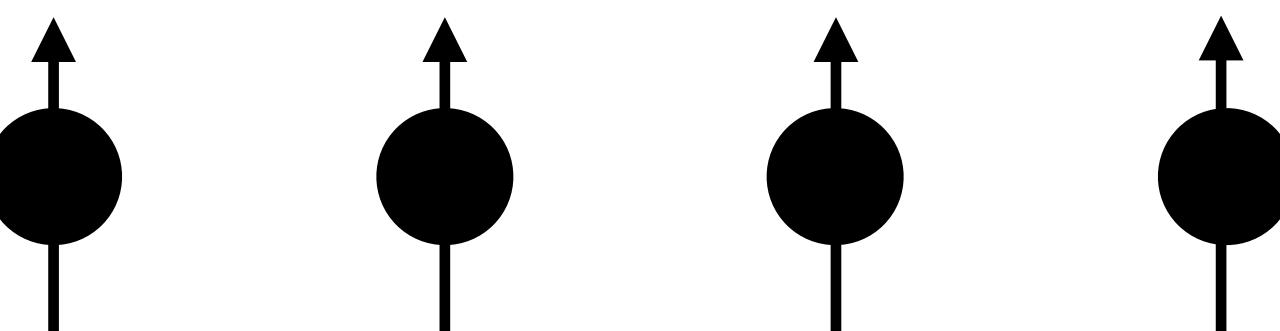
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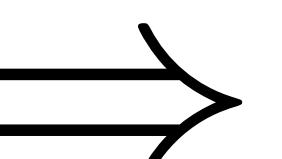
Anisotropic interactions

$$\text{perturbation} \sim \sum_{jl} g_j \mathbf{f}_j \cdot \mathbf{S}_{jl}$$



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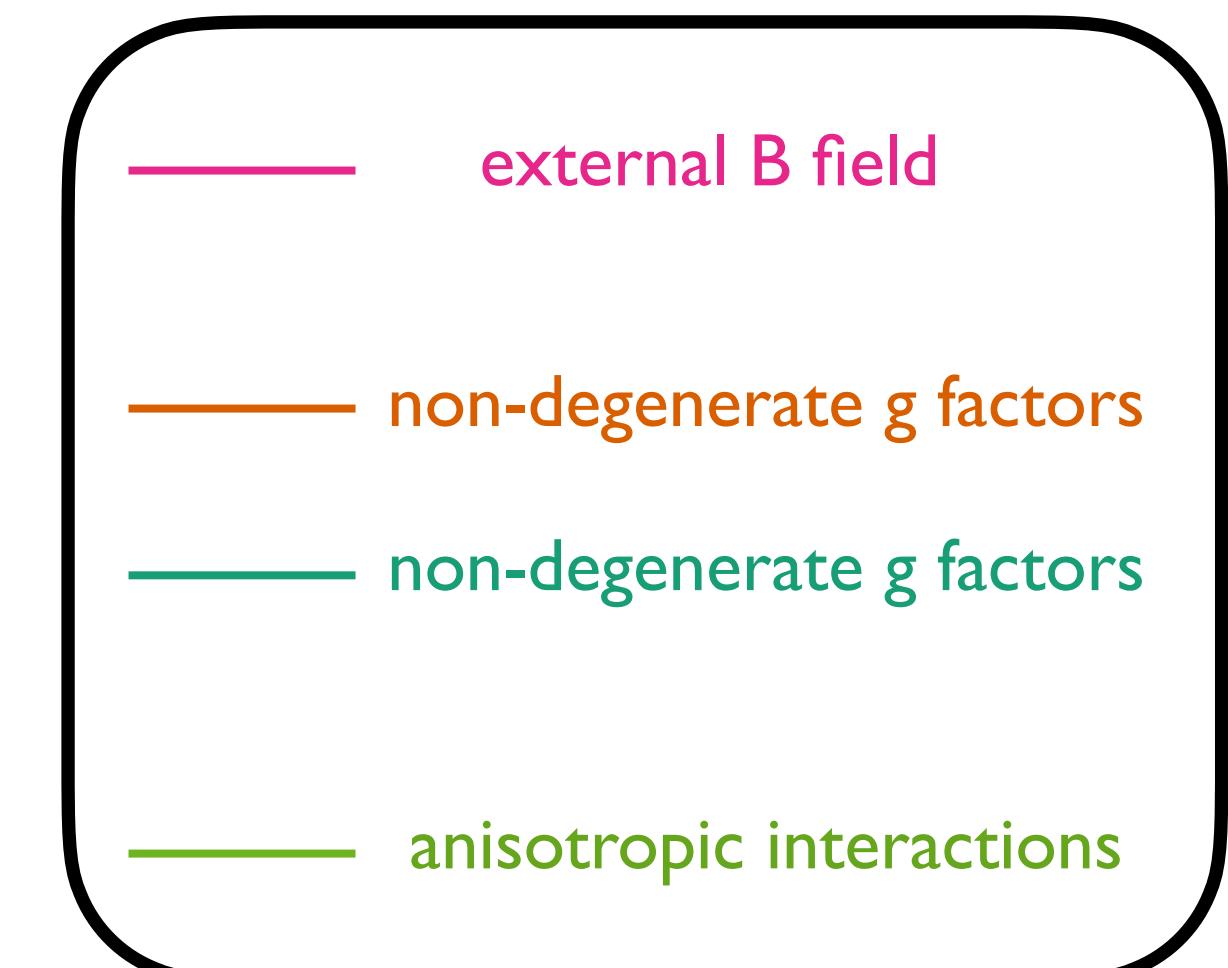
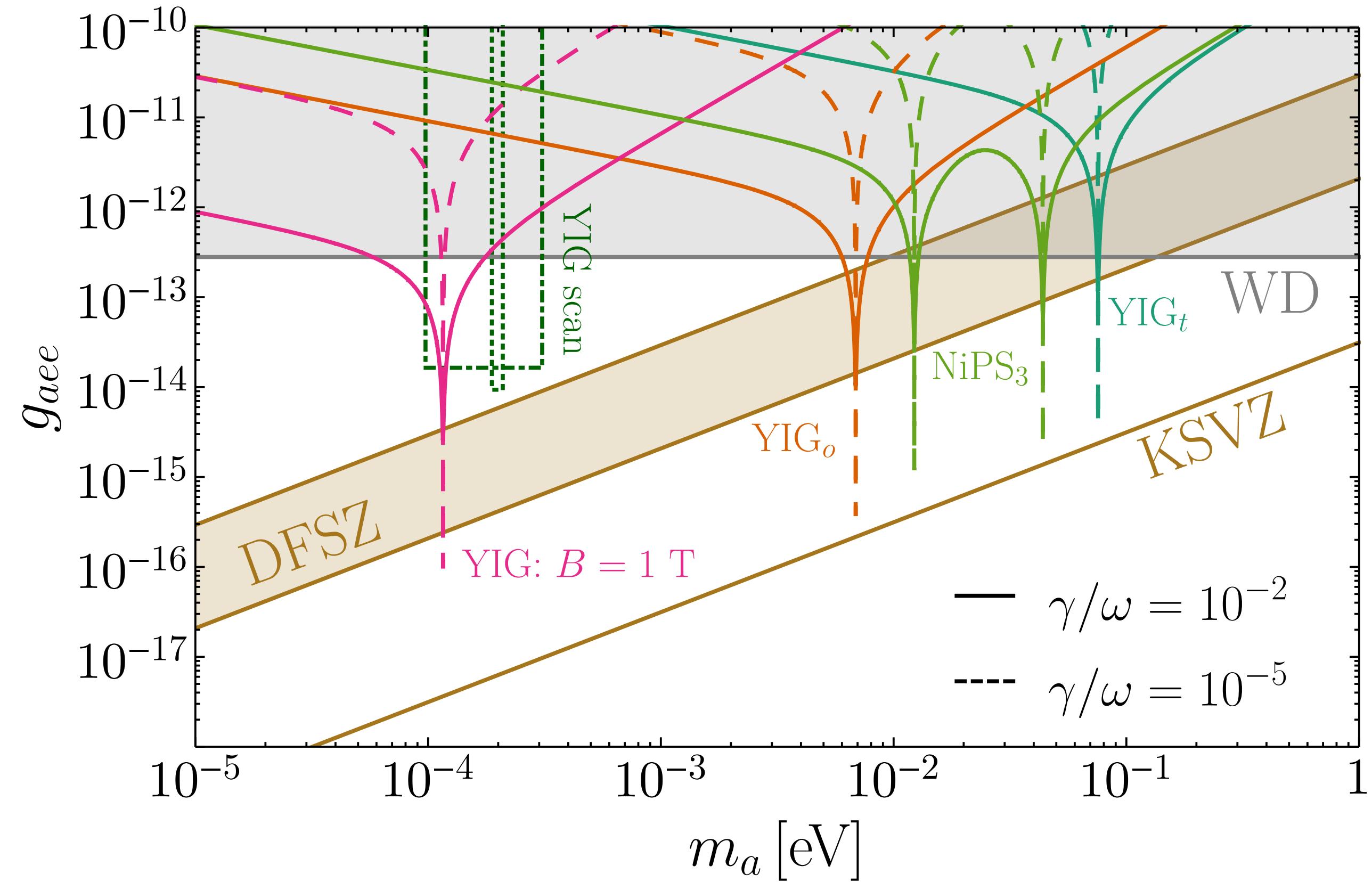


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phonon polariton and magnon excitations can probe the QCD axion DM in the $(1 \div 100)$ meV mass range

a thoughtful choice of complementary target can offer a broadband coverage

we identified two new ways to couple axion to gapped magnons

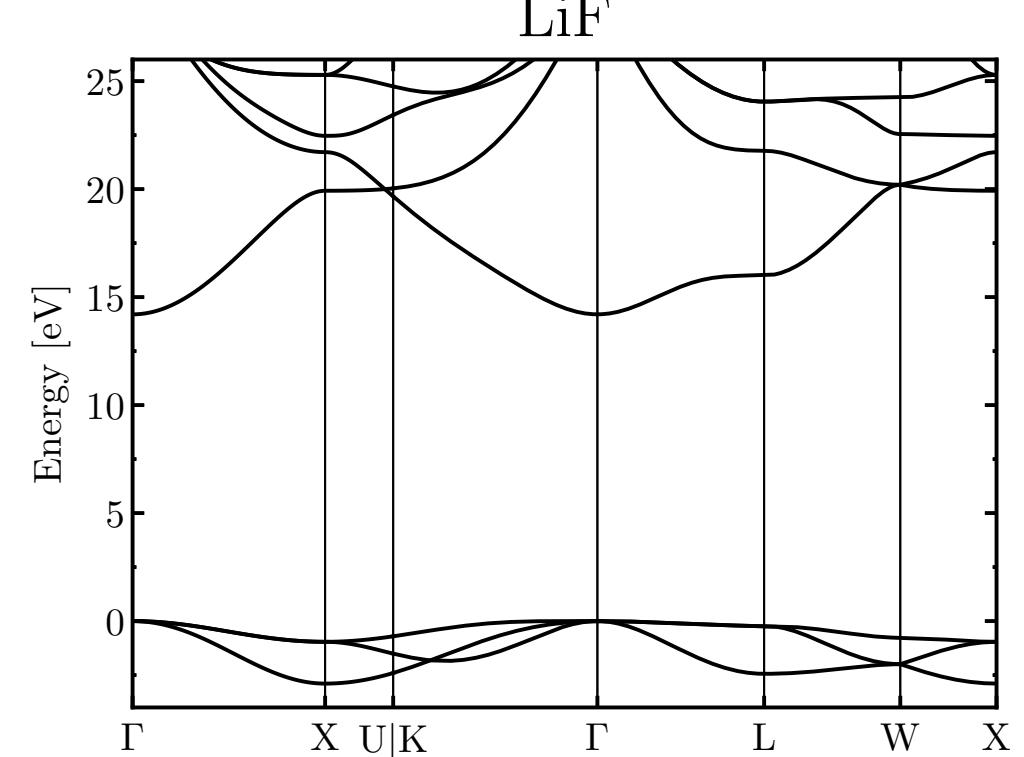
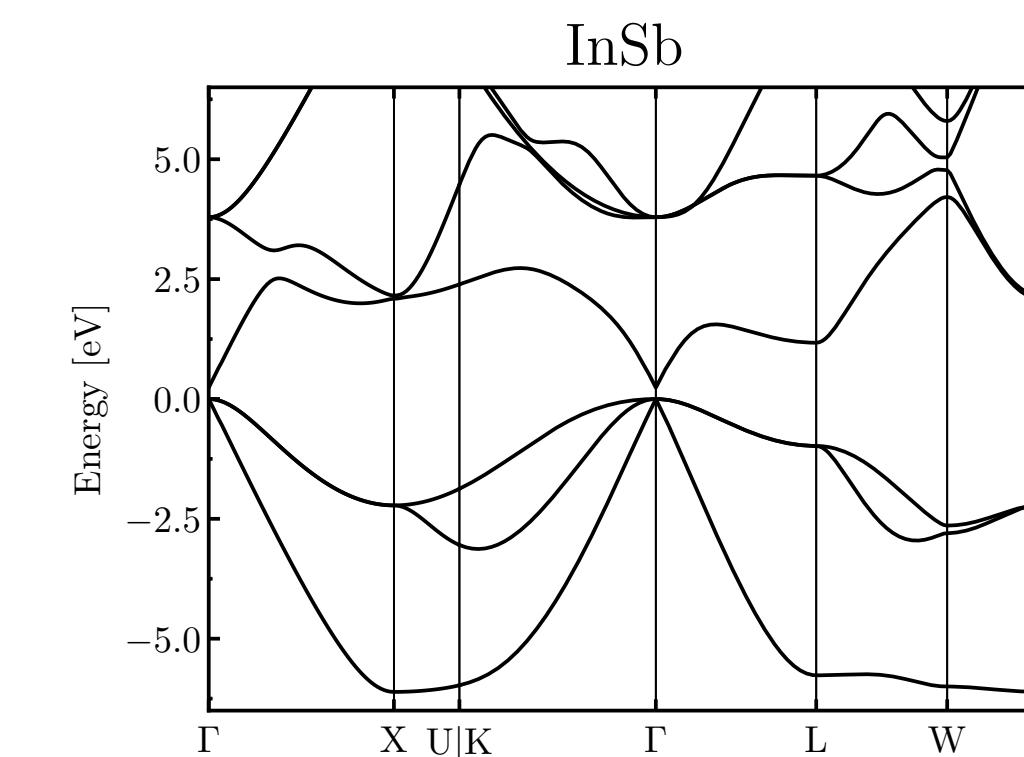
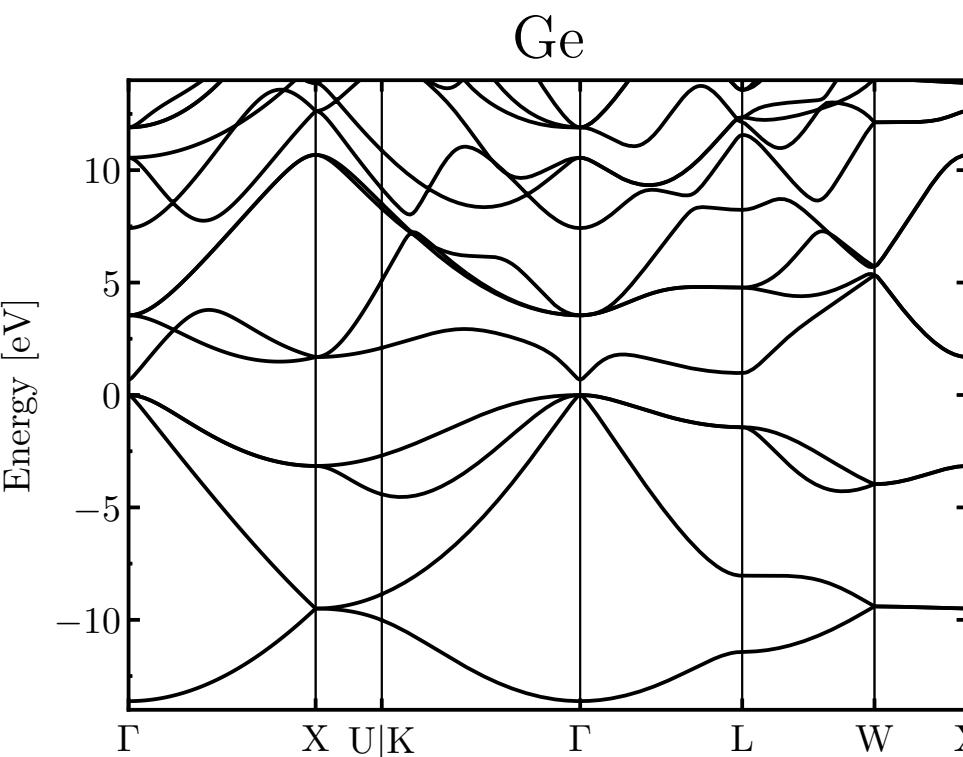
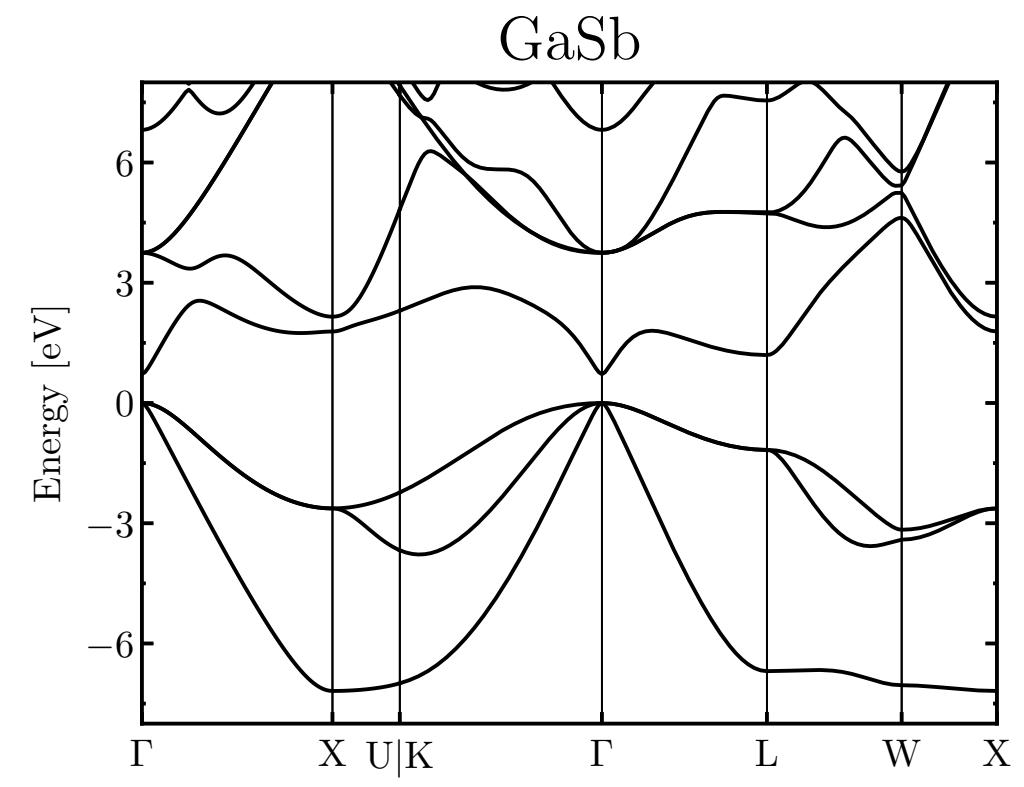
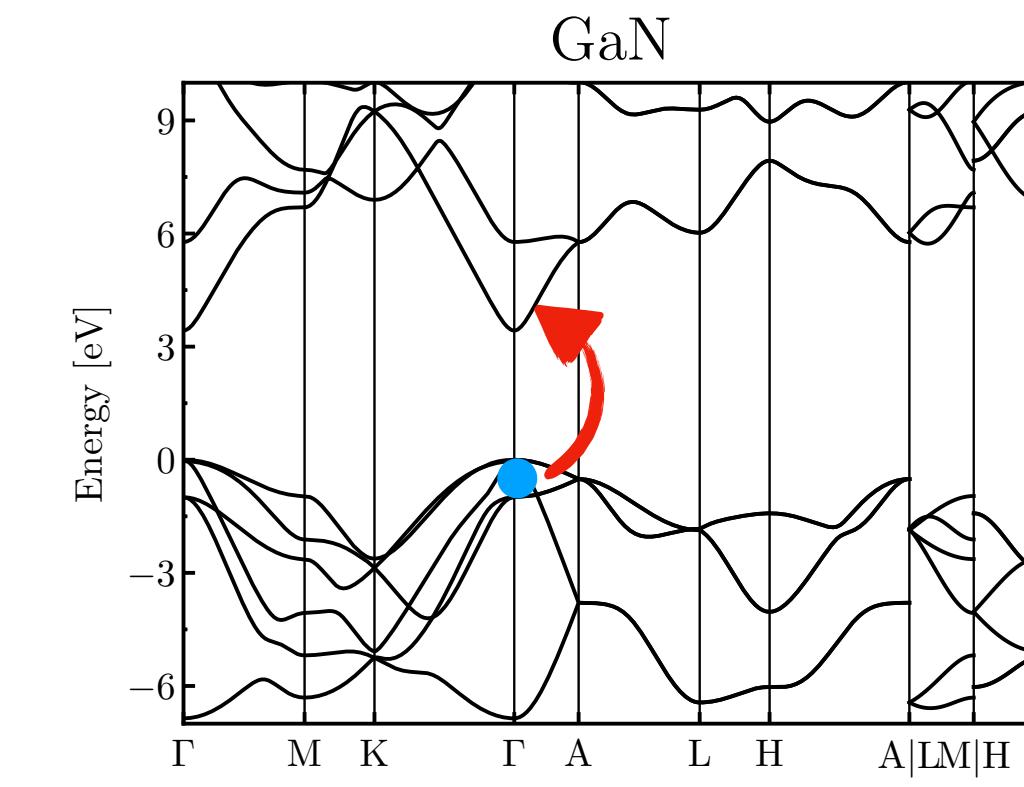
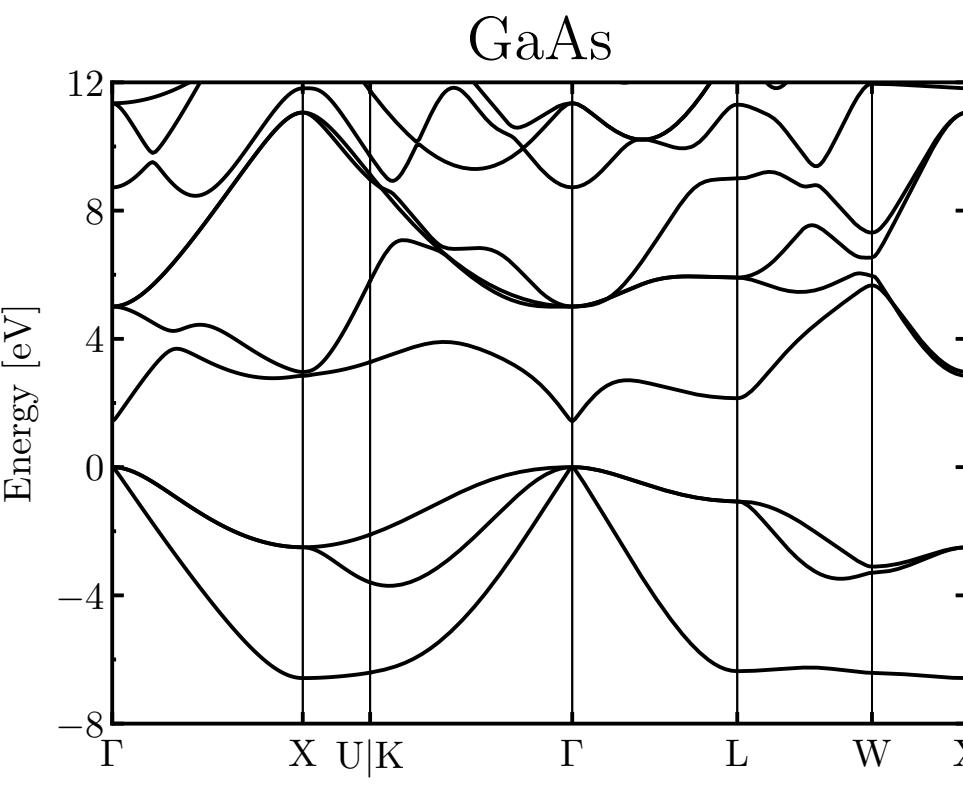
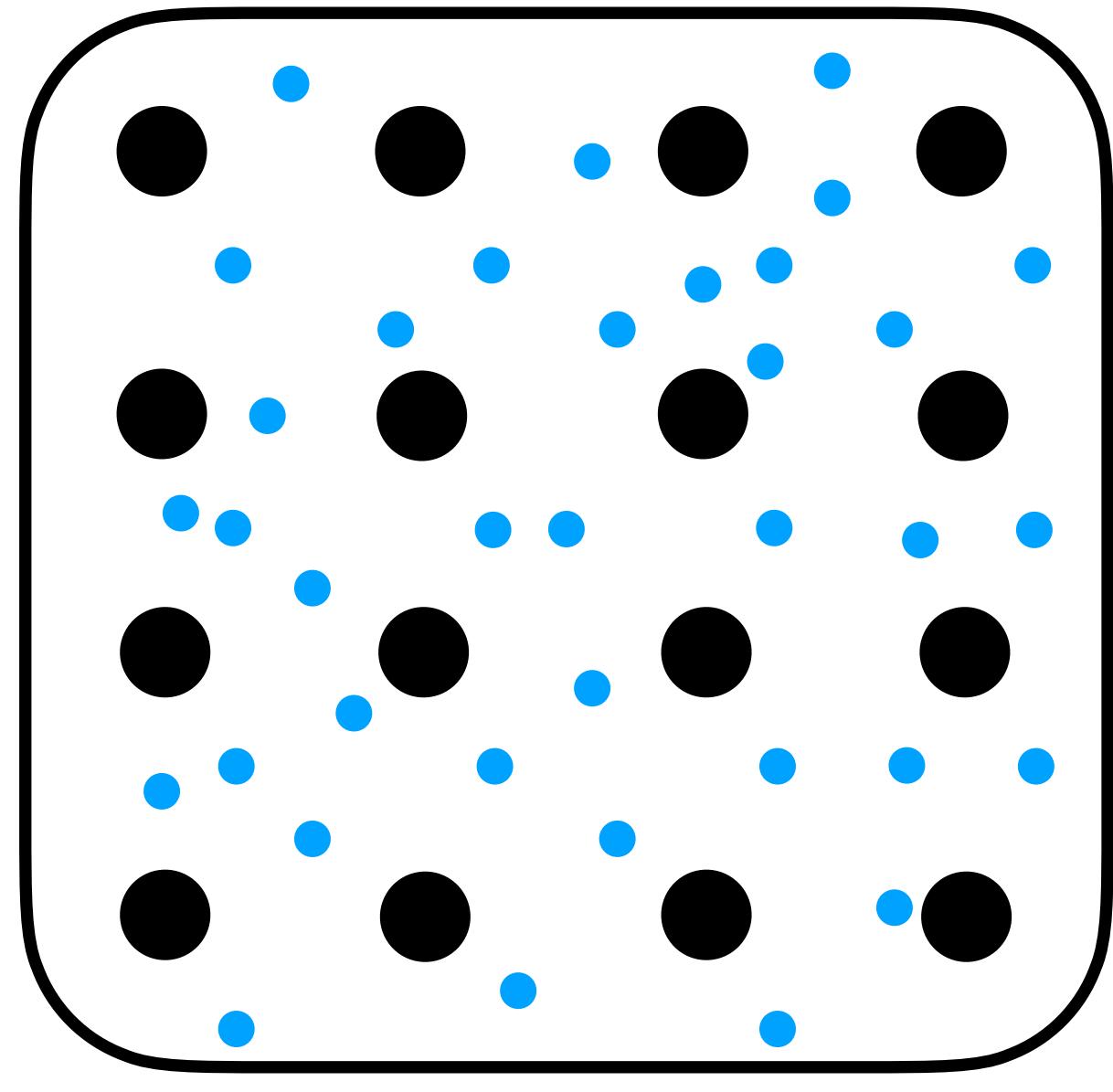
a better knowledge of magnon and phonon line-shape is needed to get more precise results

we suggest possible read-out schemes, R&D will be crucial to identify realistic experimental set-up

Backup slides

● : ions

● : electrons



band gap is too large $\mathcal{O}(\text{eV})$

figure from Griffin, Inzani, Trickle, Zhang, Zurek, 1910.10716